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SINUSOIDAL CODED MODEL

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Abstract

This article discusses a usage of modified periodic harmonic analysis to prove the periodicity of experimental data for seasonal stable isotope variations in precipitation and stream water. A proof for sinusoidal coded model is based on a hypothesis concerning the importance of the first harmonic. The utility of proposed model was demonstrated on experimental data for oxygen isotope stream water content of the Velika Morava catchment over the period 2004-2007. This study suggests an applicability of sinusoidal coded model for more accurate estimation of mean residence time without a loss of theoretical or physical significance.

Introduction

The residence time of water draining a catchment has important implications for flow pathways and storage, as well as water quality, since many biogeochemical reactions are time-dependent periodical functions. Seasonal stable isotopic variations ($^{18}O/^{16}O$ and $^{2}H/^{1}H$) in surface and groundwater have been used for the estimation of mean residence times (MRTs) of less than 5 years using regression models with annual sinusoidal components [1]. Generally, the stream $\delta^{18}O$ composition reflects the temporal pattern of the precipitation $\delta^{18}O$ composition with the signals that are significantly damped. Any claim that natural meteorological phenomena are in general periodic and could be characterized by the generalized sine-function equation produces limitations of the seasonal amplitude-attenuation (convolution integral) approach for estimating MRTs due to (1) multiyear isotopic data for surface water and groundwater; (2) accurate and representative isotopic values of basin precipitation over several years; (3) high-precision, unbiased isotopic analyses over the period of interest; and (4) sufficient time of travel for attenuation of the isotope precipitation amplitude [2].

A shape of continual and limited function should not affect to its basic periodicity. The starting point is an application of harmonic analysis [3] to a periodic function significantly different from sinusoidal shape that frequently leads to rejection of hypothesis concerning the importance of the first harmonic. In the case that raw data previously have been coded near to a sinusoidal wave shape, this problem would be overcome. The main hypothesis being tested in this paper is that variations of isotopic composition in precipitation and stream water are periodical appearances of a seasonal cycle over a 12 month period which could be fitted to a coded sine function.

Results and Discussion

Weighted mean monthly isotopic data (concerning amount of precipitation or discharge for precipitation and stream water, respectively) were transformed using Equation 1 to be varied in the interior of the interval (—1, 1)

$$\delta_N(i) = 2 \cdot \frac{\overline{\delta}_W(i) - \overline{\delta}_{WMin}}{\overline{\delta}_{WMax} - \overline{\delta}_{WMin}} - 1 \qquad (1)$$

where $\overline{\delta}_W$ is weighted mean monthly value of the feature of interest in the *i-th* month and subscripts *Min* and *Max* are refereeing to minimal and maximal values, respectively. Then, transformed data were coded in a sinusoidal shape function using permutation function along a time axis (Equation 2).

$$\delta_N^{Cod}(i) = \delta_N(P_k^{12}(i)) \qquad i = 1, 2, \ldots 12 \qquad (2)$$

$P_k^{12}(i)$ is a permutation function of the *k*-th lexical order. By suitably choosing the constants *k*, the coded function should resemble to a sinusoidal form of the resulting curve. Superscript *Cod* stands for a coded value. In order to get the most representative calculated periodic function, the process of decoding was performed on the fitted sinusoidal function δ_{Cal} using Equation (3).

$$\delta_{Cal}^{Decod}(i) = \delta_{Cal}(P_k^{12}(i)^{-1}) \qquad i = 1, 2, \ldots 12 \qquad (3)$$

Superscript (-1) designates the inverse of permutation function $P_k^{12}(i)$. Superscript *Decod* and subscript *Cal* stand for decoded and calculated fitted sinusoidal function, respectively. The other notation used is the same as previously mentioned in Equation (2).

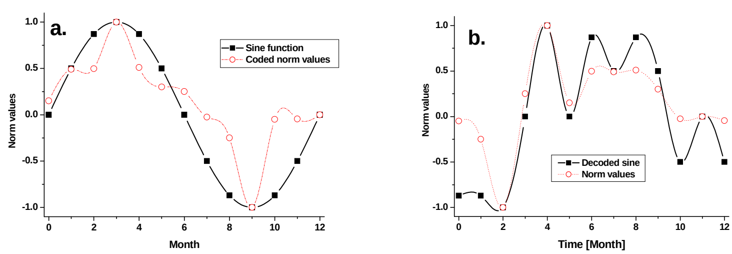

Fig. 1. Coded and decoded data for a sine function with white noise (σ^2 =1)

Figure 1a shows one original sinusoidal function (solid symbol) and derived function using proposed coded model (open symbol). Figure 1b compares the sine function returned by our coded model (solid symbol) and raw norm values (open symbol).

If the condition written in Equation 4 is satisfied, it has made considerable use of a zero hypothesis [3] concerning the periodicity of the coded model statistically significant at the 95% level

$$\frac{\sigma^2 - \overline{\sigma}_m^2}{\sigma^2} \geq 0.638 \tag{4}$$

where σ^2 is the variance for all considered data and $\overline{\sigma}_m^2$ presents average monthly variance.

The isotopic measurements of bimonthly steam water samples (solid symbol) sequentially collected at the last profile (Ljubicevski most) on the Velika Morava before confluence with the Danube span a large but variable annual range that does not accurately resemble a sine wave (Figure 2). Under the conditions of Equation 4, coded model was used to fit actual isotopic records (open symbol).

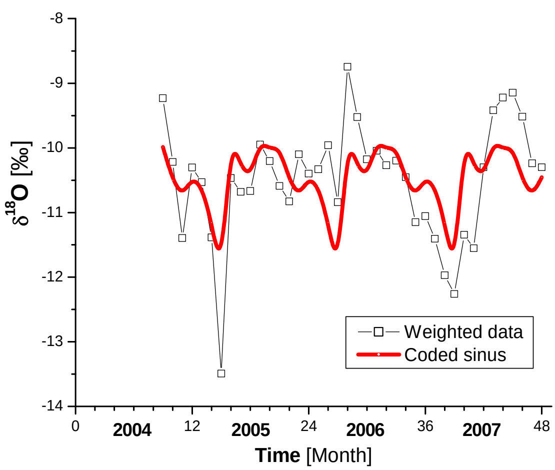

Fig. 2. Measured and simulated oxygen-18 content for stream water of the Velika Morava catchment using sinusoidal coded model over the period 2004-2007

Conclusion

In conclusion, the seasonal stable isotope signal in precipitation and stream water exhibit a periodicity of 12 months with 95% confidence limits. The proposed model provides superior fit to actual isotopic records for the Velika Morava catchment. Sinusoidal coded model can be applicable as an alternative method in order to refine our understanding of flow paths and residence times, and to help structure and validate more accurate hydrological models.

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