Thermal binary mixture convection in inclined cylindrical containers

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Convection in a fluid layer is affected by its orientation with respect to the gravitational field. In the present work, we investigate numerically pattern selection in a cylindrical cell heated from below and inclined against gravity. We are interested in analyzing the effect of inclination on pattern formation near the onset of convection. We will focus on thermophobic mixtures, i.e. mixtures in which the heavier component of the fluid is driven into the direction of lower temperature, where square patterns, roll structures and cross-roll regimes are expected to arise. In agreement with experimental observations, we will show that pattern formation in disklike cylinders is strongly affected even by very small inclinations.

INTRODUCTION

Thermal convection is an important problem with relevant implications to many geophysical flows and a multitude of technological applications. The flow patterns that are observed for simple incompressible fluid convection, are highly enriched by the consideration of binary fluid mixtures. Binary fluid convection has been used for many years as a prototypical system for the study of the transition to chaos in a fluid flow. In binary mixtures thermal convection promoted by thermal gradients may be enhanced by concentration non-uniformities sustained by the Soret effect, i.e. the generation of concentration fluxes by temperature gradients. The components of miscible ordinary two-component mixtures tend to separate in an imposed thermal gradient, and this separation in turn alters the driving force for convection. The interesting feedback loop between concentration, velocity, and temperature that takes place causes a surprising richness of spatiotemporal pattern formation even close to the onset of convection.

Tilting a cavity is a simple and intrinsic way of introducing a shear mechanism in the convection layer. The shear breaks the rotational isotropy of the layer and generates a large-scale circulatory flow. An interplay with the double-diffusive convective mechanisms is going to take place. The resulting vertically sheared motion will differentially advect properties across lateral fronts and the mixing and transport properties (heat, momentum), are going to be affected.

We will present numerical results which show that pattern selection is strongly affected by shear even when inclination is very small.

EQUATIONS AND NUMERICAL TOOLS

We consider the Boussinesq binary-fluid convection in a cylindrical cell of height H and radius R, inclined an angle α with respect to the horizontal (see Fig. 1). The cylinder is heated from below, with a temperature difference between the lids equal to ΔT. The whole boundary is impermeable and non-slip, with fixed temperature at the lids, while the lateral wall is assumed to be thermally insulated.

The density varies linearly with the temperature, T,

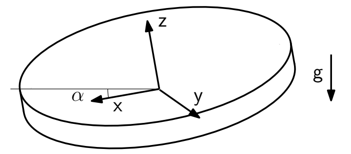

FIG. 1. Sketch showing the geometry of the cell and the choice of axis orientation.

and the concentration of the denser component, C, as

$$\rho = \rho^* \left[1 - \gamma (T - T^*) + \beta (C - C^*)\right], \qquad (1)$$

where $\gamma > 0$ and $\beta > 0$ are the thermal and concentration expansion coefficients, respectively, T^* and C^* are the mean values of T and C, respectively, and ρ^* is the density at $T = T^*$ and $C = C^*$. The mass flux $\mathbf{J}$ for a binary mixture can be written as

$$\mathbf{J} = -\rho^* D \left[C^* (1 - C^*) S_T \nabla T + \nabla C\right],$$

where D is the mass diffusivity and S_T is the Soret coefficient. The governing equations of the problem reflect the incompressibility condition, the mass and heat conservation laws, and the Navier-Stokes equations in the Boussinesq approximation.

The resulting system of nondimensional equations depends on the inclination angle α, the aspect ratio $\Gamma = R/H$, and four dimensionless parameters, namely the Rayleigh number Ra that provides a dimensionless measure of the vertical force imposed temperature difference ΔT, the separation ratio S that measures the ratio of the concentration contribution to the buoyancy force due to cross-diffusion, the Prandtl and Lewis numbers σ, τ, respectively, and the aspect ratio $\Gamma = R/H$.

The nondimensionalized system of equations and boundary conditions has been solved numerically using the algorithm described in [1]. More details about the derivation of the equations and the numerical method can be found in some previous works in the same or related configurations [2, 3].

To gain insight of the spatio-temporal properties of the patterns obtained by DNS we will use a post-processing tool known as higher order dynamic mode decomposition (HODMD) [4]. This method decomposes the thermal and concentration fields as Fourier-like expansions that can be obtained from a set of spatially discretized snapshots, which are portraits of the system for equispaced values of time.

