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Cite this article: Klimm F. 2022 Quantifying the 'end of history' through a Bayesian Markov-chain approach. *R. Soc. Open Sci.* **9**: 221131.
<https://doi.org/10.1098/rsos.221131>

Received: 31 August 2022
Accepted: 6 November 2022

Subject Category:
Science, society and policy

Subject Areas:
applied mathematics/complexity

Keywords:
democracy, Markov chain, cultural evolution,
network inference

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Electronic supplementary material is available online at <https://doi.org/10.6084/m9.figshare.c.6302899>.

Quantifying the 'end of history' through a Bayesian Markov-chain approach

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Political regimes have been changing throughout human history. After the apparent triumph of liberal democracies at the end of the twentieth century, Francis Fukuyama and others have been arguing that humankind is approaching an 'end of history' (EoH) in the form of a universality of liberal democracies. This view has been challenged by recent developments that seem to indicate the rise of defective democracies across the globe. There has been no attempt to quantify the expected EoH with a statistical approach. In this study, we model the transition between political regimes as a Markov process and—using a Bayesian inference approach—we estimate the transition probabilities between political regimes from time-series data describing the evolution of political regimes from 1800 to 2018. We then compute the steady state for this Markov process which represents a mathematical abstraction of the EoH and predicts that approximately 46% of countries will be full democracies. Furthermore, we find that, under our model, the fraction of autocracies in the world is expected to increase for the next half-century before it declines. Using random-walk theory, we then estimate survival curves of different types of regimes and estimate characteristic lifetimes of democracies and autocracies of 244 years and 69 years, respectively. Quantifying the expected EoH allows us to challenge common beliefs about the nature of political equilibria. Specifically, we find no statistical evidence that the EoH constitutes a fixed, complete omnipresence of democratic regimes.

1. Introduction

Political systems undergo constant changes, which are driven by a variety of internal and external forces [1]. Naturally, this raises the question whether there is an endpoint in this development of human societies. Many authors, among them Georg Wilhelm

Friedrich Hegel [2], Karl Marx [3] and Karl Popper [4], have been aiming to predict theoretically which kind of political system may constitute this final state of human sociocultural evolution. Francis Fukuyama popularized the term ‘end of history’ (hereafter EoH), first in a 1989 essay [5] and second in a 1992 book [6], indicating that after the defeat of fascism and communism, the Western liberal democracy may become a universal form of government. He argued that liberal democracies and the accompanying market liberalization provide a wealth for their citizens that makes transitions from liberal democracies to autocracies unlikely. These ideas are especially challenged in recent years which have seen a rise of so-called hybrid regimes, such as illiberal democracies [7,8], and a more fine-grained deterioration of democratic norms, jointly referred to as *democratic backsliding* [9–12], even though a generally agreed definition is lacking [13]. At least since the 2016 US presidential election, there also has been discussion about the impact of polarization [14], ethnic antagonism [15,16] and spread of misinformation [17] on political decision making, illustrating a rising interest in quantitative studies of the democratic process [18,19]. Complex-system approaches, in particular, have been used to find hidden structure in political data [20–24].

In this paper, we use an empirical complex-system approach to predict the EoH and quantify what the EoH would look like under the assumption that the historically observed developments of regimes are representative of the long-term behaviour of their transitions. Specifically, we use a *Markov-chain approach* to model the transition of regimes, which we characterize in an ordinal 21 point scale. Markov chains are a ubiquitous tool for statistical data analysis [25] and have been, for example, employed to speech recognition [26] and are also used by Google to rank webpages with their *PageRank* algorithm [27]. In a political science context, Markov models have been employed to investigate the democratization process [28], to study, for example, whether certain socio-economic factors (e.g. gross domestic product) [29] or international influences [30] impact the democratization process. One challenge that is commonly ignored in such transition models is that the available transition data are sparse. We tackle this challenge by using a Bayesian estimator of Markov chains to infer regime-transition probabilities (for an introduction to Bayesian data analysis methods, see [31]).

The remainder of the paper is structured as follows. First, we give a brief, intuitive primer on Markov chains and illustrate them with a simplified model of regime change. Second, we present the results of our analysis based on empirical regime change data. Third, we discuss our findings and limitations. We provide a detailed description of the data and the mathematical methods in the Methods section. The electronic supplementary material contains statistical tests and additional results.

2. A primer on Markov processes with an illustrative model of regime transitions

In this section, we provide a brief, intuitive description of Markov processes and how they can be used to model the transition between political regimes. For a more formal discussion, see the Methods section.

Assume the political regime of a country can be in one of three states¹: autocratic (A), mixed (M)² or democratic (D). The regime may change over time and we quantify its state yearly. From one year to the next, the regime has a certain probability to stay in the current state or change to one of the two other states. We assume that these transition probabilities depend exclusively on the current state, something called the *Markov property*.

In figure 1, we give an example of such a Markov process with conveniently (but unrealistically) chosen probabilities of regime changes. The discs represent the three possible states (A, M and D) and the arrows represent transitions from one to another. In the given example, a mixed regime has a 40% chance of becoming an autocracy in the following year, a 50% chance of staying a mixed regime and a 10% chance of becoming a democracy. A democracy has a probability of 90% of remaining a democracy and a 10% probability of becoming a mixed regime. The probability of becoming an autocracy is zero and therefore this transition is not possible. An autocracy has a 70% chance of remaining an autocracy, a 20% chance of becoming a mixed regime and a 10% chance of becoming a democracy.

This model is a much simplified description of the actual underlying processes which are complex socio-economical systems that are—most likely—intractable at a global scale. Nevertheless, this

¹We use the term ‘state’ exclusively to mean ‘Markov state’ and refrain from using it to describe a polity under a system of governance.

²Regimes that mix characteristics of democratic and autocratic regimes are also sometimes referred to as anocracies or semi-democracies.

