

1 Title:

2 Modeling Fuzzy Fidelity: Using Microsimulation to Explore Age, Period, and Cohort Effects in
3 Secularization

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21 Abstract

22 This article presents a microsimulation that explores age, period, and cohort effects in the decline
23 of religiosity in contemporary societies. The model implements a well-known and previously
24 empirically validated theory of secularization that highlights the role of “fuzzy fidelity,” i.e., the
25 percentage of a population whose religiosity is moderate (Voas 2009). Validation of the model
26 involved comparing its simulation results to shifts in religiosity over 9 waves of the European
27 Social Survey. Simulation experiments suggest that a cohort effect, based on weakened
28 transmission of religiosity as a function of the social environment, appears to be the best
29 explanation for secularization in the societies studied, both for the population as a whole and for
30 the proportions of religious, fuzzy, and secular people.

31 Keywords

32 demographic projection, religiosity, secularization, microsimulation, cohort effects

33 Introduction

34 What are the mechanisms that drive secularization in contemporary societies? Under what
35 conditions are populations most likely to experience a decline in religiosity? What role do age,
36 period, and cohort effects play in these processes? These are among the most contested

37 questions discussed by researchers interested in religion and demography. In this article we
38 attempt to contribute to these debates by describing the construction (and reporting on the
39 simulation experiment results) of a microsimulation model designed to simulate processes of
40 secularization hypothesized in a prominent theory of secularization (Voas 2009).

41 ***Secularization and Fuzzy Fidelity***

42 The term secularization commonly refers both to the waning power of religious institutions and
43 to the waning of religiosity at the individual level, i.e.: “a decline in the extent to which people
44 engage in religious practices, display beliefs of a religious kind, and conduct other aspects of their
45 lives in a manner informed by such beliefs” (Bruce 2002, p. 3). We focus in this article on
46 secularization at individual level, which involves a drifting away from identifying with a religion,
47 holding supernatural beliefs, attending worship services, praying, and regarding religion as
48 personally important. Here we will use the term “secular” as opposite to “religious” and the term
49 “secularity” to refer to the state of being secular.

50 Voas (2008) argues that the process of secularization (i.e., the long-term religious decline and
51 the complex of causal connections that promote it) is analogous to the demographic transition
52 (i.e., the shift to longer life expectancy and then low birth rates in the presence of economic
53 development) in a number of respects, not least in that the trends are clear but the mechanisms
54 are not. The “secular transition” comes late in the course of modernization, and it is difficult to
55 slow, stop, or reverse once it begins. Voas subsequently offers a model to illustrate how the
56 seemingly disparate and complex patterns of religious change observed across Europe could all
57 emerge from a common process of secularization. This article aims to replicate two key elements
58 in this theoretical and empirical treatment of “The rise and fall of fuzzy fidelity in Europe” (Voas
59 2009). The first is a quasi-linear downward trend in average religiosity. Although the levels of
60 religious involvement are very different across Europe (being high in Poland and low in the
61 Czech Republic, for example), decline seems to be proceeding at about the same pace across the
62 continent. The second is the way that the share of the population that is neither fully religious
63 nor wholly secular – a group Voas labels the “fuzzy faithful” – rises and then falls over a period
64 of two centuries or more.

65 In keeping with the literature on the diffusion of innovations (Kucharavy and De Guio 2011;
66 Rogers 2003), the model assumes that the rise of secularity follows a logistic (S-shaped)
67 trajectory. People do not convert from active religiosity to complete secularity in a single step.
68 The rise in the secular share of the population lags behind the decline in the religious share,
69 which makes it possible for the fuzzy faithful to become a majority. Ultimately, however, the
70 proportion in the fuzzy middle falls as the secular transition continues. Explaining these
71 processes of secularization requires attending to three effects: age, period, and cohort. Age
72 effects change religiosity in individuals at particular points in the life course (as a result, for
73 example, of having children or losing parents). Period effects have an impact on everyone alive at
74 a given time and might be associated with crises such as war, recession, or pandemics. Cohort
75 effects are generation-specific changes that are typically linked to the environment of upbringing
76 and peer interactions in teenage years.

77 ***The APC Identification Problem***

78 Because any two of age, current year, and year of birth determine the third, there is no unique
79 way (at least on the surface) to determine which of the three processes explain religious decline
80 in secularizing contexts. This is the so-called APC identification problem, which was first
81 analyzed in terms of the APC accounting model (Mason et al. 1973). For example, cohort effects
82 could be equivalently explained by combining period and age effects. The difficulty in identifying
83 these processes is exacerbated in part because data is available only for a couple of decades.
84 Fortunately, in the specific case of the secularization process, we are not helpless in the face of

85 the APC identification problem. There is now aggregate data spanning over four decades and
86 analyses of this data have led researchers to strongly favor a cohort-replacement explanation of
87 the secularization process; even though it is still logically possible that alternative explanations
88 may have produced the observed patterns (Voas 2009; Voas and Chaves 2016). It is then
89 possible to make a plausibility argument: when cohort effects explain religious decline with
90 decent fit, it is mathematically possible but sociologically implausible that apparently independent
91 age and period effects could be so perfectly synced that they produce the same result (e.g., Voas
92 & Chaves 2016).

93 But we can rarely be confident that only one or two of these effects are in play with any
94 particular demographic phenomenon, so something else needs to be done to escape the APC
95 identification problem. One approach is to use so-called ‘side information’ to guide our choice in
96 the set of feasible solutions. However, this approach relies on theoretical assumptions that are
97 rarely justified or verifiable (Reeves 2016). Another is to use non-linear models of these effects as
98 a way around their linear dependence. Assuming that all three are indeed in play, however, one
99 might wonder whether age, period, and cohort effects interact such that hidden constraints
100 might permit optimal explanations. This has led to innovations such as the APC-interaction
101 model (APC-I). Luo & Hodges (Luo and Hodges 2020) use the APC-I to handle a classic
102 instance of interaction effects with the possibility of distinctive interactions between age and
103 period. The APC-I enhancement and correction to the classical APC accounting model is an
104 example of a cautious embrace within sociology and demography of methods capable of
105 handling formally complex systems, which are characterized by non-linear interaction effects,
106 amplifying loops, and dampening processes.

107 ***Rationale for Using Microsimulation***

108 The most powerful method for understanding and explaining complex social systems is
109 computational simulation, which can be thought of as an intensification of the move Luo and
110 Hodges made in introducing the APC-I. Computational simulations can give expression to every
111 kind of interaction effect, not just the one type that appears in the APC-I. Moreover, they need
112 not be limited to linear models, unlike the APC-I. They can handle forbidding complexity in
113 terms of time periods, non-linearity of interacting variables, and underlying causal processes.
114 This latter point seems promising to social psychologists, for whom the sociologist’s traditional
115 framing of the APC identification problem is an odd abstraction from the concreteness of
116 human minds in which age, period, and cohort effects are merely facets of a complex process of
117 self-evaluation and self-transformation in rich social settings. Methods suited to handling
118 complexity, and computational simulation above all, have enormous potential to tackle seemingly
119 intractable problems, such as the APC identification problem that arises whenever sociologists
120 try to explain population change in secularizing contexts.

121 The model presented in this paper does not go so far as to articulate a causal architecture of
122 religious change within individual human minds. That is a possibility for computational
123 simulations and one that our research group hopes to realize in due course. The current model
124 has a more modest aim: to implement the potential APC processes as described in the literature
125 and demonstrate the possibility and usefulness of a model that (1) is not based on linear
126 assumptions, as APC models have tended to be; (2) includes all three processes of change
127 operative within the same artificial society; (3) promotes evaluation of the relative importance of
128 those types of change; and (4) simulates up to two and half centuries, from early modernity all
129 the way through the last several decades and onwards into the future yielding population
130 projections for religiosity. Thus, this is a proof-of-concept model, establishing that the vehicle
131 functions well even if its full power remains to be exploited. The conclusion of the analysis is
132 secondary. In fact, assuming the kind of S-curve process of decay in religiosity documented in
133 Voas 2009, the model shows that cohort effects supply the best explanation, which is a

134 conclusion broadly favored within the literature (Idler 2021; McAndrew and Richards 2020;
135 Molteni and Biolcati 2018; Stolz, Biolcati, and Molteni 2021; Voas and Chaves 2016; Brauer
136 2018). But that result should be understood not as an argument for the greater importance of
137 cohort changes so much as validation of a proof-of-concept model with almost unlimited
138 potential for deeper exploration.

