



LIDAR (laser radar)

Chap. 5A. Inversion Algorithms

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RAW-DATA PROCESSING

RAW-DATA RECORD

$$V_{\text{raw}}(R) = V(R) - V_{\text{OS}} = R'_V P(R) + n_{\text{eq}}(R) + \varepsilon_q + x_a(R) + x_s(R)$$

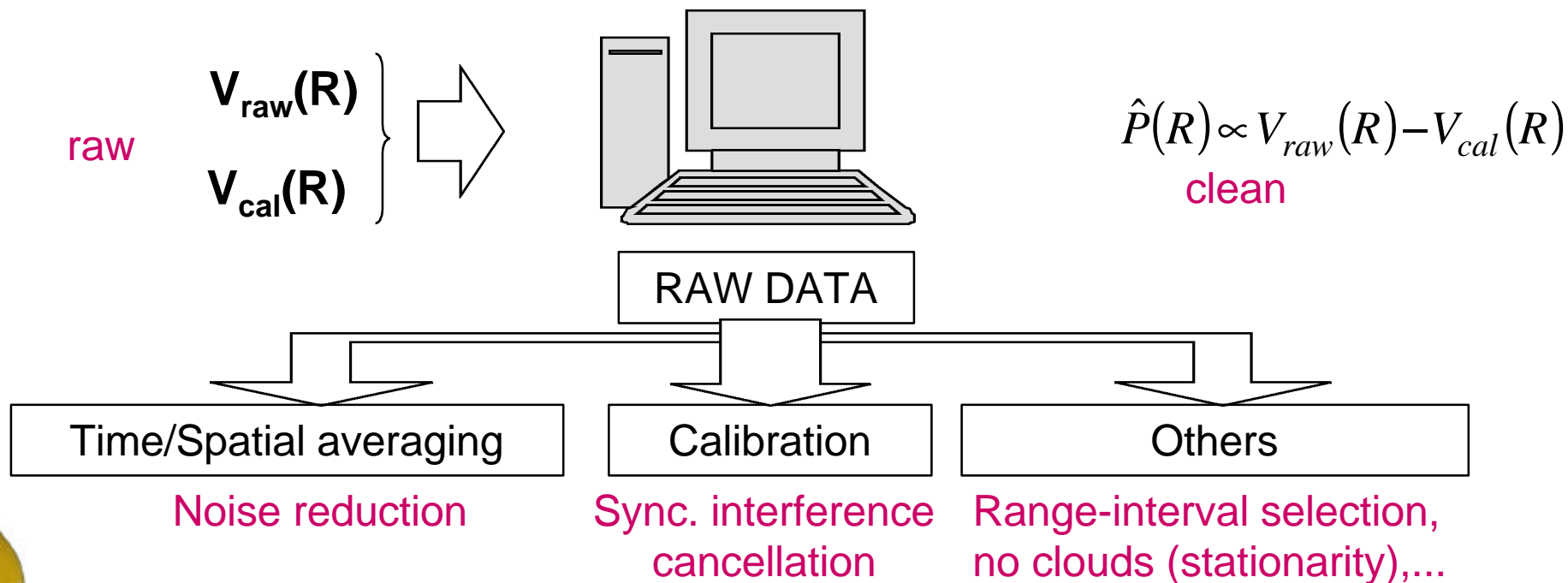
$\underbrace{\hspace{10em}}$
clean data term
 $\underbrace{\hspace{10em}}$
 $\ll n_{\text{tot}}(R)$

$$(V_{\text{OS}} = R'_V P_{\text{back}} + V_{\text{drift}} + V_{\text{user}})$$

CALIBRATION RECORD

$$V_{\text{cal}}(R) = V_{\text{cover}}(R) - V'_{\text{OS}} = n'_{\text{eq}}(R) + \varepsilon_q + x'_a(R) + x_s(R)$$

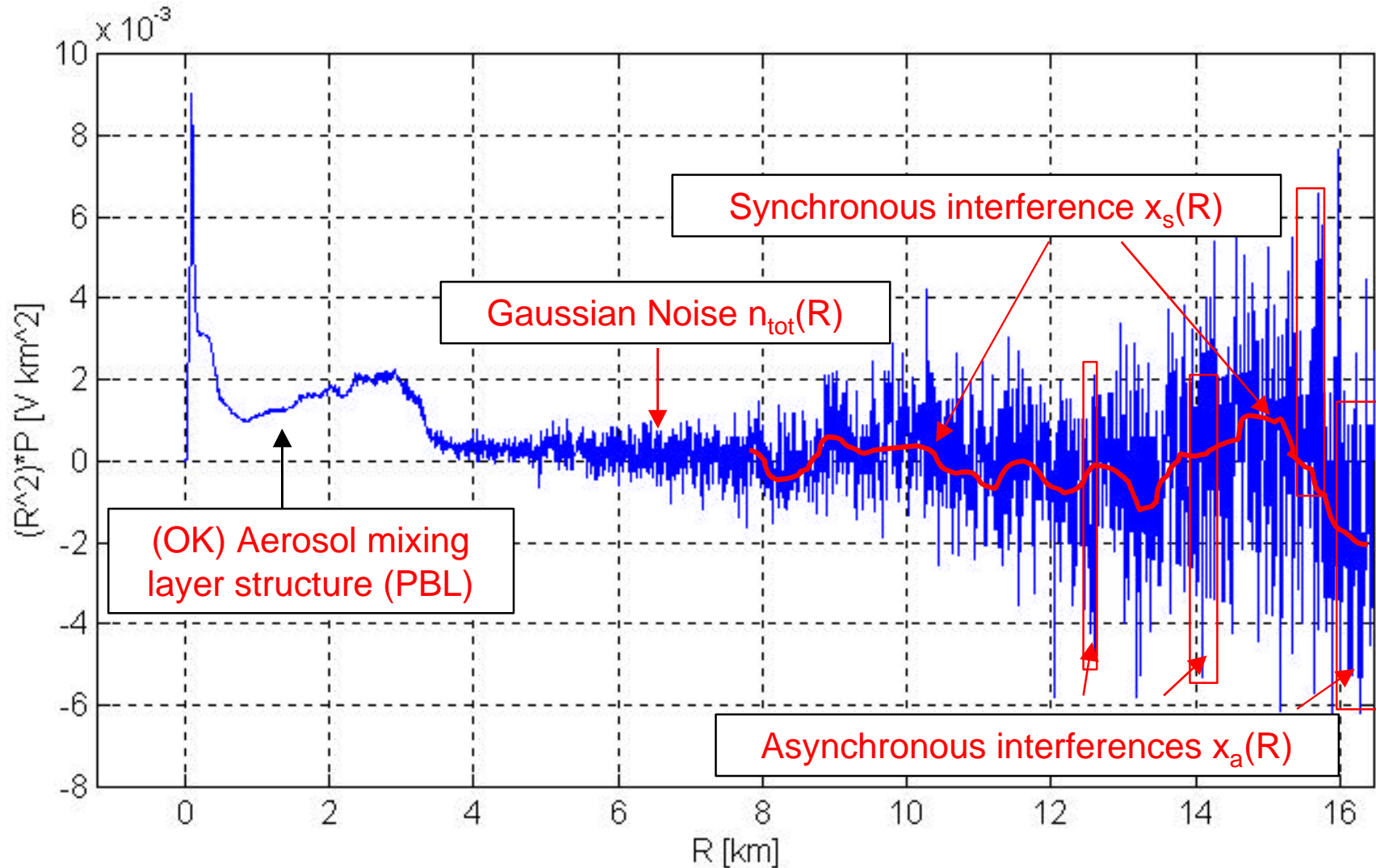
$$(V'_{\text{OS}} = V_{\text{drift}} + V_{\text{user}})$$





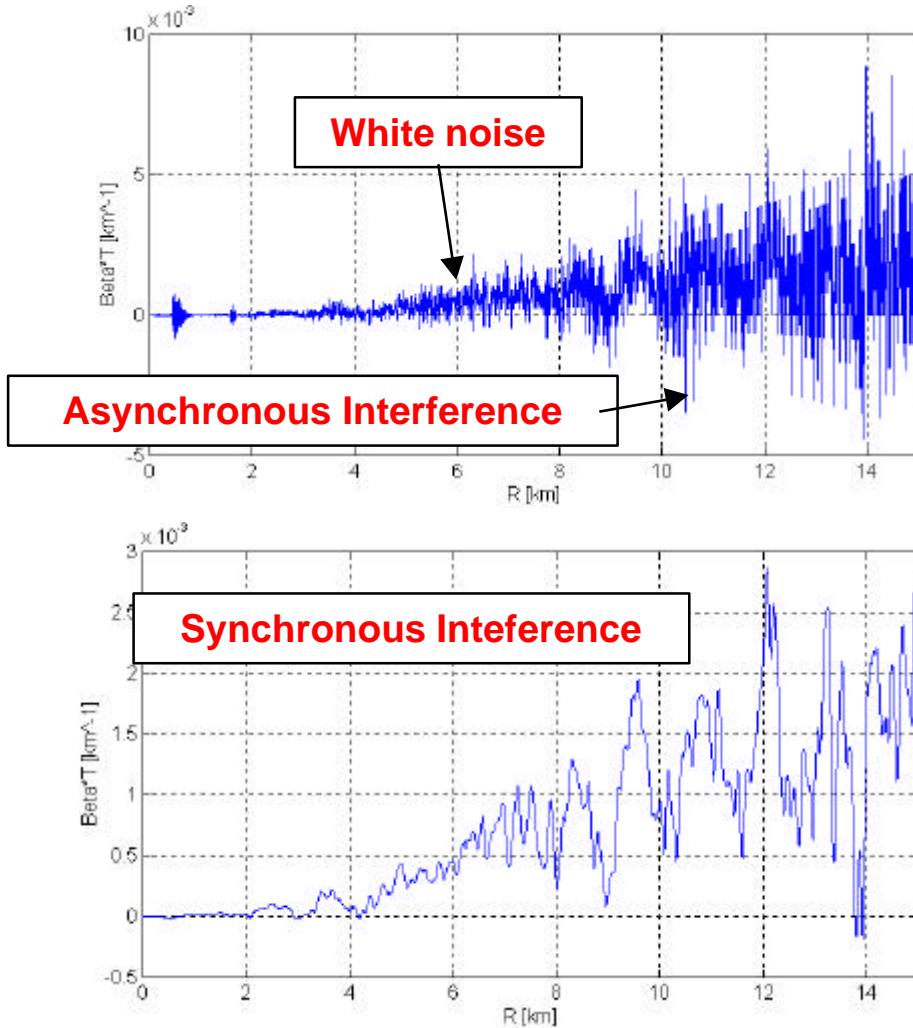
CORRUPTING NOISE SOURCES

Example of raw R²P record





CALIBRATION PROCEDURE (I)



Time-avg raw signal

- Record IP , $PPI=N_1$ pulses

$$V_{raw, \hat{O}}(R) = R'_V P(R) + n_{\hat{O}_1}(R) + x_s(R)$$

Calibration signal

- You cover the telescope aperture
- Record $IP=1$, $PPI=N_2$ pulses

$$V_{cal}(R) = n'_{\Sigma_2}(R) + x_s(R)$$

CALIBRATION PROCEDURE

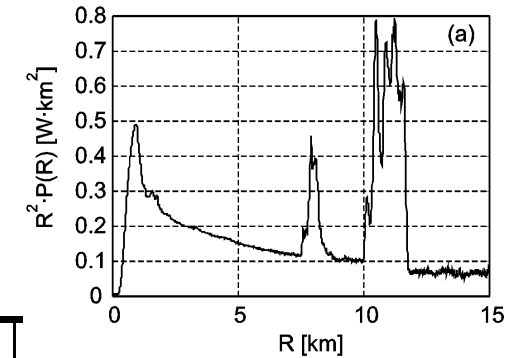
- \textcircled{R} $V(R) = V_{raw, \Sigma}(R) - V_{cal}(R)$
- Synchronous interferences cancel out

$$V(R) = R'_V P(R) + n_{\hat{O}_1}(R) - n'_{\hat{O}_2}(R)$$

CALIBRATION PROCEDURE (II)

SNR PERFORMANCE

Noise variances during the calibration differential procedure ADD UP so...



	<i>SIGNAL RETURN</i>	<i>NOISE SOURCES</i>	<i>STANDARD DEVIATION</i>	<i>COMMENT</i>
$V_{raw,\Sigma}$	$P(R) + P_{back}$	$n_{eq,\Sigma 1}, x_s, (x_a)$	$\frac{\sigma_{n,eq}}{\sqrt{N_1}}$	Meas. step PPI= N_1
V_{cal}	0 (telescope covered)	$n_{eq,\Sigma 2}, x_s, (x_a)$	$\frac{\sigma_{n,eq}}{\sqrt{N_2}}$	Calibration PPI= N_2
V_{clean}	P(R)	$n_{eq,\Sigma}, (x_a)$	$\frac{\sigma_{n,eq}}{\sqrt{N_1}} + \frac{\sigma_{n,eq}}{\sqrt{N_2}}$	Sync. Interfer cancel out

$N_2 \gg N_1$ is required to maintain measurement-step SNR !