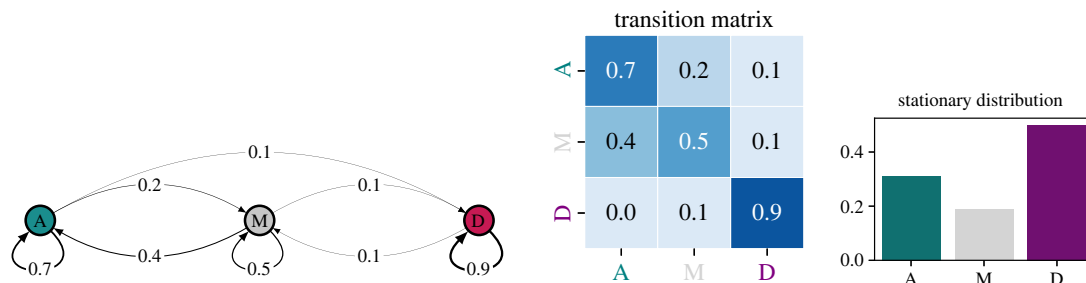


Figure 1. We discuss the transition between political regimes as a Markov chain with three states, representing autocracies (A), mixed regimes (M) and democracies (D). The transition probabilities between these states are given as arrows. These transition probabilities can also be represented in a transition matrix \mathbf{P} . The Markov chain reaches a stationary distribution in which the proportion of the regimes is not changed under the transitions. In this example, the stationary distribution consists of approximately 50% democracies, 30% autocracies and 20% mixed regimes.

abstract description allows us to make certain predictions. Here, we focus on a concept called the *stationary distribution* which represents the distribution of political regimes that is unchanged under the given transition probabilities (for mathematical details on how to compute this, see the Methods section). In the example given here, this stationary distribution consists of $5/16 \approx 31\%$ autocracies, $3/16 \approx 19\%$ mixed regimes and $8/16 = 50\%$ democracies (see bar chart in figure 1). This stationary distribution, however, does not reflect a situation in which there are no more transitions occurring. Rather, the expected transitions between the different regimes occur at rates that cancel each other out.

3. Results

We discuss the transition between political regimes with a Markov model similar to the one discussed in §2. The model, however, is more complex as it has 21 instead of three discrete characterizations of political regimes. These characterizations of political regimes are given by the POLITY2 score [32,33]. The POLITY2 score characterizes the political regimes of 195 countries on a 21-point scale from -10 (least democratic; full autocracy) to $+10$ (most democratic; full democracy) from 1800 to 2018 on a yearly basis. For details on the definition of the data, see the Method section. We separate these data into time series for each country that describe the development of its political regimes.

In figure 2a, we show the obtained time series describing $n = 193$ countries. The time series differ in their length with a mean length of $\langle L \rangle \approx 89$ years. We highlight the time series for five selected countries, which differ drastically in their POLITY2 score. The United States of America has a relatively high POLITY2 score s but shows a recent decline. Spain's POLITY2 score s has been increasing and decreasing over time but reached recently its maximum $s = +10$. Kazakhstan, a former Soviet republic, has a much lower POLITY2 score $s = -5$.

Modelling the time series of the POLITY2 score as a Markov model assumes that a country's transition probability depends exclusively on the current POLITY2 score. While this is a common assumption in many analyses of regime transitions (e.g. [29,34–37]), it is a drastic simplification. To test whether this is a reasonable assumption, we use a statistical procedure for the estimation of the order of a Markov chain from time-series data [38]. We compute the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) for Markov-chain models of order $K = 1, \dots, 7$. The information criteria identify which of different models describe data best (for details, see electronic supplementary material, 7). Our analysis yields that a Markov chain of first order (i.e. a memoryless Markov model) describes the data best. In the following, we will, therefore, assume that our data follow a memoryless Markov model.

3.1. Countries have a predominantly constant POLITY2 score but tend to become more democratic over time

We use a Bayesian mean posterior approach to estimate the transition probabilities between the 21 states from these time series under the Markov assumption. As the data of regime transitions are sparse, we use a Bayesian approach to update our belief in the transition probabilities (see Methods section). The methodological advantages of using a Bayesian approach are twofold. First, it allows us to estimate

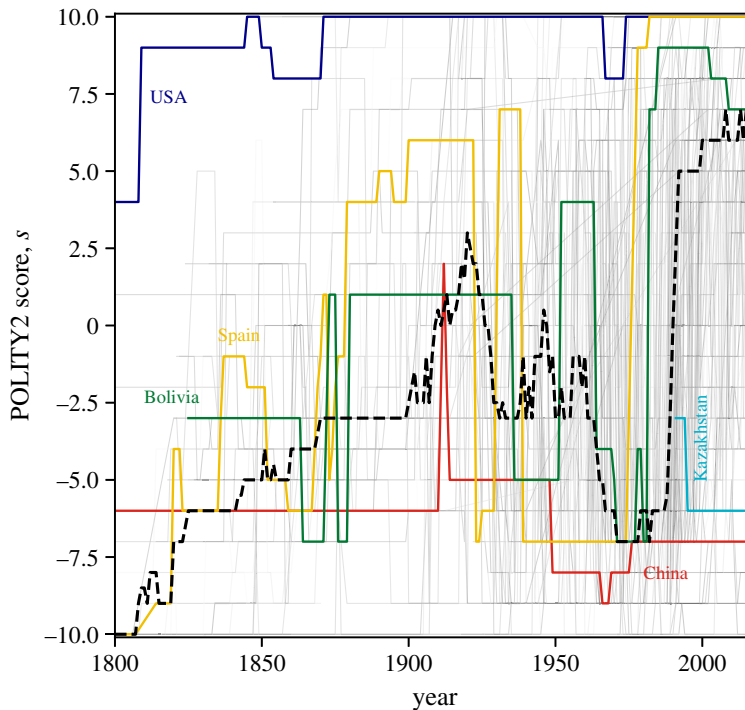


Figure 2. The POLITY2 data describe the development of countries' political systems from 1800 to 2018. The POLITY2 score s ranges from -10 (least democratic) to $+10$ (most democratic). We show the associated time series of $n = 193$ countries and highlight selected countries (USA, Bolivia, China, Kazakhstan and Spain). In addition, we illustrate the temporal development of the median POLITY2 score $\langle s \rangle$ as a dashed line.

underlying transition probabilities, even though some regime transitions occur rarely. Second, it allows us to obtain a unique stationary distribution, which we will discuss in more detail in §3.3.

