IDENTIFIERS FOR STRUCTURAL WARNINGS OF MALFUNCTION IN POWER GRID NETWORKS

by

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Although its uninterrupted supply is essential for everyday life, the electricity occasionally experiences disruptions and outages. The work presented in the current paper aims to initiate the research to design a strategy based on advanced approaches of algebraic topology to prevent such malfunctions in a power grid network. Simplicial complexes are constructed to identify higher-order structures embedded in a network and, alongside a new algorithm for identifying delegates of the simplicial complex, are intended to pinpoint each element of the power grid network to its natural layer. Results of this methodology for analysis of a power grid network can single out its elements that are at risk to cause cascade problems which can result in unintentional islanding and blackouts. Further development of the outcomes of research can find implementation in the algorithms of the energy informatics research applications.

Key words: *algebraic topology, simplicial complexes, complex systems, power grid network, power grid redundancy*

Introduction

Prevention of damages and blackouts they cause put a challenge and efforts on the development of reliable methods in energy informatics research which applies thinking and skills of information systems to increase energy efficiency, stability, and sustainability, with the aim to provide reliable power grids with an uninterrupted supply, transitioning at the same time toward the smart power grids. In order to make electricity widely and reliably accessible, it is necessary to have an infrastructure resistant to severe disruptions or unintentional islanding - an event where part completely separates from the rest of the power grid network and continue running. Non-etheless, power interruptions and power outages mostly occur due to equipment failures, hence causing the blackouts in most extreme scenarios. Furthermore, the malfunction and low reliability of the power grid indirectly leads to the fragility of other technological networks, which are highly dependent on it. Thus, disturbances in the functioning of the power grid network can have serious implications on everyday life, making all attempts to develop strategies to prevent, or when a problem happens to overcome, are of vital importance.

The work presented in current paper is motivated by the tendency to prevent a vulnerability of the power grid network and hence contribute the efficient energy management. In this context, the aim is to introduce a novel approach, based onols originating from the topological analysis of data, to the study of failure of the power grid network and contribute to the devel-

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opment of prevention strategies. So far research in complex networks was mostly based on the methods originating from the graph theory [1], with some recent applications of the topological data analysis [2]. Topological analysis of data, as a set of tools, is drawing an attention of the wide variety of practitioners, from research scientists to industry. Although there are attempts in applications of algebraic topology, in particular persistent homology [3] on power grid networks [4], in this paper we are extending an interest in less studied structures which are characterized by the connectivity of mesostructures. Namely, unlike commonpological approaches to datasets and complex networks, which are mostly based on the calculation of Betti numbers [3], that is higher-dimensional holes within the structure, here the objects of interest are non-trivial higher-dimensional chains of connectivity that are responsible for the system's functioning.

Properties of power grids are well studied by applying complex network quantities, in particular with an aim of considering the effects caused by the power failures. For example, in order to prevent the worst scenarios that can lead to cascading failures in a power grid, one must understand the nature of grid vulnerability. A good insight into the current state of the grid and potential weaknesses can be identified through the use of different static and dynamic models to predict potential cascading failure scenarios in grids [5]. Similarly, Nesti *et al*. [6] approached the modelling of blackouts of different sizes through the use of graphs with heavy-tailed sinks that represented the power demand from users and created a model of Germany's power grid that linked scale-free blackout sizes with scale-free city sizes. On the other hand, Carreras *et al*. [7] created a dynamical numerical model for blackouts in power transmission systems with minimal implementation and assumption of uniformity of the components throughout the system, with an idea to further expand it through the introduction of non-homogeneous components. European power grids are studied in [8], and modeled as the weighted graphs, with a focus on their topology and scalability. Self-organized criticality is studied by Zhao *et al*. [9] to explain the dynamic behavior and investigate the China power grid blackouts, targeting the small disturbances in the network that may trigger the chain reactions. Finally, Pagani *et al*. [10] reviewed the current state of the power grid studies performed by the statistical tools of complex network analysis in different states and regions over the world. The frequently studied grid infrastructure with these tools, at the time of review, was located in the USA, most of the European Union countries, China, and India.