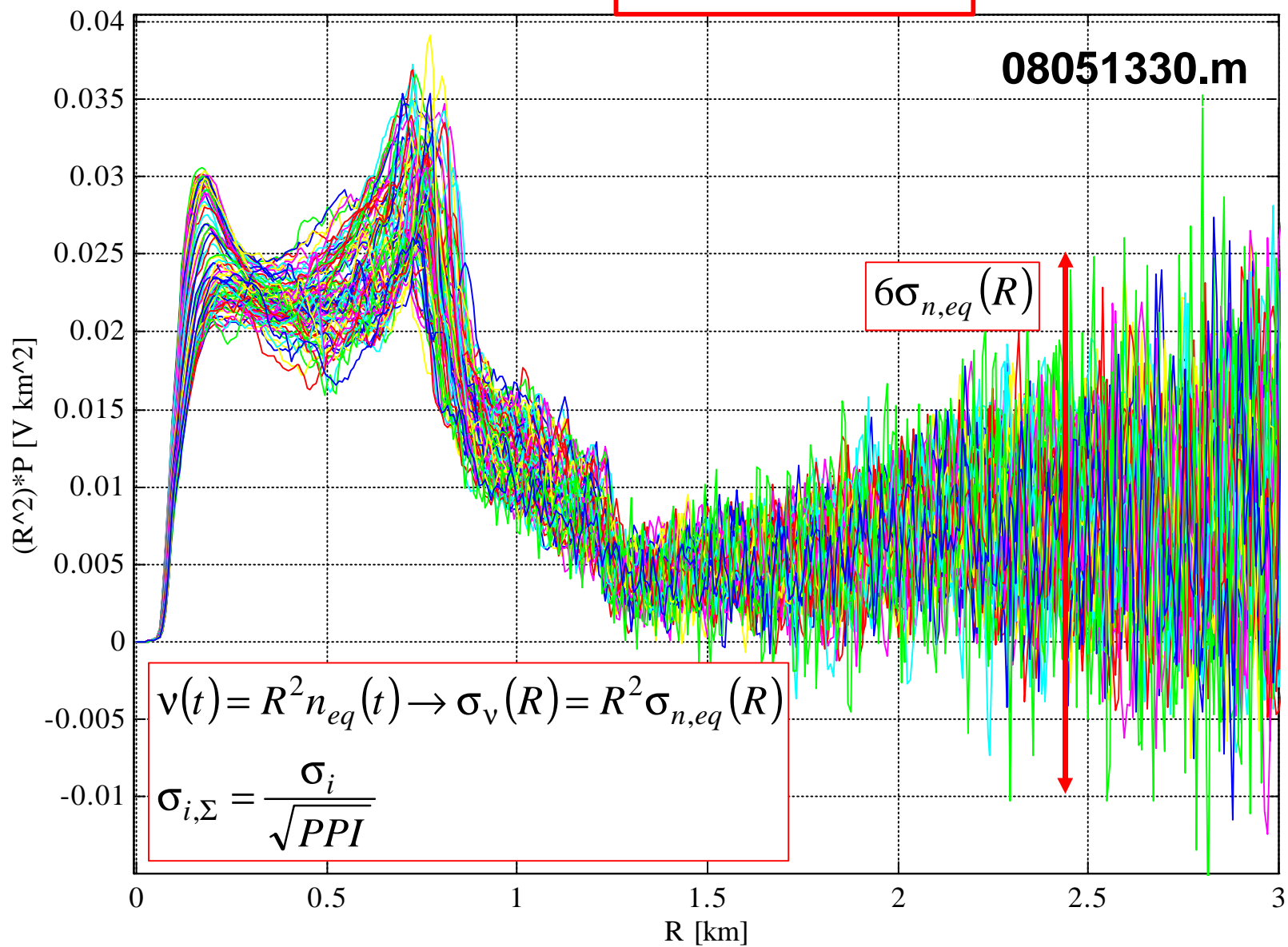
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TEMPORAL AVERAGING (I)

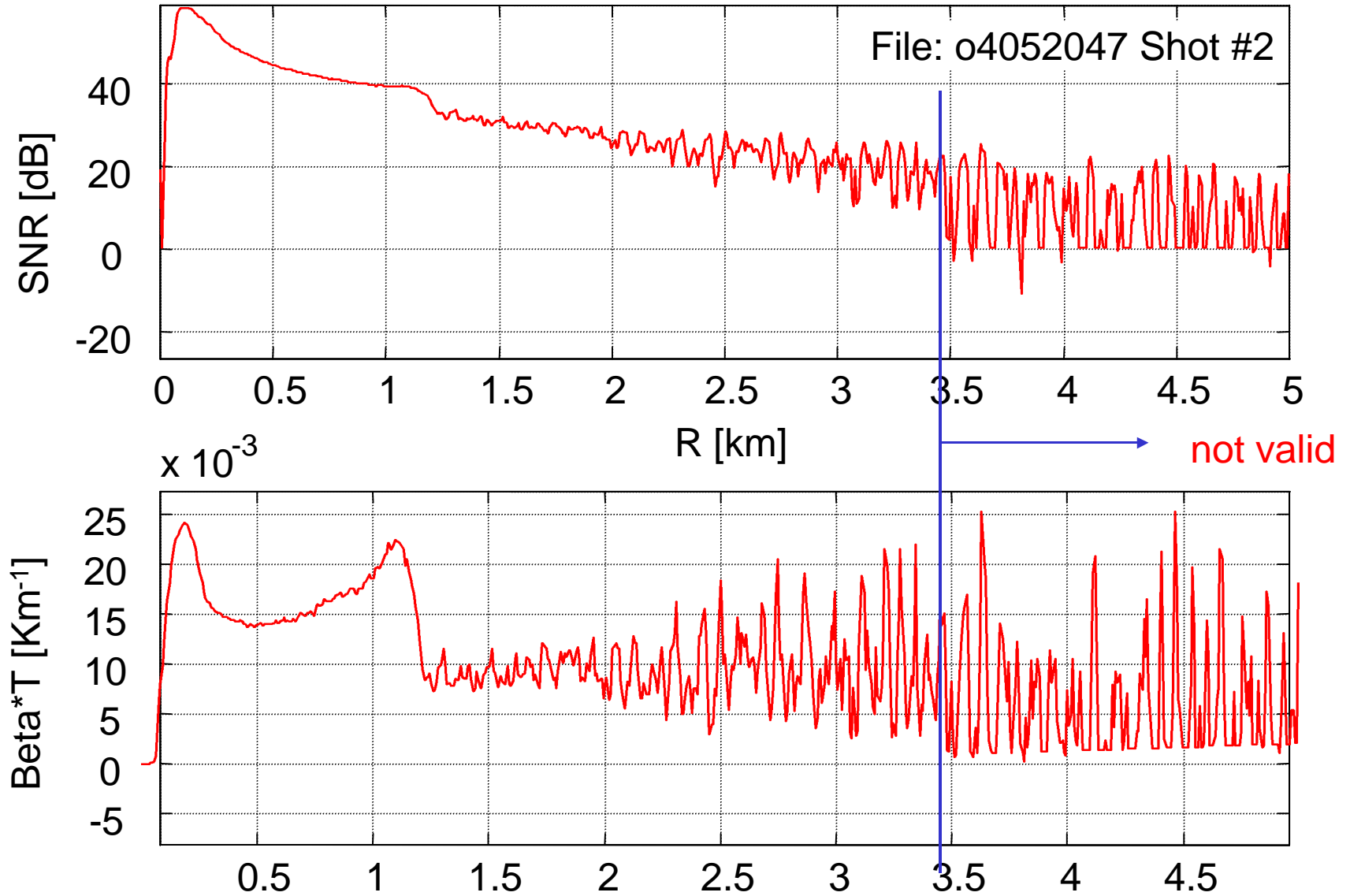
Vout. File: o8051330. #Packets: 90. #IP/shot: 80. Rmin: 0





TEMPORAL AVERAGING (II)

SNR ESTIMATION



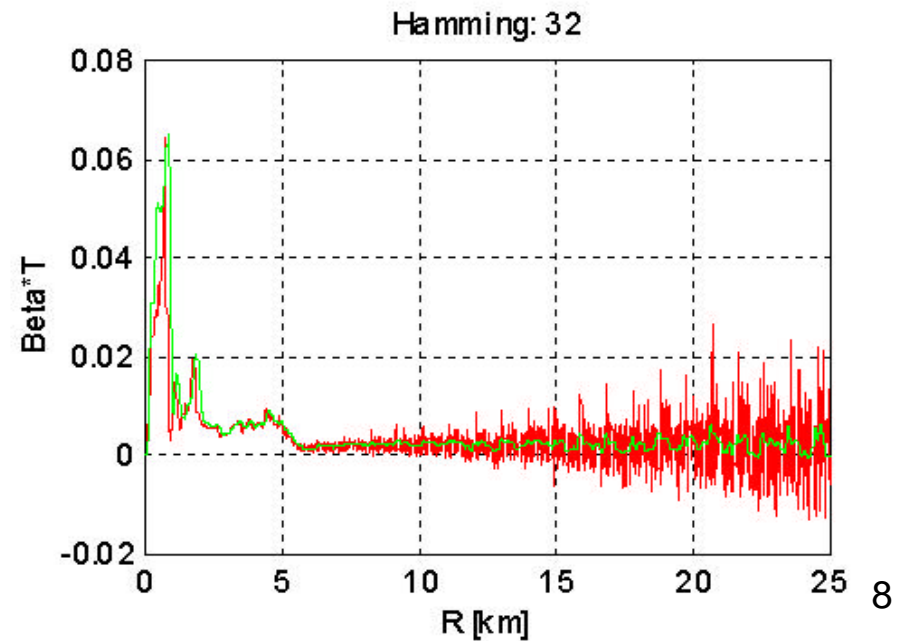
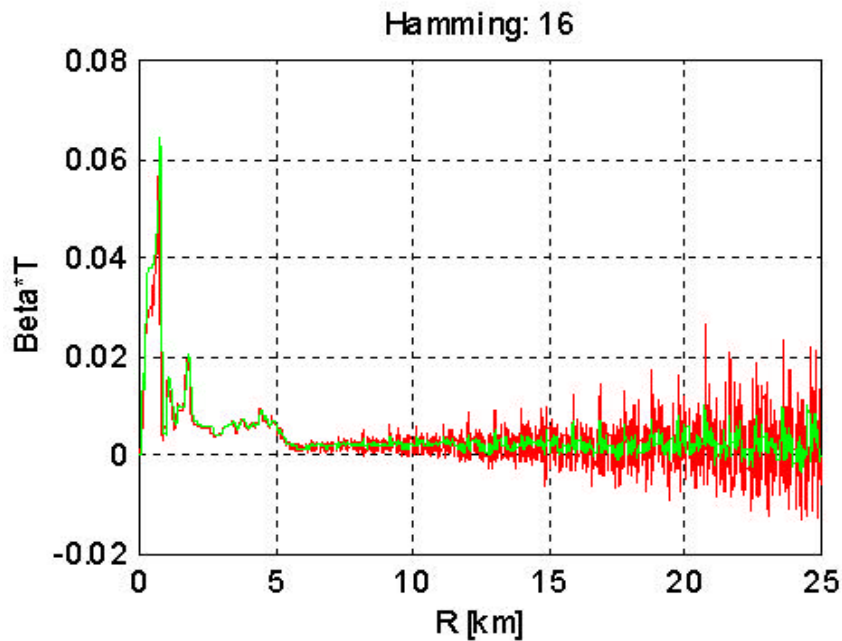
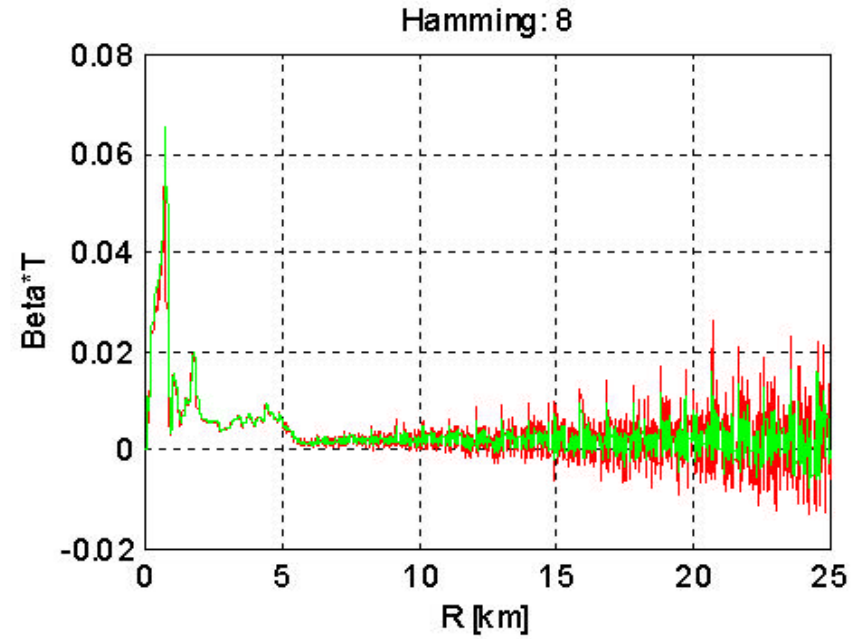
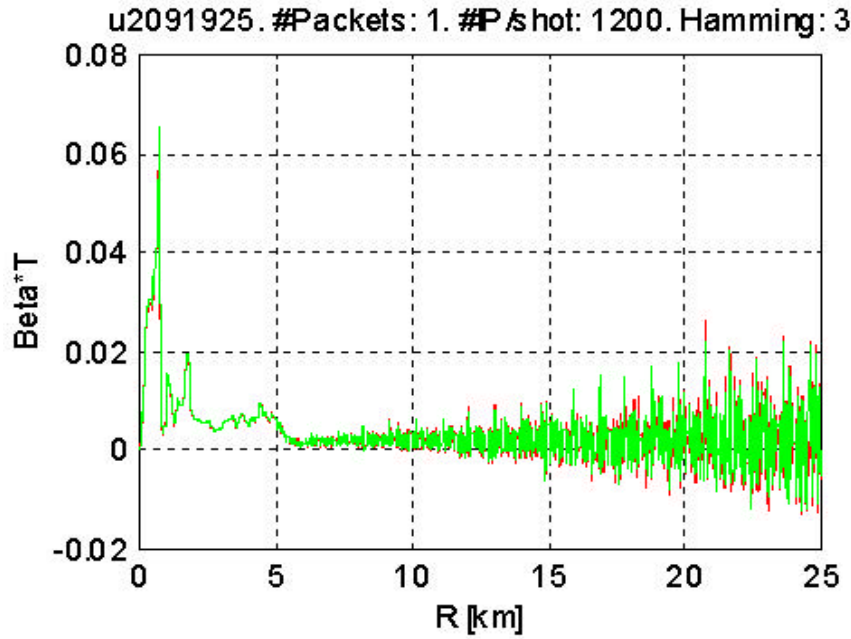