We show the transition matrix \mathbf{P} in figure 3. The matrix is dominated by its diagonal elements, which indicates that regimes have a high probability of staying at the same POLITY2 score. A country with score -10 (full autocracy), for example, has a probability of 97.5% to stay at this score in the following year. The other diagonal elements are similarly high, with the lowest being $p_{00} \approx 78\%$. The probabilities of regime changes are represented by the off-diagonal elements in \mathbf{P} with the elements above the diagonal representing an increasing POLITY2 score (i.e. becoming more democratic) and the elements below the diagonal representing a decreasing POLITY2 score (i.e. becoming less democratic). We observe that both transition directions are possible, but transitions that increase the POLITY2 score tend to be more likely. On average, across all regimes, the POLITY2 score is expected to increase by approximately $\langle p_{ij} \rangle \approx 0.4$ per year, which is in accord with earlier results on data up to the end of the twentieth century [28]. While transitions that change the POLITY2 score strongly (e.g. from -9 to $+10$ with probability 0.2%) are possible, smaller transitions (e.g. from -9 to -8 with probability 1.9%) tend to be more likely. The most likely transition that increases the POLITY2 score is from $+5$ to $+6$ and has a probability of 4.2%. The most likely transitions that decrease the POLITY2 score are from $+3$ to 0 and from -7 to -8 , both with a probability of 2.9%. An example of a fairly likely off-diagonal transition that changes the POLITY2 score strongly is the transition from $+2$ to -9 with a probability of 1.4%, a transition that occurred, for example, in 1898 in Guatemala, when Manuel Estrada Cabrera established a dictatorship [39].

3.2. Full democracies and full autocracies are more stable than mixed regimes

For each state i , we compute the expected change of the POLITY2 score as $\Delta s(i) = \sum_{j=1}^N p_{ij}(i - j)$ (see figure 4). The expected change Δs of the POLITY2 scores is close to zero for full autocracies ($s = -10$), mixed regimes ($s = 0$) and full democracies ($s = +10$). The empirically obtained $\Delta s(i)$, therefore, resembles a cubic polynomial, which we highlight through a least-squares fit. This cubic curve can be understood from the remain probabilities p_{ss} , which are the diagonal elements of the transition matrix \mathbf{P} (see inset in figure 4). We find that p_{ss} follows approximately a parabola with the extreme regimes

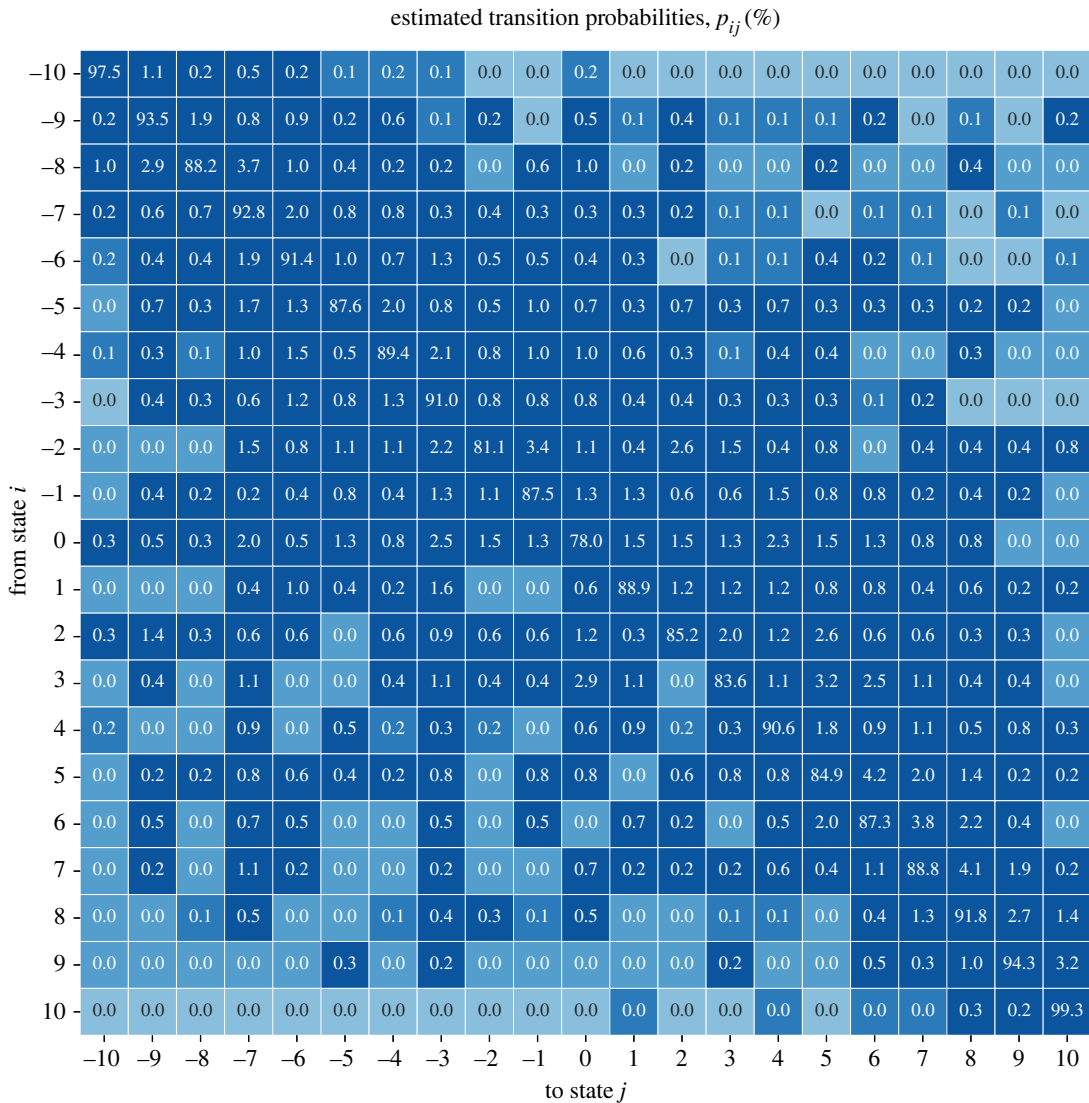


Figure 3. Countries' regimes are predominantly constant as the estimated transition matrix \mathbf{P} has high probabilities along the diagonal. Entries above the diagonal represent the probability of regime transitions that increase the POLITY2 score (i.e. more democratic) and entries below the diagonal represent regime transitions that decrease the POLITY2 score (i.e. less democratic). The transition probabilities p_{ij} are given in per cent and colour-coded from low (bright) to high (dark).

