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Stability condition of multiclass classical retrials: a revised regenerative proof*

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Abstract. We consider a multiclass retrial system with classical retrials, and present a new short proof of the sufficient stability (positive recurrence) condition of the system. The proof is based on the analysis of the departures from the system and a balance equation between the arrived and departed work. Moreover, we apply the asymptotic results from the theory of renewal and regenerative processes. This analysis is then extended to the system with the outgoing calls. A few numerical examples illustrate theoretical analysis.

Keywords: multiclass system, classical retrials, outgoing calls, stability analysis, regeneration, simulation

1. Introduction

The importance of the retrial queues to model the modern wireless telecommunication systems is well-known, for instance, see [2–4, 8], where also a comprehensive bibliography on research related to retrial queues can be found. In this work we focus on the stability analysis of a classical retrial queue, and using regenerative arguments present a new short proof of the known sufficient stability condition of

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such a system. This proof is not only much shorter than that have been used earlier (see [1, 9]) but also allows easily to cover more general retrial systems with the so-called 'outgoing' calls [11]. Although there are many papers which investigate the steady-state performance of the retrial queues, still a little attention is devoted to stability analysis outside the Markovian setting, which is the topic of this research.

In this regard, we first mention a fundamental work [1] in which a detailed stability analysis of a general $G/G/1$-type single-server retrial queue is developed. The authors study the system with a stationary input process and non-exponential retrial times, and also investigate the convergence rate to stationarity, but they do not appeal to the regenerative method. The stability of multiserver retrial systems is studied in a few papers. For instance, the paper [7] studies such a system with a finite buffer, batch Markovian arrival process, phase-type service time distribution and a general retrial rate, and stability analysis is based on the corresponding embedded Markov chains.

In the present paper, we consider the stability of a multiclass retrial $M/G/1$ queue with independent Poisson inputs of primary customers belonging to N different classes. Then we outline how this analysis is extended to a multiserver system. If an arriving class-i primary customer finds server busy, he joins the corresponding (infinite capacity) orbit i, and, after exponential time, attempts to capture server again. These attempts continue until he finds server idle. Service time as well as the retrial times are assumed to be class-dependent.

We use the regenerative approach [5, 10, 16, 17] to reprove the known stability condition, and this work thus complements our previous works [9] and [11]. In [9], the proof is based on the negative drift of the remaining work in the (single) orbit, while in the paper [11], we utilize the positive drift of the idle time of server. In a contrast, in the current analysis we observe the system at the departure instants and evaluate the idle time of servers after each departure. Then, assuming instability, the idle times decreases, and this effect, in the limit, contradicts a predefined condition. Then, according to the approach developed in [14, 15], we appeal to a characterization of the remaining regeneration time of a basic process to deduce that it is *positive recurrent*. This approach leads to a radical simplification of the stability analysis and also is extended to the system in which there are 'outgoing calls' during idle periods of the servers. The idea to consider the output process is not new of course. For instance, the analysis of $M/G/1$-type retrial system in [8] is based on the analysis of an embedded Markov chain representing the orbit size at the service completion epochs. The main feature of a retrial system from the point-of-view of stability analysis is that, in such a system, after each service completion, the server becomes idle for a random time until the beginning of the next service. This implies a loss of the server capacity after each departure, and thus the service discipline turns out to be not *work-conserving*. Fortunately, the service discipline in the retrial queueing system with classical retrials approaches the work-conserving discipline in the corresponding buffered system as the orbit size increases, and by this reason, it is *asymptotically work-conserving* [9]. This leads to the coincidence of the stability conditions in the retrial system and in the corresponding classical buffered system.

The stability analysis then is extended to the system in which the idle server initiates an *outgoing call* [11]. Although these calls are expected to increase the utilization of the server, while keeping the throughput of the primary external customers, it is intuitive that the stability condition remains the same in this case as well, and we show it below.

The main contribution of this paper is to present a new short proof of the sufficient stability condition of this model, which then allows to study analogously the system with the outgoing calls (We note that the main result of this paper has been announced in [12].) Moreover, the approach used in this paper has a promising potential to analyse stability condition of a multiserver multicalss system in which the service times are both *class- and server-dependent*. In particular, it is demonstrated in a recent paper [13] where the stability analysis of a retrial system (a modified Erlang system) with two classes of customers and c identical servers has been performed by means of the same approach.

