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Ty2 revisited

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Ty₂ revisited (abstract)

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Gallin, in [6], provides a faithful translation of Montague intensional logic [8], **IL**, into **Ty₂** (a two-sorted simple theory types) :

$$\begin{array}{ll} (x^\alpha)^* = x^\alpha & (\lambda x^\alpha. t^\beta)^* = \lambda x^\alpha. (t^\beta)^* \\ (c^\alpha)^* = c^{\mathbf{s} \rightarrow \alpha} \mathbf{x}^\mathbf{s} & (\hat{\ } t^\alpha)^* = \lambda \mathbf{x}^\mathbf{s}. (t^\alpha)^* \\ (t^{\alpha \rightarrow \beta} u^\alpha)^* = (t^{\alpha \rightarrow \beta})^* (u^\alpha)^* & (\sim t^{\mathbf{s} \rightarrow \alpha})^* = (t^{\mathbf{s} \rightarrow \alpha})^* \mathbf{x}^\mathbf{s} \end{array}$$

In [5], another translation of **IL** into **Ty₂** is proposed. This translation, which has the property of translating a closed term by a closed term, interprets extensions as terms of type $\mathbf{s} \rightarrow \alpha$, and intensions as terms of type $\mathbf{s} \rightarrow \mathbf{s} \rightarrow \alpha$. Montague's intension and extension operators correspond then to Curry's elementary cancelator (K) and duplicator (W), respectively [4]:

$$\begin{array}{ll} (x^\alpha)^\dagger = \lambda i^\mathbf{s}. x^\alpha & (\lambda x^\alpha. t^\beta)^\dagger = \lambda i^\mathbf{s} x^\alpha. (t^\beta)^\dagger i^\mathbf{s} \\ (c^\alpha)^\dagger = \lambda i^\mathbf{s}. c^{\mathbf{s} \rightarrow \alpha} i^\mathbf{s} & (\hat{\ } t^\alpha)^\dagger = \lambda i^\mathbf{s}. (t^\alpha)^\dagger \\ (t^{\alpha \rightarrow \beta} u^\alpha)^\dagger = \lambda i^\mathbf{s}. (t^{\alpha \rightarrow \beta})^\dagger i^\mathbf{s} ((u^\alpha)^\dagger i^\mathbf{s}) & (\sim t^{\mathbf{s} \rightarrow \alpha})^\dagger = \lambda i^\mathbf{s}. (t^{\mathbf{s} \rightarrow \alpha})^\dagger i^\mathbf{s} i^\mathbf{s} \end{array}$$

In this paper, we use the same kind of interpretation in order to translate hybrid logic [2] into **Ty₂**:

$$\begin{array}{ll} (p)^\ddagger = \lambda i^\mathbf{s}. p^{\mathbf{s} \rightarrow \mathbf{t}} i^\mathbf{s} & (\varphi \wedge \psi)^\ddagger = \lambda i^\mathbf{s}. ((\varphi)^\ddagger i^\mathbf{s}) \wedge ((\psi)^\ddagger i^\mathbf{s}) \\ (j)^\ddagger = \lambda i^\mathbf{s}. j^\mathbf{s} = i^\mathbf{s} & (@_j \varphi)^\ddagger = \lambda i^\mathbf{s}. (\varphi)^\ddagger j^\mathbf{s} \\ (\neg \varphi)^\ddagger = \lambda i^\mathbf{s}. \neg((\varphi)^\ddagger i^\mathbf{s}) & (\downarrow_j. \varphi)^\ddagger = \lambda j^\mathbf{s}. (\varphi)^\ddagger j^\mathbf{s} \end{array}$$

where $i^\mathbf{s}$ is a fresh variable.

Translation $(\cdot)^\ddagger$, which is such that $\mathfrak{M}, g, w \models \varphi$ if and only if $[[(\varphi)^\ddagger i]_{g[i:=w]}^{\mathfrak{M}}] = 1$, offers several advantages:

1. It allows to establish results on hybrid formulas simply by using β -reduction. For instance, among the axioms given in [2]: Axioms *Q1*, *K@*, and *Scope* appear to be mere instances of the propositional tautology $A \rightarrow A$; Axioms *Q2*, *Q3*, *Introduction*, *Label*, *Nom*, and *Swap* derive from elementary equality rules; Axioms *Self Dual*_↓, and *Self Dual*_@ are instances of the double negation rule.
2. It may be easily extended to cover the higher-order case, resulting in a rebuilding of the Hybrid Type Theory of Areces et al. [1]. A completeness theorem may then be derived as a corollary of Henkin's classical result [7].
3. It may be mixed with translation $(\cdot)^\dagger$, resulting in translations of (higher-order) Intensional Hybrid Logics [3, CHAP. 7].
4. From a more practical point of view, it provides a way of easily incorporating hybrid logic constructs in a Montague grammar.

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