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SPATIAL AVERAGING



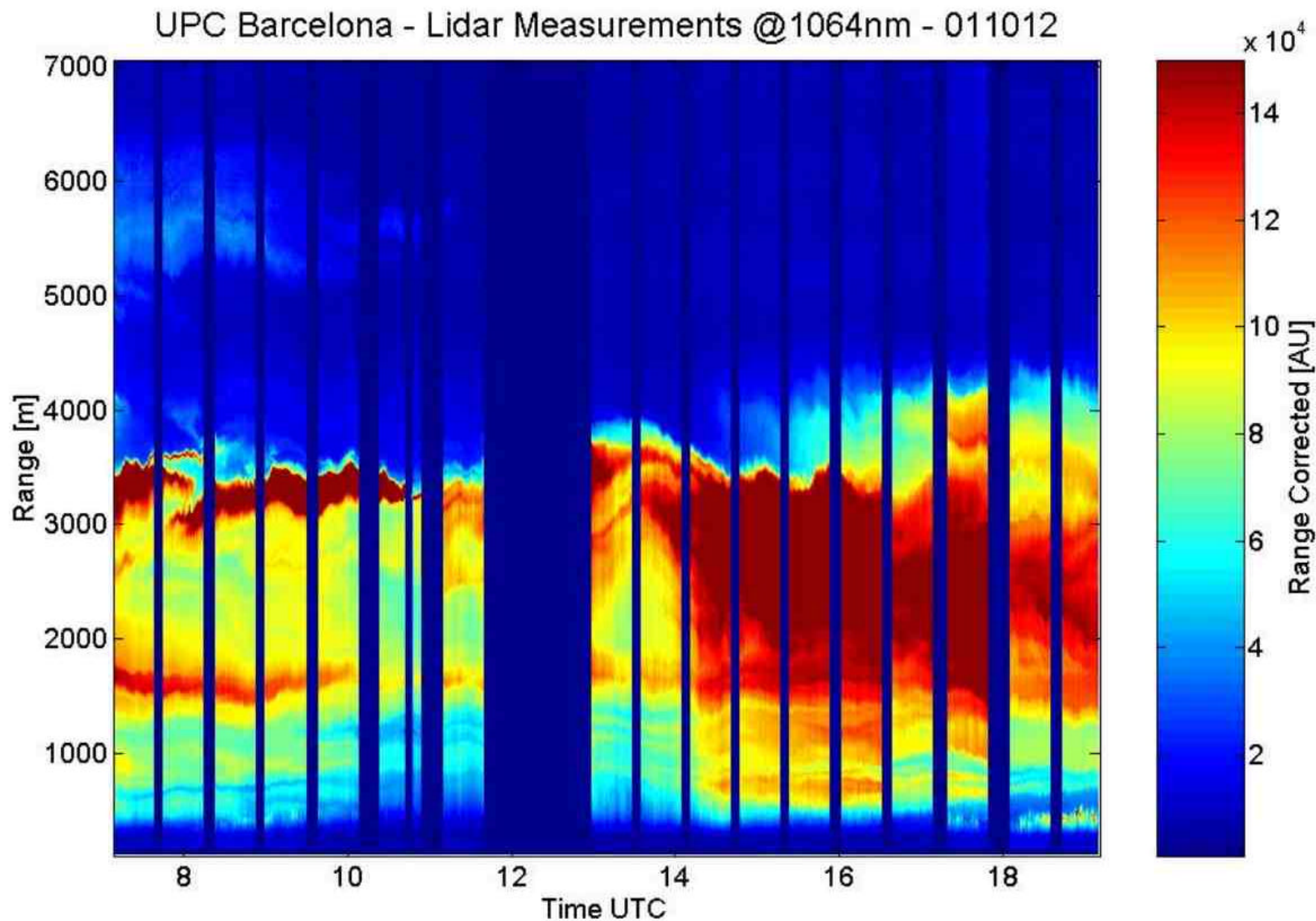


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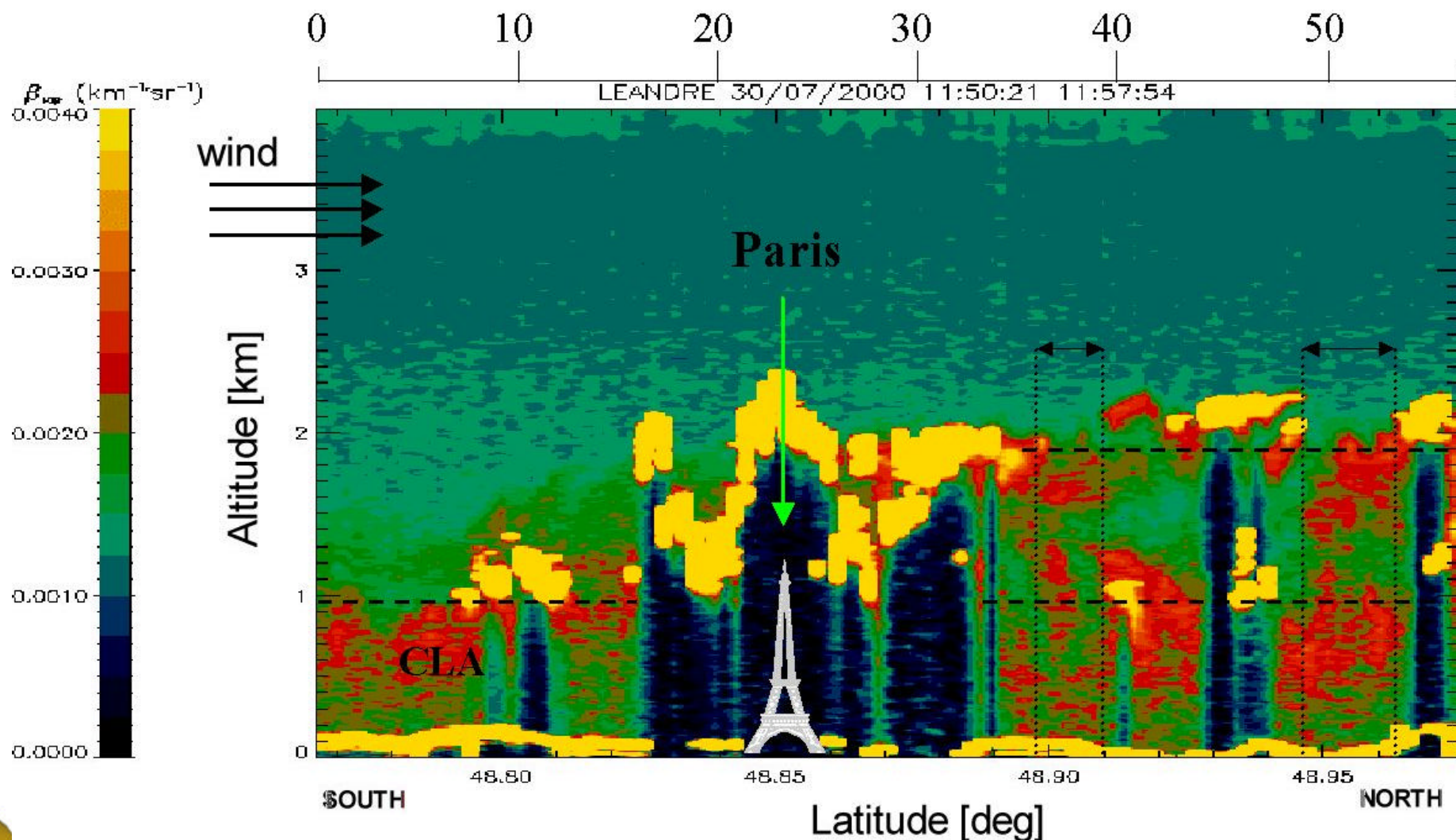
EXAMPLE (I): R²P TEMPORAL SERIES





EXAMPLE (II): AIRBORNE R²P SPATIAL SERIES

Airborne 2D lidar profiles at 532 nm on a northbound axis over Paris on 30 July, 2000 - ESQUIF 2000



SOURCE FIG.: M. Sicard (internal correspondence).



1-D INVERSION OF OPTO-ATMOSPHERIC PARAMETERS

The elastic LIDAR single-scattering equation

$$P_{I_1}(R) = \frac{K_{I_1}}{R^2} b_{I_1}(R) \times \exp\left[-2 \int_0^R a_{I_1}(r) dr\right] x(R)$$

$$b_{I_1}(R) = b_{I_1}^{aer}(R) + b_{I_1}^{mol}(R); \quad a_{I_1}(R) = a_{I_1}^{aer}(R) + a_{I_1}^{mol}(R) + a_{I_1}^{abs}(R) \approx 0$$

where:

$a_{I_1}(R)$ atmospheric optical extinction coefficient (Rayleigh+Mie) [km^{-1}]

$b_{I_1}(R)$ atmospheric optical backscatter coefficient [$\text{km}^{-1}\text{sr}^{-1}$]

– where $b(I, R) = \bar{N}(R) \frac{d\bar{s}(p)}{d\Omega}$,

– and N is the avg. density of aerosol + molecular constituents [$\text{cm}^2/\text{cm}^3\text{sr}$]

$x(R)$ optical overlap function (≈ 1 from 250 m onwards)

$P(R)$ optical range-return power [W]

Note the LIDAR *optical thickness (COT)* and related *transmissivity!* $T(I, R) = \exp[-2COT(R)]; \quad COT(R) = \int_0^R a(I, r) dr$





1-D INVERSION OF OPTO-ATMOSPHERIC PARAMETERS

DIFFERENTIAL FORMULATION OF THE LIDAR EQ.

- Let's depart from Bernouilli's form of lidar Eq.

$$\frac{dS(R)}{dR} = \frac{1}{\beta(R)} \frac{d\beta(R)}{dR} - 2\alpha(R)$$

- where

$$S(R) = \ln[R^2 P(R)]$$

- Under the assumption of a *homogeneous atmosphere*, the optical parameters are assumed to be constant over the entire lidar range

$$a(R) \approx a ; \quad b(R) \approx b$$

- This leads to the conjecture of *low fractional gradients*,

$$\frac{1}{\beta(R)} \frac{d\beta(R)}{dR} \ll 2\alpha(R)$$

- This is *unjustified under* the conditions prevailing in dense clouds, fog or where local *inhomogeneities* occur.

1-D NON-MEMORY HOMOGENEOUS ALGORITHMS

SLOPE METHOD

- Both methods assume a *homogeneous atmosphere*

$$S(R) = \ln\left(R^2 [P(R) + n(R)]\right) = \underbrace{mR + c}_{\text{ideal line}} + \underbrace{\ln\left(1 + \frac{n(R)}{P(R)}\right)}_{\text{noisy term}}$$

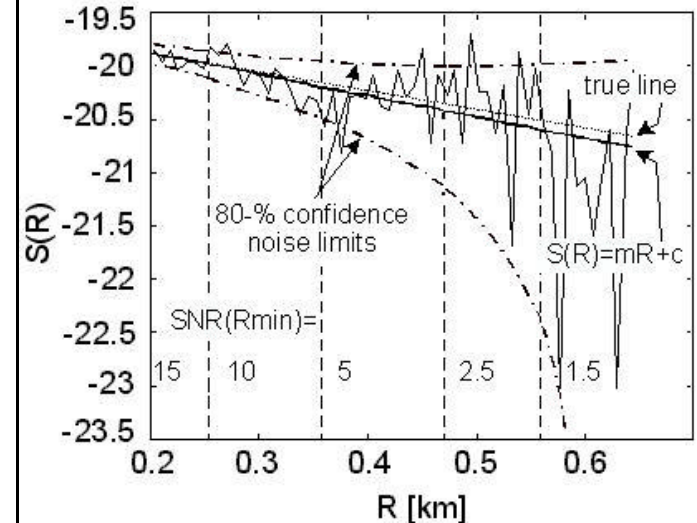
$$\hat{S}(R) = \hat{m}R + \hat{c} \quad \rightarrow \quad \hat{\alpha} = -\frac{\hat{m}}{2}; \quad \hat{\beta} = \frac{\exp(\hat{c})}{K}$$

EXPONENTIAL FITTING

- Best estimator in terms of extinction bias

$$F(R) = R^2 [P(R) + n(R)] = \underbrace{K\beta \exp(-2\alpha R)}_{\text{ideal term}} + \underbrace{R^2 n(R)}_{\text{noisy term}}$$

$$\hat{F}(R) = b \exp(-aR) \quad \rightarrow \quad \hat{\alpha} = -\frac{a}{2}; \quad \hat{\beta} = \frac{b}{K}$$