As a case study, the US Power Grid network [11] is considered, where elements and their connections play different roles in the network s functioning. Due to the lack of information about the exact roles of nodes within the considered power grid, we have artificially selected the set of delegates which moves up to higher hierarchical levels. Namely, the goal is to reconstruct the latent hierarchical structure under selection criteria that distinguish node s participation in the formation of non-trivial structure. As the criteria for delegate nodes selection the so called topological dimension [12] is chosen and it quantifies the node s influence on their local higher-order neighborhoods. Higher-order structural properties of the US Power Grid network have already been considered in [13], where the calculated quantities displayed the statistical invariance. On the other hand, regarding the power grid vulnerability [14], research so far is characterized by an interest in a lack of robustness and resilience of a system, and was focused on considerations of random, as well as selective failures, through the percolation theory rooted in numerical simulation of different scenarios.

Results suggest that the underlying network has an internal structure of the most significant delegates, see section *Methods*, rather with higher-order aggregation than at the level of the initially obtained simplicial complex from a mathematical graph. A node that naturally originates on a lower hierarchical level can be found mixed within the same simplex with nodes that belong on a higher hierarchical level, and vice versa, presenting the potential problem, hence indicating roads towards future research. Generally, the outcomes of presented research contribute to the energy informatics research, as well as to building smart grids [15]. The proposed approach offers a broad framework for applications, not necessarily restricted to power grid networks, but also such as for example, the thermal storage systems [16].

Phenomenological background and its modelling

The underlying mathematical graph of the US Power Grid network is formed by nodes with different roles in the network s functioning. These various physical objects can be distributed on different levels in the hierarchical organization of a power grid network, fig. 1. Due to the authors lack of knowledge on specific roles of nodes in the considered US Power Grid network, and with the aim to reconstruct the inherent hierarchical organization, the topological criteria are applied in order to distinguish and select the influential vertices, termed delegates. Using selected delegate vertices in a clique complex, after a few iterations, different hierarchical levels are extracted. Hence, when disruption occurs, the scope of malfunction depends on the position of the level and the structure that is formed there.

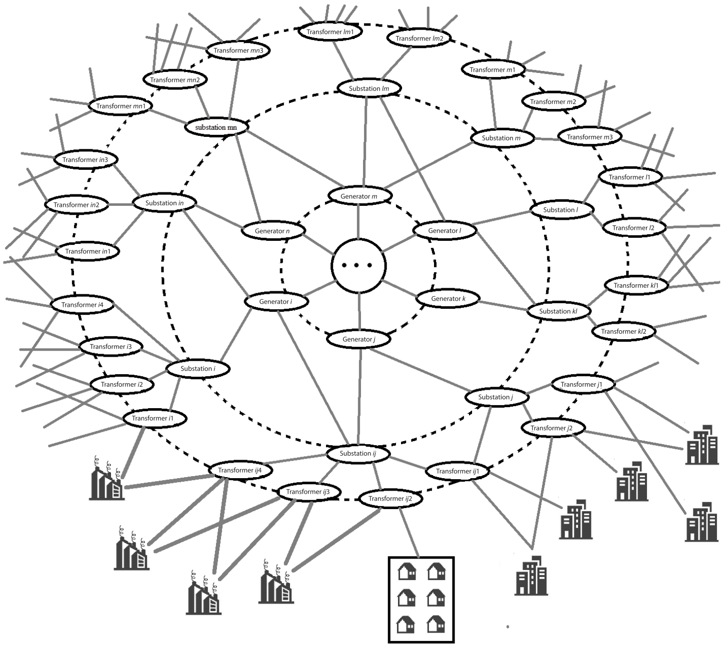

Figure 1. Depiction of a segment of a layer-architecture of an arbitrary power grid network; dashed circles display the same layer; every type of node is found at the layer to which it naturally belongs or it may cause disruption in network stability and sustainability, so the proposed layer-architecture may be of great significance