(i.e. full autocracies and full democracies) having remain probabilities p_{ss} close to 100%, resulting in small expected changes $\Delta s \approx 0$. Mixed regimes have the smallest remain probability $p_{00} \approx 78\%$, yet the expected change $\Delta s \approx 0$ as transitions that increase and decrease the POLITY2 score s tend to balance each other out. A simple mathematical model describing the resilience of extreme regimes is

$$p_{ij} = \begin{cases} p_{\text{remain}}(i) & \text{if } i = j, \\ \frac{1 - p_{\text{remain}}(i)}{2L} & \text{if } i \neq j, \end{cases}$$

with $p_{\text{remain}}(i) = p_0 + p_2(i/L)^2$, for which we compute the expected change to

$$\Delta s(i) = \frac{N}{2L} (p_{\text{remain}}(i) - 1)i = \frac{N}{2L} \left(p_0 + p_2 \left(\frac{i}{L} \right)^2 - 1 \right) i,$$

resulting in a cubic polynomial, as observed in the inferred transition matrix \mathbf{P} , indicating that the particular form of $\Delta s(i)$ is indeed driven by the large remain probabilities p_{ss} of extreme regimes.

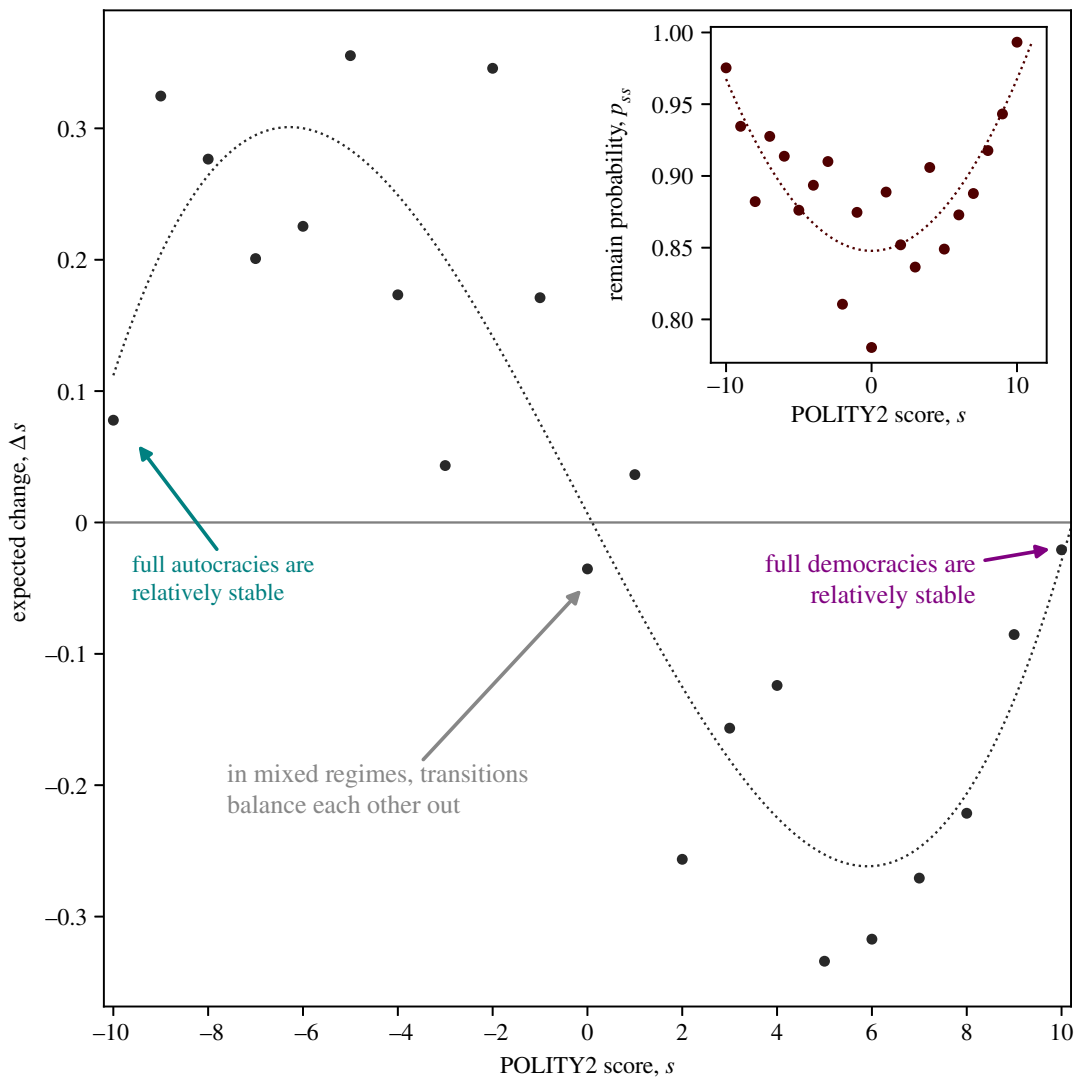


Figure 4. The expected change Δs of the POLITY2 score follows a cubic polynomial with countries being full autocracies ($s = -10$), mixed regimes ($s = 0$) or full democracies ($s = +10$) having expected changes Δs close to zero. Regimes that are autocratic ($s < 0$) are expected to increase their POLITY2 score, whereas regimes that are democratic ($s > 0$) are expected to decrease their POLITY2 score. We show the cubical trend line as a least-squares fit ($R^2 \approx 0.84$). Inset: the remain probabilities p_{ss} are smallest for mixed regimes ($s = 0$) and highest for extreme regimes ($s = -10$ and $s = 10$), resulting approximately in a quadratic polynomial (see trend line, $R^2 \approx 0.63$).

3.3. At the EoH, a plurality but no majority of countries are predicted to be democracies

Under the Markov process described by the transition matrix \mathbf{P} , countries may constantly change their POLITY2 score: some regime changes make countries more democratic and other changes make countries less democratic. As we estimated that autocracies have a higher probability of becoming democracies than democracies becoming autocracies we expect the amount of democracies to increase over time. When there are much more democracies than autocracies, however, a dynamic equilibrium is established in which the rate of democracies dying is the same as the rate of autocracies becoming democracies. Necessarily, there must be a distribution of regimes that is unchanged under the transition matrix \mathbf{P} and this is called the *stationary distribution* (see Methods section). Once this stationary distribution is reached, it does not change anymore and therefore represents a mathematical abstraction of the EoH. The stationary distribution is unique for irreducible and aperiodic Markov chains. As we used a Bayesian estimator, necessarily all transition probabilities $p_{ij} > 0$, making the obtained Markov chain irreducible and aperiodic, yielding a unique EoH. We note that methodologically different but conceptually similar ideas, in the form of the random-surfer model, have been employed by Google in the PageRank algorithm [27].