PERFORMANCE

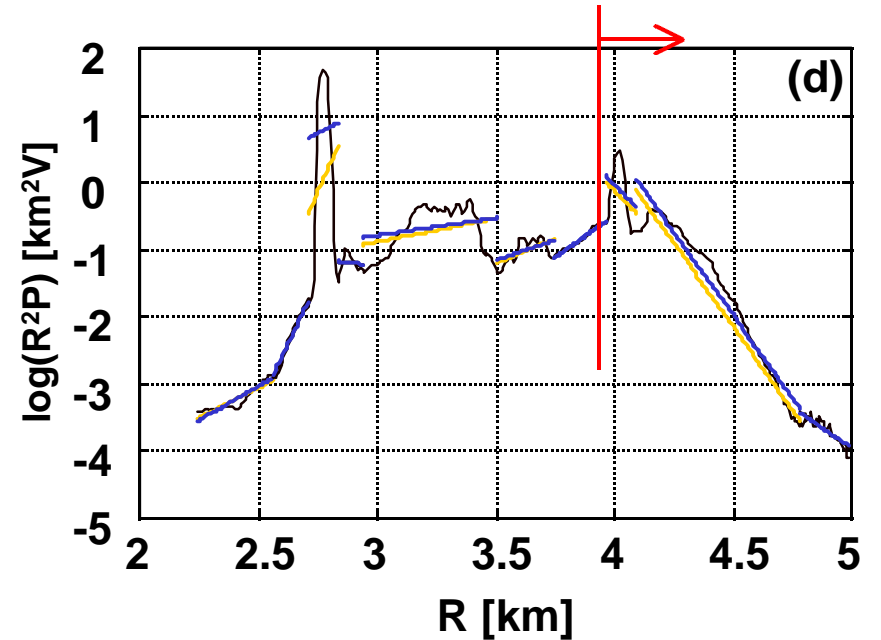
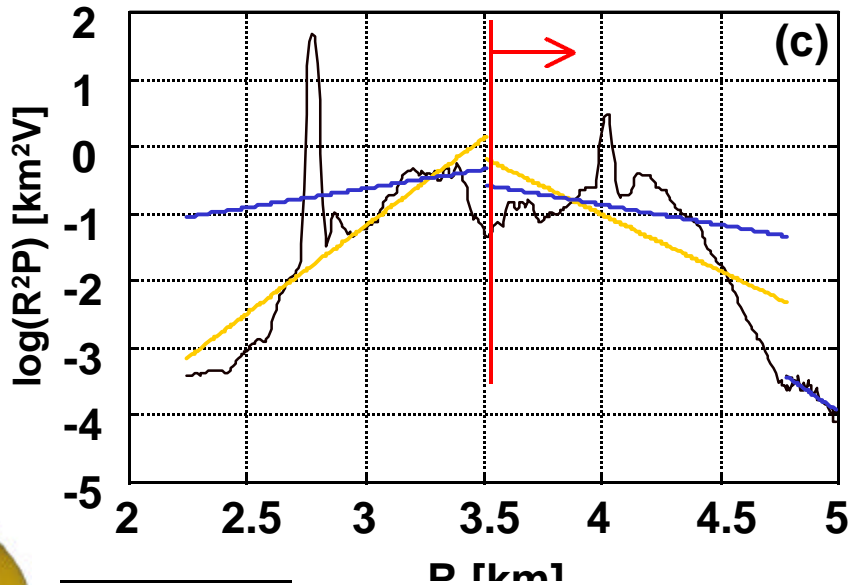
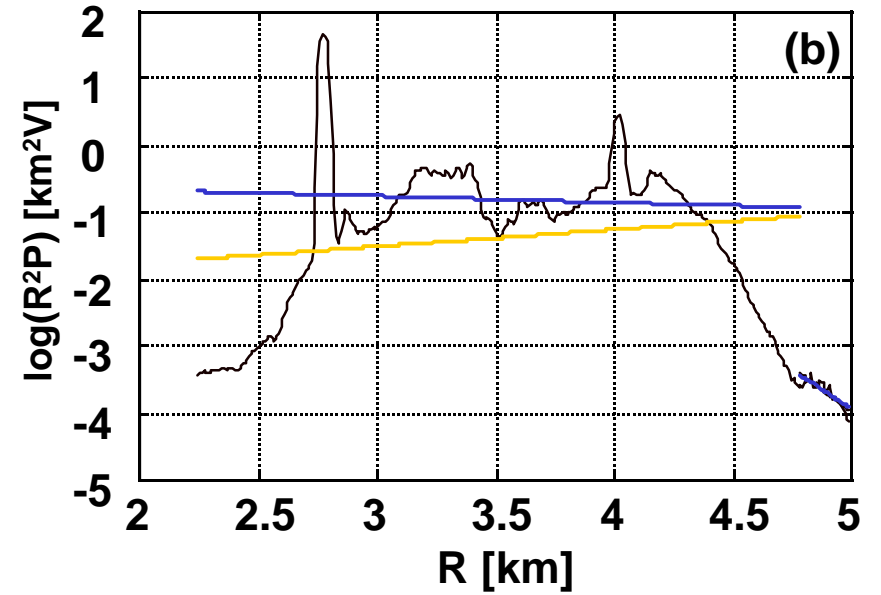
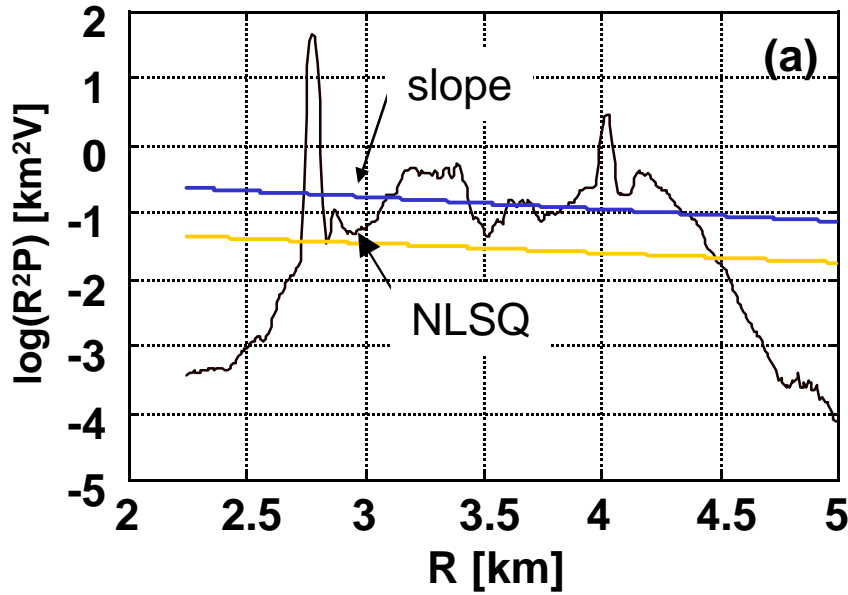
- Kunz dilemma*
- Bias*

See also: G.J. Kunz, "Probing of the atmosphere with lidar," Proc. Remote Sensing of the Propagation Environment (AGARD-CP-502), Conf. Date: 30 Sept.-4 Oct. 1991, Conf. Location: Cesme, Turkey. Publisher: AGARD, Neuilly sur Seine, France, 23, pp. 1-11 (France, 1992)

SOURCE FIG: F. Rocadenbosch et al., "Statistics Of The Slope-Method Estimator," Appl. Opt. 39(33), 6049-6057 (2000).



HIBRID METHODS: THE SLICE METHOD



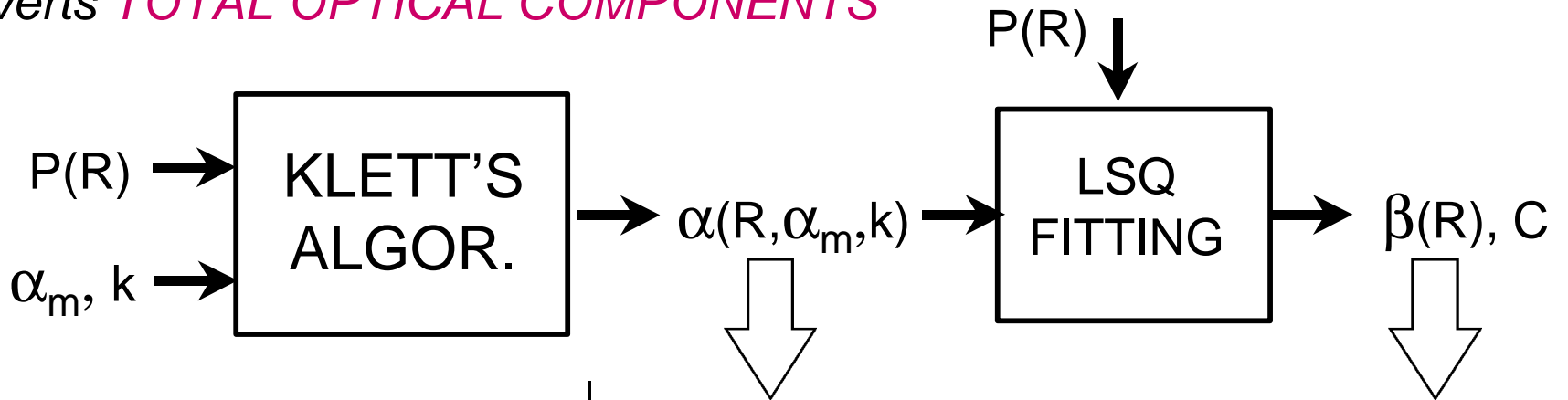
Adapted from C. Molina Martínez, PFC, ETSETB (1998).

(C) F. Rocadenbosch (UPC), 2004

(SIMPLE) KLETT'S METHOD (I)

1) SIMPLE KLETT'S METHOD

- Inverts **TOTAL OPTICAL COMPONENTS**



Basic relationships:

- Assumed correlation (eq.(1))
- User calibrations (eq.(2)):
 - boundary extinction \mathbb{R} look-up TABLE
 - $k = 1$ (struct.), 0.67 (clouds)
 - or $k=f(a)$
- Backward form (eq.(3))

$$\beta(R) = C \alpha(R)^k \tag{1}$$

$$\begin{cases} S(R) = \ln[R^2 P(R)] \\ S(R_m) = S_m \leftrightarrow a(R_m) = a_m; \quad k = ? \end{cases} \tag{2}$$

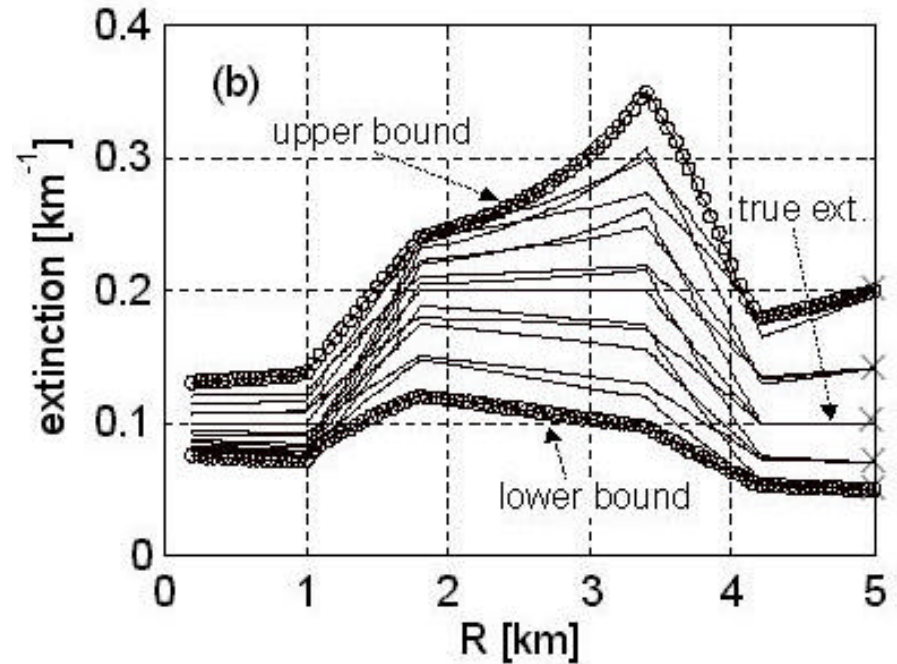
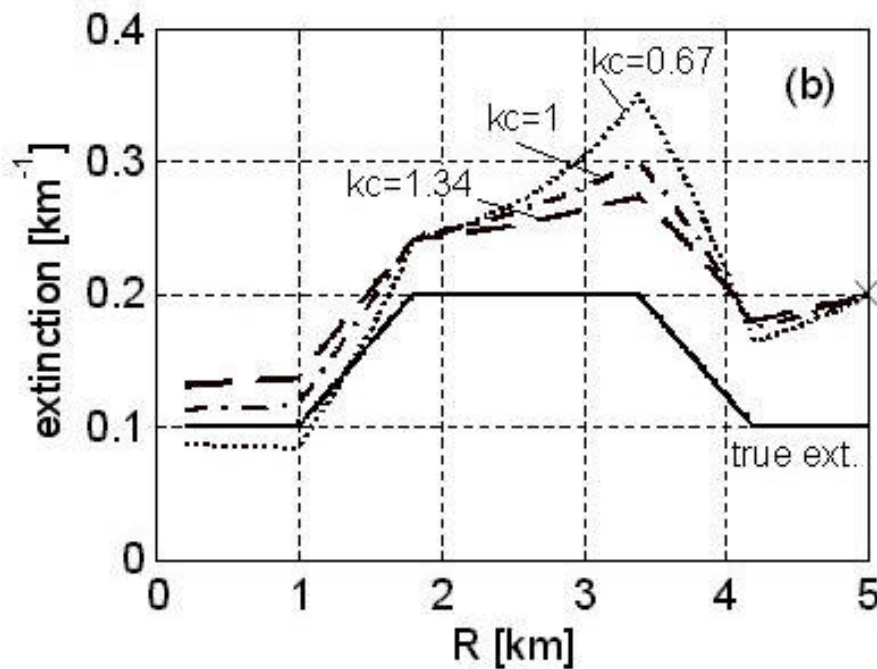
$$a(R) = \frac{\exp[(S - S_m)/k]}{a_m^{-1} + \frac{2}{k} \int_R^{R_m} \exp[(S - S_m)/k] dr} \tag{3}$$



SIMPLE KLETT (II)