139 Using agent-based models in this context combines the benefits of top-down (driven by
140 macro-level forces) and bottom-up (driven by individual-level behaviors) analysis. On the one
141 hand, complicated theories about the origins and operation of age, period, and cohort effects can
142 be represented straightforwardly. The influences can wax and wane in non-linear ways, and
143 likewise they can interact with each other. It would be extremely difficult to infer details of such
144 complexity from a statistical model. The simulation can thus be theory-driven and deductive
145 rather than wholly data-driven and inductive. Data have an important role in validating and
146 calibrating a computational simulation, but theoretical considerations are the starting point. On
147 the other hand, the outcome of the simulation ultimately depends on individual-level actions and
148 decisions. The environment can matter a great deal, but the unit of analysis is the agent rather
149 than some impersonal force. Explanation may start and end at the macro level, but it must also
150 operate via the micro level (Coleman 1994). If we want to understand the social or psychological
151 mechanisms at work, we need to track what individual agents do. This focus on agency makes
152 simulation more humanistic than might be immediately apparent (Diallo et al. 2019).

153 There is significant empirical evidence related to the age, period, and cohort effects that are
154 at work in religious change. For example, panel data from countries where there has not been
155 much aggregate movement away from religion (including highly developed countries such as
156 Israel) can help us to see whether and how religious involvement changes with age or life stage in
157 the absence of secularization (Eisenstein, Clark, and Jelen 2017). No simple story applies
158 universally: we can see clear signs of period effects (with many people drifting away from religion
159 during adulthood) in some countries and not in others, for example. And even where
160 generational replacement appears to have far greater impact than age or time, the size of the
161 generation gaps (i.e., cohort effects) will also rise and fall (Voas 2009; Molteni and Biolcati 2018;
162 McAndrew and Richards 2020; Stolz 2020; Idler 2021; Brauer 2018).

163 We aim, then, to implement our best conjectures about the proximate mechanisms of
164 religious change in a model to see what trajectories they produce, from the outset of the
165 secularization process to a point centuries later. Models generating outcomes that are at odds
166 with our real-world observations can be rejected. The objective is to identify a small number of
167 models that are consistent with 1) theories about how religiosity is or is not acquired, maintained,
168 and transmitted; and 2) data from societies at different stages in the secular transition. Ideally, we
169 will be able to identify patterns of religious change that apply to many countries, as past work
170 suggests may be possible. We also hope to find models that accommodate exceptions or
171 variations.

172 Below we provide a description of the APC processes implemented in the simulation.

173 *Period effect processes*

174 We conceive two different period-effect processes, one static and one dynamic. In the static
175 process, agents' religiosity decreases every year at a constant rate throughout their life, regardless
176 of starting religiosity, inherited parental religiosity, or age of agent. This static process is
177 capturing latent societal-level factors (e.g. improving education, existential security) that are
178 theorized to encourage decline in religiosity over time (Bruce 2011; Norris and Inglehart 2011;
179 Wildman et al. 2020) and these impact everyone living in the society. In the dynamic process,
180 agents' religiosity decreases throughout the life of the individual, regardless of starting religiosity,
181 inherited parental religiosity, or age of agent, but the degree of change is a function of religiosity.
182 Thus, the absolute decline in an agent's religiosity varies across time, where change is smallest

183 among the most religious and secular individuals, and there is a larger decline for those in the
184 middle of the religiosity spectrum. This dynamic process therefore accounts for highly religious
185 traditions that preserve their religiosity better than others, as well as a reluctance among the
186 nominally religious to reject all religion and become wholly secular (Day 2011; Smith and Denton
187 2009).

188 *Cohort effect processes*

189 Parents transmit their religiosity to their offspring with a bias towards lower religiosity values.
190 We call this a cohort effect, since after the inheritance event, when the individual reaches age 12,
191 their religiosity remains constant. This cohort process is supported by evidence that a
192 consistently large predictor of one's own religiosity is the religiosity of one's parents and there is
193 a net decline in religiosity from parents to children in secularizing societies (Cragun et al. 2018;
194 Min, Silverstein, and Lendon 2012; Storm and Voas 2012; Brauer 2018). We also consider an
195 alternative in which the size of the cohort effect depends on how religious the society is. In this
196 case, agents inherit the religiosity of their parents minus a value that reflects the current secularity
197 of the environment (i.e., the share of individuals classified as seculars). We test which of five
198 different measures provides the best fit, based on the relative frequency in the population of the
199 religious, fuzzy, secular, or non-religious (i.e., secular plus fuzzy), or on the product of the
200 religious and secular shares. The rationale behind this assumptions is that just as in the real
201 world, the social environment in the model changes over time, and the aggregate level of
202 religiosity has an impact on religious transmission and socialization of individuals in adolescence,
203 when their religious identities, beliefs and practices are being formed (Min, Silverstein, and
204 Lendon 2012; Strhan and Shillitoe 2019; Voas and Storm 2021).

205 *Age effect processes*

206 It is easily argued that age effects on their own cannot produce religious decline. Although
207 people may become more or less religious as they age, that fact would not alter the average
208 religiosity of a stationary population (Voas 2009). To explain secularization, we require period or
209 cohort effects, or some combination of the two, with age effects having at most a moderating
210 influence. Those might still be significant (if some people return to church while raising a family,
211 for example), but the central question is whether individual-level religious change occurs mostly
212 early in life (especially adolescence and young adulthood) or is spread over much of the life
213 course.

214 Based on findings from the literature, we devised three different processes by which age
215 affects agents' religiosity, independently of inheritance at age 12. In the first process, agents
216 decrease their religiosity as they become older (Lechler and Sunde 2020). In the second one, the
217 effect is reversed, i.e., the religiosity of agents increases as they become older (Argue, Johnson,
218 and White 1999; Azzi and Ehrenberg 1975; Bengtson et al. 2015; Iannaccone 1998). In the third
219 one, agents decrease their religiosity up to an age at which their religiosity starts to increase again,
220 a U-shape effect (Hayward and Krause 2013). These age effects always occur in combination
221 with period or cohort processes because (as mentioned above) age effects alone can never
222 explain secularization processes.

223 **The Fuzzy Fidelity Microsimulation**

224 *Microsimulation overview*

225 The microsimulation explores the way that different APC processes lead to a decrease in
226 religiosity over time in a stationary population. (For specificity, we adapted the initial age
227 structure and vital rates from those for Norway, as described below.) The microsimulation was

228 implemented in AnyLogic 8 University version 8.5.2. We designed this microsimulation mindful
 229 of the concerns of social and cognitive scientists of religion, particularly those interested in
 230 religious decline. The entities represented in the simulation are human agents characterized by
 231 age, generation, and religiosity. The religiosity of agents ranges between 0 and 1. We subdivide
 232 this range into three equal intervals, classifying agents as religious (R) if their religiosity (a variable
 233 ranging between 0 and 1) is ≥ 0.66 , seculars (S) if it is ≤ 0.33 , and fuzzies (F) otherwise. During
 234 the simulation, the religiosity value of each agent changes according to specific APC processes.
 235 These processes are based on theory and evidence about the age, period, and cohort effects that
 236 we find in studies of religious change. The overall decrease of agents' religiosity in each of these
 237 APC processes is an umbrella estimate representing several factors hypothesized and shown to
 238 decrease religiosity in human societies, e.g., religious socialization, existential security, pluralism,
 239 education, freedom of expression, etc. (Stolz 2020; Wildman et al. 2020; Gervais, Najle, and
 240 Caluori 2021).

241 Voas (2009) starts from a population that is 95% religious, with only 4% in the fuzzy
 242 category. In our view that distribution exaggerates the level of religious commitment in even the
 243 most traditional societies; it is more realistic to assume that an appreciable proportion of the
 244 population is slightly detached from belief and practice. We therefore assume that, although only
 245 1% of the population qualifies as secular at the beginning of the process (agreeing with Voas),
 246 15% can be regarded as fuzzy. A Weibull distribution with appropriate parameters is well suited
 247 to defining our starting point. The overall mean religiosity at the outset is 0.81; within the three
 248 categories of religious, fuzzy, and secular, the group means are 0.86, 0.56, and 0.22, respectively.
 249 The shares of these groups gradually change from one year to the next, and at the same time the
 250 average distribution of religiosity changes in a secular direction.