Methods

The notions of basic, and derived, quantities that will be introduced within this section are originating from the mathematical field of algebraic topology, more specifically, the Q-analysis [17, 18]. Let $V = \{v_1, v_2,...v_k\}$ and $S = \{s_1, s_2,...s_l\}$ be a two sets assembled by k and l elements, respectively, and suppose that exists a binary relation, λ, that by some rule associate elements of sets V and S. In other words, an element s_i from the set S is λ-related to some subset $v_{j_0}, v_{j_1},..., v_{j_q}$ from the set V, and vice versa, an element v_j is λ^{-1}-related to some subset $s_{i_0}, s_{i_1},..., s_{i_p}$ from the set S, where λ^{-1} is the inverse of relation λ. Consequently, the set S and relation λ define a subset C of the power set $P(V)$ of V, that is $C \subseteq P(V)$, and each element $\{v_{j_0}, v_{j_1},...v_{j_q}\} \in$ C, $(q \leq l)$, can be assigned to an element $s_i \in S$, labeled like $s_i\lambda v_{j_0}, s_i\lambda v_{j_1}, ..., s_i\lambda v_{j_q}$. The elements of set C will be called simplices labeled by the relation-associated elements of set S like $\sigma(s_i)$, whereas the elements from set V will be called *vertices*. This notation emphasizes the distinction between elements from set S and set of elements C to which they are λ-related. Further, elements $\sigma(s_i)$ are called *q-dimensional simplices* or just *q-simplices*, labeling them like $\sigma_q(s_i) = \langle v_{j_0}, v_{j_1},...v_{j_q}; \lambda\rangle$. The dimension of q-dimensional simplex is equal to the number of associated vertices minus 1. The r-dimensional face of a q-simplex is its subset of $r + 1$ elements, where $r \leq q$, and it is also a simplex. As a result, sets S, V and relation λ build a mathematical object called the simplicial complex $K_S(V, \lambda)$ and it represents a collection of all simplices, together with all their faces, hence building the structure by attaching simplices via their mutual faces. Dimension of a simplicial complex is determined by the maximal dimension of simplices in the simplicial complex. In geometrical representation, simplices are displayed as q-dimensional polyhedra, *e.g.* 0-simplex is represented as a point, 1-simplex as an edge, 2-simplex as a triangle, and 3-simplex as a tetrahedron, *etc*.

Mutual q-face between two simplices $\sigma_r(i)$ and $\sigma_p(j)$, having dimensions r and p, respectively, is a subset of their common $q + 1$ vertices, where $q \leq r$, p, and they also share $(q - 1)$-, $(q - 2)$-, ..., 0-face also. The latter property assigned to these two simplices is called q-nearness. The degree of connectivity defined by the *q-connectedness* is the chain of simplices between any pair of simplices in the chain of simplices such that any adjacent pair of simplices share a mutual face, is determined by the lowest q-nearness, which is the smallest dimension of shared face. The q-connectedness property between simplices in the simplicial complex K generates the relation that satisfies mathematical equivalence ϕ_q in a sense that it displays reflexivity, symmetry, and transitivity properties. Therefore, the subcomplex K_q of simplicial complex K is a set of all simplices with dimension greater or equal to q and the relation ϕ_q partitions K_q into equivalence classes of simplices based on the q-connectedness. The number of q-connected components, or *q-connectivity classes*, represents the entries of *Q-vector* [17], or the *first structure vector*, and it is given as a vector starting from the largest q dimension in descending order of entries:

$$\mathbf{Q} = \left\{Q_{\dim(K)}Q_{\dim(K)-1}\cdots Q_1Q_0\right\} \tag{1}$$

The number of simplices that have a dimension greater or equal to q represents the entries of the *second structure vector N*, and it is written in the same descending order as the Q-vector starting from the largest dimension:

$$\mathbf{N} = \left\{N_{\dim(K)}N_{\dim(K)-1}...N_1N_0\right\} \tag{2}$$

The *third structure vector* describes the degree of connectedness at all dimensions of simplicial complex and it is defined like:

$$\hat{\mathbf{Q}}_q = 1 - \frac{Q_q}{N_q} \tag{3}$$

Structure vectors describe the global properties of a simplicial complex revealing its geometrical and combinatorial aspects, as well as behavior. On the other hand, in order to characterize immediate environment and, inner hidden structure of a single simplex, other quantities can be also derived within the mathematical framework of Q-analysis. One of these quantities is the so called *node s Q-vector* [12], Q^i, that describes the environment of vertex i in a simplicial complex and it is defined:

$$\mathbf{Q}^i = \left\{ Q_{\dim(K)^i} Q^i_{\dim(K)-1} \ldots Q^i_1 Q^i_0 \right\} \tag{4}$$

where entries Q^i_q enumerate the number of q-simplices that contain the node i. *Topological dimension* $\dim Q_i = \sum_q Q^i_q$ of vertex i is derived in order to quantify the total number of simplices in which the vertex i participates. Term *dimension* originates from conjugate complex K^{-1} [17, 18], in which vertices and simplices have swapped roles – vertices are simplices and vice versa, and in conjugate complex the dimension of simplex associated to node i from underlying a simplicial complex is equal to the topological dimension of node i, $\dim Q^i = \sigma(v_i)$.