KLETT'S METHOD

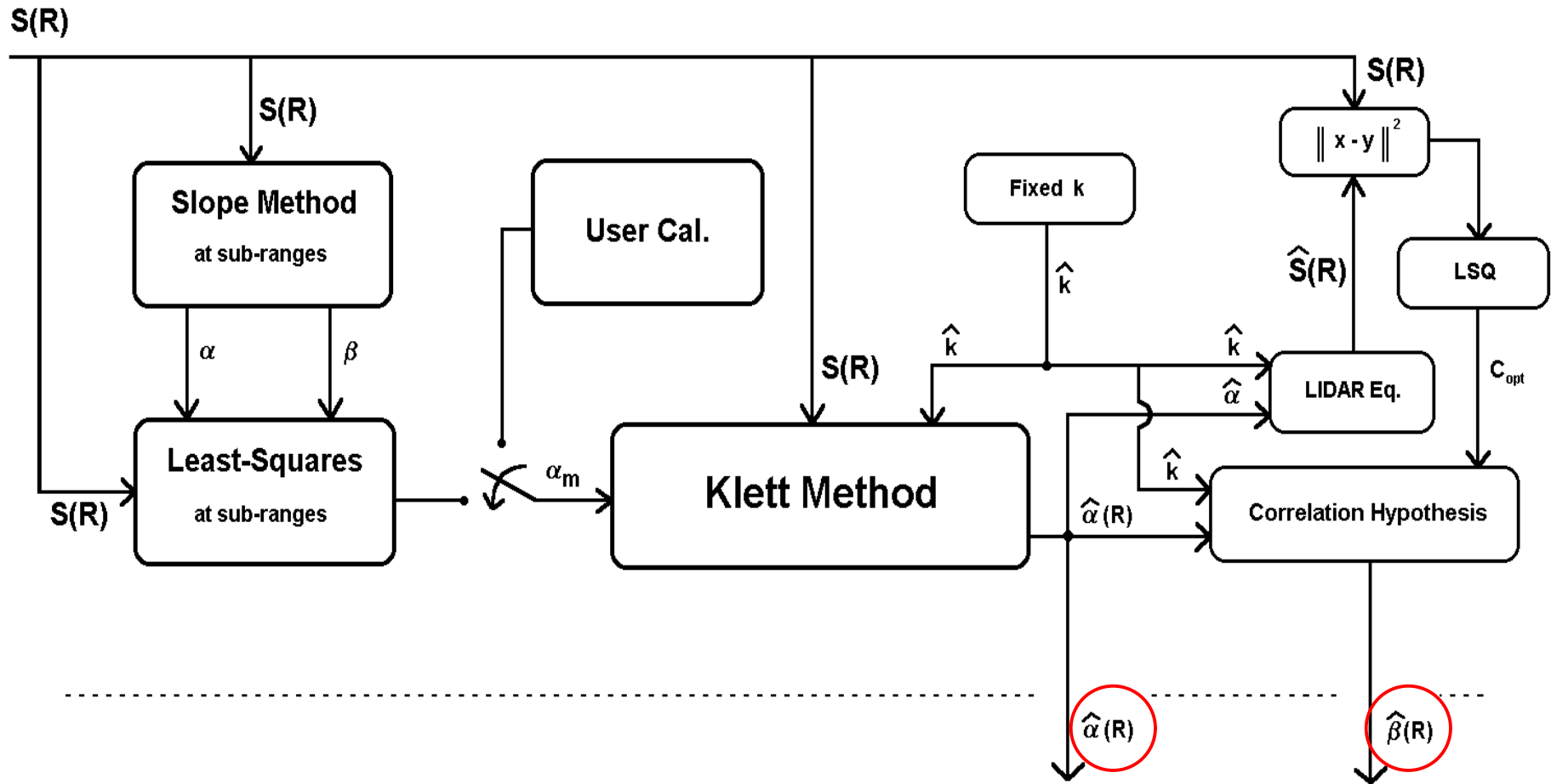
- Different extinction families arise in response to different user-calibrations in terms of:
 - 1) Boundary extinction, α_m
 - 2) Power-law backscatter-to-extinction exponent, k (kc in Fig. below)



SOURCE FIG: F. Rocadenbosch, A. Comerón, "Error Analysis For The Lidar Backward Inversion Algorithm," Appl. Opt. 38(21), 4461-4474 (1999).

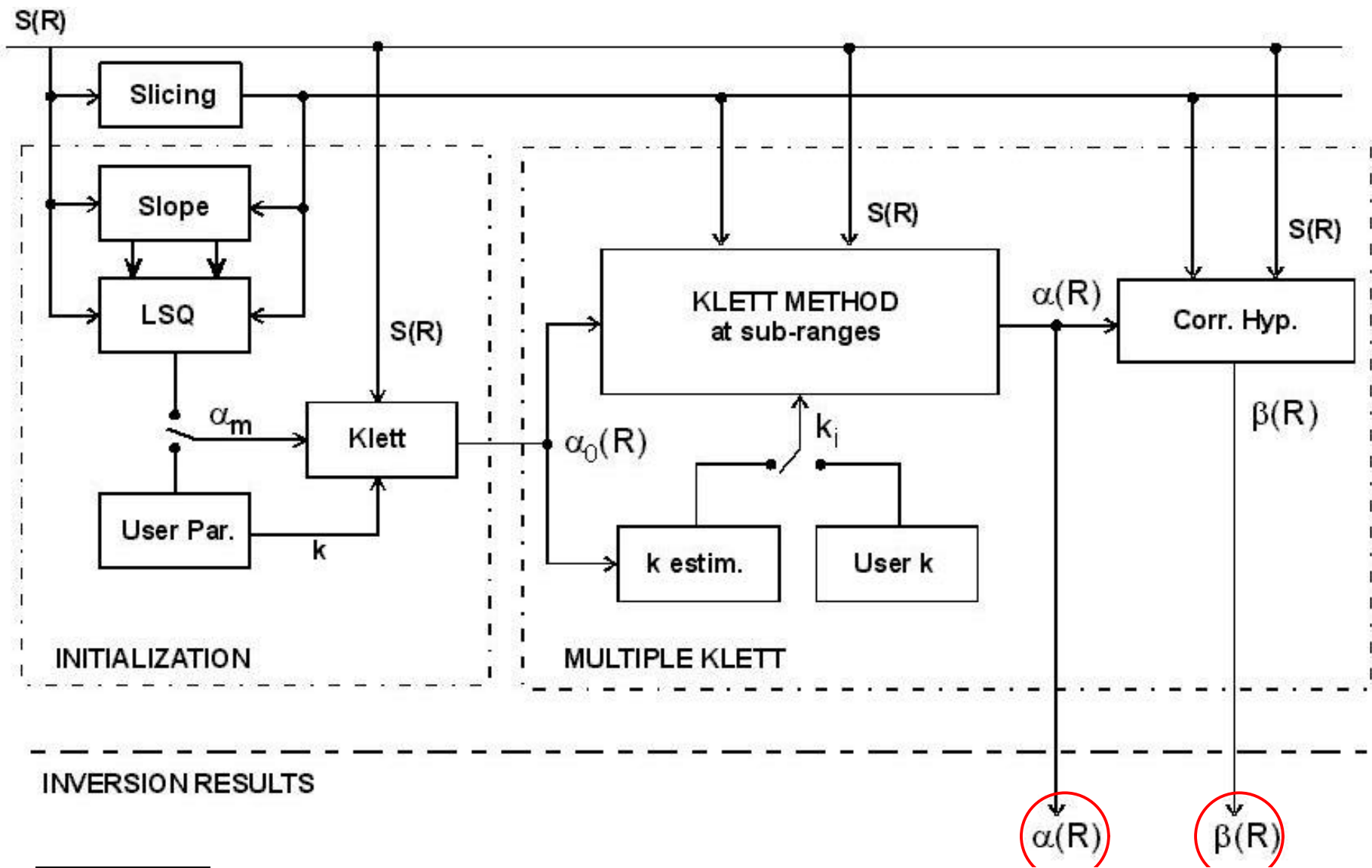
SIMPLE KLETT (III)

OPERATIONAL SIMPLE-KLETT HYBRID ALGORITHM



SOURCE FIG: C. Molina Martínez, "Creación de un entorno software para el acceso, segmentación y procesado de medidas lidar", PFC, ETSETB, 21 Jul. 1998.

HYBRID METHODS: MULTIPLE KLETT (I)

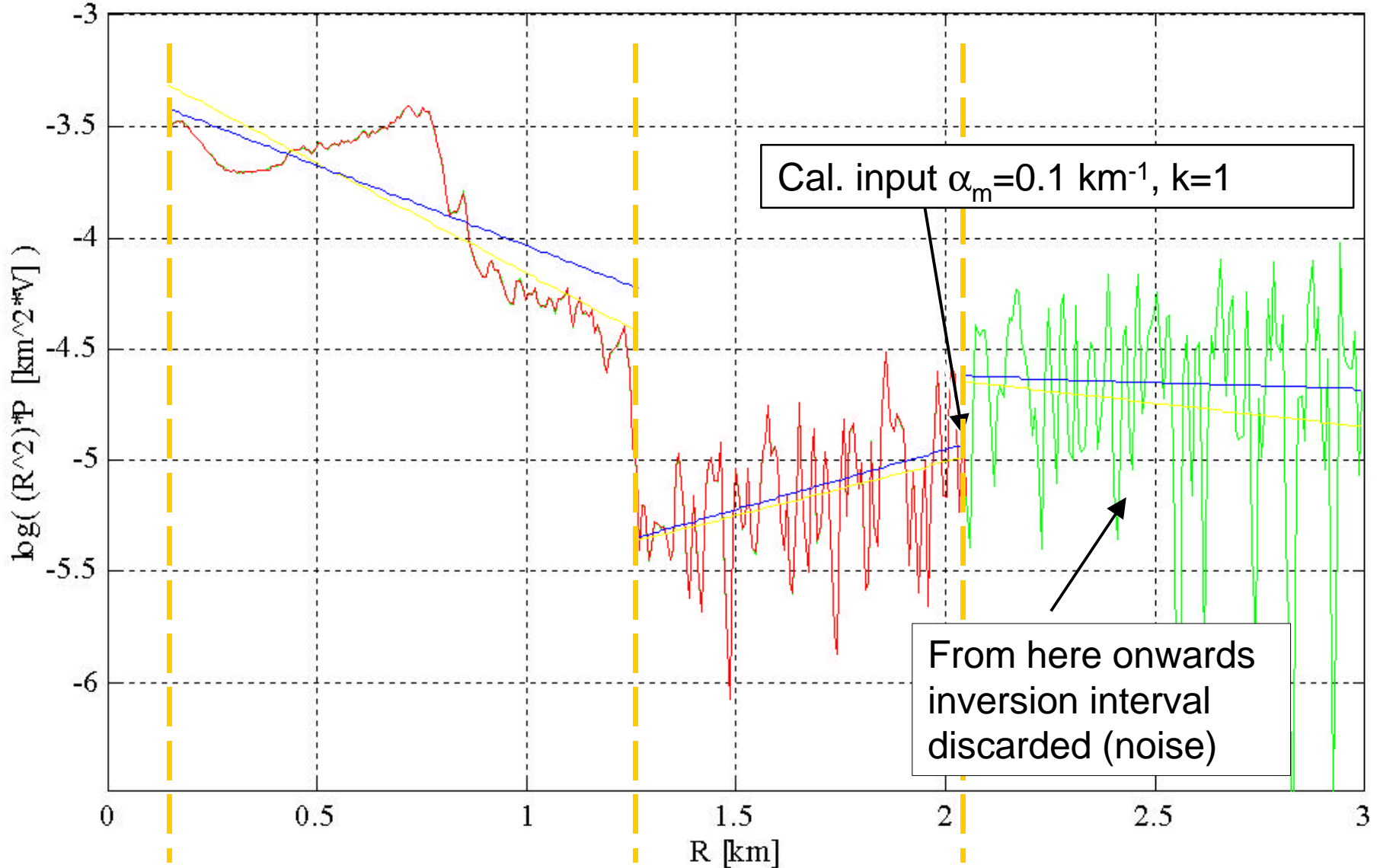


SOURCE FIG: C. Molina Martínez, "Creación de un entorno software para el acceso, segmentación y procesado de medidas lidar", PFC, ETSETB, 21 Jul. 1998.