251 The initial population is fixed at 1000 agents. The values for age, mortality, and fertility are
 252 based on statistics obtained from Statistics Norway (Statistisk Sentralbyrå;
 253 <https://www.ssb.no/en/befolkning>). The starting age distribution follows that of the Norwegian
 254 population in 1900. For simplicity, we assume no gender and, to keep the population relatively
 255 constant, the total fertility rate is fixed at 1.005 per agent throughout the simulation. Thus, every
 256 agent produces an average of 1.005 new agents during the reproductive ages of 15-49, equivalent
 257 to fertility of 2.01 children per woman. When turning 12 years old, agents born in the simulation
 258 inherit a religiosity value similar to that of their parents (see APC processes). If the parent dies
 259 before the agent turns 12, the value inherited is similar to that when the parent was last alive.
 260 Further, also for simplicity, we use a constant mortality schedule throughout, with life
 261 expectancy of approximately 80 years. In each annual time step, agents experience the following:
 262 they age by one year, die or give birth with a probability according to their age, and change their
 263 religiosity according to the APC process being applied. In all cases, the change in the agents'
 264 religiosity is deterministic and governed by the equations given in each of the following
 265 processes.

266 ***Cohort effects: simple and social influence***

267 Cohort processes are supported by evidence showing that a consistently large predictor of one's
 268 own religiosity is the religiosity of one's parents and that there is a net decline in religiosity from
 269 parents to children in secularizing societies (Cragun et al. 2018; Min, Silverstein, and Lendon
 270 2012; Storm and Voas 2012; Brauer 2018). Following this, in the model, at 12 years old agents
 271 inherit the religiosity value of their parents with a bias (eq. 1).

$$272 \quad 273 \quad 274 \quad REL_{offspring} = REL_{parent} * Bias \quad \text{eq. (1)}$$

275 where REL is the religiosity value of the offspring and parent, respectively, and $Bias$ is a value
 276 drawn from the Weibull distribution function of AnyLogic. This function takes two different

277 values: alpha, the shape parameter, and beta, the scale parameter. Its formula is given by equation
 278 2:
 279

$$f(x) = e^{-\frac{x^\alpha}{\beta}} \quad \text{eq. (2)}$$

280
 281 the values of alpha and beta are constrained within specific ranges so the distribution will be
 282 skewed and thus the religiosity values of offspring will be on average lower than those of their
 283 parents.
 284

285 We implemented an alternative cohort effect that explicitly incorporates social influence
 286 rather than simply a general downward bias. In this case, 12-year-old agents inherit the religiosity
 287 of their parents minus a constant (C) multiplied by the proportion in the population of one of
 288 the following: a) non-religious, b) religious, c) fuzzies, d) seculars, or e) religious multiplied by
 289 seculars. The whole term is then multiplied by *Noise*, a value from a normal distribution with
 290 $\mu=1$ and $\sigma=sd$ (eq. 3). Where *sd* is a parameter determined during the optimization experiments
 291 (see below).
 292

$$REL_{offspring} = (REL_{parent} - (C * Prop.agent.category)) * Noise \quad \text{eq. (3)}$$

293
 294 Recall that agents are categorized as religious, fuzzy, or secular depending on whether they
 295 are in the upper, middle, or lower third of the religiosity range. The environment changes over
 296 time as the population becomes more secular, and transmission of religiosity from parents to
 297 children tends to be increasingly affected as aggregate religiosity falls.
 298

299 ***Period effects: static and dynamic***

300 We model period effects as the loss of individual religiosity over time. At age 12, agents inherit
 301 the religiosity of their parents times some noise (value from a normal distribution with $\mu=1$,
 302 $\sigma=0.05$). Thereafter, their religiosity declines year by year according to equation 4. This static
 303 process captures latent societal-level factors (e.g. improving education, existential security) that
 304 are theorized to encourage decline in religiosity over time (Bruce 2011; Norris and Inglehart
 305 2011; Wildman et al. 2020) and impact everyone living in the society.
 306

$$REL_{t+1} = REL_t - Inhibitor \quad \text{eq. (4)}$$

307
 308 The value of the inhibitor may be a constant or a dynamic value. When dynamic, the inhibitor is
 309 a function of the agent's current religiosity, as shown equation 5. This dynamic process accounts
 310 for highly religious traditions that preserve their religiosity better than others, as well as a
 311 reluctance among the nominally religious to reject all religion and become wholly secular (Day
 312 2011; Smith and Denton 2009).
 313
 314
 315

$$Inhibitor = A * (REL_t - 0.5)^2 + C \quad \text{eq. (5)}$$

316
 317 where REL is the religiosity value of the agent at time t , C is the vertex of the quadratic function
 318 (i.e., the maximum value that the inhibitor can take), and A is a constant ($-4*C$) that keeps the
 319 boundaries of the quadratic function at 0 (Figure 1). Note that the decrease in religiosity occurs
 320 fastest when current religiosity is close to 0.5 and more slowly when the value is near the
 321 extremes of 0 or 1. This reflects that the most strongly religious families resist secularizing
 322 processes within their children most effectively, and less religious families aren't as successful in
 323

324 religious transmission (cf. Smith 2005). Further, note that in both cases (static and dynamic),
325 when the value of the inhibitor is greater than $Rel_{(t)}$, then $Rel_{(t+1)}$ is set to 0.

326

327

Fig 1 here

328 *Age effects: religiosity decreasing and increasing with age*

329 We devised three different processes: (1) agents decrease their religiosity as they become older,
330 (2) agents increase their religiosity as they become older increase, and (3) agents decrease their
331 religiosity up to an age at which their religiosity starts to increase again (U-shape effect). Note
332 that these three effects have an empirical basis (see age processes section). Hence, in the model,
333 when an agent becomes 12 years old and the decrease process is active, the age of the agent
334 modulates the value of the religiosity inhibitor; see equation 6.

335

$$336 \quad REL_{t+1} = REL_t - (Inhibitor * Age Effect) \quad \text{eq. (6)}$$

337

338 where the inhibitor is a constant value or dynamic value (defined the same way as in eq. 4 and eq.
339 5, above), and age effect is given by equation 7:

340

$$341 \quad Age\ effect = (1 - Age_{standardized})^\gamma \quad \text{eq. (7)}$$

342

343 The age of agents is standardized between 1 and 0: 1 when an agent's age is 12 years old and
344 0 when an agent's age is ≥ 100 years old. Thus, when an agent is 12 years old the age effect is
345 maximum and so is the value of the religiosity inhibitor (Fig 2). Thereafter the age effect
346 decreases as an agent gets older; this decrease is linear or non-linear depending on the value of
347 gamma (γ) (Fig. 2).

348

349

Fig 2 here

350

351 Under the influence of the second age effect process, the religiosity of agents increases as
352 they become older. Religiosity starts increasing when an agent reaches a minimum age, the age of
353 the agent then modulates the value of the religiosity enhancer; see equation 8.

354

$$355 \quad REL_{t+1} = REL_t + (Enhancer * Age Effect) \quad \text{eq. (8)}$$

356

357 where the enhancer is a constant value, and the age effect is given by equation 9:

358

$$359 \quad Age\ effect = (Age_{standardized})^\gamma \quad \text{eq. (9)}$$

360

361 In this case, age is standardized between 0 and 1: with 0 being the minimum age at which
362 religiosity starts to increase and 1 being when agents are 100 years old or older. Thus, when an
363 agent reaches the minimum age, the effect of age is minimum and so is the value of the enhancer
364 (Fig 3). Thereafter the age effect increases with age reaching its maximum value at 100 years old.
365 Depending on the value of gamma, the age effect may increase linearly or non-linearly (Fig. 3).

366

367

Fig 3 here

368

369 Finally, when the third age effect process is active, religiosity starts decreasing at age 12,
370 according to equation 6 and 7; then, when reaching a minimum age, religiosity starts increasing
371 according to equation 8 and 9.

372 *Microsimulation and APC processes combinations*

373 Table 1 summarizes the combinations of processes that were implemented in the
374 microsimulation. We defined two types of cohort effects (simple and social-influence, in five
375 variations depending on the nature of the social influence), two types of period effects (static and
376 dynamic), and three types of age effects (decreasing, increasing, and decreasing/increasing with
377 age). From the numerous combinations possible, we selected those that express fundamental
378 options for interpreting demographic process of religious change. Note that the final option in
379 Table 1 includes all five variations of social influence expressing cohort change in religiosity (H,
380 I, J, K, L).

381
382 Table 1 here

383 *Analysis of empirical data*

384 We needed to evaluate variants of the microsimulation model against data, and we did so using
385 three different approaches. The first and second approach assume that the religiosity decay is
386 logistic and calculate this decay at the cohort and population level respectively. For these
387 calculations, we used the data generated in the model of Voas (see Voas 2009 for details of the
388 data analysis), which assumes logistic decay; this model passed tests against available data so
389 there is a sturdy empirical basis for using it. The third approach assumes that the religiosity decay
390 is linear, we used data from the European Social Survey, extrapolating outwards to cover 200
391 years. These comparator models are described below.