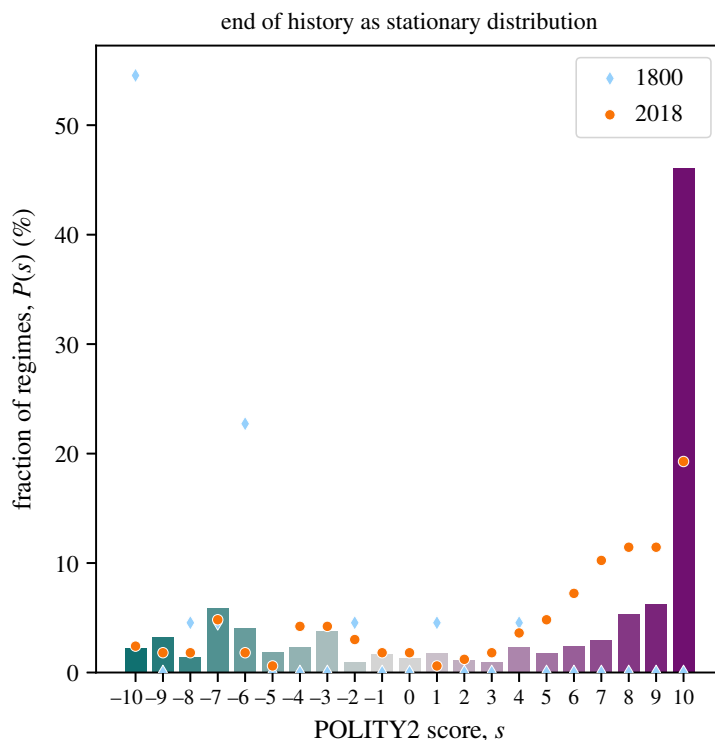


Figure 5. At the EoH, approximately 50% of countries are expected to be full democracies (i.e. POLITY2 score $s = 10$). We show the fraction of regimes that has each POLITY2 score for 1800 (blue diamonds) and 2018 (orange discs). While the fraction $P(s = 10)$ of full democracies is in 2018 considerably lower than at the EoH, the fraction of democracies with intermediate POLITY2 score $3 \leq s \leq 9$ is lower at the EoH.

We show the stationary distribution π of the estimated transition matrix \mathbf{P} in figure 5. We see that 46% of all regimes are predicted to be full democracies (i.e. $s = +10$). The fraction of full autocracies ($s = -10$) is with 2% much smaller. The fraction of countries in any state of autocracy (i.e. $s \leq 0$) is, however, with 28% sizeable. The remaining 26% of countries are in various states of partial democracies (i.e. $0 < s < 10$).

We can compare this predicted distribution at the EoH with the observed distributions in the years 1800 and 2018 (shown as blue diamonds and orange discs, respectively). We do observe that the fraction of full autocracies did indeed shrink over these two centuries, while the number of full democracies did rise. At the EoH the fraction of full democracies is expected to be considerably higher than in 2018, while the fraction of partial democracies is expected to be smaller.

To investigate the robustness of our results under different choices in the data analysis, we explore the EoH in slightly different variations in the electronic supplementary material. All of these support our finding that a plurality of countries is expected to be a full democracy. In electronic supplementary material, 2, we compute the EoH for the POLITY score, which is an older version of the POLITY2 score. In electronic supplementary material, 3, we compute the EoH for different binning of the POLITY2 score. In electronic supplementary material, 4, we investigate the *Varieties of Democracy* score. In electronic supplementary material, 5, we study the EoH for data covering different temporal subsets of the regime-transition data and find that the EoH is largely temporally invariant. In electronic supplementary material, 10, we study whether the EoH is robust under counterfactual perturbations of the time-series data (in particular no collapse of the USSR and a stronger democratization during the Arab Spring) and find that such perturbations alter the results negligibly.

3.4. The amount of autocracies is predicted to reach a maximum in 2063

Using the transition matrix \mathbf{P} , describing the Markov model, we can investigate the expected development of the distribution of regimes extrapolating from the last available data for the year 2018. We expect that the EoH in the form of the unique stationary distribution π of regimes is approached for $t \rightarrow \infty$. In figure 6, we show the expected temporal development of the distribution of POLITY2 scores over the next 800 years. Specifically, we show the fraction of full democracies (i.e. $s = +10$) and

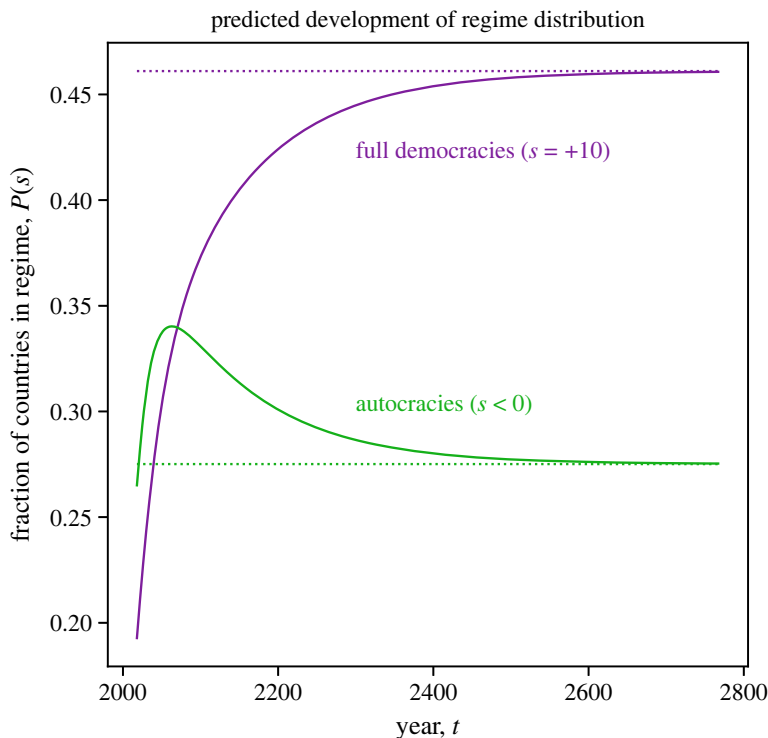


Figure 6. While the predicted fraction of full democracies increases over time and approaches the steady state, the fraction of autocracies increases for 43 years and decreases then to reach the steady state. We show the fraction of full democracies (i.e. $s = +10$) as purple line and the fraction of autocracies (i.e. $s < 0$) as a green line. Both approach their respective fractions (shown as dashed lines) at the EoH without ever reaching it but get within 1% in approximately 400 years.

the fraction of autocracies (i.e. $s < 0$). We see that both approach their respective steady-state distributions at the EoH without ever exactly reaching it. While the fraction of full democracies is continuously increasing over time, the fraction of autocracies is increasing until 2063, when it reaches a maximum of 34%, and then shrinking until it reaches the steady state of 28%. In the year 2070, the fraction of full democracies is predicted to be larger than the fraction of autocracies for the first time.