HYBRID METHODS: MULTIPLE KLETT (II)

File: o8051330. #Packet: 5. Slope(y-), LSQ(b-) & Klett(r-) processing



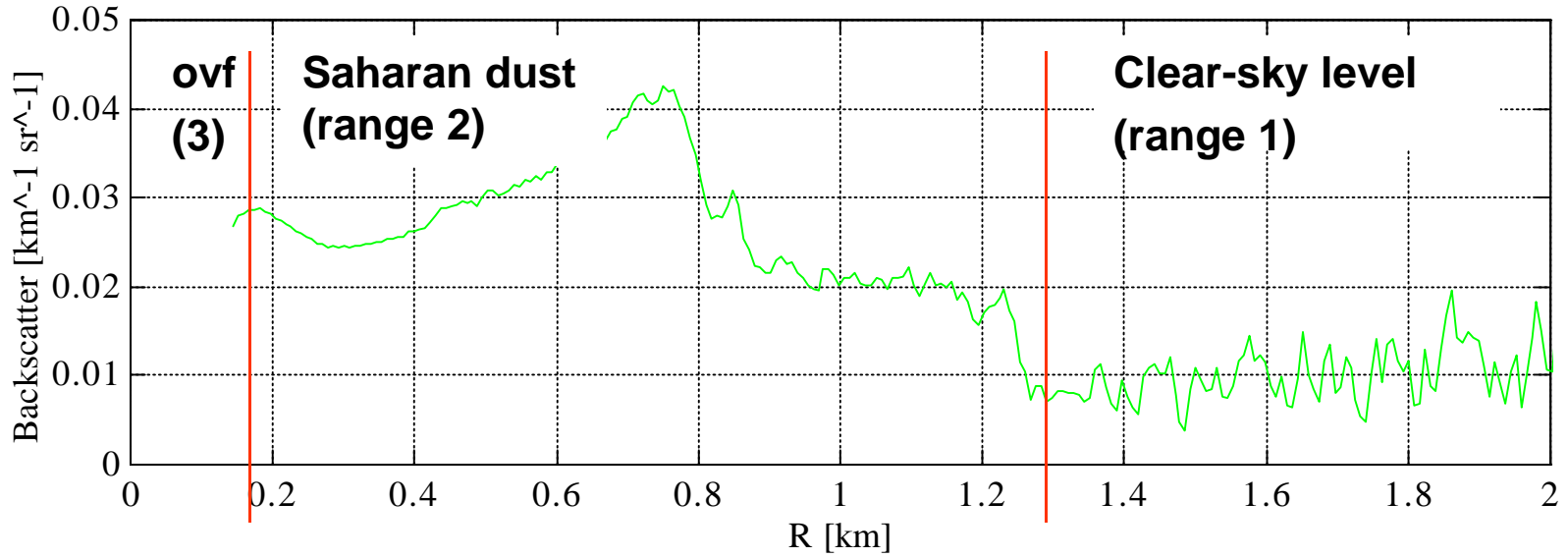


LIDAR (LASER RADAR)

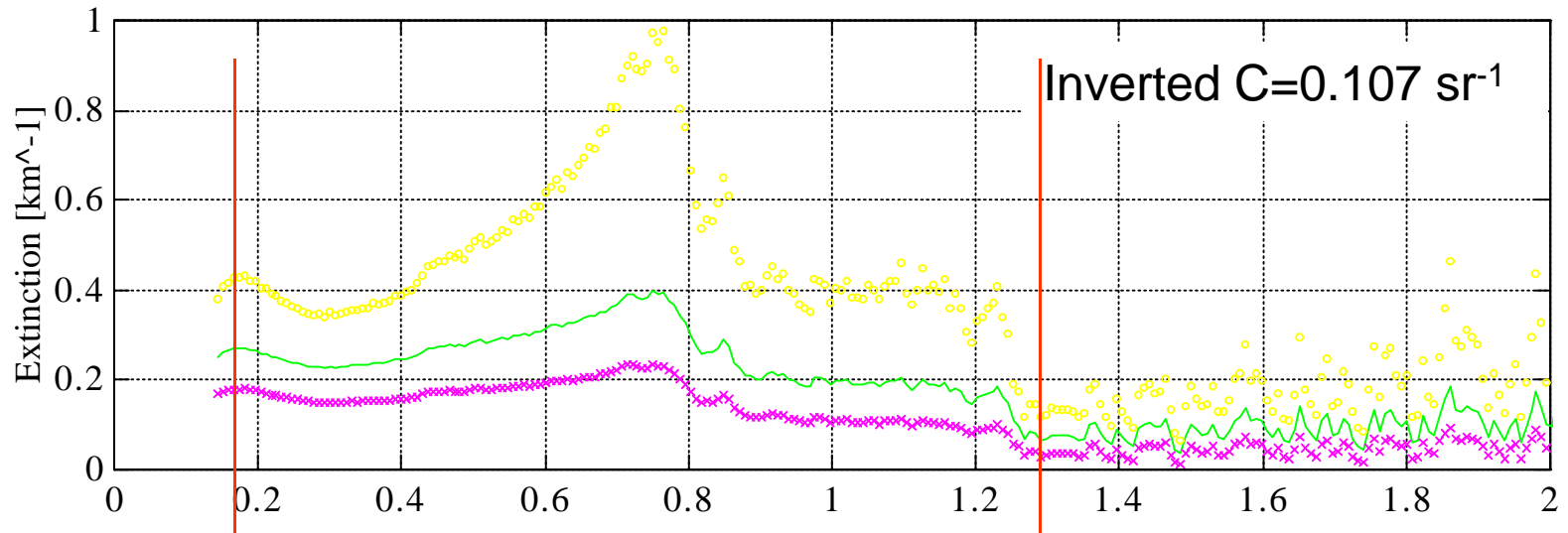
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HYBRID METHODS: MULTIPLE KLETT (III)

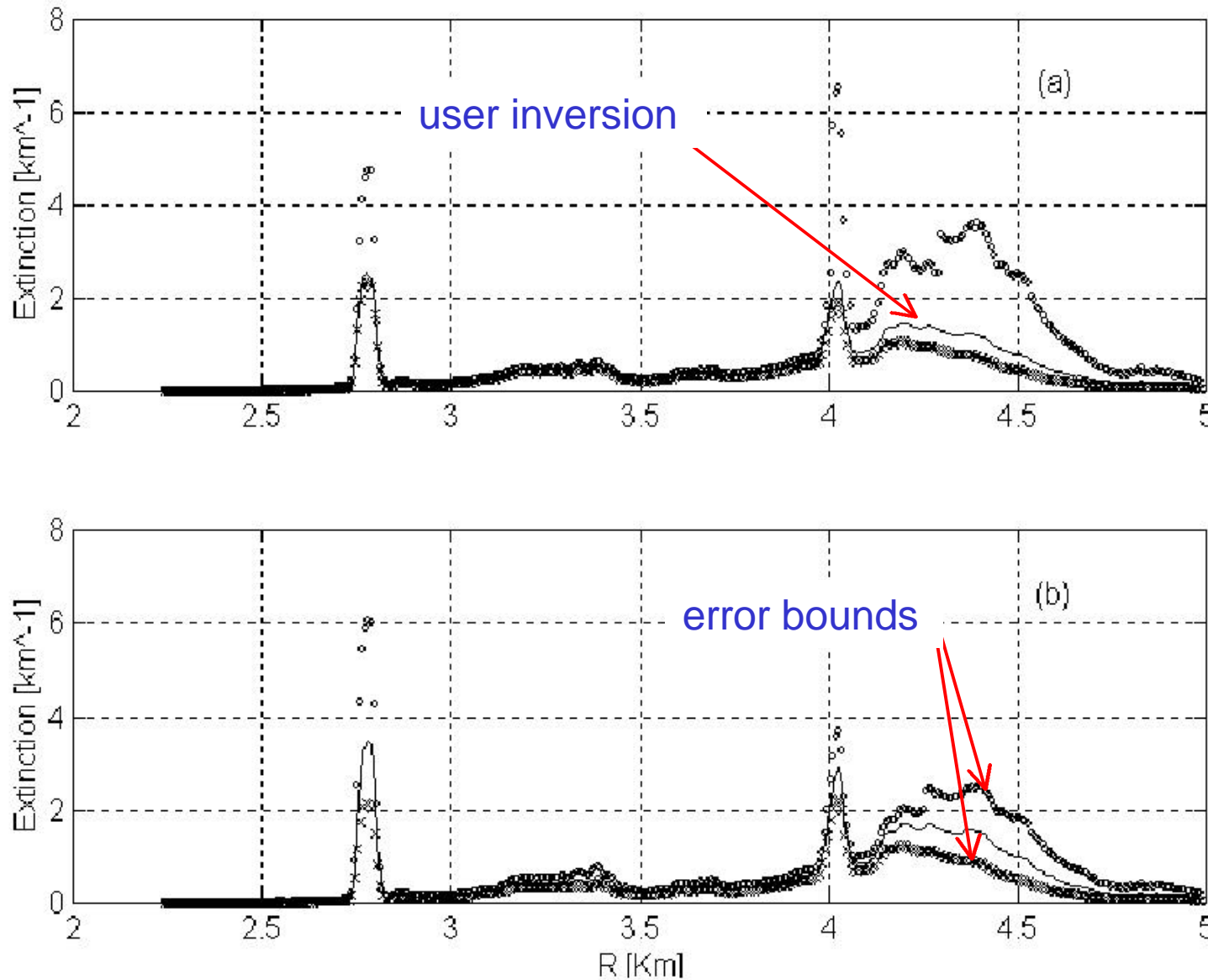


File: o8051330 Shot #5





MULTIPLE KLETT VS. SINGLE KLETT



(a) Simple Klett
(b) Multiple Klett.

Inversion example continued from slide 14

SOURCE FIG: C. Molina Martínez, "Creación de un entorno software para el acceso, segmentación y procesado de medidas lidar", PFC, ETSETB, 21 Jul. 1998.

(C) F. Rocadenbosch (UPC), 2004

FROM KLETT TO KFS

Warning Summary:

- For a given α_m calibration, the simple Klett algorithm **DOES NOT** guarantee inversion of a **reasonable C** (e.g. look-table lidar ratio)
- The α_m calibration is **always guessed** using a “visual rule-of-thumb”
- F. Rocadenbosch, A. Comerón, “Error Analysis For The Lidar Backward Inversion Algorithm.” Applied Optics **38** (21), 4461-4474 (1999). → enables to **estimate envelope error bounds given a priori errors in a_m , k**.

2) VARIABLE KLETT'S METHOD

- Inverts **TOTAL OPTICAL COMPONENTS**
- Eq.(3) is solved iteratively, assuming a spline approximation of the **a-b-look-up table** of the form \textcircled{R}
- Range-dependent correlation ratio \textcircled{R}

$$C = \frac{\mathbf{b}_{tot}}{\mathbf{a}_{tot}} = f(\mathbf{a}) \quad [sr^{-1}]$$

$$\mathbf{b}(R) = C(R)\mathbf{a}(R)^k ; \quad k = 1$$

3) KLETT-FERNALD-SASANO

- Distinguishes between **MOLECULAR** and **AEROSOL** components

$$\beta_M(z) = \frac{P(z)z^2 \exp\left\{+2 \int_z^{z_0} [S_M(u) - S_R(u)] \beta_R(u) du\right\}}{\beta_M(z_0) + \beta_R(z_0) + 2 \int_z^{z_0} S_M(u) P(u) u^2 \exp\left\{2 \int_u^{z_0} [S_M(v) - S_R(v)] \beta_R(v) dv\right\} - \beta_R(z)}$$

User inputs:

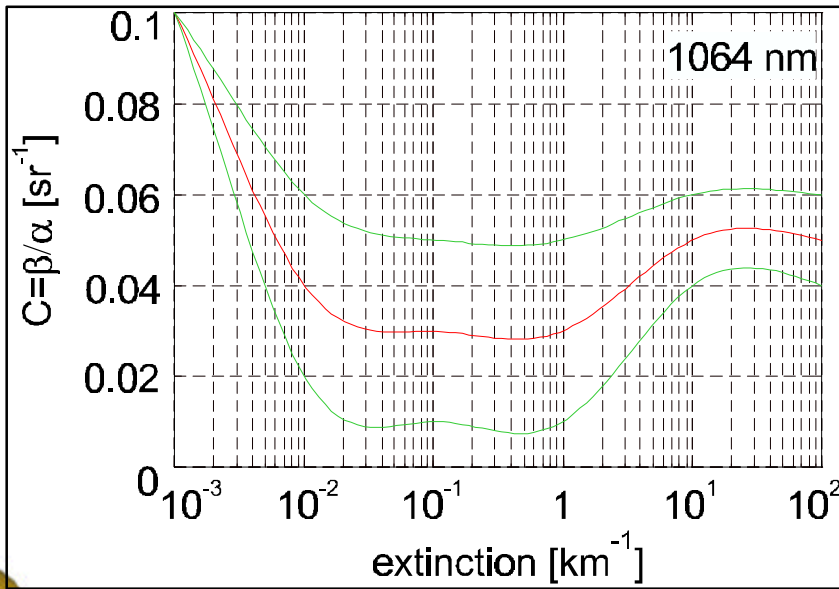
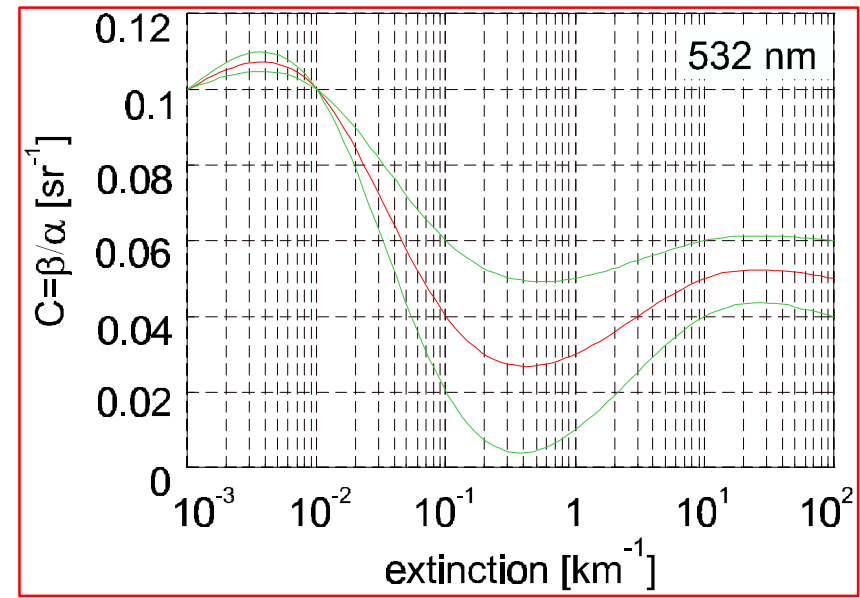
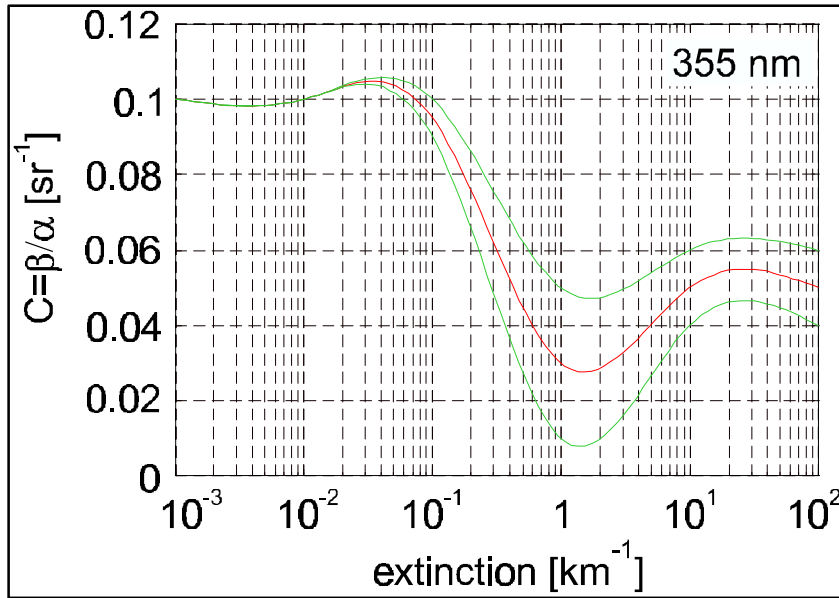
- Aerosol backscatter boundary calibration, $\beta_M(z_0) \ll \beta_R(z_0)$
- Aerosol variable lidar ratio, $S_M(z)$?
- Rayleigh extinction
 - Height-dependent $P(z)$, $T(z)$ and $n(l, z) = f[l, P(z), T(z)]$
 - Depolarization ® King Factor

$$S_M = \frac{a_{Mie}}{b_{Mie}}$$

$$\alpha_R \approx \alpha_R^{sca}(z, \lambda, P, T) = \frac{8\pi^3 [n_{air}^2(P, T, \lambda) - 1]^2}{3\lambda^4 N_s^2(z, P, T)} KingF(\lambda); \quad S_R = \frac{\alpha_R}{\beta_R} = \frac{8\pi}{3}$$



