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SETTING THE OPTIMAL SHEET THICKNESS DISTRIBUTION FOR PLASTICS THERMOFORMING BY MULTI-OBJECTIVE OPTIMIZATION

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1. Introduction

Thermoforming is a thermoplastic processing technique commonly used in the rigid packaging industry. The process comprises a heating stage, which aims at allowing the sheet to acquire the required deformability, a deformation stage, in which the sheets conform to the mould surface, and, finally, a cooling stage, which allows the part to be extracted from the mould without distorting. Since there are several processing variables associated with those stages, optimizing the thermoforming process is a complex task. In this work, a multi-objective optimization evolutionary algorithm is proposed to optimize the plastics thermoforming process. For that purpose, the thickness distribution of the final part was optimized considering that it is manufactured from uniform temperature sheets with different thickness distributions, such as constant and spline and concentric profiles. The aims were to minimize the sheet volume, as it implies less material use; assure a minimum value for the part thickness distribution, to avoid hindering its mechanical behavior; and minimize the thickness heterogeneity, i.e., the difference between the thickness of the part and a reference thickness. The Pareto optimal solutions found by the algorithm correspond to different thickness profiles for the three different sheet shapes. In all cases, an improvement of the different profiles along the successive generations of the evolutionary algorithm was obtained, which are related to the objectives considered. Moreover, the initial sheet thickness distribution was found to clearly influence the optimization process. The results obtained for these three different initial sheet shapes indicate that the proposed methodology is valid, providing solutions with physical meaning and with great potential to be applied in more complex cases.

2. Thermoforming optimization

In plastic thermoforming, the most important objective when a part is being produced is to obtain a uniform final thickness, since, due to the specificities of the process, different thicknesses at different regions of the part can be achieved. The regions where more deformation occurs during the shaping phase will have a smaller thickness. Thus, it is possible to change the thickness of the original sheet to compensate for this effect. In the present study, a square cup was thermoformed with constant temperature, a female mould and three types of sheets, as illustrated in Figure1.

Figure 1- Types of sheets that can be used: (A) constant thickness; (B) spline thickness variation on x-direction and (C) concentric thickness variation (adapted from [2] under an open access Creative Common CC BY license).

The aim is to determine the sheet thickness profile in order to: i) minimize the initial sheet volume, as it implies less material use (f_1); ii) minimize the minimum thickness found in the cells of the mesh used in the modelling calculations without hindering its mechanical behaviour, as it is related with the capacity of the polymer sheet deformability, representing indirectly a measure of the thickness heterogeneity (f_2); and iii) minimize the thickness heterogeneity, *i.e.*, the difference between the thickness of the part and a reference thickness (f_3): $f_3 = \frac{1}{M}\sum_{i=1}^{M}\frac{|t_0-t_i|}{t_0}$, where M is the number of points located in a line defining the centre of the cup in direction x, t_0 is a reference thickness defined by the user and t_i are the thicknesses of the M points. The following constraints in decision variables and objectives (dimensions in meters) are imposed: $2.0 \times 10^{-3} \leq x_i \leq 4.0 \times 10^{-3}$ and $Minimum\ thickness \geq 1.0 \times 10^{-4}$ where x_i is sheet thickness of the constant thickness along the *x-direction*. The thickness along the *x-direction* is then imposed using a spline variation based on 10 control points, or the thickness of 5 control points determining the concentric thickness variation, from the centre to the border.

Three bi-objective problems were considered with objectives f_1 and f_2, (Cases 1 to 3), one for each case of sheet thickness variation. Two other bi-objective problems using objectives f_1 and f_3, were also considered (Cases 4 and 5), respectively, with spline and concentric variations.

3. Optimization results

The multi-objective evolutionary algorithm used in this work is based on the SMS-EMOA [1] and implemented in MATLAB. Considering the characteristics of the multi-objective optimization problem being solved, different configurations and search mechanisms can be adopted for SMS-EMOA. Gaussian mutation with covariance matrix adaptation was selected as the variation procedure since the problem being solved comprises continuous variables. Simultaneously, the hypervolume metric was chosen to provide well-distributed alternative solutions in the objective space. The configuration, including the parameters values of the algorithm, considers the computational effort required to compute the objective function values. The population size was set at 20 individuals. The selection is done using a uniform distribution and variation is performed by the CMA evolution strategy operator, which is designed to work with real number representations. The maximum number of generations was set to 20.