3.5. The expected lifetime of a democracy is threefold that of an autocracy

As established earlier, the expected steady state at the EoH is not characterized by an absence of regime change, but rather there is an equilibrium between democracies becoming autocracies and autocracies becoming democracies. We can aim to characterize these fluctuations at the EoH. Specifically, we can numerically estimate the expected lifetime of democracies and the expected time until an autocracy becomes a full democracy as *hitting times* for random walks (see Methods section).

For this, we simulate the trajectories of $r = 10^5$ political regimes that start as full democracies (i.e. in state $s = +10$). We simulate the transitions as Markov process given by the inferred transition matrix \mathbf{P} . We compute the time t until these countries become an autocratic regime (i.e. reach a state $s < 0$). We estimate the median lifetime $t_{1/2}^{\text{autocracy}}$ of a full democracy of 244 years. In the same way, we may estimate the median lifetime $t_{1/2}^{\text{autocracy}} \approx 69$ years of a full autocracy by starting $r = 10^5$ political regimes in the state $s = -10$ and compute the time until a democracy (i.e. $s > 0$) is reached. The lifetimes for both types of regimes have relatively large standard deviations with 62 years and 310 years for autocracies and democracies, respectively.

More generally, we may estimate the survival function $S(t)$ of political regimes, which gives the probability that a regime survives past time t . In figure 7, we show the Kaplan–Meier [40] estimator of the survival function $S(t)$ for these simulated regimes (see Methods section). We find that the full autocracies' survival probability $S(t)$ drops considerably faster than the one of full democracies. The survival probabilities fall below 1% after 287 and 1450 years for full autocracies and full democracies, respectively.

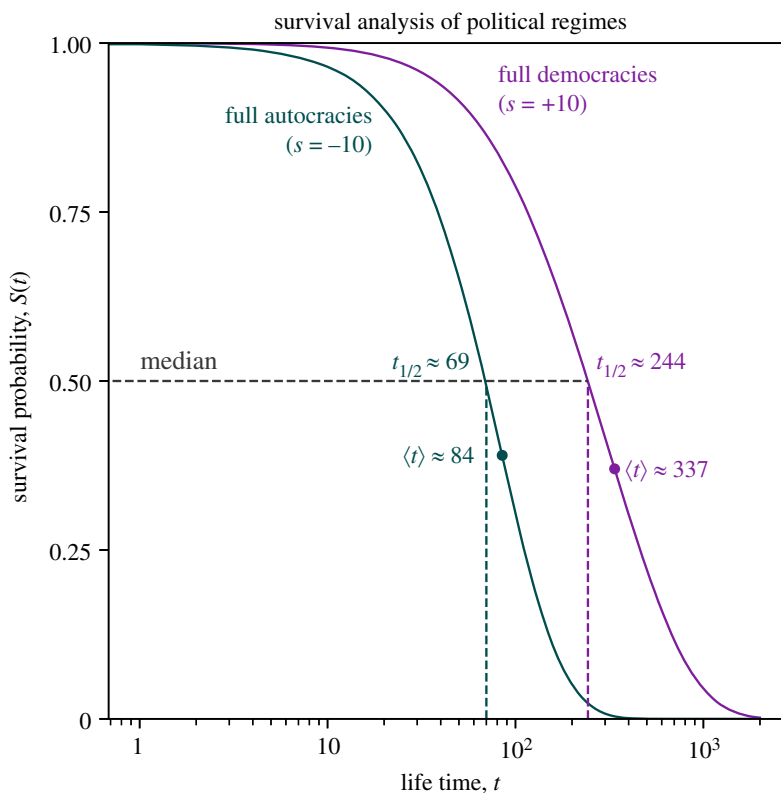


Figure 7. The survival probability $S(t)$ for full autocracies drops quicker than for full democracies. We estimate the median lifetime of both types of regimes to $t_{1/2}^{\text{autocracy}} \approx 69$ years and $t_{1/2}^{\text{democracy}} \approx 244$ years, respectively. The survival probabilities $S(t)$ of both regimes were estimated with a Kaplan–Meier estimator applied to 10^5 time series. We may also determine the mean lifetimes of regimes from the transition matrix directly by computing hitting times of the Markov chain explicitly. We obtain mean lifetimes of full autocracies and full democracies of $\langle t \rangle^{\text{autocracy}} \approx 84$ and $\langle t \rangle^{\text{democracy}} \approx 337$, respectively.

The Markov-chain model allows us to compute the mean lifetimes of regimes from the transition matrix directly. For this, we compute hitting times $\tau_{i,A}$ by solving the associated system of linear equations (see Methods section). The mean time until a full democracy becomes an autocracy for the first time is then, for example, $\tau_{+10,s<0}$ and we obtain $\tau_{+10,s<0} = \langle t \rangle^{\text{democracy}} \approx 337$. Analogously, we compute the mean lifetime of a full autocracy to $\tau_{-10,s>0} = \langle t \rangle^{\text{autocracy}} \approx 84$, indicating that the mean lifetime of a full democracy is fourfold the lifetime of a full autocracy. We verify these analytical expressions by comparing them with the numerical estimates $\hat{\tau}$ and find that they differ by less than a year for both autocracies and democracies. For both types of regimes, the mean lifetime and the median lifetime are of similar magnitude, yet $\langle t \rangle > t_{1/2}$, which indicates that a small number of the simulated regimes have much larger lifetimes than the majority of regimes. In the electronic supplementary material, 7, we show that an empirical cumulative distribution function yields virtually indistinguishable results from the Kaplan–Meier estimator.