The rest of this paper is organized as follows. Section 2 describes the model and the regenerative structure of a basic stochastic process. In Section 3, we give the main balance equation and present the new proof of the stability condition. In Section 4, the stability analysis is extended to the system with the outgoing calls. To illustrate the theoretical results, some numerical results based on stochastic simulation are included in Section 5.

2. Description of the model

We consider a multiclass retrial $M/G/1$ queueing system with N independent Poisson inputs of primary customers, and for each class i, denote by τ_i a generic (exponential) interarrival time, with rate $\lambda_i = 1/\mathsf{E}\tau_i$, by $\{S_n^{(i)}\}$ the iid service time (with generic time $S^{(i)}$ with rate $\mu_i = 1/\mathsf{E}S^{(i)}$), and by γ_i the rate of (exponential) class-i retrial time, $i = 1, \dots, N$. If a class-i primary customer finds server busy, he joins the corresponding (infinite capacity) orbit i and, after the exponential time with rate γ_i, the customer attempts to capture server. He continues his attempts until finds the server idle. Denote by $\gamma_0 = \min \gamma_i$. This rule is called 'classical retrial policy', and if the orbit size equals N, then the retrial rate at this instant is lower bounded by $\gamma_0 N$ by the memoryless property of the exponential distribution.

To describe the regenerative structure of the system, we denote by $Q(t)$ the total number of customers in the system at instant t^-, let $\{t_k\}$ be the arrival instants of the superposed input (Poisson) process and $Q(t_k) =: Q_k$. Then the regeneration instants $\{T_n\}$ of the process $\{Q(t),\, t \ge 0\}$ are recursively defined as

$$T_{n+1} = \inf_k (t_k > T_n : Q_k = 0),\ n \ge 0, \tag{2.1}$$

$(T_0 := 0)$, and the regeneration instants of the embedded process $\{Q_n\}$ are

$$\theta_{n+1} = \inf(k > \theta_n : Q_k = 0),\ n \ge 0\ \ (\theta_0 := 0). \tag{2.2}$$

The generic regeneration period, that is the distance between two arbitrary adjacent regeneration points, is denoted by T (for continuous-time construction (2.1)) and by θ (for discrete-time construction (2.2)). If the mean $\mathsf{E}T < \infty$ then the regenerative process $\{Q(t)\}$ (and the queueing system) is called *positive recurrent* (stable), and it implies the existence of the stationary process $Q(t) \Rightarrow Q$, $t \to \infty$ ($\Rightarrow$ denotes convergence in distribution). If $\mathsf{E}T = \infty$ then the system is called *null-recurrent* or unstable. By the (stochastic) equality $T =_{st} \tau_1 + \cdots + \tau_\theta$, it follows by the Wald's identity that $\mathsf{E}T = \mathsf{E}\theta\,\mathsf{E}\tau$, where both sides of the equality are finite/infinite simultaneously [6], implying that both processes $\{Q(t)\}$ and $\{Q_n\}$ are positive recurrent/null-recurrent simultaneously.

3. Stability analysis

Denote by

$$\rho_i = \lambda_i/\mu_i,\ \rho = \sum_{i=1}^{N} \rho_i. \tag{3.1}$$

Below we present a new and short proof of the following statement.

Theorem 3.1. *If $\rho < 1$ then the (initially idle) system under consideration is positive recurrent, $\mathsf{E}T < \infty$.*

Proof. Let $\{d_k,\ k \geq 1\}$ be the departure instances of the served customers leaving the system. Denote by $V_i(t)$ the total workload which class-i customers bring in the system in time interval $[0,\ t]$ and let $V(t) = \sum_i V_i(t)$. Moreover denote by $W(t)$ the remaining work in all orbits at instant t, and let $I(t)$ be the total idle time of the server in interval $[0,\ t]$. (All processes we consider are right-continuous with left-hand limits [15].) Finally, denote by

$$W(d_n) = W_n,\ V(d_n) = V_n,\ I(d_n) = I_n,\ n \geq 1.$$

By assumption, the first customer arrives at instant $t_1 = 0$ in the empty system (it is called *zero initial state*), and we obtain the following balance equation

$$W_n = V_n - d_n + I_n,\ n \geq 1. \tag{3.2}$$

We notice that the arrived in the interval $[0,\ d_n]$ class-i work can be written as

