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GAUSS–MANIN CONNECTIONS FOR p-ADIC FAMILIES OF NEARLY OVERCONVERGENT MODULAR FORMS

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ABSTRACT. — We interpolate the Gauss–Manin connection in p-adic families of nearly overconvergent modular forms. This gives a family of Maass–Shimura type differential operators from the space of nearly overconvergent modular forms of type r to the space of nearly overconvergent modular forms of type $r + 1$ with p-adic weight shifted by 2. Our construction is purely geometric, using Andreatta–Iovita–Stevens and Pilloni's geometric construction of eigencurves, and should thus generalize to higher rank groups.

RÉSUMÉ. — Nous obtenons l'interpolation de la connexion de Gauss–Manin en familles p-adiques de formes modulaires quasi-surconvergentes. Ceci donne une famille d'opérateurs différentiels à la Maass–Shimura qui envoie l'espace de formes modulaires quasi-surconvergentes de type r dans celui de formes modulaires quasi-surconvergentes de type $r + 1$ et de poids p-adique augmenté par 2. Notre méthode est purement géométrique, elle utlise les constructions géométriques des courbes de Hecke dues à Andreatta–Iovita–Stevens et Pilloni, et devrait donc se généraliser aux groupes de rang supérieur.

1. Introduction

In this article, we seek to combine two important tools in arithmetic: the nearly holomorphic modular forms of Shimura and the p-adic families of modular forms of Hida and Coleman–Mazur. The former are an integral part of Shimura's study of the algebraicity of values of automorphic L-functions while the latter have become a ubiquitous tool in number theory with recent applications including the proof of Serre's conjecture, the proof of the Fontaine–Mazur–Langlands conjecture for $GL(2)$, and the construction of automorphic Galois representations to name a few. The p-adic

theory of nearly holomorphic modular forms has recently emerged from the work of Darmon–Rotger [5] and Skinner–Urban (see [10]) with applications to the Birch–Swinnerton-Dyer and Bloch–Kato conjectures for elliptic curves and to the theory of Stark–Heegner points. Since it is sufficient for their purposes, these authors rely on q-expansions in their definition of nearly overconvergent modular forms. Our approach, via the work of Andreatta–Iovita–Stevens [1] and Pilloni [8], is geometric: we give a geometric definition of nearly overconvergent modular forms of arbitrary p-adic weight and construct p-adic families of Gauss–Manin connections varying over the weight space. This provides a robust theory that is amenable to generalization.

In order to state our main theorem, let us introduce the following notation: for an affinoid algebra A, a continuous character $\chi_A : \mathbb{Z}_p^\times \to A^\times$, a non-negative integer r, and a positive rational number v, let $M_r^{\dagger,v}(\chi_A)$ denote the Banach A-module of nearly overconvergent modular forms of type r, weight χ_A, and radius of convergence v. Our main result is then the following.

THEOREM. — *For sufficiently small v, we have an A-linear map*

$$\nabla^{\chi_A,r} : M_r^{\dagger,v}(\chi_A) \to M_{r+1}^{\dagger,v}(\chi_A\chi_{\mathrm{cycl}}^2),$$

such that for each point $x \in \mathrm{Spm}(A)$ that corresponds to a classical weight, the specialization of $\nabla^{\chi_A,r}$ to x restricts to the classical Gauss–Manin connection.

This theorem is also proved in [9] using a subtle argument heavily dependent on the q-expansion principle. Our construction (which was worked out independently) is purely geometric and hence can be easily generalized to similar situations whenever there is a geometric construction like the ones given by Andreatta–Iovita–Stevens [1] and Pilloni [8].

We now briefly sketch our construction of $\nabla^{\chi_A,r}$. Let X be the modular curve over $\mathbb{Q}_p$, with cuspidal subscheme C, and let E be the universal generalized elliptic curve over X. Let $\omega \subseteq \mathcal{H}$ be the sheaf of relative 1-differentials and the relative de Rham cohomology sheaf of $E \to X$, respectively. For integers $0 \leqslant r < k$, the Gauss–Manin connection gives rise to a connection $\nabla^{k,r} : \omega^{k-r} \otimes \mathrm{Sym}^r \mathcal{H} \to \omega^{k+1-r} \otimes \mathrm{Sym}^{r+1} \mathcal{H}$. We choose a splitting of the Hodge filtration $\mathcal{H} = \omega \oplus \omega^{-1}$. Then, the Gauss–Manin connection $\nabla : \mathcal{H} \to \mathcal{H} \otimes \Omega_X^1(\log C)$ can be reconstructed from a differential operator $\partial : \omega \to \omega \otimes \Omega_X^1(\log C)$. We interpret ∂ as a connection on the associated principal $\mathbb{G}_m$-bundle for ω, which we further restrict to a connection on the principal bundle for the rigid balls inside $\mathbb{G}_m$ appearing in

the construction of Andreatta–Iovita–Stevens [1] and Pilloni [8]. Then, we obtain our family version of ∂ by applying the formal dictionary between principal bundles and vector bundles associated to the representation of the monodromy group. The family of Gauss–Manin connections can be then reconstructed from the family version of ∂ as in Katz's paper [7].

One of the applications of our result is the construction of the p-adic Rankin–Selberg convolution over the product of two Coleman–Mazur eigencurves, which was in fact the original motivation of this paper. We now refer to [9] for this construction.

We also note that our method of construction should be quite general. For instance, we expect a similar construction for the case of Siegel modular forms making use of the work of Andreatta–Iovita–Pilloni [2] (or its Hilbert–Siegel and PEL variants).

Acknowledgments

2. A geometric construction of nearly overconvergent modular forms

We first review the geometric construction of families of nearly overconvergent modular forms. This approach essentially follows from the work of Andreatta, Iovita, and Stevens [1] and, independently, Pilloni [8]. The advantage of this construction is that nearly overconvergent modular forms are on the nose sections of certain vector bundles.

2.1. Weight spaces

We fix a prime number p. Let $\mathbb{C}_p$ denote the completion of a fixed algebraic closure of $\mathbb{Q}_p$. It is equipped with a valuation $\mathtt{v} : \mathbb{C}_p^\times \to \mathbb{R}$ normalized so that $\mathtt{v}(p) = 1$. Let $|\cdot| = p^{-\mathtt{v}(\cdot)}$. Put $q = 4$ if $p = 2$ and $q = p$ if $p \neq 2$.[1] We can then write $\mathbb{Z}_p^\times = (\mathbb{Z}/q\mathbb{Z})^\times \times (1 + q\mathbb{Z}_p)^\times$.