ASSUMPTIONS ON LIDAR RATIO

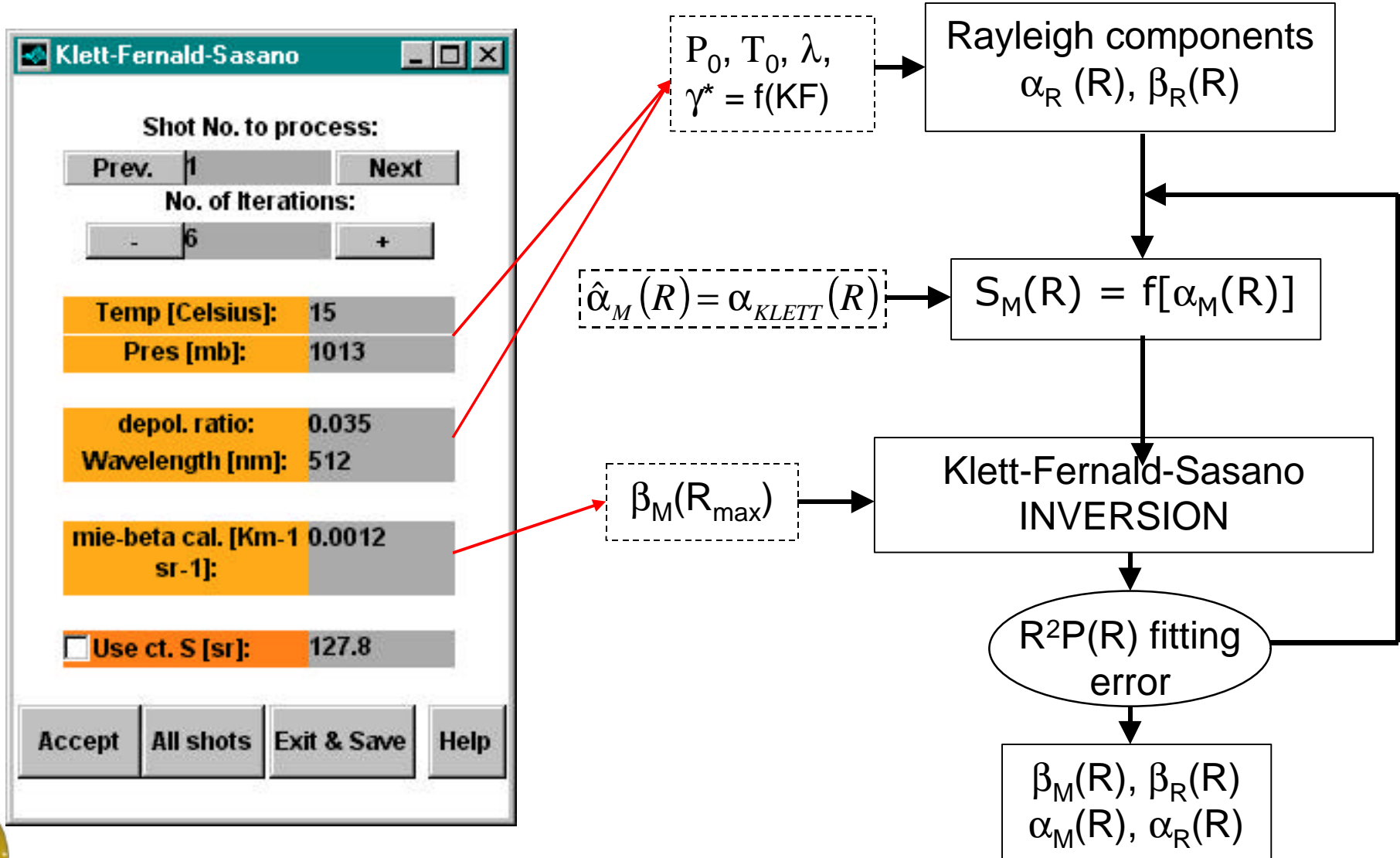


We plot $1/S_M$ (look-up table)

- Note that $S_M = \frac{a_{Mie}}{b_{Mie}} [sr] \neq \frac{1}{C}$
- Need to cross-examine with retrievals from co-operative instrumentation (e.g. Sunphotometer) or multiangle profiles to adjust S_M

KFS (II)

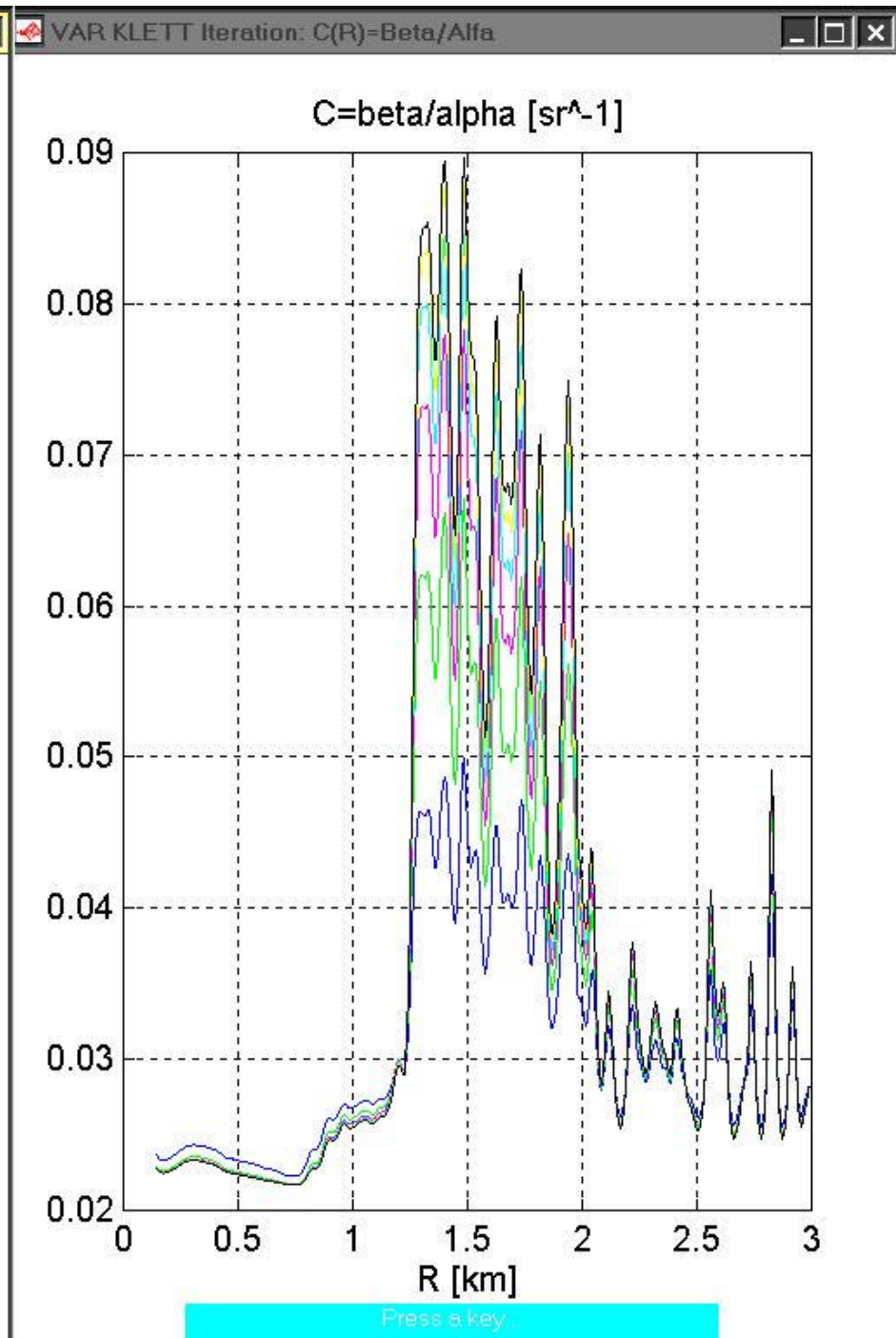
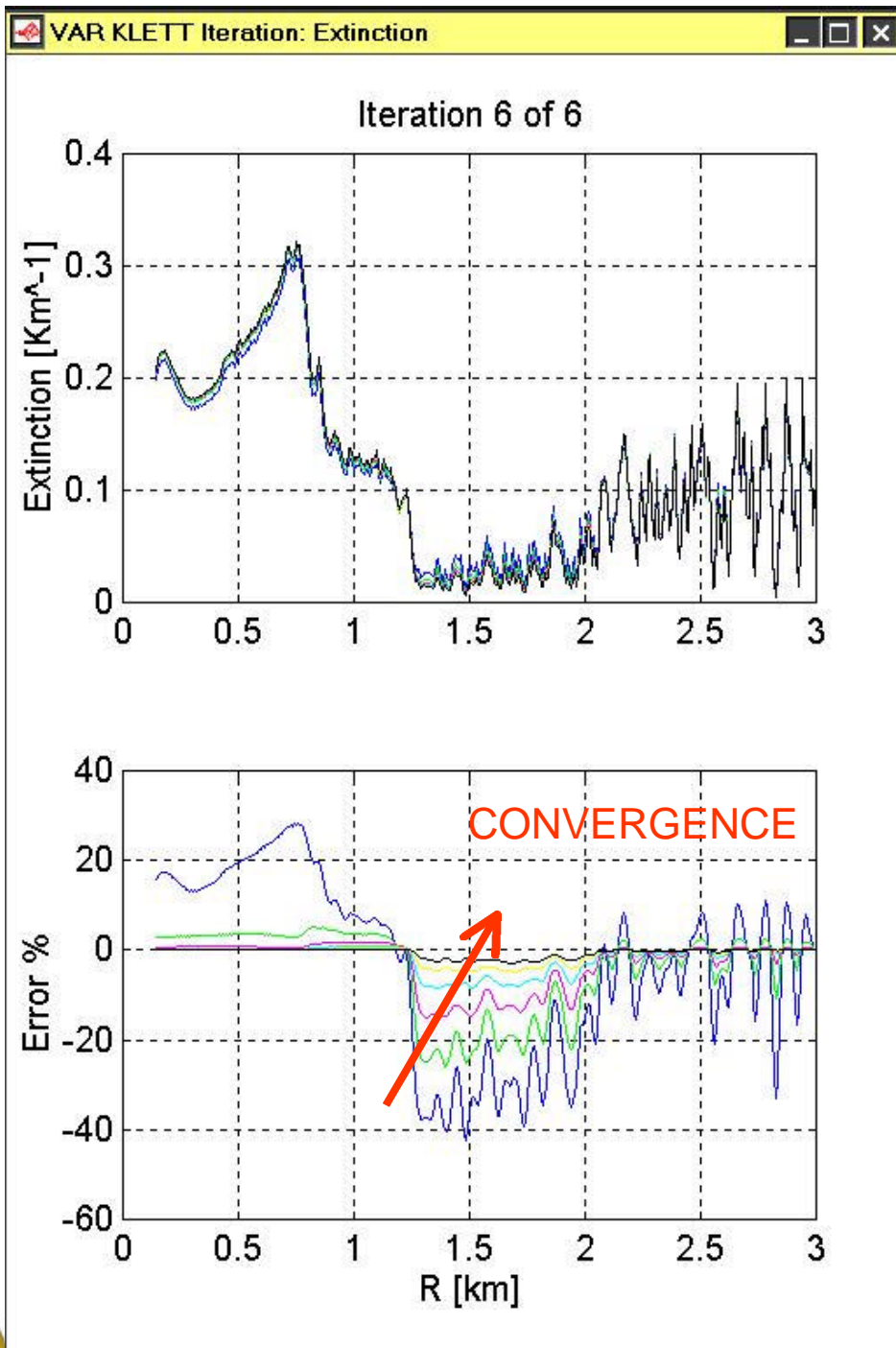
3) VARIABLE KLETT-FERNALD-SASANO (Continued)





LIDAR (LOGS) RADAR

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COMPARISON BETWEEN KLETT'S AND KFS' KERNELS

SINGLE-ANGLE: Most elastic lidar inversion algorithms can be summarised in kernel-function form as

Klett's method:
$$\mathbf{a}_M(R) = G_{Klett} \left[R^2 P(R), \mathbf{a}_{tot}(R_0), S_{tot}(R) \right] - \mathbf{a}_R(R),$$