392 For the case that religiosity decay is logistic, the projected dynamics of the rise and fall of R-
393 F-S shares over 200 years are shown in Figure 4. The basic concept is that the secular transition
394 starts when the religious share of the population begins to decline, slowly at first, then more
395 rapidly, and slowly again as it approaches a floor. The change in religious share (RS) is given by
396 equation 10:

397
398
$$RS = \frac{0.88}{1+e^{-3.15 * e^{0.03 * Year}}} \quad \text{eq. (10)}$$

399 The wholly secular share (SS) rises from an initial level of just 1%, following the logistic
400 trajectory given by equation 11:

401
402
$$SS = \frac{1}{1+e^{4.6 * e^{-0.025 * Year}}} \quad \text{eq. (11)}$$

403
404 The slight lag between these two trends generates the rise of the fuzzy share (FS = 1 – [RS +
405 SS]), which ultimately declines as more complete secularity takes hold (Figure 4). The R-F-S
406 curves relate to birth cohorts, following Voas (2009), and hence we take these graphs as
407 representing 40 5-year cohorts.
408

409
410 Fig 4 here

411
412 The shares of the religious, fuzzy, and secular can be used in conjunction with the average
413 religiosity within each group to calculate the mean religiosity of the whole population. We
414 assume that when the process begins, average religiosity within each category is higher than the
415 midpoint, at 0.86, 0.56 and 0.22 for the religious, fuzzy, and secular groups respectively. During
416 the following two centuries, the shift towards lower religiosity means that these values gradually
417 decline. The largest drop is in the fuzzy group, where average religiosity falls from 0.56 to about
418 0.46. Multiplying the share of each group by the average religiosity within it gives us the overall
419 mean religiosity by birth cohort (Figure 5).

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Fig 5 here

Additionally, we also calculated the decay of religiosity at the population level. This calculation was done in two different ways. First, using the shares and mean religiosity values of R-F-S agents, we calculated the mean religiosity of each cohort as described above. We then calculated moving averages, where each average covers ten 5-year cohorts or 50 years of age (to include adults from age 25 to 74). Note that the initial pace of decline is lower because of the inertia from older generations (Figure 6).

Fig 6 here

For the case that religiosity decay is linear, we used data from the 15 countries that participated in all 9 waves of the European Social Survey (ESS 2018). Detail information on the ESS can be found at <https://www.europeansocialsurvey.org/>. First, we calculated a continuous variable called religiosity index using three questions from the ESS. These three questions were also used in the study by Voas 2009: (1) self-declared religiosity (SDR), “Regardless of whether you belong to a particular religion, how religious would you say you are?” ; (2) Attendance, “Apart from special occasions such as weddings and funerals, about how often do you attend religious services nowadays?”; and (3) Pray, “Apart from when you are at religious services, how often, if at all, do you pray?”. The questions had a 11-, 7- and 7-point scale, respectively; thus, we transformed SDR to a 7-point scale ($SDR_7 = 0.6 * SDR_{11} + 1$). The sum of these answers constituted the religiosity index, ranging from 3 (non-religious) to 21 (very religious). Next, using this religiosity index, we calculated the average religiosity of the population per country and wave, and the average religiosity of the fifteen countries per wave (Table S1 in supplementary information). These calculations show that the average religiosity of the population is decreasing in all countries (Fig 7a). Then, using this data, we performed a linear regression, and found that among these European countries the average religiosity of the population decreases linearly by 0.103 every two years (ESSs were done every two years). Finally, we transformed the religiosity index [3,21] to the religiosity scale used in the model [0,1], and using the initial average religiosity of the population in the model (0.81) as the intercept and the slope from the linear regression, adjusted to the [0,1] scale, we extrapolated the religiosity decay for a period of 250 years. The resulting religiosity decay is shown in Figure 7b. Note that the period covered by the nine ESS waves is only a small portion of the whole range, so the ESS data are consistent with both the logistic-decay and linear-decay hypotheses. Our purpose here is not to evaluate the ESS data but to employ it to generate a credible version of the linear-decay hypothesis that we can use to evaluate the microsimulation alternatives.

Fig 7 here

Optimization of microsimulation parameters

The main goal of the microsimulation is to find, for each combination of APC processes in Table 1, the right parameter values (listed in Table 1) leading to output that mimics the religious decline observed across cohorts or at the population level (Figures 5-7). To do so, we used the optimization engine of AnyLogic v 8.5.2. The optimization engine allows the user to explore many combinations of parameter values with the goal of identifying values that produce the best result, as defined by a particular function. In our case, we try to minimize the residual sum of squares (RSS) between the values obtained from the model and the target religiosity decay curve at: (a) the cohort level, logistic decay with S-shaped curve (Figure 5); (b) the population level,

468 logistic decay with S-shaped curve (Figure 6); and (c) the population level, linear decay (Figure
469 7b).

470 To calculate the RSS, we collected the average religiosity (at the cohort or population level as
471 appropriate) from each optimization experiment and compared these values to the
472 corresponding target. For each APC process (Table 1) and target curve, we ran five optimization
473 experiments. We then took the combination of parameter values that produced the lowest RSS
474 and reran the model 100 times, overlaying the target curve with the output of these 100 runs. We
475 thereby established the degree of success with which each APC process could reproduce the
476 target curves for the decline in average religiosity. Similarly, we compared the output of each
477 APC process with the expected changes in R-F-S proportions (Figure 4). Note that the
478 parameters were optimized to produce the best fit with average aggregate religiosity, so the
479 degree to which each proposed solution reproduced the changing breakdown of religious, fuzzy,
480 and secular serves as a form of validation.

481 Results

482 *Targeting logistic decay of religiosity at the cohort level*

483 The best fit was produced by the cohort effect taking account of social environment (H-L in
484 Table 2). These processes generated RSS values below 0.052, except when the social
485 environment was represented by the proportion of religious population (I in Table 2). Among
486 the different social environments, the best fits were produced when the social environment was
487 represented by the proportion of non-religious (i.e., secular plus fuzzy) or fuzzy agents (H and J
488 in Table 2). Of the other APC processes, the best fits were produced by a static period with U-
489 shaped age effect and a cohort with age effect (C and G respectively in Table 2), but they were
490 not as good as the cohort and social environment effects. All other APC processes produced a
491 much worse fit.

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493
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Table 2 here

495 Figure 8 shows the overlay between the cohort target curve and the trajectories of 100 model
496 runs using the combination of parameter values producing the best fit for each of the APC
497 processes. The trajectories in Figure 8 corroborate the results in Table 2: the best fits are
498 produced by the cohort (social environment) effects, particularly when the social environment is
499 represented by the proportion of non-religious or fuzzy agents.

500 Figure 9 compares the output of these 100 models runs with the dynamics of the R-F-S
501 shares derived from Voas (2009). Here as well, the best fit is produced by the cohort effect when
502 the social environment is represented either by the proportion of fuzzies or non-religious agents.
503 The overlap is not perfect; when using the non-religious proportion as social environment, the
504 fit for the religious category is very good, but less so for fuzzies and seculars. There is a slightly
505 higher proportion of fuzzies around 150 years and a slightly lower proportion of seculars during
506 the first 100 years of the run. In the case of the cohort effect with the fuzzy proportion defining
507 the social environment, the proportion of religious individuals appears lower and that of fuzzies
508 higher during the first 100 years of the run. Overall, however, both processes reproduce the R-F-
509 S dynamics well, especially considering that the parameter values of these processes were not
510 optimized to fit these dynamics. Regarding all other APC processes, none of them performs as
511 well as the two just described.

512

513 Fig 8 here

514 Fig 9 here

515 The values of the parameters producing the best fit for each of the APC processes are shown in
516 Table S2 (Supplementary Information). The C and SD values for the cohort process with social
517 environment represented by the proportion of non-religious agents are 0.172 and 0.058. Hence,
518 when this process is activated, the maximum decrease in religiosity from parent to offspring is a
519 bit higher than 0.172 (depending on the value of *noise*, eq. 3), but only when all agents are
520 categorized as secular or fuzzy. In other words, such a decrease will only happen when nearly the
521 whole population has become non-religious, which takes 200 years. On the other hand, in the
522 cohort process with the social environment represented by the proportion of fuzzies, the values
523 of C and SD are 0.187 and 0.115 respectively. In contrast to the previous case, the value of C and
524 thus the maximum decrease in religiosity from parent to offspring (eq. 3) will never be reached
525 because the proportion of fuzzies is always well short of 1. Here the maximum decay in
526 religiosity is reached after around 150 years, when the proportion of fuzzies is at its peak (Figure
527 9). Thereafter, the decrease in religiosity from parent to offspring lessens with time.