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[1] The confusion with $q = e^{2\pi i z}$ later should be minimal.

We put $\Gamma = \mathrm{Gal}(\mathbb{Q}(\mu_{p^\infty})/\mathbb{Q})$, where μ_{p^∞} denotes the collection of all p-power roots of unity. We use the *cyclotomic character* $\chi_{\mathrm{cycl}} : \Gamma \to \mathbb{Z}_p^\times$ to identify these two groups.

Let $\mathcal{W}$ denote the rigid analytic space over $\mathbb{Q}_p$ whose K-points are $\mathcal{W}(K) = \mathrm{Hom}_{\mathrm{cont}}(\Gamma, K^\times)$ for any complete field extension K of $\mathbb{Q}_p$. We have an isomorphism

$$
\mathcal{W} \xrightarrow{\ \cong\ } (\widehat{\mathbb{Z}/q\mathbb{Z}})^\times \times B(1, 1^-)
$$
$$
\chi \longmapsto \big(\ \chi|_{(\mathbb{Z}/q\mathbb{Z})^\times},\ \chi(\exp(q))\ \big),
$$

where $B(1, 1^-)$ denotes the open unit ball of radius 1 centered at 1.

A *classical character* of Γ is a character of the form $\epsilon\chi_{\mathrm{cycl}}^k : \Gamma \to K^\times$ with $k \geqslant 2$ an integer and ϵ a *finite order* continuous character of Γ. We will use (ϵ, k) to denote such a character and also the associated point on $\mathcal{W}$. The *p-conductor* of such ϵ is p^n, where $n = n(\epsilon)$ is the maximal *positive integer* such that ϵ factors through $(\mathbb{Z}/p^{n-1}q\mathbb{Z})^\times$. (In particular, if ϵ is trivial, our convention says that $n(\epsilon) = 1$.) We remark that the set of classical characters is Zariski dense in the weight space $\mathcal{W}$.

Let A be an affinoid algebra over $\mathbb{Q}_p$. (A good example to keep in mind is the ring of analytic functions on an affinoid subdomain of the weight space $\mathcal{W}$.) For a continuous character $\chi_A : \mathbb{Z}_p^\times \to A^\times$, its *weight* is defined to be

$$
\mathrm{WT}(\chi_A) := \lim_{a \to 1} \frac{\log(\chi_A(a))}{\log a} \in A.
$$

We sometimes view it as a function on $\mathrm{Spm}(A)$. When $\chi_A = (\epsilon, k)$ is a classical character, its weight is simply k.

2.2. Nearly algebraic modular forms

Let $X \to \mathrm{Spec}\,\mathbb{Z}_p$ denote the compactified modular curve with hyperspecial level at p over $\mathrm{Spec}\,\mathbb{Z}_p$. We always take the tame level subgroup to be sufficiently small so that X is a fine moduli space of generalized elliptic curves. We fix such a tame level throughout the paper. Let C denote the cusp subscheme. Let $\pi : E \to X$ be the universal generalized elliptic curve. Put $\tilde{C} = \pi^{-1}(C)$; it is a divisor of E with simple normal crossings. Let $e : X \to E$ be the unit section and put $\omega := e^*\Omega^1_{E/X}$. The Kodaira–Spencer isomorphism gives

$$
\Omega^1 := \Omega^1_{X/\mathbb{Z}_p}(\log C) \cong \omega^{\otimes 2}.
$$

We use $\mathcal{H}$ to denote the relative de Rham cohomology $\mathrm{R}^1\pi_*(\Omega^\bullet_{E/X}(\log \tilde{C}))$.[2]

For integers $k \geqslant 2$ and $r \in [0, \frac{k}{2} - 1]$, following Urban [9], we define the *nearly algebraic modular forms* of weight k and type r to be

$$M_{k,r} := H^0(X_{\mathbb{Q}_p}, \omega^{k-r} \otimes \mathrm{Sym}^r \mathcal{H}).$$

When $r = 0$, this recovers the space of usual modular forms M_k, which we call *classical modular forms* in this paper. These nearly algebraic modular forms are closely related to the nearly holomorphic modular forms of Shimura. We refer to [9] for a discussion of the relation between the two.

2.3. The ball fibration

We now review a construction given in [1] and [8]. We put $T = \mathrm{Spec}(\oplus_{n\in\mathbb{Z}_{\geqslant 0}}\omega^{-n})$ and $T^\times = \mathrm{Spec}(\oplus_{n\in\mathbb{Z}}\omega^n)$. The space T may be viewed as the physical line bundle over X associated to ω and $T^\times$ as the $\mathbb{G}_m$-torsor that defines the line bundle T. Let X_{rig} denote the analytification of $X_{\mathbb{Q}_p}$ (as a rigid analytic space in the sense of Tate). Let T_{rig} be the analytification of $T_{\mathbb{Q}_p}$. Let $T^\times_{\mathrm{rig}}$ be the analytification of $T^\times_{\mathbb{Q}_p}$; it is a $\mathbb{G}_{m,\mathrm{rig}}$-torsor over X_{rig}.

There is a continuous map $\deg : X_{\mathrm{rig}} \to [0,1]$, given by the valuation of the truncated Hasse invariant. More precisely, we fix a lift $\tilde{h} \in H^0(X, \omega^{p-1})$ of the Hasse invariant; for $x \in X_{\mathrm{rig}}$, $\deg(x) = \min\{\mathrm{v}(\tilde{h}(x)), 1\}$.[3] For any $v \in [0,1] \cap p^\mathbb{Q}$, $X(v) := \{x \in X_{\mathrm{rig}} \mid \deg(x) \leqslant v\}$ is a subdomain of X_{rig}. In particular, $X(0)$ is the tube of the ordinary locus of the special fiber of X.

