Promoting lab engagement in experimental compressible flow modelling

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ABSTRACT

The present work depicts the development of an experimental equipment that reveals compressible fluid dynamics, while collecting data from an incompressible flow like water in an open-channel. It consists of an extensive theoretical framework followed by a practical analysis, the aim of which was to trigger the hydraulic jump, both normal and oblique, in order to illustrate its hydro-gasdynamic analogy with a shock wave, occurring in supersonic compressible flows.

The assembly, called “water table”, arises from the necessity of economical alternatives to expensive supersonic wind tunnels in the experimental study of compressible flows. Thus, a canal based on a Laval nozzle was constructed where water flow could experiment a hydraulic jump. Through its visual and experimental perception, fellow interested could more easily understand the physics and engineering behind this phenomenon.

Multiple design alternatives were evaluated considering environmental, economic, functional and aesthetic factors. A low-cost implementation was critical in the design process. The measurements revealed that the geometry of the nozzle and the wedges designed as obstacles to cause obliquity were the most influential elements in the formation of a hydraulic jump in the set-up. Regarding the experimental variables, the upstream and downstream

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heights had the highest relevance. Therefore, their manipulation and analysis could lead to further educational investigations.

This research is a step forward to support students in the understanding of compressible flow principles by providing an in-house experimental set-up. The equipment is an opportunity of carrying out lab measurements, which certainly guides to a major commitment in the field.

1 INTRODUCTION

1.1 Scope

The experimental study of compressible flow dynamics has been of growing interest since the 1940s, for both experts and students of fluid mechanics due to its learning difficulty, since this type of flow is not very common and easy to visually detect. Unfortunately, its experimental simulation presents the disadvantage of requiring expensive equipment such as a supersonic wind tunnel that not all universities can afford[1-5]. Therefore, the main goal of this paper is to describe the design, manufacture and experimentation of an equipment that illustrates the hydro-gasdynamic analogy between the shock wave, compressible flow characteristic, and the hydraulic jump. From the set-up students can collect data while experimenting with water easily and economically.

The purpose of this work is not only to show the evidence of theoretical formulas throughout an experimental modelling, but also to develop it by means of an inexpensive and easy-to-manufacture experimental assembly. Hence, it demonstates support for students' knowledge of compressible flow principles and, as a result, the physics and engineering education major engagement.

1.2 Theoretical context

The objective of the theoretical context is to elucidate the analogy between the dimensionless Mach (Ma) and Froude (Fr) numbers because they are both based in describing the fluid velocity, compressible and incompressible flow, respectively.

An extensive explanation of the theoretical development could be detailed and followed by fellow interested, but it is not the purpose of this short concept paper and it can be found in many fluid mechanics books[7]. In preference, the most relevant formulas are going to be presented.

On the basis of Bernoulli's equation applied between the stagnation state and a generic point of a reservoir open to the atmosphere of fluid depth connected to a hydraulic canal, the dimensionless Froude number (Fr) can be defined as:

$$Fr = \frac{v}{\sqrt{gH}} \tag{1}$$

Where v is the velocity, g gravity and H the height of the generic point. If an analogous analysis is carried out on a streamtube of compressible flow like air, where the steady-state conservation of energy equation is applied, and it is compared to the previous one according to an incompressible flow, a certain similarity between the ratios of velocities arises. At this point, the first analogy between Mach (Ma) and Froude numbers is revelead, if the hydraulic current is seen as a hypothetical gas flow of $\gamma = 2$. Consequently, both dimensionless numbers can be equally estimated[8].

$$Ma \cong Fr = \sqrt{2\left(\frac{H_0}{H} - 1\right)} \qquad (2)$$

Additionally, Mach number is defined by using the sound speed (a) and the fluid velocity (u):

$$Ma = \frac{u}{a} \qquad (3)$$

From the equations of perfect gas, isentropic flow and the continuity equation results the hydro-gasdynamic analogy for the ratio of the throat area and the local area in a diffuser of compressible flow. From the following formula, A denotes the surface and the superscript "$*$" represents sonic conditions[7].

$$\frac{A}{A^*} = \frac{1}{Ma}\left[\frac{2}{3}\left(1 + \frac{1}{2}Ma^2\right)\right]^{3/2} \qquad (4)$$

Furthermore, the analogy between bidimensional supersonic compressible flow and supercritical incompressible flow can be studied. A shock wave can form an oblique angle when the oncoming flow collides with an obstacle, like a wede of angle 2θ. Under particular conditions for an incompressible flow, the analogous oblique hydraulic jump arises[2,3,6-8]. Thus, the following equation outlines the mathematical relation between their variables. It may be considered that β is the angle between the shock wave and the incident supersonic current, subindex 1 refers to supersonic flow and δ is the angle between the hydraulic jump and the oncoming flow.

$$\theta = \delta - tan^{-1}\left[\frac{2tan\delta}{-1 + \left(1 + 8{Fr_1}^2 sin^2\delta\right)^{\frac{1}{2}}}\right] \qquad (5)$$

2 METHODOLOGY

To begin with the experimental set-up building process and analysis, a deep study and evaluation of design alternatives, materials and dimensions was carried out. The first 3D design incorporated two reservoirs (upstream and downstream) and the noteworthy water canal that followed a Laval's nozzle criteria.