$$V_i(d_n) = \sum_{k=1}^{A_i(d_n)} S_k^{(i)},$$

where $A_i(d_n)$ is the number of class-i arrivals in $[0,\ d_n]$, $i = 1, \ldots, N$, $n \geq 1$. It is easy to find, using the Strong Law of Large Numbers and the property of the *cumulative processes* [17] that with probability 1 (w.p.1)

$$\lim_{n\to\infty} \frac{V_i(d_n)}{d_n} = \rho_i,$$

regardless of whether the system is stable or not, implying

$$\lim_{n\to\infty} \frac{V_n}{d_n} = \rho. \tag{3.3}$$

(Indeed it follows from theory of cumulative processes, that convergence in mean in (3.3) holds as well.) Now we assume, by a contradiction, that the system is null-recurrent that is $\mathsf{E}\theta = \infty$. (Then $\mathsf{E}T = \infty$ as well, see [6].) Note that θ is a generic number of arrivals and departures within a regeneration period of the system. By a characterization of the renewal process [15], it then follows from $\mathsf{E}\theta = \infty$ that the remaining regeneration time

$$\theta(n) := \inf_k(\theta_k - n : \theta_k - n > 0), \tag{3.4}$$

at instant n up to the next regeneration instant, increases to infinity *in probability*, that is

$$\theta(n) \Rightarrow \infty,\ n \to \infty. \tag{3.5}$$

Denote $Q(d_n) = Q_n$. Using a proof by contradiction we can show (see [15]) that $Q_n \not\Rightarrow \infty$ implies $\mathsf{E}\theta < \infty$. Thus it follows from (3.5) that $\mathsf{E}\theta = \infty$, and then we obtain as well

$$Q_n \Rightarrow \infty,\ \ n \to \infty. \tag{3.6}$$

Denote by $\Delta_k = I(d_{k+1}) - I(d_k)$ the idle time of server between the kth and $(k+1)$th departures. We note that, provided $Q_k \geq n$, the mean idle time of server after the kth departure is upper bounded by the constant

$$C_n := 1/(\lambda + n\gamma_0), \tag{3.7}$$

and $C_n \to 0$ as $n \to \infty$. (Recall that γ_0 is the minimal retrial rate.) This shows that, if $Q_k \geq n$, then $\mathsf{E}\Delta_k \leq C_n$ can be done arbitrarily small for n large enough $(n \geq k)$. Then one can show that for an arbitrary $\varepsilon > 0$

$$\mathsf{E}I_n \leq \varepsilon n(1 + 1/\lambda) + L,$$

where L is a constant. (For details see formulas (21)-(25) in the paper [13].) This implies that, under assumption (3.6),

$$\lim_{n\to\infty} \frac{\mathsf{E}I_n}{n} = 0. \tag{3.8}$$

We assume that if the nth customer entering server belongs to class i, then we assign the service time $S_n^{(i)}$ from the corresponding iid sequence $\{S_n^{(i)}\}$ initially intended for this class of customers. (In other words, we omit not used elements of this sequence.) Now we consider the 'minimal' service times realized by the server,

$$S_n^{(0)} = \min_{1\leq i\leq N} S_n^{(i)},\ n \geq 1.$$

These times constitute an iid sequence $\{S_n^{(0)}\}$ with generic element $S^{(0)}$. Now we define a random walk

$$\widehat{d}_n = \sum_{k=1}^{n} S_k^{(0)}, \ \ n \geq 1. \tag{3.9}$$

An important observation is that $\widehat{d}_n \leq d_n$, $n \geq 1$. Now we can write

$$\frac{I_n}{d_n} \leq \frac{I_n}{\widehat{d}_n} = \frac{I_n}{n}\frac{n}{\widehat{d}_n}, \ \ n \geq 1.$$

On the other hand, we have, from the renewal theory, that w.p.1,

$$\lim_{n\to\infty} \frac{n}{\widehat{d}_n} = \frac{1}{\mathsf{E}S^{(0)}}. \tag{3.10}$$