For $n \in \mathbb{Z}_{\geqslant 1}$ and $v \in [0, \frac{1}{p^{n-2}(p+1)})$, the universal generalized elliptic curve E over $X(v)$ has a canonical subgroup C_n of order p^n. For each closed point $x \in X(v)(K)$, we use $C_{n,x}$ to denote the corresponding canonical subgroup of the (generalized) elliptic curve E_x at the point x. We may choose formal models $\mathfrak{C}_{n,x}$ and $\mathfrak{E}_x$ of both objects over $\mathcal{O}_K$. We have a Hodge–Tate map

$$(2.3.1) \qquad \mathrm{HT} : \mathfrak{C}^D_{n,x}(\mathcal{O}_{\mathbb{C}_p}) \to e^*(\Omega^1_{\mathfrak{C}_{n,x}/\mathcal{O}_K}) \otimes_{\mathcal{O}_K} \mathcal{O}_{\mathbb{C}_p},$$

where $\mathfrak{C}^D_{n,x}$ denotes the Cartier dual of $\mathfrak{C}_{n,x}$.

Using the Hodge–Tate map,[4] Pilloni [8, Théorème 3.2] showed that there exists an open rigid subdomain $F_n \hookrightarrow T_{\mathrm{rig}} \times_{X(v)} C^D_n$ such that, for

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[2] Here, $\Omega^1_{E/X}(\log \tilde{C}) = \Omega^1_{E/\mathbb{Z}_p}(\log \tilde{C})/\pi^*\Omega^1_{X/\mathbb{Z}_p}(\log C)$.

[3] The notation deg refers to the degree of the corresponding finite flat p-group scheme in the sense of [6].

[4] Technically speaking, just knowing the description of Hodge–Tate map over closed points is not enough to prove the existence of F_n. Pilloni [8] needed a version of the Hodge–Tate map over a formal scheme, just as in [1]. However, this subtlety does not matter to our discussion, so we ignore it.

any closed point $x \in X(v)(K)$, we have

$$
\begin{aligned}
F_n|_x(\mathbb{C}_p) = \big\{ (y,\omega) \in \mathfrak{C}_{n,x}^D(\mathbb{C}_p) \times \big(e^* \Omega_{\mathfrak{E}_x/\mathcal{O}_K}^1 \otimes_{\mathcal{O}_K} \mathcal{O}_{\mathbb{C}_p} \big) \\
\text{with } \mathrm{HT}(y) = \omega|_{e^* \Omega_{\mathfrak{C}_{n,x}/\mathcal{O}_K}^1 \otimes_{\mathcal{O}_K} \mathcal{O}_{\mathbb{C}_p}} \big\}
\end{aligned}
$$

where $F_n|_x$ denotes the fiber of F_n above $x \in X(v)(K)$.

Let $(C_n^D)^\times$ denote the union of the connected components of C_n^D formed by local generators of C_n^D. We put $F_n^\times = F_n \times_{C_n^D} (C_n^D)^\times$. When $v < \frac{p-1}{p^n}$, Pilloni [8, Proposition 3.5] proved that the natural morphism $F_n^\times \to F_n \to T_{\mathrm{rig}}$ factors through $T_{\mathrm{rig}}^\times$ and is an open immersion. In a more explicit form, over $x \in X(v)(\mathbb{C}_p)$, we may choose a generator of ω over X and hence identify T_{rig} with $\mathbb{A}^1_{\mathbb{C}_p,\mathrm{rig}}$. Then

$$
F_n^\times|_x(\mathbb{C}_p) = \coprod_{m \in (\mathbb{Z}/p^n\mathbb{Z})^\times} x_m h^{1/(p-1)} + p^n h^{-\frac{p^n-1}{p-1}} \mathcal{O}_{\mathbb{C}_p} \subset \mathbb{C}_p,
$$

where $h \in \mathbb{C}_p$ is some element with $\mathrm{v}(h) = \deg(x)$ and $\{ x_m \mid m \in (\mathbb{Z}/p^n\mathbb{Z})^\times \}$ is some fixed set of lifts of $(\mathbb{Z}/p^n\mathbb{Z})^\times$ to $\mathbb{Z}_p^\times$.

From this, it is easy to see (and is also explained in [8, §3.4] and [1, §3.2] implicitly) that $\mathrm{pr}_n : F_n^\times \to X(v)$ is a torsor for the open subgroup $G_n \subseteq X(v) \times \mathbb{G}_{m,\mathrm{rig}}$ given by

$$
(2.3.2)
$$
$$
G_n(\mathbb{C}_p) = \big\{ (x,a) \in X(v)(\mathbb{C}_p) \times \mathbb{C}_p^\times \mid |a - m| \leqslant p^{-n} p^{p^n \deg(x)/(p-1)}
$$
$$
\text{for some } m \in \mathbb{Z}_p^\times \big\}.
$$

Remark 2.4. — The following viewpoint of the above construction was communicated to us by Emerton. The line bundle ω defines a $\mathbb{G}_m$-torsor over (the entirety of) X; this is why we can consider integer powers of ω and define modular forms with integer weights. The dual Tate module of the p^∞-canonical subgroup of E only exists over the ordinary locus $X(0)$; it gives rise to a $\mathbb{Z}_p^\times$-torsor over $X(0)$. This is why Hida can consider p-adic families of modular forms parameterized by $\mathbb{Z}_p[\![\mathbb{Z}_p^\times]\!]$. Over the ordinary locus, the Hodge–Tate map is an isomorphism. It identifies the aforementioned dual Tate module as a $\mathbb{Z}_p^\times$-subtorsor of the $\mathbb{G}_m^\times$-torsor $T^\times$. As we include some supersingular locus, i.e. we work over $X(v)$, the Hodge–Tate map fails to be an isomorphism. Instead, we see a torsor $F_n^\times$ for the group G_n, which is a group sitting between $\mathbb{Z}_p^\times$ and $\mathbb{G}_{m,\mathrm{rig}}$.

