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MATHEMATICAL MODEL OF THE FIXED OXYGEN BREATHING APPARATUS WITH A CIRCULAR AIRWAY SCHEME

EVGENIY POTAPENKO, STEPAN EKHILEVSKIY, OKSANA GOLUBEVA
Polotsk State University, Belarus

The research deals with the influence of the CO_2 *breakthrough on the dynamic sorption activity of the regenerative cartridge of fixed oxygen breathing apparatus with a circular airway scheme. It is illustrated that the breakthrough return during breath at the later stages of the apparatus operation is essential and depending on the apparatus model or its operating regime noticeably (up to 10%) reduces its protection term.*

The atmosphere in the fixed oxygen breathing apparatus regenerates in the process of the exhaled air filtration through the regenerative cartridge with porous granules of potassium superoxide-based oxygen-containing product. As a result of the CO_2 chemisorption in the proportion close to ideal the oxygen necessary for breathing is produced

$$4KO_2 + 2CO_2 = 2K_2CO_3 + 3O_2 + 360\ \text{kJ}\ , \tag{1}$$

Regenerating process modeling is a classical task of sorption dynamics (see [1-3]) that traces the evolution of admixture break through the absorber layer. It is usually solved by using mathematical physics methods if there are stationary boundary conditions upon entering the filter [4]. However, the apparatus with a circular airway scheme, besides the invariable component set by the apparatus operation regime, adds the CO_2 breakthrough that steadily increases as the regenerative cartridge resources exhausts. In other words, a variable absorber concentration upon entering the absorber layer takes place. The research [5] suggests an appropriate formalism that analytically describes the dynamic sorption activity with a variable absorber concentration upon entering the filter and comes to the following equation system:

$$-\omega'_{\xi}(\xi,\tau) = e^{-\tau}\left[e^{-\xi}\omega_0(0) + \int_0^{\tau} e^{\tau} d_{\tau}\,\omega(\xi,\tau)\right], \qquad \tau > 0, \tag{2}$$

$$u(\xi,\tau) = e^{-\tau}\int_0^{\tau} e^{\tau}\omega(\xi,\tau)\,d\tau, \qquad \tau > 0, \tag{3}$$

where τ and ξ are dimensionless time and coordinate (the penetration depth in the absorber layer) respectively, $\omega(\xi,\tau)$ – reduced concentration CO_2, $\omega_0(0)$ – its initial value upon entering the filter, $u(\xi,\tau)$ – waste product ratio;

The solution (2) can be presented as a series

$$\omega(\xi,\tau) = e^{-\xi-\tau}\sum_{n=0}^{\infty}\frac{f_n(\tau)}{n!}\xi^n\ , \tag{4}$$

with its coefficients connected by the recurrent correlation

$$f_{n+1}(\tau) = \int_0^{\tau} f_n(\tau)\,d\tau\ , \tag{5}$$

and knowing that

$$f_0(\tau) = e^{\tau}\omega_0(\tau) \tag{6}$$

allows to calculate any $f_n(\tau)$ up to any number. The formula (6) for $f_0(\tau)$ follows from the series (4) and the boundary condition

$$\omega(0,\tau)=\omega_0(\tau)\,. \tag{7}$$

Using computer calculations the correlations (2) – (7) allow to describe the CO_2 chemisorption in the regenerative cartridges of the breathing apparatus with a circular airway scheme[1]. To do so $\omega_0(\tau)$ in (7) in accordance with the described above should be replaced[2] with

$$\omega_0(\tau)=1+\omega(\eta,\tau)\,, \tag{8}$$

where η is a dimensionless cartridge length. As a result a self-consistent problem of defining the required function $\omega(\xi,\tau)$ appears. In order to solve it an iterative procedure with a small parameter $\omega(\eta,\tau)$ is used.