Then it follows from (3.8) that, as $n \to \infty$,

$$\frac{I_n}{n} \Rightarrow 0.$$

In turn, then there exists a subsequence $n_k \to \infty$, $k \to \infty$, such that

$$\lim_{k\to\infty} \frac{I_{n_k}}{n_k} = 0, \tag{3.11}$$

w.p.1, see [6]. Now we return to the balance equation (3.2) written for the subsequence $\{n_k\}$:

$$W_{n_k} = V_{n_k} - d_{n_k} + I_{n_k}, \ \ n \geq 1. \tag{3.12}$$

Note that $\widehat{d}_{n_k} \to \infty$ w.p.1 and, as in (3.10),

$$\lim_{k\to\infty} \frac{n_k}{\widehat{d}_{n_k}} = \frac{1}{\mathsf{E}S^{(0)}}.$$

Thus, w.p.1, as $k \to \infty$,

$$\frac{I_{n_k}}{d_{n_k}} = \frac{I_{n_k}}{n_k}\frac{n_k}{d_{n_k}} \leq \frac{I_{n_k}}{n_k}\frac{n_k}{\widehat{d}_{n_k}} \to 0. \tag{3.13}$$

Now we divide both sides of (3.12) by d_{n_k} and let $k \to \infty$. Because

$$\lim_{k\to\infty} \frac{W_{n_k}}{d_{n_k}} \geq 0 \tag{3.14}$$

(this limit exists because the r.h.s. limit of (3.12) exists), then we obtain that

$$\rho \geq 1, \tag{3.15}$$

implying a contradiction with the assumption $\rho < 1$. In other words,

$$Q(d_n) \not\Rightarrow \infty, \, n \to \infty,$$

and there exists a subsequence $\{z_k\}$, $z_k \to \infty$ and some $\varepsilon > 0$ and constant $C < \infty$, such that

$$\inf_k \mathsf{P}(Q(d_{z_k}) \le C) \ge \varepsilon.$$

We note that a 'regeneration' condition

$$\min_{1 \le i \le N} \mathsf{P}(\tau > S^{(i)}) > 0$$

in this system holds automatically because the input process is Poisson, see [15]. Then by a standard method [15] we can show that the remaining regeneration time (3.4) (measured in the number of the arrivals/departures within a cycle)

$$\theta(n_k) \not\Rightarrow \infty, \tag{3.16}$$

which in turn implies that the mean number of arrivals/departures within a regeneration cycle $\mathsf{E}\theta < \infty$. Finally,

$$\mathsf{E}T = \mathsf{E}\theta\, \mathsf{E}\tau < \infty, \tag{3.17}$$

and the proof of Theorem 3.1 is hereby completed. □

Remark 3.2. The prove given above is radically shorter and more intuitive than that have been obtained in previous works [9, 11, 15]. This also relates to the 2nd step of the stability analysis describing the so-called 'unloading' of the system after the step (3.16).

Remark 3.3. It follows from (3.17) that, under assumption $\rho < 1$, the continuous-time processes and the embedded processes (both at the instants $\{t_n\}$ and $\{d_n\}$) are positive recurrent.

Assume that the system has $m \ge 1$ identical servers. In this system definition of the regeneration points remains the same as in (2.1). In this case, in the balance equation (3.2), d_n is replaced by md_n and the total idle time of all servers in interval $[0, d_n]$ becomes

$$I_n = \sum_{j=1}^{m} \sum_{k=1}^{n-1} \Delta_k^{(j)},$$

where $\Delta_k^{(j)}$ denotes the idle time of server j after the kth departure. The extension of stability analysis to this case is straightforward, and the proof of the following Theorem 3.4 follows mainly the same lines as the proof of Theorem 3.1.

Theorem 3.4. *If $\rho < m$ then the (initially idle) system is positive recurrent, $\mathsf{E}T < \infty$.*

4. Extensions to the system with outgoing calls

Now we consider a single-server system with *outgoing calls*. Keeping the main notation, we assume that, provided the server is idle, it generates an outgoing class-i call with rate ν_i, and there are K different types of such calls. Denote the total rate by $\nu = \sum_{i=1}^{K} \nu_i$. The service times (durations) of class-i calls are iid $\{Z_n^{(i)}, n \geq 1\}$ with the mean $\mathsf{E}Z^{(i)} < \infty$, $i = 1, \dots, K$. (We omit serial index denoting a generic element of an iid sequence.) In this system the balance equation, again considered at the departure instants $\{d_n\}$, becomes