2.5. Locally analytic characters

Let A be an affinoid algebra and $\chi_A : \mathbb{Z}_p^\times \to A^\times$ a continuous character. Then there exists an integer $n \geqslant 3$ such that $\lambda_{A,n-1} := \chi_A(\exp(p^{n-1})) \in A$ satisfies $|\lambda_{A,n-1} - 1| < p^{-1/(p-1)}$.[5] For any such n, the character χ_A extends by continuity to a *locally analytic character*

$$\chi_A^{\mathrm{an}} = (\chi_A, \lambda_A): \qquad \mathbb{Z}_p^\times \cdot (1 + p^{n-1}\mathcal{O}_{\mathbb{C}_p})^\times \longrightarrow (A\widehat{\otimes}\mathbb{C}_p)^\times$$

$$x \cdot z \longmapsto \chi_A(x) \cdot \lambda_{A,n-1}^{\log(z)/p^{n-1}},$$

where $x \in \mathbb{Z}_p^\times$ and $z \in (1 + p^{n-1}\mathcal{O}_{\mathbb{C}_p})^\times$. In other words, this defines a family of representations of the rigid analytic group

$$\overline{G}_{n-1} = \coprod_{m \in (\mathbb{Z}/p^{n-1}\mathbb{Z})^\times} B(x_m, p^{1-n})$$

parametrized by $\mathrm{Spm}(A)$, where $B(x_m, p^{1-n})$ denotes the union of the closed disks of radius p^{1-n} centered at the chosen representatives $x_m \in \mathbb{Z}_p^\times$ as m varies over all classes in $(\mathbb{Z}/p^{n-1}\mathbb{Z})^\times$.

2.6. Overconvergent sheaves

Keep the notation as above. Choose $v \in (0, \frac{p-1}{p^n}) \cap \mathbb{Q}$. The representation χ_A^{an} of $\overline{G}_{n-1}$ induces a line bundle $\mathcal{V}$ over $X(v) \times \mathrm{Spm}(A)$ on which $\overline{G}_{n-1} \times X(v)$ acts. Our assumption on v ensures that G_n is a subgroup of $\overline{G}_{n-1} \times X(v)$ by the description (2.3.2). We may thus consider the induced action of G_n on $\mathcal{V}$.

Let $\mathrm{pr}_n \times \mathrm{id} : F_n^\times \times \mathrm{Spm}\, A \to X(v) \times \mathrm{Spm}\, A$ denote the map induced by pr_n. Following [8, §5.1], we define the *modular sheaf* with character χ_A to be

$$\omega^{\chi_A} := \left((\mathrm{pr}_n \times \mathrm{id})_* \mathcal{O}_{F_n^\times \times \mathrm{Spm}\, A} \otimes_{\mathcal{O}_{X(v) \times \mathrm{Spm}\, A}} \mathcal{V}\right)^{G_n}.$$

Since $F_n^\times$ is a G_n-torsor, ω^{χ_A} is a locally free sheaf over $X(v) \times \mathrm{Spm}(A)$ of rank one.

For any such χ_A, we define an *overconvergent modular form with character* χ_A to be a section of ω^{χ_A} over $X(v) \times \mathrm{Spm}(A)$ for some v as above. We put

$$M^{\dagger,v}(\chi_A) = \Gamma\big(X(v) \times \mathrm{Spm}(A), \omega^{\chi_A}\big) \quad \text{and} \quad M^\dagger(\chi_A) = \varinjlim_v M^{\dagger,v}(\chi_A).$$

---

[5] Here, we did not optimize the choice of n for simplicity of the presentation; see [8, Section 2.1] for a careful discussion of the optimal bound.

More generally, for $r \in \mathbb{Z}_{\geqslant 0}$, we define the space of *nearly overconvergent modular forms* to be

$$M_r^{\dagger, v}(\chi_A) = \Gamma\big(X(v) \times \mathrm{Spm}(A), \boldsymbol{\omega}^{\chi_A \chi_{\mathrm{cycl}}^{-r}} \otimes \mathrm{Sym}^r \mathcal{H}\big)$$

and

$$M_r^{\dagger}(\chi_A) = \varinjlim_v M_r^{\dagger, v}(\chi_A).$$

We have natural inclusions $M_{r'}^{\dagger, (,v)}(\chi_A) \hookrightarrow M_r^{\dagger, (,v)}(\chi_A)$ if $r' \leqslant r$, and $M_0^{\dagger, (,v)}(\chi_A) = M^{\dagger, (,v)}(\chi_A)$.

When $\chi_A = \chi_{\mathrm{cycl}}^k : \mathbb{Z}_p^\times \to K^\times$ is the k-th power of the cyclotomic character, $\boldsymbol{\omega}^{\chi_{\mathrm{cycl}}^k}$ is the restriction of the usual modular sheaf $\omega^k|_{X(v)}$ for $v \in (0,1)$ sufficiently close to 0 ([8, Proposition 3.6]). In this case, we put

$$M_k^{\dagger, v} = M^{\dagger, v}(\chi_A) \quad \text{and} \quad M_k^{\dagger} = M^{\dagger}(\chi_A).$$

These are the *overconvergent modular forms of weight k* in the usual sense; they contain the classical modular forms M_k as a subspace.

Similarly, for an integer $r \geqslant 0$, we define the space of *nearly overconvergent modular forms* of weight k to be

$$M_{k,r}^{\dagger, v} = M_r^{\dagger, v}(\chi_A) \quad \text{and} \quad M_{k,r}^{\dagger} = M_r^{\dagger}(\chi_A).$$

These contain the space of nearly algebraic modular forms $M_{k,r}$ as a subspace when $k \geqslant 2r + 2$.

It is clear from the construction that, when $\chi = (\epsilon, k)$ is a classical character, our construction agrees with that of [9].

Remark 2.7. — One can define Hecke actions on nearly overconvergent modular forms as in [8, §4]. We refer to *loc. cit.* for details.

3. The Gauss–Manin connection in a p-adic family

We now give the construction of the Gauss–Manin connection in a p-adic family over the weight space.

3.1. Gauss–Manin connections

The relative de Rham cohomology over the modular curve X fits into a canonical exact sequence

(3.1.1) $$0 \to \omega \to \mathcal{H} \to \omega^{-1} \to 0.$$

Equivalently, $\mathcal{H}$ admits a Hodge filtration given as follows: $\mathrm{Fil}^i\mathcal{H} = 0$ for $i > 1$; $\mathrm{Fil}^1\mathcal{H} = \omega$; and $\mathrm{Fil}^i\mathcal{H} = \mathcal{H}$ for $i \leqslant 0$. The filtration naturally induces a filtration on the symmetric power $\mathrm{Sym}^k\mathcal{H}$ for $k \geqslant 1$. In particular, for an integer $r \in [0, k]$, $\mathrm{Fil}^{k-r}\mathrm{Sym}^k\mathcal{H} \cong \omega^{k-r} \otimes \mathrm{Sym}^r\mathcal{H}$.