KFS's method:
$$\mathbf{a}_M(R) = F_{KFS} \left[R^2 P(R), \mathbf{a}_M(R_0), S_R, S_M(R) \right] - \mathbf{a}_R(R)$$

Requires:

- User *boundary calibration*, $\mathbf{a}_M(R_0)$
- User-given *aerosol lidar ratio*, $S_M(R) = \frac{\mathbf{a}_M(R)}{\mathbf{b}_M(R)}$
- *Press&Temp. profiles*, _____



SUMMARY KEYS:

- *Homogeneous* (Slope, NLSQ) vs. *inhomogeneous* atmosphere (Klett's and KFS' variants)
- Retrieval of *total* (Klett) vs. *aerosol&molecular* components (KFS)
- KFS is the state-of-the-art *widely accepted operative algorithm*



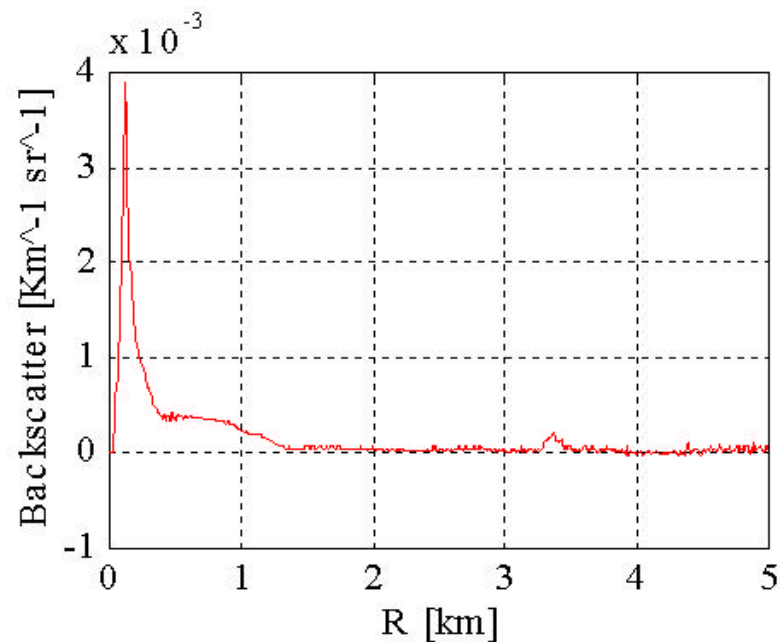
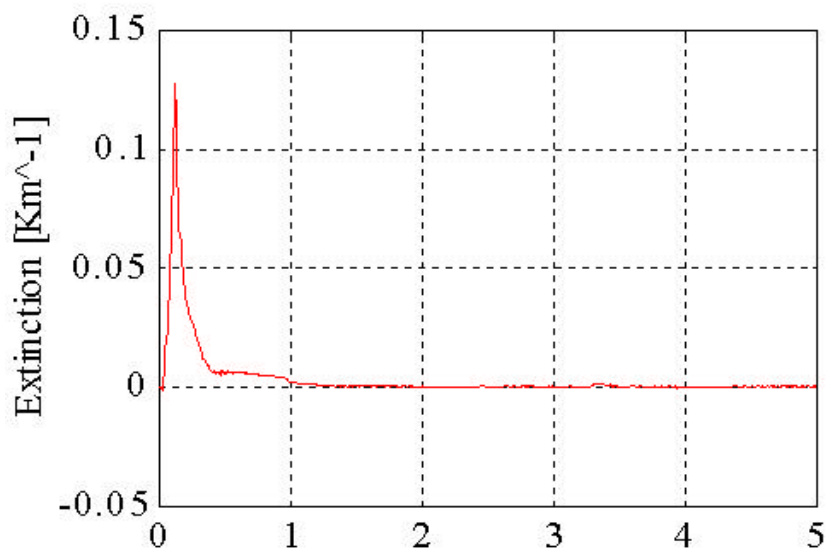
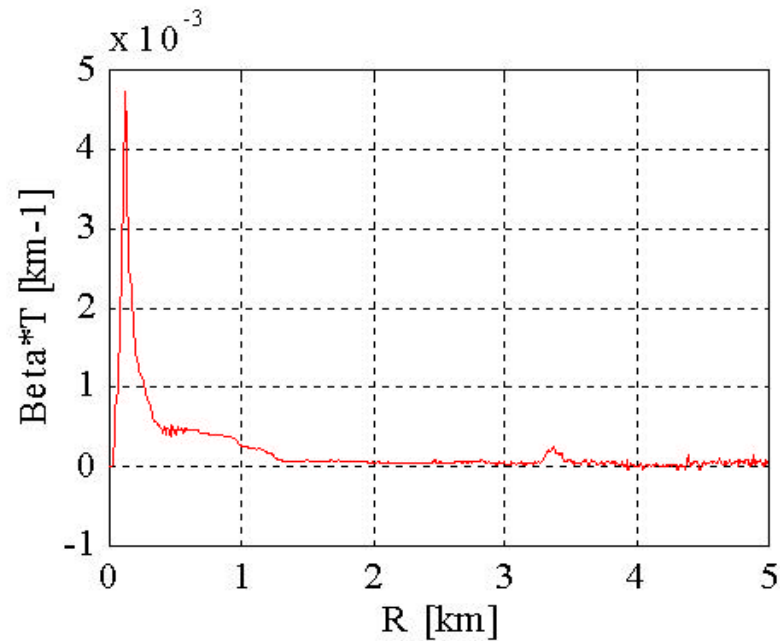
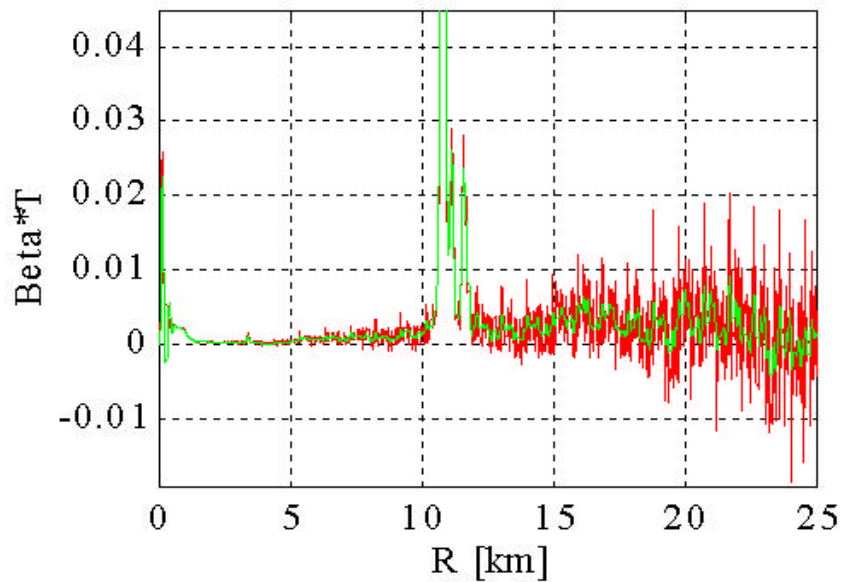
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A FEW INVERSION WARNINGS: KFS (CASE I)

Vout. File: u1091950. #Packets: 1. #IP/shot: 36000.



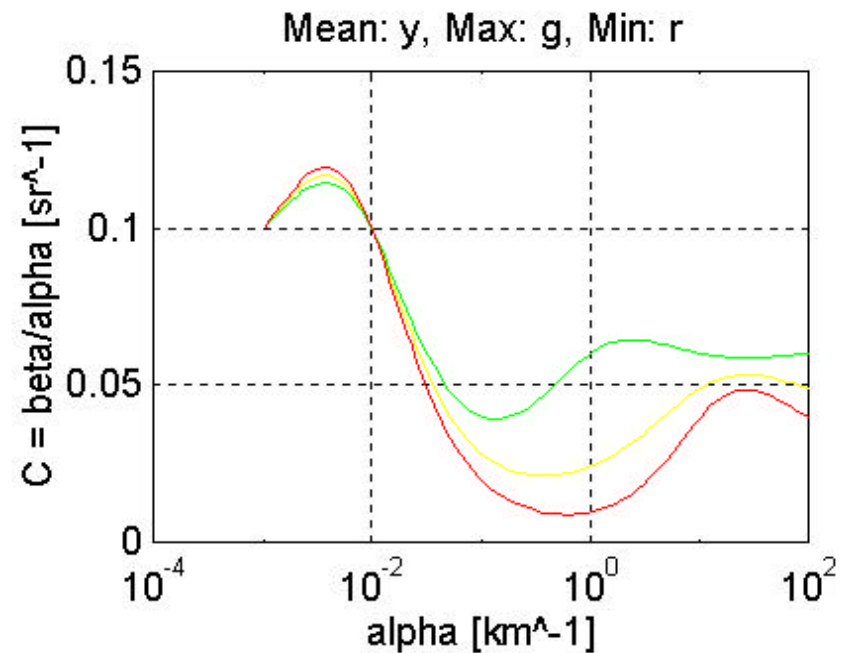
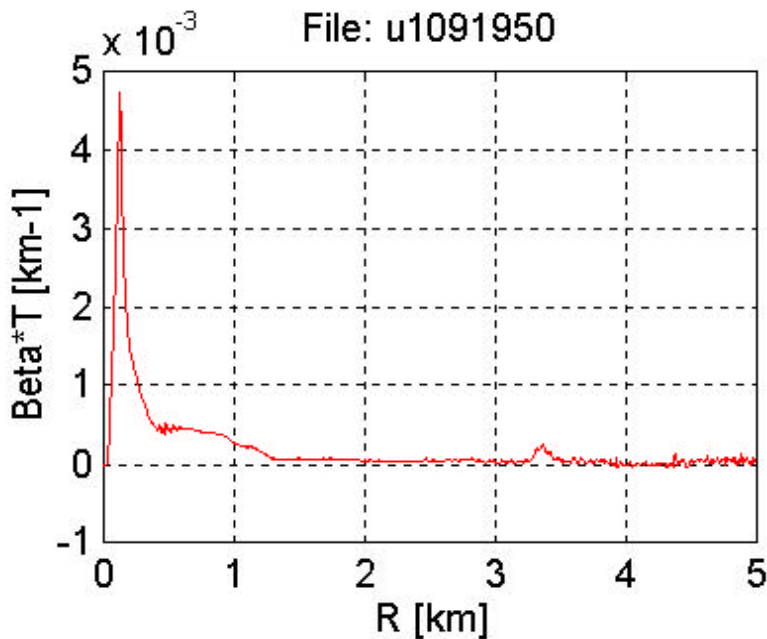
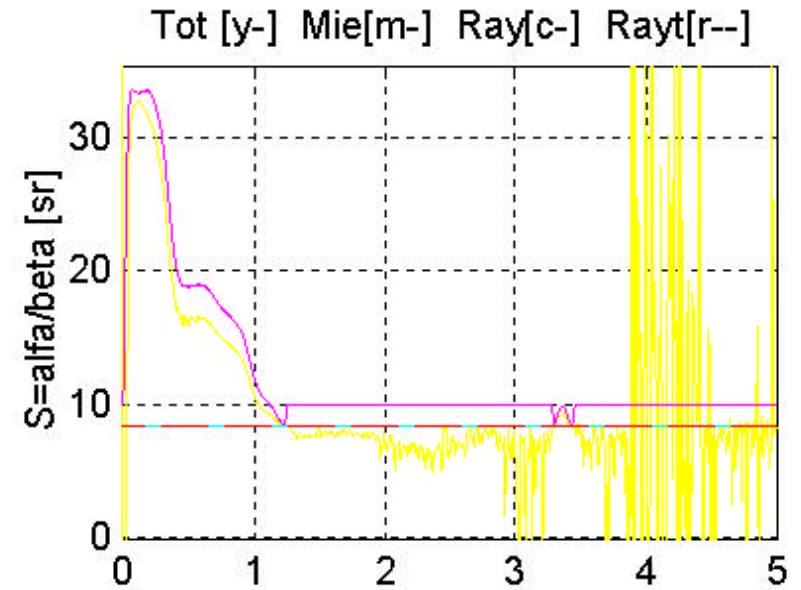
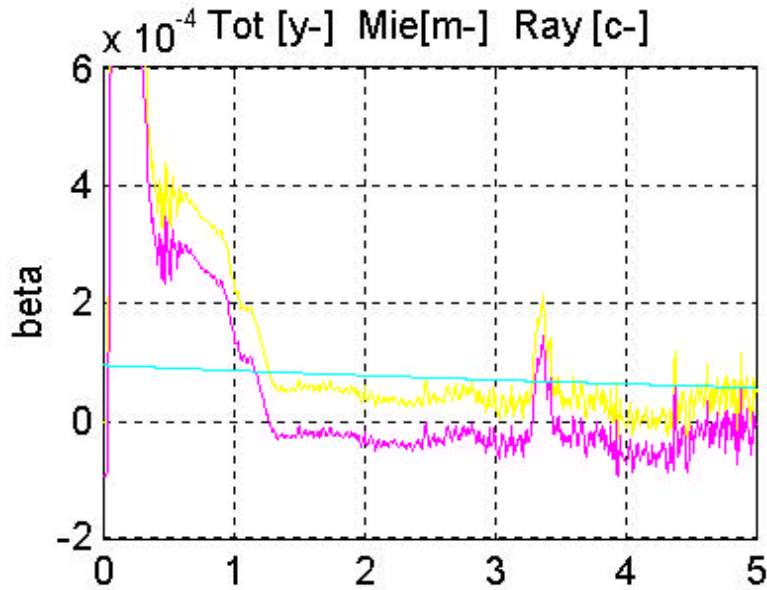


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A FEW INVERSION WARNINGS: KFS (CASE I)





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A FEW INVERSION WARNINGS: KFS (CASE II)