Alongside the aforementioned, in this work two important properties of a simplicial complex are known as *eccentricity* and *vertex significance of simplex* [19] are calculated. Let us define $\hat{q}$ as a dimension of a q-simplex $\sigma(\mathrm{s}_i)$ and $\check{q}$ as a dimension at which it is joint to connectivity class for the first time, *i.e.* the dimension of a minimal face that attaches simplex $\sigma(\mathrm{s}_i)$ to other simplices. Now, eccentricity can be defined:

$$ecc\left[\sigma(i)\right] = \frac{\hat{q} - \check{q}}{\hat{q} + 1} \tag{5}$$

which takes values between 0 and 1 and quantifies the degree of integration of a simplex in a complex. High values of eccentricity suggest low integration of simplex in local environment, whereas low values imply high integration of simplex in the simplicial complex. On the other hand, the vertex significance quantifies the significance of simplices with the respect to vertices that build them. Therefore, let weight θ_i be the number of simplices that contain the vertex i, and $\Delta(i) = \sum_{j\in i}\theta_j$ represents the sum of θ_j for simplex $\sigma(i)$, where j goes through all vertices in $\sigma(i)$. Vertex significance of simplex is defined:

$$vs\left[\sigma(i)\right] = \frac{\Delta(i)}{\max(\Delta)} \tag{6}$$

where $\max(\Delta)$ is the maximal value from Δ, and it represents the normalization constant so that vertex significance of simplex has maximal value 1, *i.e.* most significant simplex has $vs = 1$.

In this paper, the inquiry of the considered phenomena is divided in two parts, one considering graph theory measures and the second part represent algebraic topology analysis of so called clique complex [20] constructed from the underlying mathematical graph G (object which consider only pairwise interaction between vertices). In graph theory, a mathematical graph is presented via the adjacency matrix $A(G)$ which is a square matrix ($n \times n$) with rows and columns presenting vertices and its elements are commonly $A_{ij} = 1$ if vertices i and j are connected by an edge, otherwise $A_{ij} = 0$ (if an edge exists, element of adjacency matrix can also be the weight of an edge). Measures from the graph theory that are used are: n is the number of vertices in mathematical graph, e – the number of edges in mathematical graph, $\langle k\rangle$ – the average degree of nodes (vertex s degree – total number of edges connected to a node), cc – the average clustering coefficient (the clustering coefficient is calculated as the ratio between the number of triangles around a vertex and the maximum number of triangles that could possibly be formed around that same vertex), d – the diameter (maximal distance between any pairs of vertices), $\langle p\rangle$ – the average path length (vertex s path length - average distance from a vertex to all other vertices), and *modul* – the modularity (represents a measure of the structure of mathematical graph – determines the strength of division of a graph into communities or subgraphs).

Adjacency matrix is defined by binary relation between vertices of a mathematical graph. On the other hand, the incidence matrix $\Lambda[K(G)]$ which defines the clique complex is the $n \times m$ matrix, where m is the number of cliques in the underlying mathematical graph. In the clique complex $K(G)$, vertices are the same as vertices of the mathematical graph, G, and simplices are identified as cliques (maximal fully connected subgraphs) which also represents the relation that form clique complex from the binary related mathematical graph. Bron-Kerbosch algorithm [21] is used for the detection of maximal cliques in the underlying mathematical graph.

For the purpose of the presented research, new criteria for layer-reconstruction of a simplicial complex will be introduced. A *delegate* of a simplex is a vertex with the highest value of the node's topological dimension. After determining all delegates within an underlying simplicial complex, a higher layer of a simplicial complex is constructed by connecting those delegates whose underlying simplices share a mutual face. This criterion can be applied multiple times in order to determine the inherent layers of vertices and simplices in an underlying complex.