Figure 2 shows the Pareto optimal solutions for all the cases studied. In Case 3 (concentric spline) the optimization converges to five non-dominated solutions, identified by black dots. The sheet and final part thickness profile of solutions Ps1, Ps4 and Ps7 are illustrated in Figure 3 (left). The black dots identify the location of the points used to generate the symmetrical concentric spline represent by a dashed line, the decision variables. As can be seen in the graphs the thickness profiles perpendicular to the spline when moving from solutions Ps1 to Ps7 the final part profile is more uniform. The final part thickness profile of four solutions for Case 2

(Spline), Pc1, Pc2, Pc4 and Pc5 are represented in Figure 3 (centre). Again, from solution Pc1 to solution Pc5 the profile obtained is more uniform. Also, it is important to note that in this case, the part thickness profile is the same in all directions, as the sheet thickness presents an axisymmetric distribution. Figure 3 (right) shows the part thickness profile of the unique solution found for Case 1, Pf1. The profile obtained is very similar to that of solutions Ps7 and Pc5 in the previous cases.

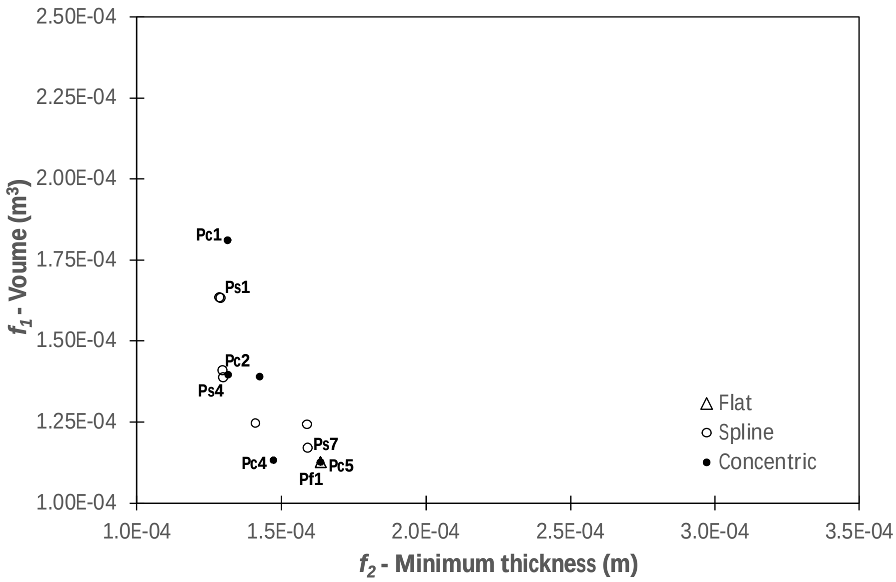

Figure 1- Non-dominated solutions for all cases: flat thickness, spline and concentric spline (with permission from [2] under an open access Creative Common CC BY license).

Finally, Figure 4 shows the non-dominated solutions for Cases 4 and 5, considering spline and concentric sheet thickness variation and t_0 equal to 0.5 mm. As previously, the gaps between the solutions are due to the problem constraints, related to limited search space. It is clear that, as expected, the concentric variation produces much better results concerning the uniformity of the thickness. Solutions P’s3 and P’c3 are the same as those obtained previously, i.e., solutions Ps7 and Pc5.

4. Conclusions

In this paper, a multi-objective optimization strategy was proposed to deal with the process forming phase. The aim was to determine a better initial sheet thickness distribution that allows the production of parts with the least amount of material while assuring the appropriate characteristics of the final part. The optimization process allowed a reduction of approximately 30% in the volume of the material used.