MAIN RESULTS

We take as reference values for the simulations those of the experiment [5], where a isobutylbenzene–n-dodecane at 50% weight fraction binary mixture is used. The aspect ratio of the cell will be fixed to values around $\Gamma \approx 5$. Inclination will be varied.

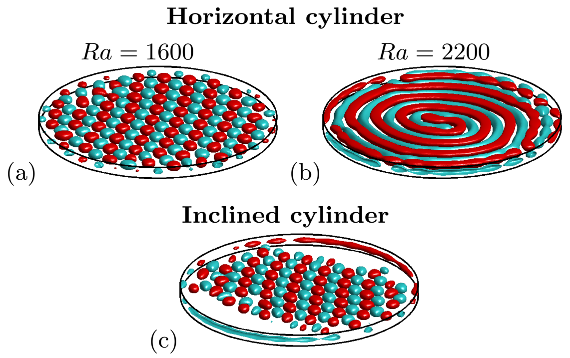

FIG. 2. (Top) Spiral convection and stationary square pattern in non-inclined cylinder. (Bottom) Establishment of large-scale shear flow in a slightly inclined cylinder.

Without inclination, the onset of convection for a pure fluid takes place $Ra_c = 1700$, but this value decreases to $Ra_c = 61.9$ for a $S = 0.13$ thermophobic mixture. Convection starts very early in the form of irregular large squares and, as Ra increases, the size of the cells decreases and the convective patterns become progressively more ordered until regular quasi-stationary square patterns are reached (Fig.2(a)). These structures are typical of the Soret regime, in which convection is primarily driven by solutal buoyancy, and are replaced by convective structures characteristic of the Rayleigh regime, where thermal driving dominates, such as spiral patterns (Fig.2(b)). When an inclination as small as $\alpha = 0.024\ rad$ is introduced there a large-scale shear flow (LSSF) is naturally generated in the cell. This circulation flow overcomes completely the square patterns obtained in the Soret regime for the horizontal cylinder (Fig.2(c)).

Coexisting with the LSSF, superhighway convection patterns (SHC) have been observed in experiments [5] and obtained numerically [2, 6]. These peculiar patterns show a number of parallel thermal lanes, each containing aligned coherent structures that counter-propagate in adjacent lanes.

We have obtained several types of SHC states by varying slightly the aspect ratio of the cell. SHC states can be periodic and quasiperiodic orbits, but they can also remain irregular SHC modulated states for extremely long times in wide regions of the parameter space. These irregular orbits seem to exhibit multiple nearly-heteroclinic connections, visiting vicinities of unstable periodic solu-

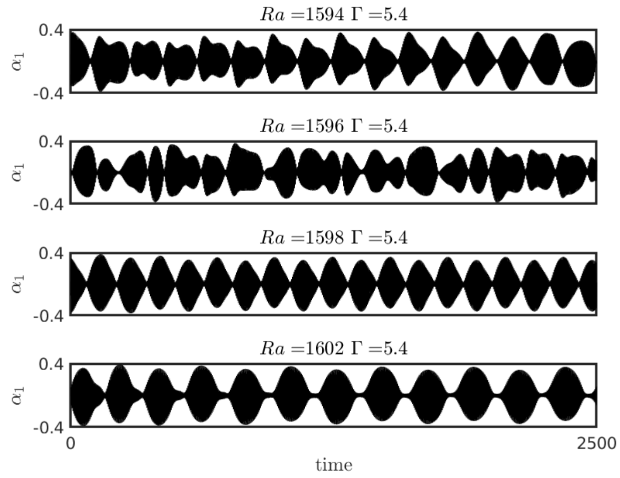

FIG. 3. Time series for different types of modulated SHC states obtained for different values of Ra in a $\Gamma = 5.4$ cell.

tions originated in a Neimark-Sacker bifurcations. As an example, Fig. 3 shows the temporal series corresponding to different types of modulated SHC states obtained for different values of Ra in a $\Gamma = 5.4$ cell.

CONCLUSIONS

We present and analyze the spatio-temporal features of relevant patterns appearing in double-diffusive convection in inclined cylinders. Different values of the aspect ratio, $\Gamma \approx 5$, and of inclination are considered. The parameter values of the mixture are also varied.

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