Recall that the relative de Rham cohomology $\mathcal{H}$ admits a Gauss–Manin connection $\nabla : \mathcal{H} \to \mathcal{H} \otimes \Omega^1$, or more generally $\nabla^k : \mathrm{Sym}^k\mathcal{H} \to \mathrm{Sym}^k\mathcal{H} \otimes \Omega^1$. For an integer $r \in [0, k]$, Griffiths transversality implies that the Gauss–Manin connection on $\mathrm{Sym}^k\mathcal{H}$ induces a differential map

$$\nabla^{k,r} : \omega^{k-r} \otimes \mathrm{Sym}^r\mathcal{H} \cong \mathrm{Fil}^{k-r}\mathrm{Sym}^k\mathcal{H}$$

$$\xrightarrow{\nabla^k} \mathrm{Fil}^{k-r-1}\mathrm{Sym}^k\mathcal{H} \otimes \Omega^1 \cong \omega^{k-r+1} \otimes \mathrm{Sym}^{r+1}\mathcal{H}.$$

When $k \geqslant 2r + 2$, taking global sections over $X_{\mathbb{Q}_p}$ gives rise to a differential operator

$$\nabla^{k,r} : M_{k,r} \to M_{k+2,r+1}.$$

We call it *the classical Gauss–Manin connection*.

3.2. Splitting of the Hodge filtration

Our goal is to consider the variation of $\nabla^{k,r}$ as k varies p-adically with r fixed. For this, we first choose a splitting of the Hodge filtration (3.1.1). We will then show that two different such choices result in equivalent p-adic interpolations.[6]

A *splitting* of the Hodge filtration is a homomorphism of coherent sheaves $\eta : \mathcal{H} \to \omega$ which is a left inverse of the natural embedding. This is equivalent to giving an embedding $\omega^{-1} \cong \mathcal{H}/\omega \to \mathcal{H}$ that is a right inverse of the natural projection. Given a splitting η, one may write $\mathcal{H} = \omega \oplus \omega^{-1}$. Then, the Gauss–Manin connection on $\mathcal{H}$ can be written as

$$\begin{array}{rccc} \nabla & : & \mathcal{H} = \omega \oplus \omega^{-1} & \longrightarrow & \mathcal{H} \otimes \Omega^1 \cong \omega^3 \oplus \omega \\ & & (x, y) & \longmapsto & (\nabla_1(x) + \nabla_2(y), \nabla_3(x) + \nabla_4(y)). \end{array}$$

We now analyse each component ∇_i of ∇.

- We put $\partial = \nabla_1$, it is a connection on ω given by

$$\partial : \omega \hookrightarrow \mathcal{H} \xrightarrow{\nabla} \mathcal{H} \otimes \Omega^1 \xrightarrow{\eta \otimes \mathrm{id}} \omega \otimes \Omega^1.$$

---

[6] In fact, to p-adically interpolate Gauss–Manin connections, we need only choose the splitting of the Hodge filtration Zariski locally on X. We choose a global splitting here to simplify the presentation. However, it would be certainly interesting to know if one can entirely avoid the splitting of Hodge filtration in the following construction.

For $k \in \mathbb{Z}$, let $\partial^{\otimes k}$ denote the induced connection on the k-th power ω^k. (Negative powers are understood as duals.) Alternatively, using the Kodaira–Spencer isomorphism, we may view $\partial^{\otimes k}$ as a first-order differential operator $\omega^k \to \omega^{k+2}$. Our first step later on will be to p-adically interpolate $\partial^{\otimes k}$.

- For ∇_2, we observe that the first term of the equality $y \otimes \nabla(a) + a\nabla(y) = \nabla(ay)$ lies in $\omega \cong \omega^{-1} \otimes \Omega^1 \subseteq \mathcal{H} \otimes \Omega^1$. Hence, ∇_2 is in fact $\mathcal{O}_X$-linear and so is given by multiplication by a section λ of ω^4 over X.

- For ∇_3, we note that the identification between Ω^1 with ω^2 is given by the Kodaira–Spencer isomorphism. Thus, ∇_3 is simply the identity map by definition.

- For ∇_4, we observe that $\wedge^2\mathcal{H}$ is the trivial line bundle with the trivial connection. This implies that ∇_4 is the connection $\partial^{\otimes(-1)}$ on ω^{-1}.

Example 3.3. — An example of a splitting of the Hodge filtration was given by Katz [7, A1.2]; he constructed a so-called "canonical splitting" using the equation for the universal elliptic curve.

For $k \in \mathbb{Z}_{\geqslant 1}$, we put $\sigma_k(n) = \sum_{d|n} d^k$. Using this splitting, the action of the differential operator on a form f of weight k is given in terms of q-expansions by

$$\partial(f) = q\frac{df}{dq} + \frac{kE_2f}{12} \quad \text{and} \quad \lambda = E_4 = 1 + 120\sum_{n\in\mathbb{N}} \sigma_3(n)q^n.$$

where $E_2 = 1 - 24\sum_{n\in\mathbb{Z}_{\geqslant 1}} \sigma_1(n)q^n$ and $q = e^{2\pi i z}$. Note that E_2 is a p-adic modular form, but not an overconvergent one [4]. The modular form E_4 is an Eisenstein series.

LEMMA 3.4. — *Let η' be another splitting of the Hodge filtration. Then $\eta' - \eta$ induces a natural homomorphism of coherent sheaves $\mathcal{H}/\omega \cong \omega^{-1} \to \omega$, which is given by multiplication by some section α of ω^2. Let ∂' and λ' be the associated differential operator and section of ω^4 defined as above with respect to the splitting η'. Then, we have*

$$\partial' = \partial + \alpha \quad \text{and} \quad \lambda' = \lambda - \alpha^2 - \partial^{\otimes 2}(\alpha).$$

Proof. — Looking at the change of basis matrix, we have

$$\begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \partial & \lambda \\ 1 & \partial^{\otimes(-1)} \end{pmatrix} \begin{pmatrix} 1 & -\alpha \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \partial' & \lambda' \\ 1 & \partial'^{\otimes(-1)} \end{pmatrix}.$$

From this, it is clear that $\partial' = \partial + \alpha$. Multiplying out the matrix product, we have

$$\lambda' = \lambda + \alpha\partial^{\otimes(-1)} + (\partial + \alpha) \circ (-\alpha) = \lambda - \alpha^2 + \alpha\partial^{\otimes(-1)} - \partial \circ \alpha.$$

Noting that $\partial \circ \alpha = \partial^{\otimes 2}(\alpha) + \alpha\partial^{\otimes(-1)}$, the second formula of the lemma then follows. □

The following lemma is essentially [7, A1.4]; it is a simple computation of tensors.