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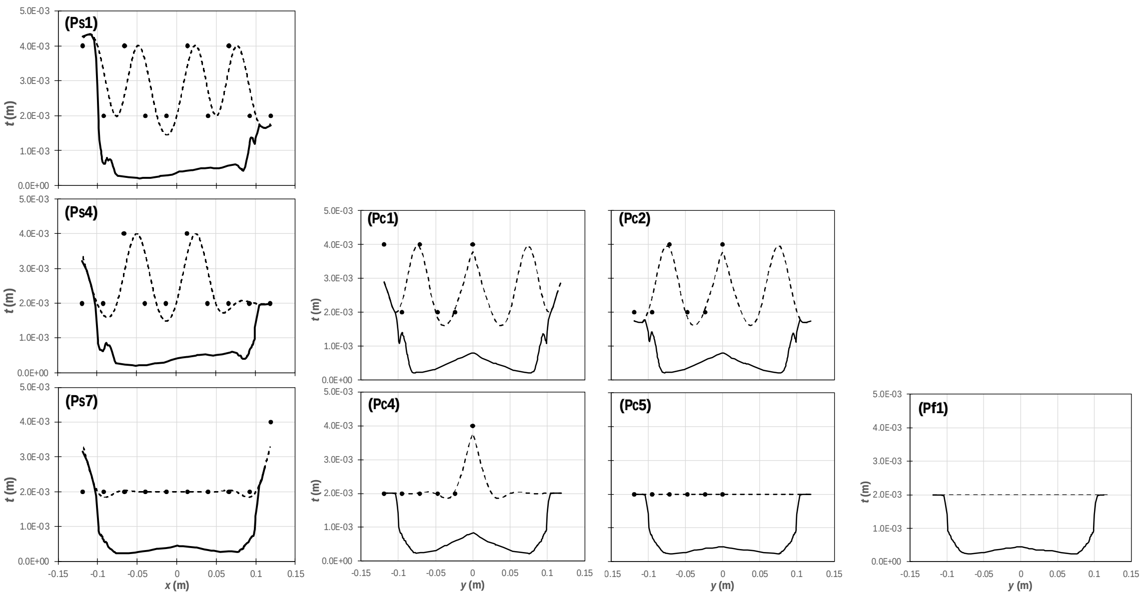

Figure 3- Thickness profile for solutions Ps1, Ps4 and Ps7 (left), Pc1, Pc2, Pc4 and Pc5 (centre) and Pf1 (right): black points are the decision variables, the dashed line is the spline and the continuous line the part thickness profile a x=0 ().

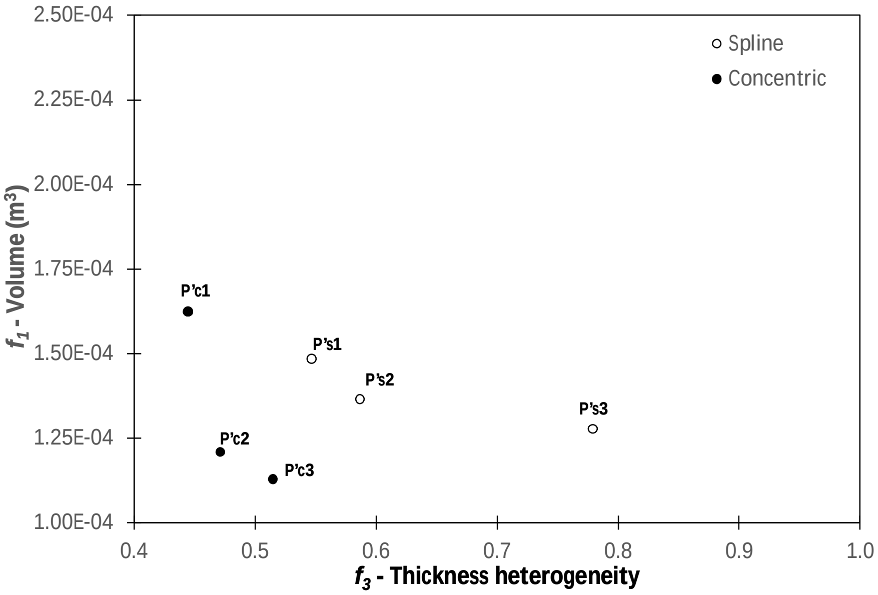

Figure 4- **N**on-dominated solutions for Cases 4 (Spline) and 5 (Concentric), t_0 equal to 0.5 mm.

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