528 ***Targeting logistic decay of religiosity at the population level***

529 When targeting the S-shaped decay in religiosity at the population (rather than cohort) level, the
530 best fit was again produced by the cohort and social environment effect (H-L in table 3),
531 particularly when using the proportion of fuzzies or non-religious individuals to characterize the
532 social environments (H and J in Table 3). These processes generated RSS values below 0.052 and
533 as low as 0.012. None of the other APC processes generated a good fit, and in fact all the RSS
534 values were above 0.131 (Table 3). Comparing the 100 model runs with the target curve
535 confirmed the results (Figure S2 in Supplemental Information).

536 Figure S3 (Supplemental Information) shows the overlap between the trajectories of 100
537 model runs for the R-F-S shares and the projections from Voas (2009). In contrast to the
538 previous results, none of the APC processes produces a good fit (though the same cohort with
539 environment effect solutions are the least unsatisfactory).

540 ***Targeting linear decay of religiosity at the population level***

541 When targeting linear decay in religiosity at the population level, the best fit was produced by a
542 static period effect (a in Table 3). This process generated RSS values as low as 0.005. All other
543 processes performed much worse (Table 3). The results are best illustrated in the overlap
544 between the 100 model trajectories and the linear decay curve (Figure S4 in Supplemental
545 Information). Turning to the dynamics of the R-F-S shares, however, none of the APC processes
546 generated a good fit. All show a large disparity between the model results and the projections by
547 Voas (2009) (Figure S5 in SI).

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Table 3 here

550 **Discussion**

551 This paper presents a computational model as a proof of concept that microsimulations can be
552 used effectively to investigate complex demographic processes such as secularization.
553 Microsimulations can easily express alternative theories of demographic change and enable
554 scholars to evaluate those alternatives against data when available. Microsimulations even offer
555 leverage against the APC identification problem by permitting non-linear interactions among age,
556 period, and cohort effects, after which procedures of the kind demonstrated here allow us to
557 identify the best explanations for a demographic process.

558 It is important to note that the decline of religiosity in the microsimulation is generated by a
559 simple rule: children receive their religiosity from parents and the transmission of parents'
560 religiosity is moderated by the social environment. This reflects a macro-micro feedback loop,
561 micro in the sense that religiosity is transmitted at the individual level from parents to children
562 and macro because the social environment influences the way both parents and children
563 maintain and pass on their religiosity. Under these conditions, the environment appears to have a
564 homogenous effect in the whole population, i.e., the effect of the environment is the same for all
565 individuals. Interestingly, this process would produce differences between societies if they
566 experience different environmental effects, but would not produce differences within the society,
567 i.e., at the individual level. This is what it is usually found in studies supporting existential
568 security theory, where differences in religiosity are apparent across societies with different GDP,
569 but much less so across individuals of the same society with different socio-economic status
570 (Norris and Inglehart 2011; Stolz 2020).

571 It is also important to note that the microsimulation is not capable of identifying the triggers
572 of secularization, nor can secularization be stopped in these models. Hence, something else may
573 be needed if we want to explore what may hinder societies from secularizing. However, this issue
574 is out of the scope of our current study; but see (Wildman et al. 2020), where it is considered in a
575 simulation).

576 Though framed primarily as a proof-of-concept exercise to demonstrate the value of
577 microsimulations in demography of religion and non-religion, the model we have presented is
578 robust enough to make a substantive contribution to the understanding of secularization. When
579 we entertained the hypothesis of linear decay in religiosity, the microsimulation identified a static
580 period effect as the best explanation of the data model, which makes good sense and helps to
581 validate the microsimulation. But a static period effect – and indeed, any of the putative
582 candidates for explaining linear decay of religiosity – could not produce anything close to the
583 correct proportions of religious, fuzzy, and secular people over time observed in the data. This
584 suggests that linear decay is a poor hypothesis and that we are better off with the logistic-decay
585 hypothesis. In light of this, our findings show substantively that Voas' interpretation of cohort
586 replacement, based on weakened transmission of religiosity as a function of the social
587 environment, appears to be an excellent explanation, both for the population as a whole and for
588 the proportions of religious, fuzzy, and secular people.

589 At the very least, our findings are persuasive support for the claim that secularization is
590 primarily a cohort process. Further exploration of the rich space of model variants possible
591 within this microsimulation could no doubt fine-tune the fit even further and demonstrate how
592 period and age effects play supplementary roles to the dominant cohort effect. That task is for
593 future work.

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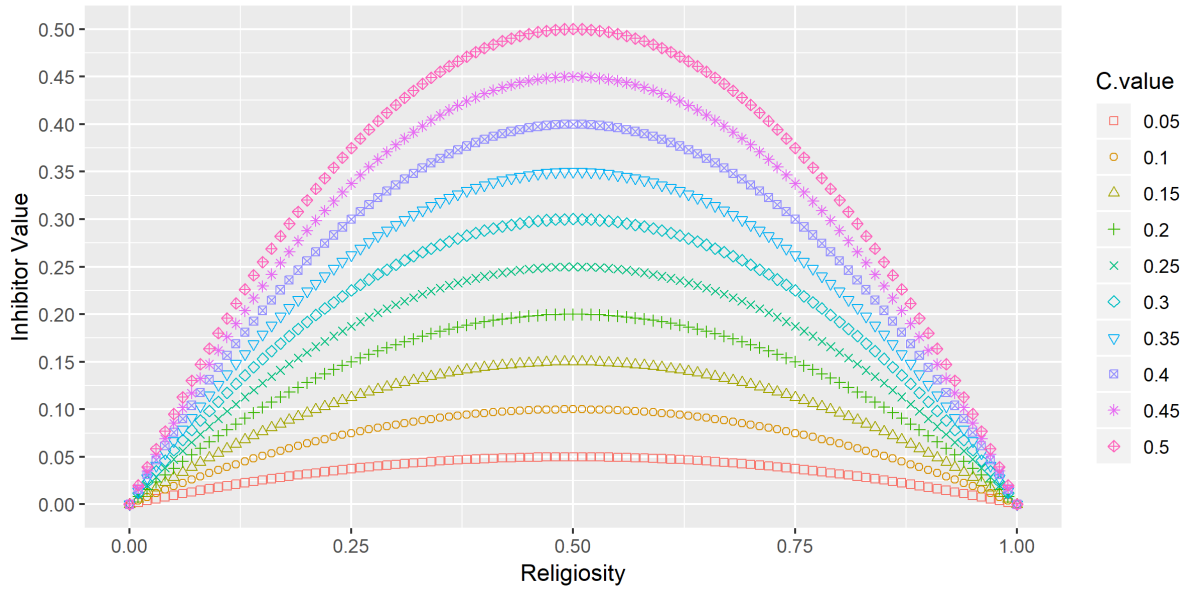
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Tables and Figures



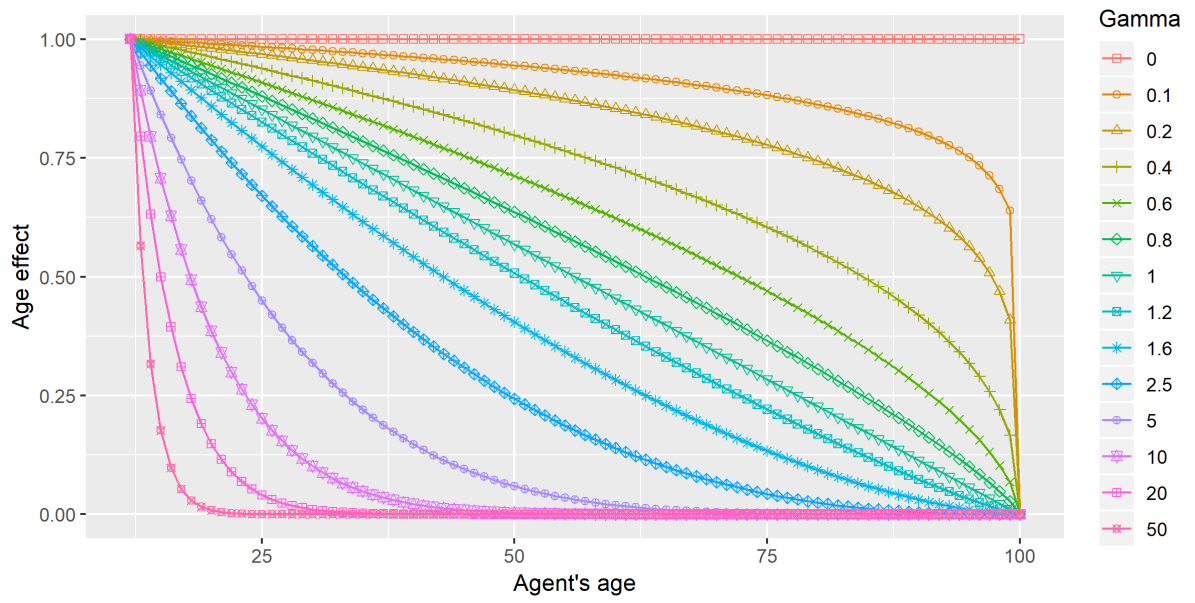
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Figure 1. Values taken by the inhibitor (y-axis) according to the agent's religiosity value (x-axis) and different values of C (points' color and shape).