Results

The network considered in this paper is the power grid of the Western States of the USA initially studied in [11], whereas the dataset is openly available in the KONECT project*. Nodes are considered to be either as a generator, a substation or a transformer units, while links represent power supply lines. The network is formed by $n = 4941$ nodes and $e = 6594$ edges. As previously mentioned, different kind of elements in power grid network naturally operate at different hierarchical layers, hence it is preferable to identify them. In order to do that, the first step is to construct a clique complex from the underlying mathematical graph of the power grid network. Since this network has three different types of nodes, three iterations of algorithm are applied to extract delegates, fig. 2. Table 1 presents graph theory measures mentioned in the previous section for the original, as well as mathematical graphs of delegates after three iterations when the criteria for layer-reconstruction is applied.

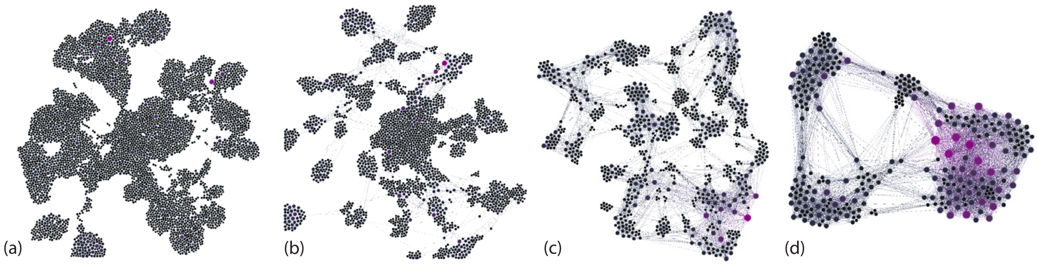

Figure 2. (a) Original power grid network from [11], (b)-(d) networks after 1-3 iterations of algorithm for creating networks of delegates, respectively

These results suggest that the size of network is shrinking through keeping only nodes that should exist at higher layers. From the presented results, the number of links stays more or less the same, which means delegate nodes that are connected at higher layers are connected via other nodes from lower layers and they represent some kind of binders in the underlying network. In this case, generators are some kind of network binders, but there is no strongly distinguishable community structure among them, as the *modularity* for network $D3$ suggests.

* http://konect.cc/networks/opsahl-powergrid/

Table 1. Common network measures for the original power grid networks and delegate networks after 1, 2, and 3 iterations of layer-reconstruction

	Original	*D*1	*D*2	*D*3
n	4941	2373	926	298
e	6594	7710	6462	5271
$\langle k \rangle$	2.67	6.50	13.96	35.38
cc	0.11	0.52	0.63	0.73
d	46	25	13	6
$\langle p \rangle$	18.99	10.05	5.40	2.73
modul	0.93	0.9	0.82	0.59

On the other hand, in the underlying original network, smaller communities are formed around generators and those clusters have denser inter-connection compared to the connection between clusters. From the tab. 2, it can be inferred that the values of clustering coefficient of these networks indicate strong pseudo-connection between nodes that belong to higher layers, that is generators and substations in *D*2 network, and among generators in *D*3 network even though they are not directly connected. The presence of those pseudo-connections between nodes that naturally exist on the same layer preserves the stability and sustainability of a power grid network and keeps it from creating unintentional islanding and severe disruptions in the infrastructure, hence having a role of some kind of bypass. When the random failure of a fraction of nodes occurs, it is likely that still active delegate nodes will take the role of connections that will keep the significant portion of the network in function.

Regarding the results obtained for structure vectors, fig. 3, it can be concluded that inner-connections and degree of connectedness increases as simplicial complexes progress toward higher layers of the power grid network. First two simplicial complexes of the underlying network of delegates display small changes compared to the original power grid network. On the other hand, the structure of simplicial complex obtained in the last iteration reveals hidden properties that simplicial complexes in previous iterations do not possess, and it is another confirmation of the existence of previously mentioned pseudo-connections of generators. Therefore, the network of delegates which may have a role of the network of generators presents a backbone of the power grid network. In this sense, the intertwine pseudo-connections, which