In order to get a null approximation in (8) the breakthrough $\omega(\eta,\tau)=0$ should be considered completely negligible. This leads back to the stationary boundary condition $\omega(0,\tau)=1$, with its solution for the recurrent correlation (6) written analytically

$$f_n(\tau)=e^{\tau}-\sum_{k=0}^{n-1}\frac{\tau^k}{k!} \qquad (n=1,\ 2,\ \ldots)\,. \tag{9}$$

Having substituted (9) into (4) we get the known result (e.g. (10) in [6])

$$\omega0(\xi,\tau)=e^{-\xi}\left[1+\sum_{n=1}^{\infty}\frac{\xi^n}{n!}\left(1-e^{-\tau}\sum_{k=0}^{n-1}\frac{\tau^k}{k!}\right)\right]. \tag{10}$$

The next step of the iterative procedure corresponding to the first approximation $\omega1(\xi,\tau)$ is the substitution of (10) into (8) and a numerical implementation of the recurrent procedure (5). To do so a special program was written in the MathCAD package environment with its main fragments illustrated in fig. 1.

The breakthrough-time diagram drawn using this program (fig. 2) is predictable and allows reasonable interpretation. In the beginning when the CO_2 breakthrough is small, the graphs drawn for the open (curve 1) and the circular (curve 2) schemes of the airway are practically identical. But as the cartridge resource exhausts the breakthrough in the circular scheme grows at a greater rate, which is natural since the CO_2 molecules that avoided chemisorption return during breath increasing the CO_2 amount in the exhaled air. As it develops the process moves farther away from the one which takes place in the open scheme. As a result the critical CO_2 breakthrough time $\tau_{\kappa p}$ is reduced by 11.6%. It equals in the order of values to the value received in [7] using approximate approach, based on the hourglass principle according to which the apparatus operation term is determined by the amount of the entered CO_2 molecules [8].

The second iteration (corresponding to the approximation $\omega2(\xi,\tau)$) deals with the substitution of $\omega2(\xi,\tau)$ into (8) instead of $\omega(\eta,\tau)$. The result of the subsequent implementation of the recurrent procedure (5) is also graphically represented in fig.2 (curve 3). The critical CO_2 breakthrough time has reduced by 0.9% which is hardly distinguished by eye. However, it raises a question of the convergence of the applied iterative procedure within the current breakthrough change range.

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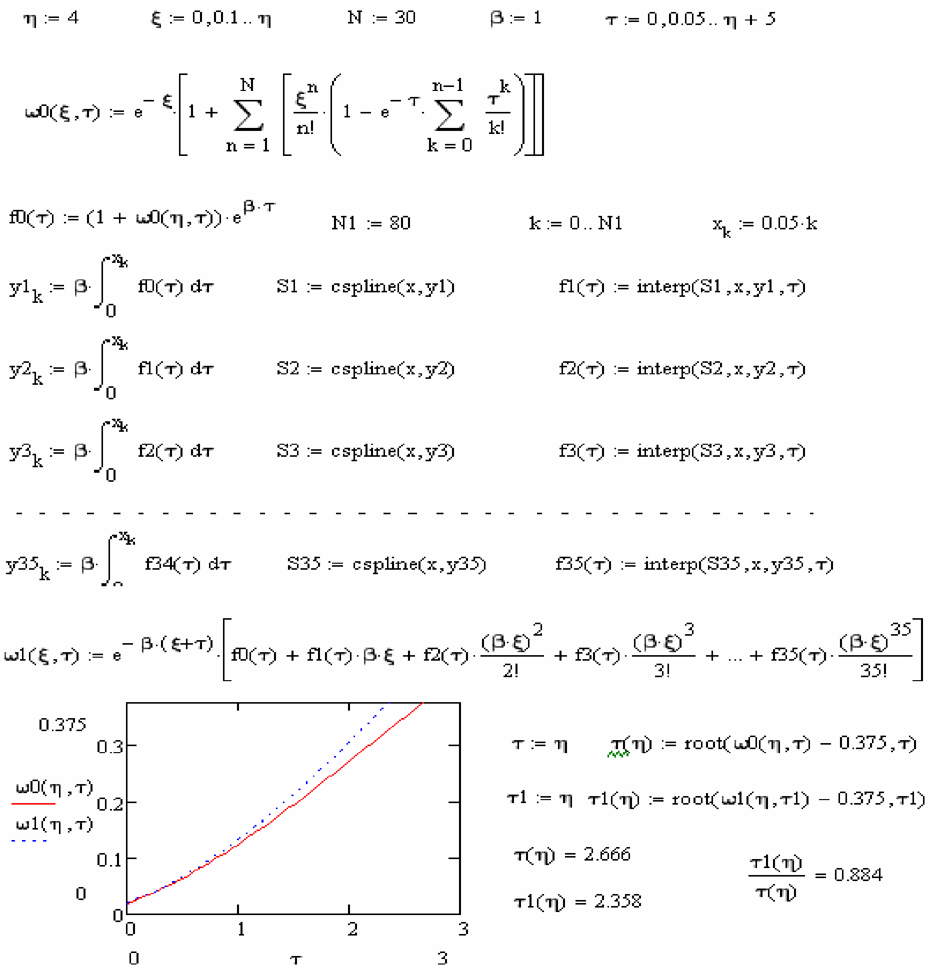
[1] In terms of application the fact that the equation (2) was received without considering the steady increase of $\omega_0(t)$ is also important. It means that the correlations (2) – (6) can also describe the admixture desorption with its dilution upon entering the filter.