The next step was the definition of different materials and configurations of the initial 3D designed model. The alternative called "A" consisted of two prefabricated plastic reservoirs and the water canal printed in 3D, made of polylactic acid (PLA). Whereas "B" was made up of methacrylate and "C" of stainless steel. All of them were evaluated following the weighted sum technique, marking each alternative according to the Saaty scale and a variety of criteria (see Table 1).

Criterion	**Weight (%)**	**Alternatives**		
		A	**B**	**C**
Environmental impact	20	8	5	6
Cost	35	9	4	4
Functionality	30	5	8	8
Aesthetic finish	15	7	9	8
Weighted score		**7.30**	6.15	6.20

Table 1. Weighted sum evaluation

As it is denoted in the resulting weighted scores, alternative “A” was the one developed for the experiment. In addition to the three main components aforementioned, three wedges were also printed in 3D and tacked on to cause obliquity.

According to Figure 1 Left), the hydraulic flow of the final experimental set-up starts in a water pump, from which it reaches the upstream reservoir through valves and pipes. Across the nozzle throat, the flow accelerates achieving first the critical state and finally the supercritical one. Then, the hydraulic jump makes the water return to its slow subcritical regime. From the mentioned figure also should be noticed the main variables of the work, which should be emphasized by students when recreating the experiment. There are two independent variables: e, the width of the throat, and b, the width of the nozzle’s diffusing part. The width of the throat was adjusted from 0.07 m to 0.03 m in steps of 0.01 m by means of two adjustable plates. Each of these widths generated a specific Froude number, which could be analytically obtained through the measured heights (dependent variables) H_0 and H, shown previously in equation (2). Finally, b was used to fix the point of the nozzle’s diffusing ending part, right after the hydraulic jump occurred.

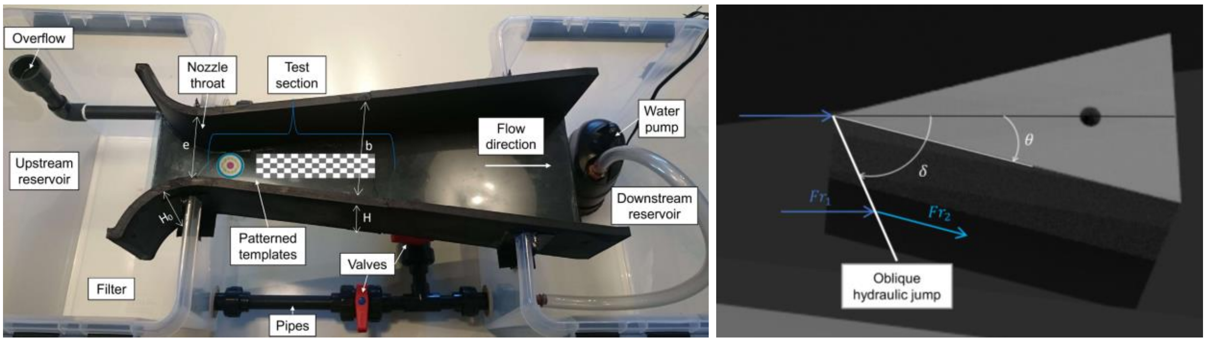

Figure 1. Left) Experimental set-up design and variables; Right) Wedges design and variables

Moreover, the wedges of angles 7.5°, 15° and 20° were of height 0.04m and length 0.08m. The independent variables of the second stage of the experiment are also the widths e and b, by adding the angle of the wedges θ. Hence, the angle of the oblique hydraulic jump δ caused in each case is the dependent variable (see Figure 1 Right)).

3 RESULTS

3.1 Normal hydraulic jump

For each of the fixed five widths of the nozzle’s throat, two Froude numbers were calculated from equations (1) and (2), respectively. In addition, theoretical and experimental values of A/A* were obtained through equation (4) (see Table 2).

e (m)	Ma=Fr (eq 2)	Fr (eq 1)	Relative error (%)	A/A* (eq 4)	A/A* (b/e)	Relative error (%)
0.07	1.593	1.482	6.96	1.168	1.214	3.84
0.06	1.558	1.242	20.27	1.151	1.417	18.78
0.05	1.420	1.035	27.12	1.091	1.700	35.82
0.04	1.183	0.763	35.47	1.020	2.125	52.02
0.03	1.085	0.528	51.38	1.005	2.833	64.56

Table 2. Mach and Froude numbers results

If the results are observed, it can be seen that Froude numbers from equation (1) are greater than one only for nozzle throat widths above 0.04 m, which indicates the experimental formation of a hydraulic jump for this range of width values. This result is consistent with the fact that the error between the results from both equations increases when the width decreases, from a 7% ($e = 0.07$ m) to a 51% ($e = 0.03$ m).