LEMMA 3.5 (Katz). — *For integers $k \geqslant 1$ and $r \in [0, k]$, the chosen splitting η induces an isomorphism $\omega^{k-r} \otimes \mathrm{Sym}^r \mathcal{H} \cong \oplus_{a=0}^r \omega^{k-2a}$. Under this identification, the Gauss–Manin connection $\omega^{k-r} \otimes \mathrm{Sym}^r \mathcal{H} \to \omega^{k-r+1} \otimes \mathrm{Sym}^{r+1} \mathcal{H}$ is given by the sum over the maps*

$$\nabla^{k,r} : \quad \omega^{k-2a} \longrightarrow \omega^{k-2a} \quad \bigoplus \quad \omega^{k-2a+2} \bigoplus \omega^{k-2a+4}$$
$$f \longmapsto \big((k-a)f, \quad \partial^{\otimes(k-2a)}(f), \quad a\lambda f \big).$$

From this lemma, we see that the p-adic interpolation of the Gauss–Manin connection will essentially follow from the p-adic interpolation of the connections on "p-adic powers" of ω.

3.6. Connection on $M^\dagger(\chi_A)$

We will use the interplay between connections on vector bundles and connections on principal bundles to construct the p-adic interpolation of the Gauss–Manin connection.

Let $\pi_{T^\times} : T^\times \to X$ denote the natural projection. Recall that, after choosing a splitting of the Hodge filtration, we have a connection $\partial : \omega \to \omega \otimes_{\mathcal{O}_X} \Omega^1$. This naturally induces a connection

$$(3.6.1) \qquad \tilde{\partial} : \quad \pi_{T^\times,*}\mathcal{O}_{T^\times} - - \succ \big(\pi_{T^\times,*}\mathcal{O}_{T^\times}\big) \otimes_{\mathcal{O}_X} \Omega^1$$
$$\quad \| \qquad\qquad\qquad\qquad \|$$
$$\bigoplus_{k\in\mathbb{Z}} \partial^{\otimes k} : \quad \bigoplus_{k\in\mathbb{Z}} \omega^k \longrightarrow \big(\bigoplus_{k\in\mathbb{Z}} \omega^k\big) \otimes_{\mathcal{O}_X} \Omega^1.$$

This may be viewed as a connection on the $\mathbb{G}_m$-torsor $T^\times$ over X. Hence the connection is equivalent to a $\mathbb{G}_m \times X$-equivariant $\mathcal{O}_{T^\times}$-linear map

$$(3.6.2) \qquad \partial_{T^\times} : \Omega^1_{T^\times/\mathbb{Z}_p}(\log \pi_{T^\times}^{-1}(C)) \longrightarrow \pi_{T^\times}^*\Omega^1$$

which splits the natural map $\pi_{T^\times}^*\Omega^1 \hookrightarrow \Omega^1_{T^\times/\mathbb{Z}_p}(\log \pi_{T^\times}^{-1}(C))$.

We may first take the analytification of $T_{\mathbb{Q}_p}^\times$ as well as the map (3.6.2) (tensored with $\mathbb{Q}_p$), and then restrict the map to $F_n^\times$. We thus obtain an $\mathcal{O}_{G_n}$-equivariant (because G_n is a subgroup of $\mathbb{G}_{m,\mathrm{rig}} \times X(v)$) $\mathcal{O}_{F_n^\times}$-linear map

$$\partial_{F_n^\times} : \ \Omega^1_{F_n^\times/\mathbb{Q}_p}(\log \mathrm{pr}_n^{-1}(C)) \to \mathrm{pr}_n^* \Omega^1_{X(v)/\mathbb{Q}_p}(\log C)$$

which splits the natural map $\mathrm{pr}_n^* \Omega^1_{X(v)/\mathbb{Q}_p}(\log C) \hookrightarrow \Omega^1_{F_n^\times/\mathbb{Q}_p}(\log \mathrm{pr}_n^{-1}(C))$. This gives the connection on the $G_n \times X(v)$-torsor $F_n^\times$ over $X(v)$. In particular, pre-composing $\partial_{F_n^\times}$ with the natural map $\mathcal{O}_{F_n^\times} \xrightarrow{d} \Omega^1_{F_n^\times/\mathbb{Q}_p}(\log \mathrm{pr}_n^{-1}(C))$ and then pushing forward along pr_n induces a natural $\mathcal{O}_{G_n}$-equivariant map

$$(3.6.3) \qquad \tilde{\partial}_{F_n^\times} : \ \mathrm{pr}_{n,*}\mathcal{O}_{F_n^\times} \longrightarrow \mathrm{pr}_{n,*}\mathcal{O}_{F_n^\times} \otimes_{\mathcal{O}_{X(v)}} \Omega^1_{X(v)/\mathbb{Q}_p}(\log C).$$

Now, let A be an affinoid algebra and $\chi_A : \mathbb{Z}_p^\times \to A^\times$ a locally analytic character as in § 2.5. We may tensor (3.6.3) with $\mathcal{V}$ and then take the G_n-invariant sections. This gives a natural connection

$$\partial^{\chi_A} : \boldsymbol{\omega}^{\chi_A} = \left((\mathrm{pr}_n \times \mathrm{id})_* \mathcal{O}_{F_n^\times \times \mathrm{Spm}\, A} \otimes_{\mathcal{O}_{X(v) \times \mathrm{Spm}\, A}} \mathcal{V}\right)^{G_n}$$

$$\xrightarrow{\tilde{\partial}_{F_n^\times}} \left((\mathrm{pr}_n \times \mathrm{id})_* \mathcal{O}_{F_n^\times \times \mathrm{Spm}\, A} \otimes_{\mathcal{O}_{X(v)}} \Omega^1_{X(v)/\mathbb{Q}_p}(\log C) \otimes_{\mathcal{O}_{X(v) \times \mathrm{Spm}\, A}} \mathcal{V}\right)^{G_n}$$

$$= \boldsymbol{\omega}^{\chi_A} \otimes_{\mathcal{O}_{X(v)}} \Omega^1_{X(v)/\mathbb{Q}_p}(\log C).$$

Taking global sections over $X(v) \times \mathrm{Spm}(A)$ induces a natural map

$$\partial^{\chi_A} : M^{\dagger,v}(\chi_A) \to M^{\dagger,v}(\chi_A) \otimes_{\Gamma(X(v),\mathcal{O}_{X(v)})} \Gamma(X(v), \Omega^1_{X(v)/\mathbb{Q}_p}(\log C))$$

$$\cong M^{\dagger,v}(\chi_A \chi_{\mathrm{cycl}}^2),$$

and $\partial^{\chi_A} : M^\dagger(\chi_A) \to M^\dagger(\chi_A \chi_{\mathrm{cycl}}^2)$.