4. Conclusion

In this paper, we used a Markov-chain approach to estimate the transition probabilities between political regimes from time-series data covering more than two centuries. We found that the most extreme regimes (i.e. full autocracies and full democracies) have the highest probabilities to persist. Using the estimated transition probabilities allowed us to quantify the distribution of political regimes at the EoH as a stationary distribution of the Markov chain. We find that approximately 49% of countries are predicted to become full democracies with a median lifetime of about 244 ± 310 years. Autocracies make up 26% of regimes and have a median lifetime of 69 ± 62 years.

Analysing the predicted temporal development from 2018 until the EoH, we find a steady increase of full democracies and a steady decline in the number of partial democracies. Surprisingly, we also detect

an increase of autocracies for the next 50 years, which is followed by a decline thereof. This development is mainly driven by a current large number of partial democracies which are more likely to become autocracies than full democracies over the short term, even though they might become full democracies in the long term. This indicates that the currently observed democratic backsliding might be a harbinger of further incline of partial democracies becoming autocracies, even if the EoH is characterized with a larger number of full democracies.

In our approach, we treated all countries the same and did not consider country-specific factors that could influence democratization. Other studies have demonstrated that there is evidence for the ‘modernization hypothesis’ (i.e. that countries that do economically well tend to undergo liberal democratic transitions) [29,41–43], influence of state legacies on democratization [44–46] and a correlation between scientific production and democracy [47]. The modernization hypothesis in particular could mean that the transition probabilities are not fixed but rather are a function of a country’s prosperity, which in turn makes the EoH dependent on economic development. Such approaches that take into account additional socio-economical factors can be fruitful and lead to a deeper understanding of the drivers of democratization. Nevertheless, we refrain from doing so in this study, because this would require us to also predict the development of these external factors until the EoH, which would lead to a much more complex model with many parameters, making it likely intractable. Similarly, it has been shown that the spread of regime change can be influenced by international treaties (e.g. defensive alliance) [48], an effect we refrain from including in our model. To some extent, we believe that the strength of our model is not its (potential) accuracy of prediction but rather its simplicity. While the prediction of quantities, such as the time until the EoH, should not be mistaken as an exact forecast, they nevertheless give indications about the order of magnitudes of the expected outcomes. To touch on the international variability, we compute EoHs for specific countries or regions and find a large variability, with some regions being largely democratic and others being dominated by autocracies (see electronic supplementary material, 1).

Independent of the exact predicted values, our findings support a view of the EoH in which there is not a single, omnipresent type of political regime but rather a broad range of regimes that span all values across the POLITY2 scale. In particular, we find no statistical evidence for a universality of full liberal democracies as suggested by Francis Fukuyama, although we predict them to represent a plurality of political regimes. Yet, in accordance with Francis Fukuyama, we find that full democracies are the most stable type of regime.

In our model, we treat each country’s regime as an entity independent of others. It is known, however, that there are considerable influences between countries that might lead to drastic changes in many countries in a short amount of time, a so-called ‘democratic domino theory’. This occurred, for example, during the decline of European democracies in the 1930s or the fall of the Soviet Union at the end of the Cold War and there is some empirical evidence for such spatio-temporal effects [49,50], raising the question to what extent such synchronized dynamics might perturb the EoH. The influence of such synchronization events might also lead to more complex dynamical processes which are not fully described by our stationary Markov chain, making the investigation of more advanced models a fruitful endeavour.

Our data cover predominantly the last two centuries, which—despite some considerable setbacks—have been a success story of liberal democracies, most likely driven by a drastically rising average income. The analysis of this time period most likely leads to an overestimation of transition probabilities that increase the POLITY2 score. With reliable time-series data that cover more years, we would be able to extend the analysis presented here. Furthermore, we might update the estimation of the transition matrix with more recent data, once available, indicating a need for further empirical studies of regime transitions and their long-term progression.

5. Material and methods

5.1. Markov chain

We analyse the time series of countries’ POLITY2 scores as a finite sequence of random variables $X = (X_1, X_2, \dots, X_n)$. Let \mathcal{X} be the state space of random variables, i.e. the set of values that each variable can take. In our case this is the POLITY2 score s and therefore $X_i \in \mathcal{X} = [-10, +10]$ for all i . The size $N = 21$ of the state space is the number of possible POLITY2 scores. We treat these time series as discrete-time *Markov chains*.

A finite discrete-time Markov chain is a sequence of random variables (X_1, X_2, \dots, X_n) that fulfil the Markov property.

The Markov property is also called ‘memoryless property’ because it requires that the transition probability only depends on the current state. Specifically, the probability $P(X_{t+1} = y | X_t = x_t, X_{t-1} = x_{t-1}, \dots, X_1 = x_1)$ of being in state x conditioned on the whole history of states only depends on the last state, i.e. $P(X_{t+1} = y | X_t, \dots, X_1) = P(X_{t+1} = x | X_t = x_t) = p_{x_t y}$.

We can describe a Markov chain through a *transition matrix* \mathbf{P} that describes the probability of a transition from one state to another. Specifically, \mathbf{P} is a non-symmetric $N \times N$ matrix, where N is the number of states (in our case the 21 possible POLITY2 scores) and entry $p_{ij} \in [0, 1]$ indicates the probability of transition from state i to state j . The rows of \mathbf{P} each sum to 1 (i.e. $\sum_{j=1}^N p_{ij} = 1$) because the probabilities are normalized.

5.2. Bayesian estimation of Markov process from time-series data

In our application, we do not have the transition matrix \mathbf{P} given *a priori*. Rather, we want to estimate it from observed time series. Let $\mathbf{X}^{(1)}, \mathbf{X}^{(2)}, \dots$ be a set of sequences of random variables, each representing the time series of political regime characterization for one country.

Let n_{ij} be the number of times that we observe a transition from state i to state j in the time series. In a frequentist approach the maximum-likelihood estimation [51] of the transition probability from state i to state j is then

$$\hat{p}_{ij} = \frac{n_{ij}}{n_{i+}} = \frac{n_{ij}}{\sum_{j=1}^N n_{ij}}, \quad (5.1)$$

where n_{i+} is the total number of observed transitions starting in state i . This estimator is a so-called consistent estimator.

For our purposes, it can be beneficial to use a Bayesian approach to estimate the transition matrix. One reason is that just because we have never observed a certain transition, we do not expect that the probability of this transition to occur in future is zero. Rather we would like to assign it a small but finite probability. From a Bayesian approach this follows naturally by combining prior beliefs with the observed data.