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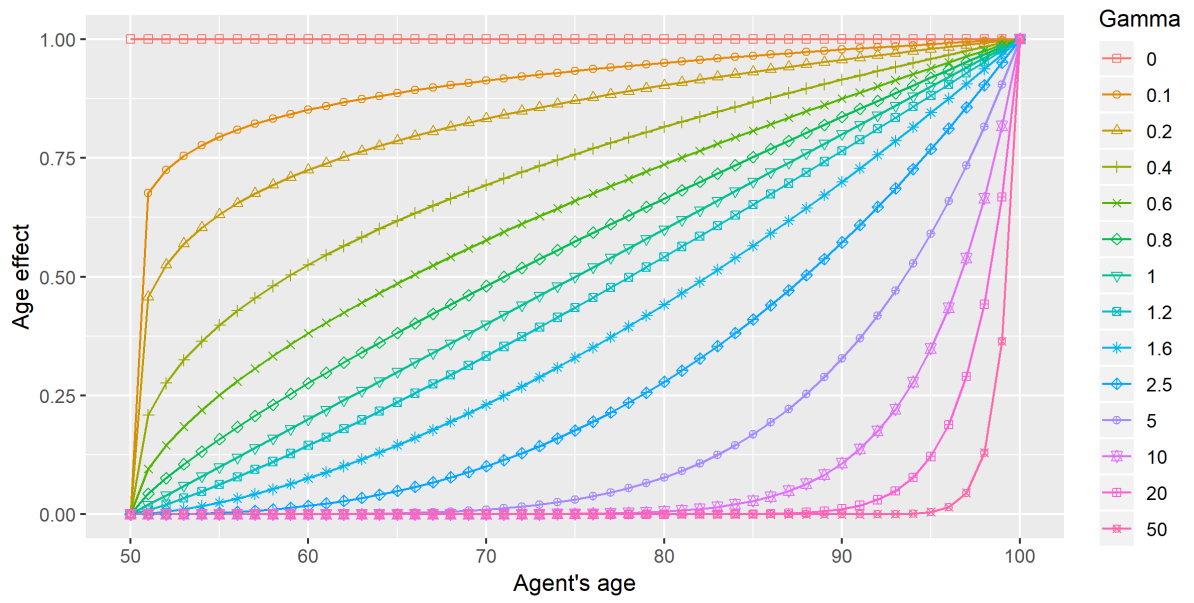
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Figure 2. Age effect (y-axis) values according to the agent's age (x-axis) and different values of gamma (points' shape and color), for use in the age-effect process where religiosity decreases with age.

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Figure 3. Age effect (y-axis) values according to the agent's age (x-axis) and different values of gamma (points' shape and color), for use in the age-effect process where religiosity increases with age. In this example fifty years old is the minimum age at which religiosity starts to increase.

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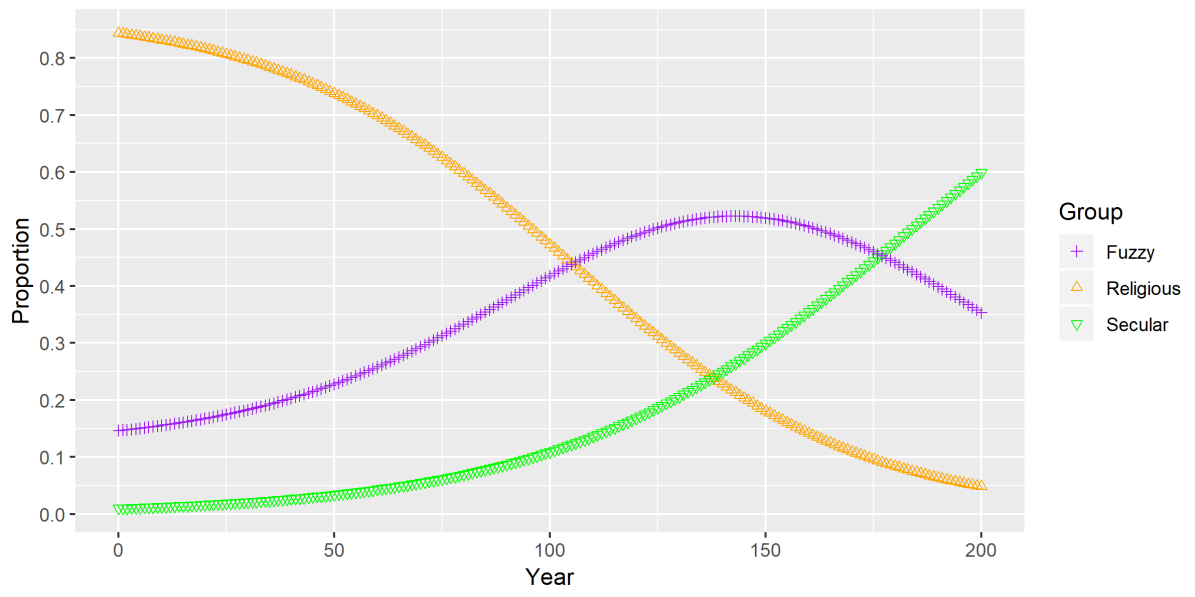


Figure 4. Dynamics of the proportions of religious, secular, and fuzzy people at the cohort level. Y-axis represents proportions and x-axis represents time in years.

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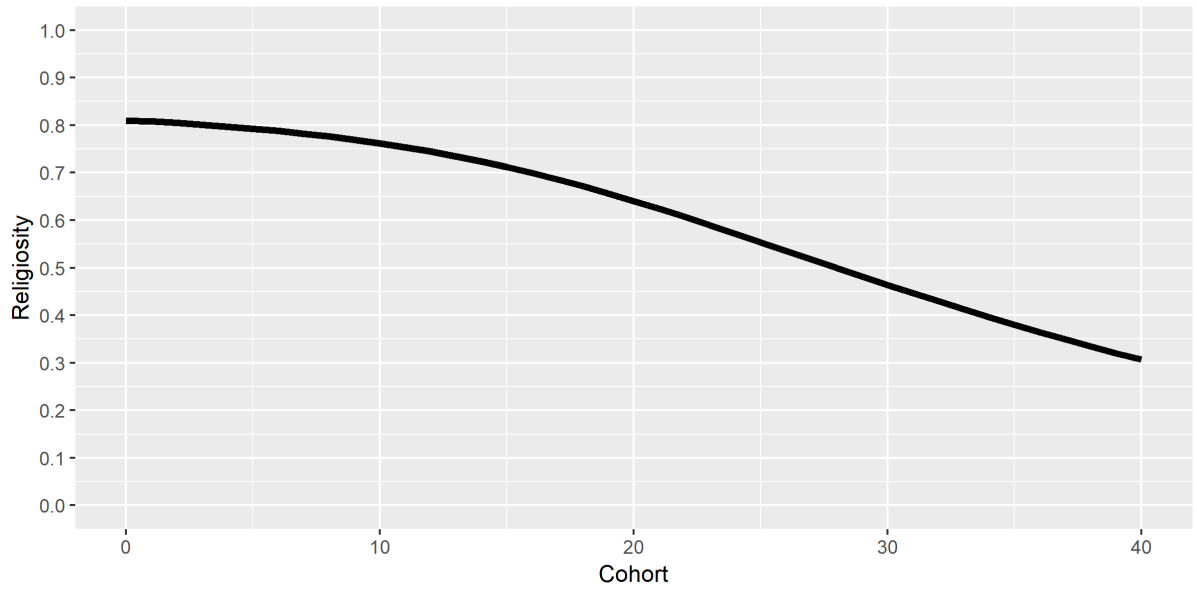


Figure 5. Religiosity decay among cohorts.