When the character χ_A is χ_{cycl}^k, it is easy to see that $\partial^{\chi_{\mathrm{cycl}}^k}$ is the restriction of $\partial^{\otimes k}$ on ω^k to $X(v)$.

LEMMA 3.7. — *If we change the splitting of Hodge filtration as in Lemma 3.4, then the corresponding family of connections ∂^{χ_A} and ∂'^{χ_A} are related by*

$$\partial'^{\chi_A} = \partial^{\chi_A} + \mathrm{WT}(\chi_A) \cdot \alpha.$$

Proof. — This can be proved by specializing to classical characters as in Proposition 3.9 below. But we prefer a down-to-earth proof which we hope will give the reader a better intuition for the construction.

Let η' be another splitting of Hodge filtration and let α be as in Lemma 3.4. Let ∂', $\tilde{\partial}'$, $\partial'_{T^\times}$, $\partial'_{F_n^\times}$, and $\tilde{\partial}'_{F_n^\times}$ be the differential operators or connections constructed using η' in place of η. We analyse their difference with the original one.

By Lemma 3.4, $\partial' - \partial = \alpha$ can be viewed as a section of $\omega^2 \cong \Omega^1$. Then $\tilde{\partial}' - \tilde{\partial}$ is multiplication by $k\alpha$ on the direct summand ω^k. In other words, the action of $\tilde{\partial}' - \tilde{\partial}$ is given by the action of the Lie algebra of $\mathbb{G}_m$ multiplied with α. It is clear that this property continues to hold for the differences of the other differential operators. In particular, $\partial'^{\chi_A} - \partial^{\chi_A}$ is given by the action of the Lie algebra multiplied by α. This is the statement of the lemma. □

3.8. The family of Gauss–Manin connections on $M_r^\dagger(\chi_A)$

Keep χ_A and v as above. Recall that the character χ_A gives rise to a weight function $\mathrm{WT} = \mathrm{WT}(\chi_A)$ on $\mathrm{Spm}(A)$. Following Lemma 3.5, we define the *family of nearly overconvergent Gauss–Manin connections* to be

$$\nabla^{\chi_A,r} : M_r^{\dagger,v}(\chi_A) = \bigoplus_{a=0}^{r} M^{\dagger,v}(\chi_A \chi_{\mathrm{cycl}}^{-2a})$$

$$\longrightarrow M_{r+1}^{\dagger,v}(\chi_A \chi_{\mathrm{cycl}}^2) = \bigoplus_{a=0}^{r+1} M^{\dagger,v}(\chi_A \chi_{\mathrm{cycl}}^{2-2a})$$

given by sending $f \in M^{\dagger,v}(\chi_A \chi_{\mathrm{cycl}}^{-2a})$ to

$$(3.8.1) \quad (\,(\mathrm{WT} - a)f,\ \partial^{\chi_A \chi_{\mathrm{cycl}}^{-2a}}(f),\ a\lambda f\,)$$
$$\in M^{\dagger,v}(\chi_A \chi_{\mathrm{cycl}}^{-2a}) \oplus M^{\dagger,v}(\chi_A \chi_{\mathrm{cycl}}^{2-2a}) \oplus M^{\dagger,v}(\chi_A \chi_{\mathrm{cycl}}^{4-2a}).$$

Taking $v \to 0^+$, this defines $\nabla^{\chi_A,r} : M_r^\dagger(\chi_A) \to M_{r+1}^\dagger(\chi_A \chi_{\mathrm{cycl}}^2)$. When $\chi_A = \chi_{\mathrm{cycl}}^k$ is a classical character and $k \geqslant 2r+2$, the Gauss–Manin connection $M_{k,r}^\dagger \to M_{k+2,r+1}^\dagger$ is compatible with the algebraic Gauss–Manin connection $\nabla_{k,r} : M_{k,r} \to M_{k+2,r+1}$.

PROPOSITION 3.9. — *The family of nearly overconvergent Gauss–Manin connections $\nabla^{\chi_A,r}$ defined above does not depend on the choice of the splitting η of the Hodge filtration.*

Proof. — We can check this by hand using Lemmas 3.4 and 3.7 and the expression (3.8.1). This amounts to checking a matrix equality. We leave the details to the interested reader.

Alternatively, we can check the independence using an abstract argument as follows. Note that when $\chi_A = \chi_{\mathrm{cycl}}^k$ for an integer $k \geqslant 2r+2$, $\nabla^{\chi_A,r}$ is the same as the Gauss–Manin connection $\nabla^{k,r}$ and hence is independent of

the choice of splitting of the Hodge filtration. In general, using the functoriality of the construction, we may assume that $\mathrm{Spm}(A)$ is a geometrically connected subdomain of $\mathcal{W}$ containing infinitely many characters of the form above. Since $M_r^{\dagger,v}(\chi_A)$ is potentially orthogonalizable in the sense of Buzzard [3], the operator $\nabla^{\chi_A,r}$ is determined by its specializations to these classical characters. Hence $\nabla^{\chi_A,r}$ is independent of the choice of splitting of the Hodge filtration. $\square$

Remark 3.10. — It is clear from the construction that the family of nearly overconvergent Gauss–Manin connections $\nabla^{\chi_A,r}$ commutes with the action of Hecke operators.

3.11. q-expansions

Neither [1] nor [8] elaborated on the q-expansion attached to overconvergent modular forms in an explicit way. We include a short discussion for completeness.