$$V_n + \mathsf{Z}_n = W_n + d_n - I_n, \tag{4.1}$$

where Z_n is the workload generated by the outgoing calls in the interval $[0, d_n]$. It is assumed that the service of an outgoing call is not interrupted by a newly arrived external customer. Note that in this case the upper bound C_n in (3.7) is modified as follows:

$$C_n = \frac{1}{\lambda + n\gamma_0 + \nu}.$$

Now we denote

$$S_n^{(0)} = \min_{1 \leq i \leq N} S_n^{(i)}, \qquad Z_n^{(0)} = \min_{1 \leq i \leq K} Z_n^{(i)} \; n \geq 1,$$

and redefine the instants $\widehat{d}_n$ (see (3.9)) as

$$\widehat{d}_n = \sum_{k=1}^{n} \min\{S_k^{(0)}, Z_k^{(0)}\}, \;\; n \geq 1.$$

As above, $d_n \geq \widehat{d}_n \to \infty$. Assume again that convergence (3.6) holds true. Then, as in Section 3, there exists a subsequence $d_{n_k} \to \infty$, $k \to \infty$, satisfying (3.11) (not necessary the same one). Rewrite the balance equation (4.1) as

$$V_{n_k} + \mathsf{Z}_{n_k} = W_{n_k} + d_{n_k} - I_{n_k},$$

and show that, as $k \to \infty$,

$$\frac{\mathsf{Z}_{n_k}}{d_{n_k}} \leq \frac{\mathsf{Z}_{n_k}}{\widehat{d}_{n_k}} \to 0 \quad \text{w.p.1.} \tag{4.2}$$

Denote by N_{n_k} the number of events in the Poisson process with rate ν in time interval $[0, I_{n_k}]$. Then it is easy to see that $N_{n_k} \geq_{st} \widehat{N}_{n_k}$, where $\widehat{N}_{n_k}$ is the number of actual outgoing calls generated in interval $[0, d_{n_k}]$. It is because some calls, among (maximally possible) number N_{n_k}, are 'lost' (if server transmits another outgoing call), and in result $\widehat{N}_{n_k}$ in general turns out to be less than N_{n_k}. Denote by v_n the work which call n brings in the system, $n \geq 1$. Then v_n is (stochastically) upper bounded as

$$v_n \leq_{st} Z_n^{(1)} + \cdots + Z_n^{(K)} =: \mathcal{Z}_n,$$

where iid $\{\mathcal{Z}_n\}$ have generic element $\mathcal{Z}$ with mean $\mathsf{E}\mathcal{Z} = \sum_{i=1}^{K} \mathsf{E}Z^{(i)} < \infty$. It now follows from above the following stochastic inequality

$$\mathsf{Z}_{n_k} = \sum_{j=1}^{\widehat{N}_{n_k}} v_j \leq_{st} \sum_{j=1}^{N_{n_k}} \mathcal{Z}_j. \tag{4.3}$$

We note that, on the event $\{\sup_k I_{n_k} < \infty\}$, it follows that $\sup_k \mathsf{Z}_{n_k} < \infty$ as well (because the number of the events in any finite interval is finite w.p.1), implying $\mathsf{Z}_{n_k} = o(\widehat{d}_{n_k})$, $k \to \infty$. Otherwise, on the event $\{\lim_{k\to\infty} I_{n_k} = \infty\}$, we can write, by (4.3)

$$\frac{\mathsf{Z}_{n_k}}{\widehat{d}_{n_k}} \leq \frac{1}{N_{n_k}} \sum_{j=1}^{N_{n_k}} \mathcal{Z}_j \, \frac{N_{n_k}}{I_{n_k}} \, \frac{I_{n_k}}{\widehat{d}_{n_k}},$$

and now (4.2) follows from (3.13) because, by the Strong Law of Large Numbers,

$$\lim_{k\to\infty} \frac{1}{N_{n_k}} \sum_{j=1}^{N_{n_k}} \mathcal{Z}_j = \mathsf{E}\mathcal{Z} < \infty,$$

and by the renewal theory,

$$\lim_{k\to\infty} \frac{N_{n_k}}{I_{n_k}} = \nu < \infty.$$

It now follows that (4.2) holds and, as in (3.14) (in notation (3.1)), we arrive to the contradictory condition (3.15). The above analysis can be summarized as the following statement.