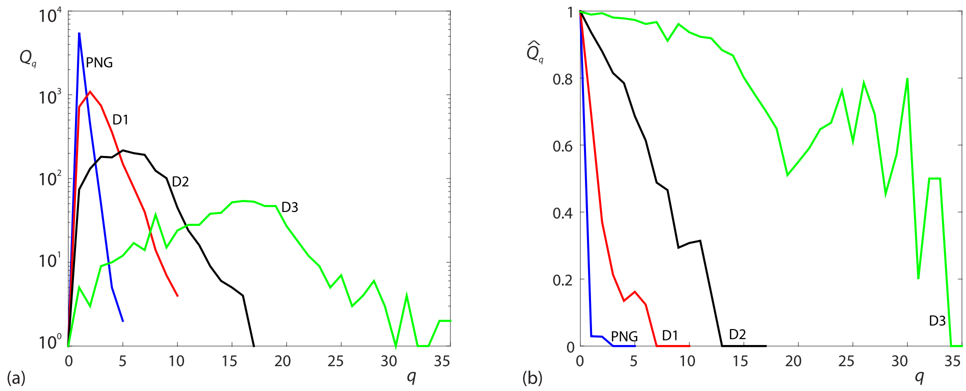

Figure 3. First (a) and third (b) structure vectors of the power grid network and networks of delegates

form bigger simplices, suggest high stability regarding disruptions and islanding in the infrastructure. If there was a poor connection of the base of one power grid network, there would be a high risk of creation of unintentional islanding and blackout of parts of a power grid.

From the fig. 4, the simplicial complex of $D3$ network has less average eccentricity, and higher average topological dimension and vertex significance than networks at other 3 layers, as expected, and it is complementary to previous results. If carefully observed, topological dimension and vertex significance of simplices in simplicial complexes $D2$ and $D3$ have rapid decline intersecting $D1$around 500[th] and 200 [th] ranked node, respectively. This can suggest that some of the nodes and simplices constructed by these nodes, do not naturally belong to that layer, therefore, they should be restrained in lower layers. Tails of previous rankings demand further inspection because if node, that naturally lives on lower layers, is progressing to the higher layers as delegate, it can be the one that has potential of causing damage to the whole cluster to which it belongs, and therefore, the power grid network s integrity and stability.

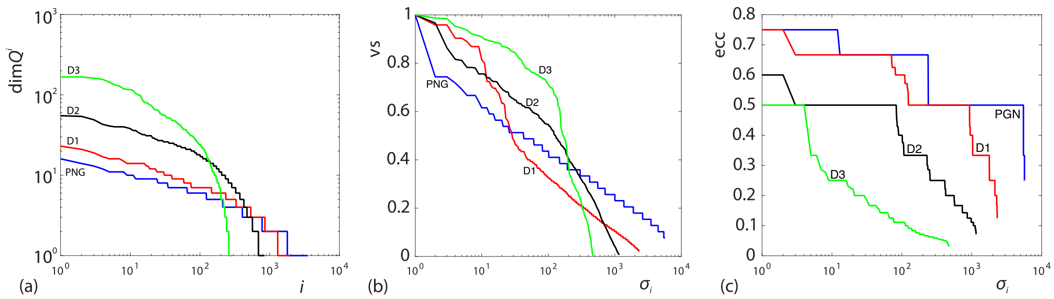

Figure 4. Ranked topological dimension (a), vertex significance of simplex (b), and eccentricity (c) of the power grid network and networks of delegates

Conclusion and discussion

Results reveal latent embedment of the most significant delegates with higher-order aggregation than at the level of the initially obtained simplicial complex from an original network. Thus, they represent the backbone of a power grid network, and higher-order clustering of the same delegates via pseudo-links refers to the high stability of the network.

Topological analysis of local architecture should classify structural layers where nodes naturally belong. The study of the tails of rankings and local properties of simplicial complex of delegates in a power grid network can reveal whether there are potential issues that can affect network stability when nodes are located at the inaccurate layer. In other words, if a node that naturally originates in a lower hierarchical layer is found mixed within the same simplex of delegates with nodes that belong on a higher layer, and vice versa, it would present a potential issue in power grid stability and can cause cascade disruptions ensuing blackouts and islanding. Therefore, this model and algorithm have great potential for testing power grid stability whether it is a network in construction or an original network, and can suggest nodes reconnecting in order to achieve a network with a long term stability and sustainability.

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