Consider the Tate curve $E = \mathrm{Tate}(q)$ over $\mathbb{Z}_p((q))$; it comes equipped with a canonical differential $\omega_{\mathrm{can}} = \frac{dt}{t}$ and its d.s.k "dual" η_{can} as defined in [7, A1.3.14].

The Tate curve admits a canonical subgroup $C_n = \mu_{p^n} \subseteq E[p^n]$. Its dual is canonically isomorphic to $\mathbb{Z}/p^n\mathbb{Z}$ over $\mathbb{Z}_p[\![q]\!]$. The Hodge–Tate map for the Tate curve is

$$\mathrm{HT} : C_n^D \cong (\mathbb{Z}/p^n\mathbb{Z})_{\mathbb{Z}_p((q))}$$
$$\longrightarrow \Omega^1_{C_n/\mathbb{Z}_p((q))} \cong \Omega^1_{E/\mathbb{Z}_p((q))} \otimes_{\mathcal{O}_E} \mathcal{O}_{C_n} = \mathcal{O}_{C_n} \cdot \omega_{\mathrm{can}}.$$

It sends the canonical generator $1 \in \mathbb{Z}/p^n\mathbb{Z}$ to ω_{can}. Moreover, this isomorphism extends to an isomorphism over $\mathbb{Z}_p[\![q]\!]$. Let $R = \mathbb{Q}_p\langle p^{-1}q\rangle$ be the ring of analytic functions on the disk at a cusp of radius p^{-1};[7] so we view $\mathrm{Spm}(R)$ as an affinoid subdomain of $X(v)$ for any v. We put $z = \omega_{\mathrm{can}}$ so that $T^\times \times_X \mathrm{Spec}\, R = \mathrm{Spec}\, R[z, z^{-1}]$. The description of the Hodge–Tate map implies that

$$F_n^\times \times_{X(v)} \mathrm{Spm}\, R \cong \coprod_{m \in \mathbb{Z}/p^n\mathbb{Z}} B_R(x_m, p^{-n}),$$

where $B_R(x_m, p^{-n})$ denotes the relative disk over R around $z = x_m$ of radius p^{-n}.

---

[7] We could have worked with a larger disk or even with $\mathbb{Z}_p[\![q]\!][\frac{1}{p}]$, but the latter does not fit into the language of rigid analytic space.

Now, let $\chi_A : \mathbb{Z}_p^\times \to A^\times$ be a continuous character. Then for some $n \geqslant 3$, we have $|\chi_A(\exp(p^{n-1})) - 1| < p^{-1/(p-1)}$. We put $\lambda = \chi_A(\exp(p^n)) \in A^\times$. It follows that we can construct the overconvergent sheaf using this n. Let $G_{n,R}^\circ$ denote the identity component of $G_{n,R} := G_n \times_{X(v)} \mathrm{Spm}(R)$ and let $F_{n,R}^{\times,\circ}$ be the connected component of $F_{n,R}^\times := F_n^\times \times_{X(v)} \mathrm{Spm}(R)$ that contains ω_{can}. We consider the following section of $\mathcal{V} \otimes_{\mathcal{O}_{X(v) \times \mathrm{Spm}\, A}} \mathcal{O}_{F_{n,R}^{\times,\circ}}$

$$z^{\log(\lambda)/p^n} = 1 + \sum_{i=1}^{\infty} (z-1)^i \otimes \binom{\log(\lambda)/p^n}{i}.$$

It is $G_{n,R}^\circ$-equivariant in the sense that, for $g \in 1 + p^n \mathbb{Z}_p[\![q]\!]$,

$$g(z^{\log(\lambda)/p^n}) = (g^{-1}z)^{\log(\lambda)/p^n} \chi(g) = z^{\log(\lambda)/p^n} \chi(g) g^{-\log(\lambda)/p^n} = z^{\log(\lambda)/p^n}.$$

Using the action of $G_{n,R}$, we can extend this section to a $G_{n,R}$-equivariant section of $\mathcal{V} \otimes_{X(v) \times \mathrm{Spm}(A)} \mathcal{O}_{F_{n,R}^\times}$. In other words, we have constructed an explicit section $\omega_{\mathrm{can}}^{\chi_A}$ of $\boldsymbol{\omega}^{\chi_A}$ over $\mathrm{Spm}(R \widehat{\otimes} A)$.

Thus, for any nearly overconvergent modular form $f \in M_r^{\dagger,v}(\chi_A)$, we may evaluate f on this Tate curve and write

$$f = \mathbf{f}_0(q)\omega_{\mathrm{can}}^{\chi_A}\omega_{\mathrm{can}}^r + \mathbf{f}_1(q)\omega_{\mathrm{can}}^{\chi_A}\omega_{\mathrm{can}}^{r-1}\eta_{\mathrm{can}} + \cdots + \mathbf{f}_r(q)\omega_{\mathrm{can}}^{\chi_A}\eta_{\mathrm{can}}^r$$

for some $\mathbf{f}_0, \ldots, \mathbf{f}_r \in A[\![q]\!]$. This gives a natural morphism

(3.11.1) $$M_r^{\dagger,v}(\chi_A) \longrightarrow A[\![q]\!][Y]$$
$$f \longmapsto \mathbf{f}_0(q) + \mathbf{f}_1(q)Y + \cdots + \mathbf{f}_r Y^r.$$

functorial in the character χ_A. When χ_A is a classical character and $r = 0$, the map $f \mapsto \mathbf{f}_0(q)$ recovers the q-expansion map of overconvergent modular forms. When the modular curve X is geometrically connected, as in the proof of usual q-expansion principle, $\mathrm{Spec}(R)$ has Zariski dense image in $X_{\mathbb{Q}_p}$; so passing to the q-expansion (3.11.1) is equivalent to taking completion at the corresponding cusps and is hence injective. Thus, one often uses the q-expansion to indicate the corresponding nearly overconvergent modular form.

Example 3.12. — Assume that we choose the splitting of the Hodge filtration to be the one given by Katz as in Example 3.3. In terms of q-expansions, $\partial^{\chi_A} : M^\dagger(\chi_A) \to M^\dagger(\chi_A \chi_{\mathrm{cycl}}^2)$ is then given by

$$\mathbf{f} \mapsto q\frac{d}{dq}\mathbf{f} + \frac{\mathrm{WT}(\chi_A)E_2\mathbf{f}}{12}.$$

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