[2] The way the amount of in the breathed air influences the gas exchange in the organism and, correspondingly, the amount of CO_2 in the exhaled air is a complicated issue. That is why the concentration additivity is an idealization as it is and can only be accurately implemented in a breathing simulator.



$\eta := 4 \qquad \xi := 0, 0.1 .. \eta \qquad N := 30 \qquad \beta := 1 \qquad \tau := 0, 0.05 .. \eta + 5$

$$\omega0(\xi,\tau) := e^{-\xi}\cdot\left[1 + \sum_{n=1}^{N}\left[\frac{\xi^n}{n!}\cdot\left(1 - e^{-\tau}\cdot\sum_{k=0}^{n-1}\frac{\tau^k}{k!}\right)\right]\right]$$

$f0(\tau) := (1 + \omega0(\eta,\tau))\cdot e^{\beta\cdot\tau} \qquad N1 := 80 \qquad k := 0 .. N1 \qquad x_k := 0.05\cdot k$

$y1_k := \beta\cdot\int_0^{x_k} f0(\tau)\, d\tau \qquad S1 := \mathrm{cspline}(x,y1) \qquad f1(\tau) := \mathrm{interp}(S1,x,y1,\tau)$

$y2_k := \beta\cdot\int_0^{x_k} f1(\tau)\, d\tau \qquad S2 := \mathrm{cspline}(x,y2) \qquad f2(\tau) := \mathrm{interp}(S2,x,y2,\tau)$

$y3_k := \beta\cdot\int_0^{x_k} f2(\tau)\, d\tau \qquad S3 := \mathrm{cspline}(x,y3) \qquad f3(\tau) := \mathrm{interp}(S3,x,y3,\tau)$

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$y35_k := \beta\cdot\int_0^{x_k} f34(\tau)\, d\tau \qquad S35 := \mathrm{cspline}(x,y35) \qquad f35(\tau) := \mathrm{interp}(S35,x,y35,\tau)$

$$\omega1(\xi,\tau) := e^{-\beta\cdot(\xi+\tau)}\cdot\left[f0(\tau) + f1(\tau)\cdot\beta\cdot\xi + f2(\tau)\cdot\frac{(\beta\cdot\xi)^2}{2!} + f3(\tau)\cdot\frac{(\beta\cdot\xi)^3}{3!} + \ldots + f35(\tau)\cdot\frac{(\beta\cdot\xi)^{35}}{35!}\right]$$

$\tau := \eta \qquad \tau(\eta) := \mathrm{root}(\omega0(\eta,\tau) - 0.375,\tau)$

$\tau1 := \eta \qquad \tau1(\eta) := \mathrm{root}(\omega1(\eta,\tau1) - 0.375,\tau1)$

$\tau(\eta) = 2.666 \qquad \frac{\tau1(\eta)}{\tau(\eta)} = 0.884$

$\tau1(\eta) = 2.358$

Fig. 1. The program to calculate the evolution of the CO_2 break through the regenerative cartridge of the fixed oxygen breathing apparatus with a circular airway scheme