Theorem 4.1. *If $\rho < 1$ then the initially idle retrial system with the outgoing calls is positive recurrent.*

5. Simulation

The purpose of the numerical result presented below (and based on the stochastic discrete-event simulation) is to demonstrate the *asymptotically work-conserving property* meaning that, as the orbit sizes increase, the dynamics of the service process becomes similar to that in the classic buffered system [9]. By this reason we consider the border of the stability region, that is $\rho = 1$, in which case the system becomes unstable. (An exception is the experiment shown on Fig. 7.) Also we analyze stability of the multiserver system (to illustrate Theorem 3.4) and we demonstrate simulation results for a three-server system. More exactly, we consider two-class retrial systems with input rates $\lambda_1 = 20/3$, $\lambda_2 = 10/3$ and with 1 and 3 servers, respectively.

To keep value $\rho = 1$, we take service rate $\mu = 10$ for 1-server system, and $\mu = 10/3$ for each server in 3-server system. Fig. 1 demonstrates a decreasing of the expected idle periods $\mathsf{E}\Delta_k$, as k increases, for exponential (Exp) and Pareto

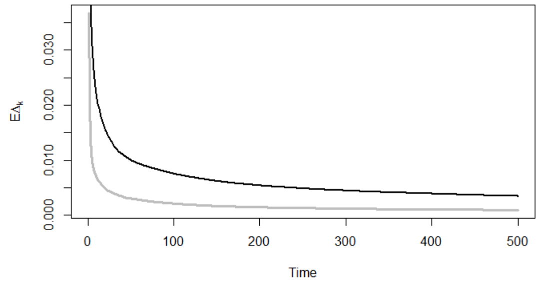

Figure 1. The expected idle time $\mathsf{E}\Delta_k$ vs. simulation time: Exp. service time (grey) and Pareto service time with $\alpha = 2$ (black); retrial rates $\gamma_1 = \gamma_2 = 30$.

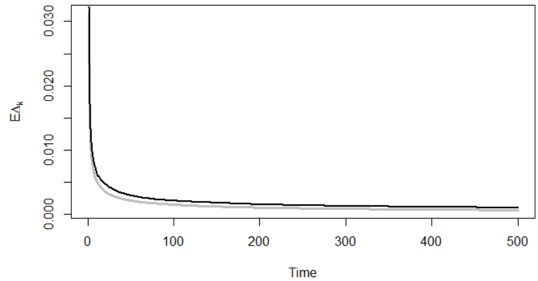

Figure 2. The expected server idle time in the system with outgoing calls vs. simulation time: Exp. service time (grey), Pareto service time (black); $\gamma_1 = \gamma_2 = 30$, $\nu = 30$.

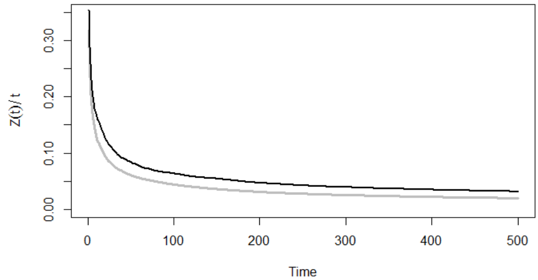

Figure 3. The workload of the outgoing calls with Weibull service time vs. modeling time: Exp. service time (grey), Pareto service time (black); $\gamma_1 = \gamma_2 = 30$, $\nu = 30$.

service time distribution

$$F(x) = 1 - \left(\frac{x_0}{x}\right)^{\alpha}, \quad x \geq x_0, \tag{5.1}$$

where parameter $x_0 = 0.05$ for 1-server system, and $x_0 = 0.15$ for 3-server system,

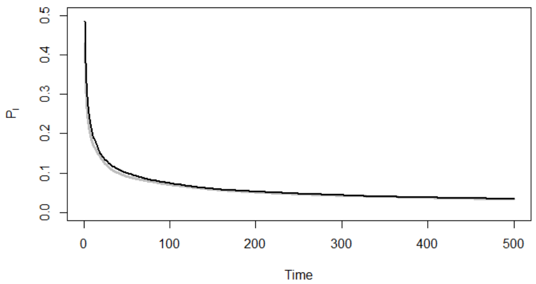

Figure 4. The idle server probability P_I in the original system vs. simulation time: 1 server (grey) and 3 servers (black); $\gamma_1 = \gamma_2 = 30$.