We assign each row of the transition matrix a Dirichlet prior with equal weighting of the states such that $\alpha_{ij} = 1/N = 1/21$ for all (i, j) , such that each transition has the same probability. We update the prior belief with the observed data and obtain a posterior mean estimate:

$$\hat{p}_{ij}^{(\text{Bayes})} = \frac{n_{ij} + \alpha_{ij}}{\sum_{j=1}^N (n_{ij} + \alpha_{ij})}. \quad (5.2)$$

The posterior mean estimate $\hat{p}_{ij}^{(\text{Bayes})}$ converges to frequentist maximum-likelihood estimation \hat{p}_{ij} for large amounts of data. Only when data are scarce does the Bayesian estimate add corrections to the frequentist approach [52]. For example, a transition $i \rightarrow j$ that is never observed such that $n_{ij} = 0$ still receives a small finite probability $p_{ij} > 0$. We show the transition matrix estimated with a frequentist approach in electronic supplementary material, 6. We note that the used estimator resembles the rule of succession, as introduced by Pierre-Simon Laplace [53].

5.3. Stationary distribution

Let $\mathbf{x}(t) = (x_1, x_2, \dots, x_N)$ be a state vector that indicates the distribution among the N states at time t . We can compute the evolution of a Markov chain by multiplying a state vector $\mathbf{x}(t)$ with the transition matrix \mathbf{P} such that the state vector at time $t + 1$ is

$$\mathbf{x}(t + 1) = \mathbf{x}(t)\mathbf{P}. \quad (5.3)$$

A *stationary distribution* $\boldsymbol{\pi}$ is a state vector that does not change under the multiplication with the transition matrix \mathbf{P} such that

$$\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{P}. \quad (5.4)$$

That is, once a Markov chain reaches a stationary distribution, it stays there. Every irreducible and aperiodic Markov chain has a unique stationary distribution (for details, see [25]). Both conditions are

fulfilled in our case. In particular, the irreducibility is necessarily (i.e. independent of the actual time-series data) fulfilled because we use the Bayesian estimator (equation (5.2)). The frequentist estimator (equation (5.1)) could lead to a reducible Markov as some transitions are never observed.

We can compute the stationary distribution $\boldsymbol{\pi}$ as the eigenvector of the transition matrix \mathbf{P} with eigenvalue 1. There are many numerical algorithms able to calculate the eigenvectors and we use the QR algorithm as used by standard PYTHON linear algebra library.

5.4. Hitting time

The *hitting time*, also called first passage time, is the mean number of steps τ_{ij} it takes a random walker starting at node i to visit a target set of nodes A for the first time [54]. For a Markov chain, we can compute this explicitly by solving the system of linear equations

$$\tau_{i,A} = \begin{cases} 1 + \sum_{j \in S} p_{ij} \tau_{j,A} & \text{if } i \notin A \\ 0 & \text{if } i \in A. \end{cases}$$

We also may estimate it numerically by simulating a large number of random walks starting from node i and choosing random transitions in accordance with the transition matrix \mathbf{P} . We stop the random walk when we reach the target set of nodes A for the first time. The number of steps taken until this happens is called the length of the walk. Assume that we have simulated r random walks starting at i and ending at j and $L(w)$ indicates the length of walk w . We then estimate the hitting time to

$$\hat{\tau}_{ij} = \frac{\sum_{w=1}^r L(w)}{r}. \quad (5.5)$$

5.5. Kaplan–Meier estimator

We estimate the survival function $S(t)$ of political regimes from simulated regime-transition time series. In particular, we simulate $r = 10^5$ political regimes and obtain their lifetimes as hitting times. We use the non-parametric Kaplan–Meier estimator [40]

$$\hat{S}(t) = \prod_{i: t_i \leq t} \left(1 - \frac{d_i}{n_i} \right), \quad (5.6)$$

where d_i are the number of political regimes dying at time t_i and n_i are the number of regimes that have survived until time t_i .

5.6. Data

We extract time-series data from ‘POLITY5: Political Regime Characteristics and Transitions, 1800–2018’ [32], which is available at <https://www.systemicpeace.org/polityproject.html>. Specifically, we use the Revised Combined Polity Score (POLITY2) score, which is a composite indicator. It is computed by subtracting the AUTOC score from the DEMOC score. The AUTOC score is an 11-point scale (0–10), which combines measurements of the competitiveness of political participation, the regulation of participation, the openness and competitiveness of executive recruitment, and constraints on the chief executive. The DEMOC score is an 11-point scale (0–10), which combines measurement data concerning the ‘Competitiveness of Executive Recruitment’, the ‘Openness of Executive Recruitment’, ‘Constraint on Chief Executive’ and the ‘Competitiveness of Political Participation’. Accordingly, the resulting unified polity scale (POLITY2) ranges from -10 (fully autocratic) to $+10$ (fully democratic) and has 21 different values $s \in [-10, +10]$.

The data cover 219 years and 195 countries. We treat each country’s trajectory as a separate time series. A time series is an ordered vector $\mathbf{S} = (s_1, s_2, \dots, s_{t'})$ in which s_i indicates the score at time t . The length t' of a time series is the number of elements in the vector. As some countries cease to exist or emerge during this period, the time series vary in their length.

Data accessibility. All code and data are available in the repository <https://github.com/floklimm/democracyMarkovChain> and <https://zenodo.org/record/7278302#Y2QcJy8w1f0>. Data and relevant code for this research work are stored in GitHub: <https://github.com/floklimm/democracyMarkovChain> and are archived within the Zenodo repository: <https://zenodo.org/badge/latestdoi/475380368> [55].

The data are provided in the electronic supplementary material [56].

Conflict of interest declaration. I declare I have no competing interests.

Funding. F.K. is supported as an Add-on Fellow for Interdisciplinary Life Science by the Joachim Herz Stiftung, by the EPSRC (Centre for Mathematics of Precision Healthcare; EP/N014529/1), and by the Max-Planck Society.

Acknowledgements. We thank Nick S. Jones, Benjamin F. Maier, Tammo Rukat and Martin Vingron for fruitful discussions. We thank Luka Petrović and Ingo Scholtes for advice regarding the Markov order of paths in graphs. We are grateful to the comments by two anonymous reviewers, whose recommendations greatly improved the manuscript.

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