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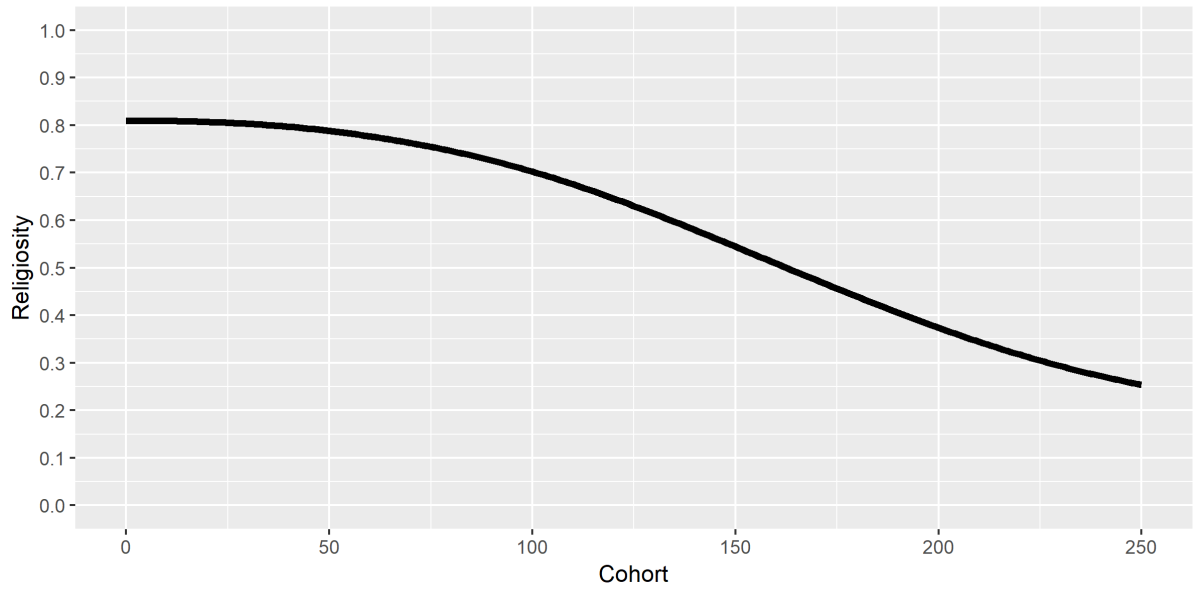
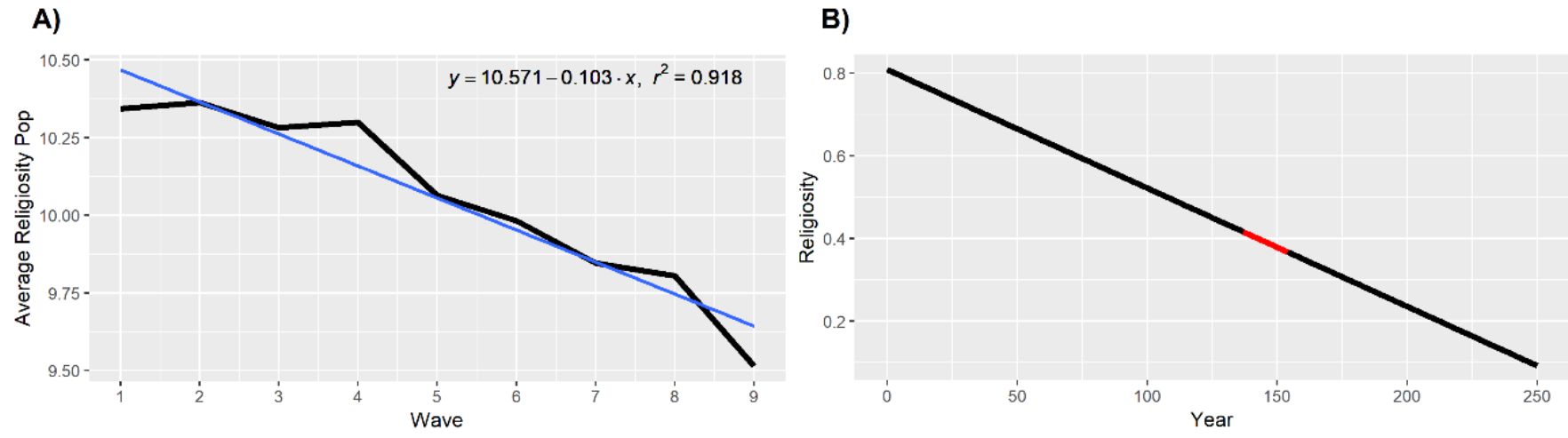


Figure 6. Religiosity decay at the population level

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 709 Figure 7 A) Religiosity decay at the population level from the 15 countries in the 9 waves of the ESS; B) Religiosity decay at the population level extrapolated from the linear regression in (A) for a period of 250 years; in red, the stretch
 710 of religiosity decay calculated from the ESS data in (A).
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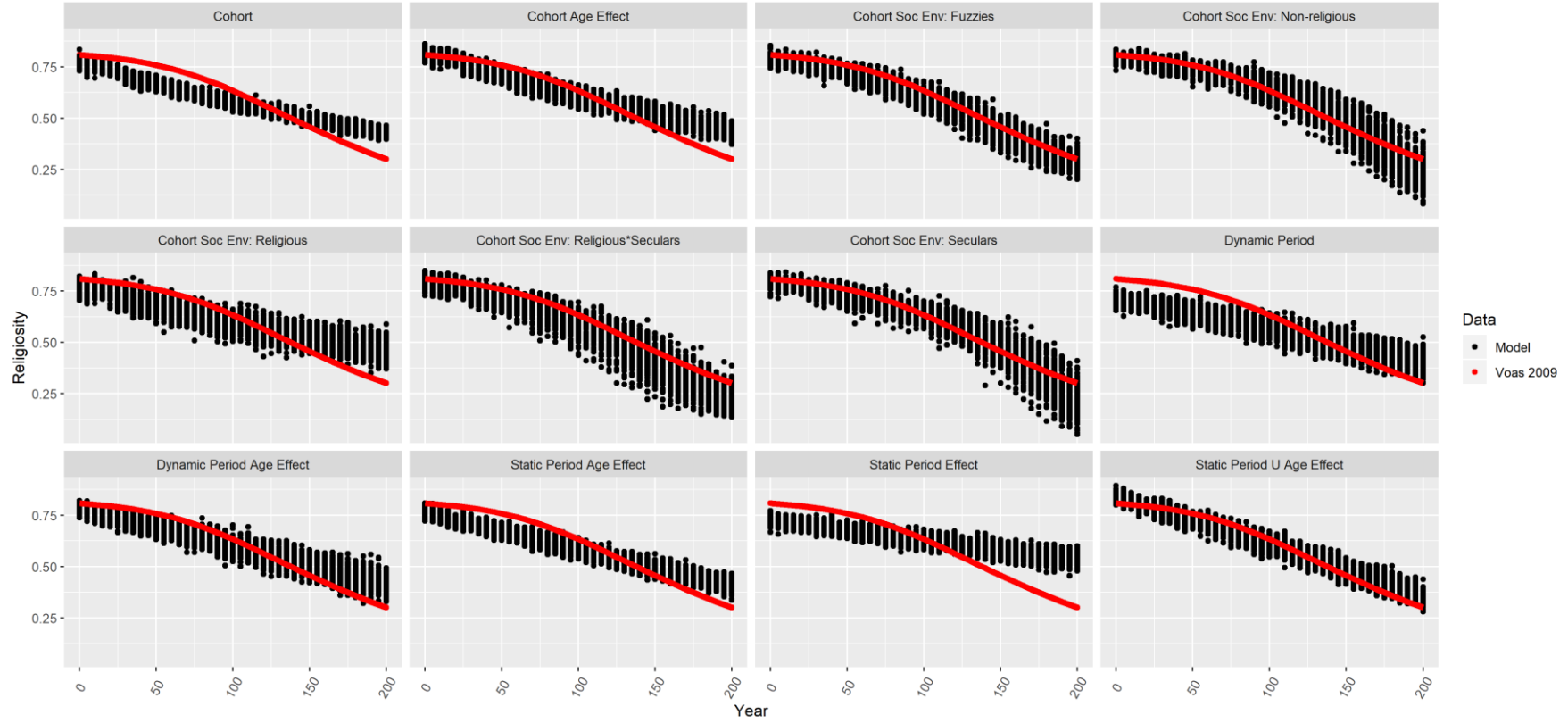


Figure 8. Trajectories of 100 model runs for each APC process (black) and the religiosity decay at the cohort level as target curve (red). See text for details