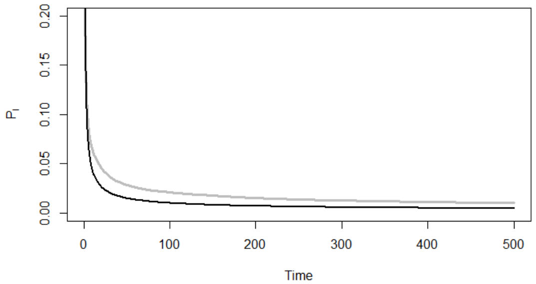

Figure 5. The idle server probability P_I vs. simulation time in the system with outgoing calls: 1 server (grey) and 3 servers (black); $\gamma_1 = \gamma_2 = 30$, $\nu = 30$.

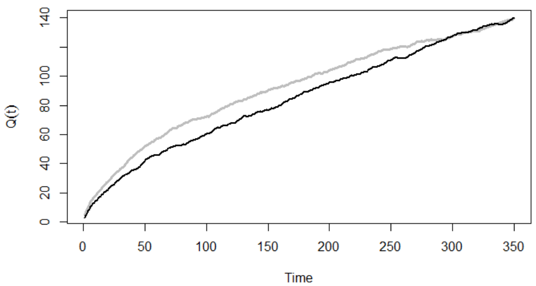

Figure 6. The orbit size in retrial system (grey) and buffer size in the buffered system (black), with Pareto service time, vs. simulation time; $\gamma_1 = \gamma_2 = 10$.

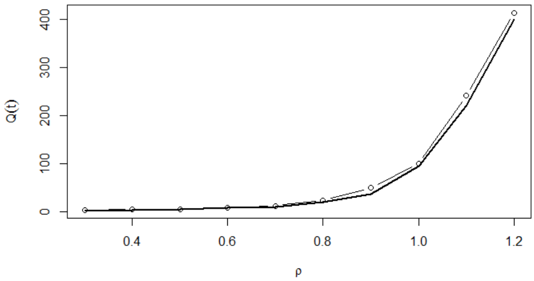

Figure 7. The orbit size in the original retrial system (dotted line) and the buffer size in the buffered system (solid line), with Pareto service time, vs. traffic intensity ρ; $\gamma_1 = \gamma_2 = 10$.

and parameter $\alpha = 2$ in all cases. Fig. 2 shows a similar result for the system with one class of the outgoing calls with rate $\nu = 30$. The service times of the outgoing calls have Weibull distribution

$$F(x) = 1 - \exp\Big\{ - \Big(\frac{x}{b}\Big)^a \Big\}, \ x \geq 0,$$

with the shape parameter $a = 0.9$, while the scale parameter $b = 0.1$ for 1-server system and $b = 0.3$ for 3-server system, respectively.

Thus Fig. 1 and Fig. 2 confirm the limit (3.11) and the asymptotic work-conserving property both for the original one -server retrial system and for the corresponding system with the outgoing calls, when $\rho = 1$. Fig. 3 describes the same effect expressed as a vanishing fraction of the workload generated by the outgoing calls, if $\rho = 1$, confirming the limit (4.2). Similar results given on Fig. 4, Fig. 5 show that the idle time fraction ('idle time probability') goes to zero in all considered systems, but this convergence is faster in the system with outgoing calls. Finally, we compare the orbit size in the retrial system and the buffer size in the corresponding buffered system with Pareto service time (5.1) (see Fig. 6, Fig. 7). In particular, Fig. 6 shows that (for $\rho = 1$) the orbit size initially increases faster than the buffer size but then they behave similarly. Fig. 7 also demonstrates the proximity between the orbit size and buffer size for the different values of the traffic intensity ρ (modeling time is 200 slots). Nevertheless, the orbit size is slightly larger than the buffer size, and it is an expected result.

6. Conclusion

In this work, we develop a new and short regenerative proof of the stability condition of a multiclass retrial system with classical retrials. Unlike previous proofs, we focus on the departures of customers and, provided the number of orbital customers increases, show a contradiction with a predefined negative drift condition, which turns out to be a sufficient stability condition. This approach is then extended to the system with outgoing calls, and it has a promising potential in the stability analysis of more general retrial systems. Some numerical examples based on the simulation are given which illustrate the asymptotically work-conserving property of the system with classical retrials.

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