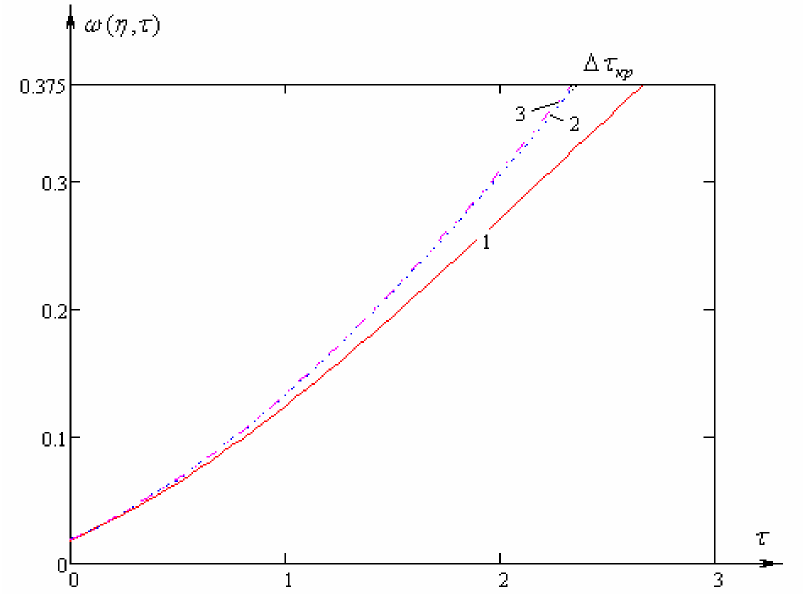

Fig. 2. The evolution of the CO_2 break through the regenerative cartridge of the fixed oxygen breathing apparatus connected via: 1 – the open scheme; 2 – the circular scheme (the first approximation), 3 – the circular scheme (the second approximation)

Due to the breathing physiology the carbon dioxide poisoning starts when its amount in the inhaled air reaches 1.5%. Normally, a person exhales air with a 4% amount of the carbon dioxide, i.e. the condition for the critical CO_2 breakthrough is

$$\omega(\eta, \tau_{кр}) = 1,5/4 = 0,375\,. \tag{11}$$

It means that if $\tau \leq \tau_{кр}$ then the adjustments to $\omega_0(\tau)$ (see (8)) appearing during the iterative procedure can be evenly estimated by the members of a descending geometric sequence with the denominator $q = 0.375$ which converges if $q<1$.

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EVALUATION OF THE RESOURCE USAGE EFFICIENCY INCREASE OF THE FIXED OXYGEN BREATHING APPARATUS

EVGENIY POTAPENKO, STEPAN EKHILEVSKIY, OKSANA GOLUBEVA
Polotsk State University, Belarus

The research shows that the CO_2 critical breakthrough time when using a heterogeneously fitted regenerative cartridge connected via the open scheme provides an understated breathing apparatus protection increase evaluation. It is better to employ the average oxygen-containing product usage along the full length of the cartridge by the time of the CO_2 critical breakthrough, because the breakthrough decrease due to the dependence of the product granules diameter on the granules occurrence depth evolves in time by itself. It is shown that the suggested evaluation in terms of quantity equals the one determined by the protection term for the circular airway scheme where the CO_2 breakthrough returns during breath.

The current fixed oxygen breathing apparatus have a substantial reserve for protection resource usage efficiency increase of the regenerative cartridge. For instance, a 2-litre cylinder for P12 respirator designed for a 4-year protection term is filled with 550g of compressed oxygen. The PX-4 apparatus is designed for the same term. Its regenerative cartridge contains 3.7kg of oxygen-containing product 90% of which is potassium superoxide. Taking into account the formula KO_2 of the chemical compound and the molecular weight of its elements we get 0,9.3,7.32/71=1,5kg of oxygen. If we apply the stoichiometry of the reaction

$$4KO2 + 2CO2= 2K2CO3 + 3O2 + 360\text{ kJ}\,, \tag{1}$$