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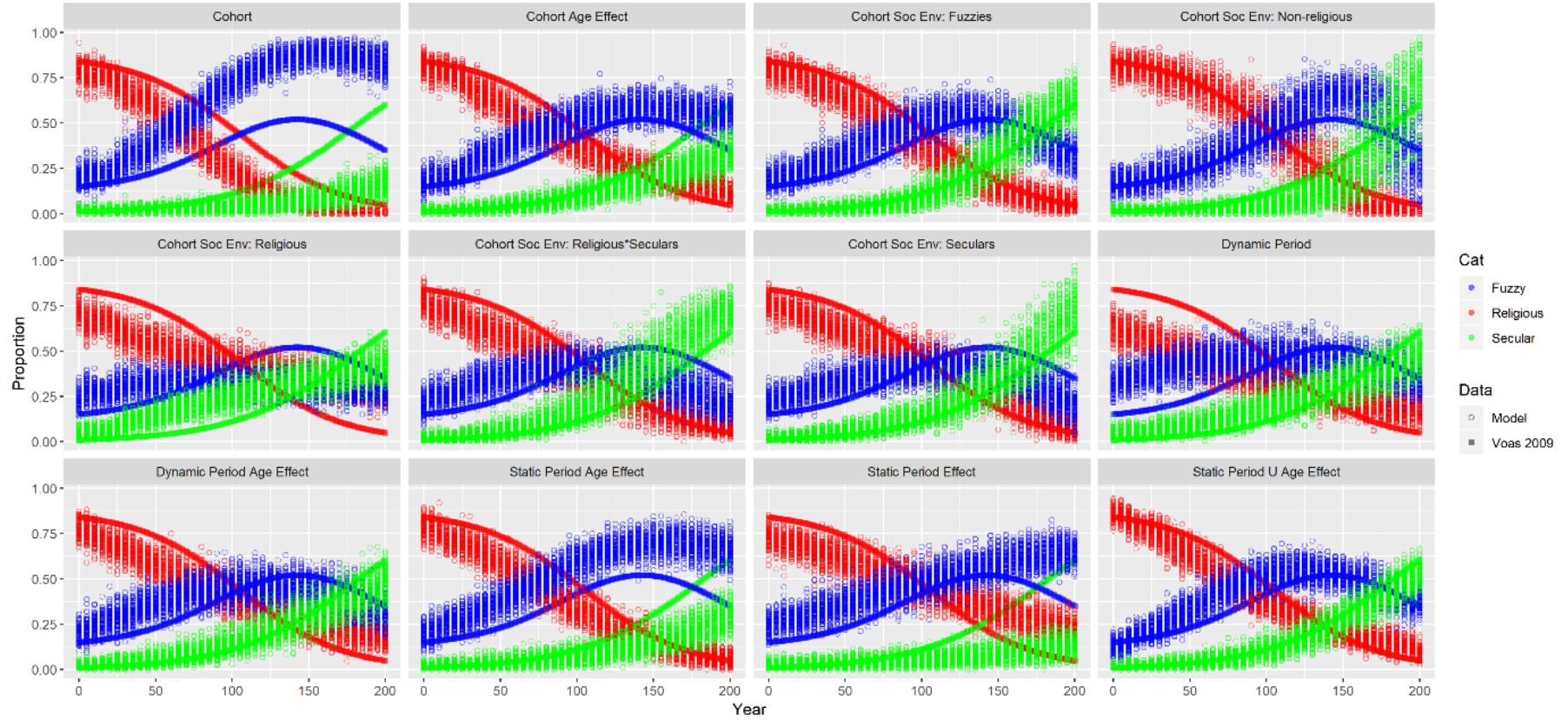


Figure 9 Trajectories of 100 model runs for the dynamics R-F-S shares according to each APC process (hollow dots) and the projections according to Voas 2009 (filled squares). Values of the model parameters were optimized by targeting the religious decay at the cohort level. Cat = category.

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| Code | APC process | Equation type and figures | Parameters optimized |
|-----------------------|--|--|---|
| A | Static period effect: religiosity decay is constant every year | Equation 4: inhibitor is a constant | 1. Inhibitor value |
| B | Static period effect with age effect (decreasing): Religiosity decay depends on inhibitor and decreases with age | Equation 4, 6 and 7; fig 2: Inhibitor is a constant modulated by agent's age | 1. Inhibitor value 2. Gamma value (age effect) |
| C | Static period effect with U-shape age effect (decreasing): Religiosity decreases up to a certain age and then increases – U age effect | Equation 4, 6 and 7; fig 2: Inhibitor is a constant modulated by agent's age. Equation 8 and 9; fig 3: Enhancer is a constant modulated by agent's age. | 1. Inhibitor value 2. Gamma value (first age effect) 3. Inflection age, religiosity stops decreasing and starts increasing 4. Enhancer value 5. Gamma value (second age effect) |
| D | Dynamic period effect: decay value is a quadratic function of the agents' religiosity | Equation 4 and 5: Inhibitor is dynamic | 1. C value (max inhibitor value) |
| E | Dynamic period effect with age effect (decreasing): decay value is a quadratic function of agents' religiosity and decreases with age | Equations 4, 5, 6 and 7; fig 2: Inhibitor is dynamic and modulated by agent's age. | 1. C value (max inhibitor value) 2. Gama value (age effect) |
| F | Cohort effect (simple): inheritance is biased towards lower than parental religiosity. | Equation 1: Inheritance with bias. | 1. alpha (shape) and beta (scale) values of the Weibull distribution |
| G | Cohort effect (simple) with age effect (increasing): inheritance is biased towards lower than parents' religious values and at a certain age religiosity starts to increase | Equation 1, 2, , 8, 9; fig 3: Inheritance with bias. Enhancer is a constant modulated by agent's age. | 1. alpha (shape) and beta (scale) values of the Weibull distribution 2. Age at which religiosity starts increasing 3. Enhancer value 4. Gamma value (age effect) |
| H I J K L | Cohort effect (social environment): Religiosity inherited from parents, minus an inhibitor reflecting the religiosity of the population. | Equation 3: Inheritance with noise. Inhibitor is dynamic. | 1. C value (max inhibitor value when all the agents are religious) 2. SD, standard deviation of the normal distribution |

Table 1. The eight combinations of age, period, and cohort effects tested.

| APC processes | RSS values range |
|--|-------------------------|
| A) Static period effect | [0.280-0.298] |
| B) Static period effect with age effect (decreasing) | [0.133-0.158] |
| C) Static period effect with U-shape age effect (decreasing) | [0.040-0.133] |
| D) Dynamic period effect | [0.219-0.241] |
| E) Dynamic period effect with age effect (decreasing) | [0.082-0.131] |
| F) Cohort effect (simple) | [0.145-0.156] |
| G) Cohort effect (simple) with age effect (increasing) | [0.052-0.068] |
| H) Cohort effect (social environment using proportion of non-religious) | [0.021-0.031] |
| I) Cohort effect (social environment using proportion of religious) | [0.171-0.206] |
| J) Cohort effect (social environment using proportion of fuzzies) | [0.022-0.044] |
| K) Cohort effect (social environment using proportion of seculars) | [0.038-0.052] |
| L) Cohort effect (social environment using proportion of religious*seculars) | [0.033-0.037] |

Table 2. Results of five optimization experiments per APC process targeting the religiosity decay curve at the cohort level.

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| APC processes | RSS values range | |
|--|------------------|---------------|
| | S-shape decay | Linear decay |
| A) Static period effect | [0.674-2.232] | [0.005-0.058] |
| B) Static period effect with age effect (decreasing) | [0.317-0.365] | [0.122-0.348] |
| C) Static period effect with U-shape age effect (decreasing) | [0.332-0.370] | [0.130-0.145] |
| D) Dynamic period effect | [0.414-0.592] | [0.071-0.090] |
| E) Dynamic period effect with age effect (decreasing) | [0.131-0.184] | [0.195-0.225] |
| F) Cohort effect (simple) | [0.419-0.505] | [0.329-0.383] |
| G) Cohort effect (simple) with age effect (increasing) | [0.353-0.422] | [0.345-0.361] |
| H) Cohort effect (social environment using proportion of non-religious) | [0.029-0.052] | [0.459-0.555] |
| I) Cohort effect (social environment using proportion of religious) | [0.387-0.716] | [0.530-0.668] |
| J) Cohort effect (social environment using proportion of fuzzies) | [0.013-0.031] | [0.458-0.552] |
| K) Cohort effect (social environment using proportion of seculars) | [0.053-0.090] | [0.490-0.613] |
| L) Cohort effect (social environment using proportion of religious*seculars) | [0.026-0.059] | [0.495-0.583] |

723 *Table 3. Results of five optimization experiments per APC process targeting the two religiosity decays at the population level: s-shape and linear decay.*

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