3.2 Oblique hydraulic jump

According to the fact that the best results were obtained from the throat widths of 0.07m, 0.06m and 0.05m, the second stage of the experiment was conducted to measure nine cases, and in five of them was possible to see experimentally an angle δ of the oblique hydraulic jump. Heights H_0 and H were also measured in order to obtain Froude numbers (through equation (2)) and consecutively theoretical δ values from equation (5), calculated by using MAPLE™ program[9]. Analogous to the behaviour of compressible flow, it occurs a detached shock wave (DS). This means that under these conditions, angle θ and Froude number, a δ value between 0 and 90° does not exist.

e (m)	θ (º)	Fr (eq 2)	δ (measured) (º)	δ (eq 5) (º)	Relative error (%)
0.07	7.5	1.573	63	51.07	23.36
0.06	7.5	1.536	50	52.93	5.54
	15	1.434	72	DS	-
0.05	7.5	1.408	58	63.81	9.11
	15	1.336	54	DS	-

Table 3. Angles δ results

3.3 Photo tracking

Photo tracking was utilised as means to physically capture the hydraulic jump. Figure 2 Left) shows an example of it when the width of the throat is 0.05 m and Figure 2 Right) depicts an oblique hydraulic jump, which corresponds to a throat width of 0.07 m and a wedge of 7.5°.

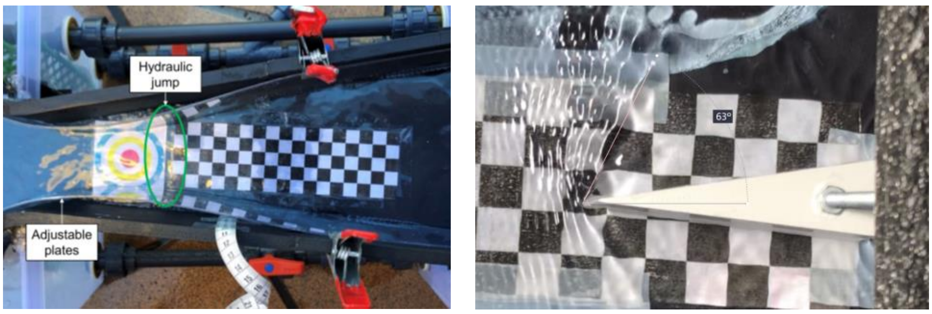

Figure 2. Left) Photographic capture of a hydraulic jump; Right) Photographic capture of obliquity

4 DISCUSSION AND CONCLUSION

From the theoretical study and experimental data obtained, it is concluded that under the suitable boundary and geometrical conditions, a compressible flow in a water table can be modelled by students on hydro-gasdynamic analogy.

The fact that an undular jump was visually detected and that the results were in agreement with the theoretical framework is outstanding. It adds even more value to this work the low cost of the constructed experimental set-up presented in this paper, being of 329 € (around 369 $), which is orders of magnitude less than a supersonic wind tunnel[6]. Thus, the set-up is easily reusable and inexpensive for fluid physics education.

Regarding the experiment itself, when the angle of the nozzle is higher, making its width smaller, the probability of what it could be called "stall" becomes higher, and this phenomenon can produce alterations in the flow path[10]. From the nozzle geometry designed in this experimental study, the theory confirms the presence of a "two-dimensional stall" for widths values lower than 0.05 m, which would justify the high error values. Hence, a plausible explanation is that hydraulic jump occurs for wide widths, whereas stall can appear as a restriction for narrow widths. Then, it is of interest for a future investigation to design a nozzle of higher experimental widths, to try to reduce this phenomenon and better visualise the hydraulic jump[11].

Finally, this study has found the cause-and-effect relationship between the results obtained and their meaning, which brings a sense of fulfilment. One of the major lessons learned is that despite the difficulties encountered and low budget invested, the presented work proves valuable. From the point of view of a fluid mechanics student, who has carried out the experiment, I believe that it is more illustrative and formative to physically see results with theoretical background rather than memorising random formulas. Therefore, it could be included in physics and engineering bachelors as a laboratory practice so that more students could gain the same enriching learning and engagement, and carry on with the proposals for future research in the field[12].

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