



ELSEVIER

# *Journal of Policy Modeling*

A Social Science Forum of World Issues

Antonio Maria Costa  
*Editor-in-Chief*

Dominick Salvatore  
*Co-Editor*

*Board of Editors*

Shinichi Ichimura  
George Braga de Macedo  
Ronald I. McKinnon  
Edmund S. Phelps  
Alberto Quadrio Curzio  
Jeffrey D. Sachs  
Karl P. Sauvant  
Douglas O. Walker  
Larry E. Westphal

*Board of Advisers*

Jagdish Bhagwati  
Robert Dorfman  
Hendrik Houthakker  
Lawrence Klein  
Wassily Leontief†  
Edmond Malinvaud  
Robert Mundell  
William Nordhaus  
Joseph E. Stiglitz  
Jan Tinbergen†

**Administration**

**Creation Date :** 10-07-2012 06:51  
**Paper Status :** [Approved For JPM Publication](#)  
**Final Draft :** yes  
**EditAuthor**  
**EditName:** Claudio Socci  
**Author:**

**Full Name**

Maurizio Ciaschini  
Rosita Pretaroli  
Francesca Severini

**Contact Details**

[ciasco@unimc.it](mailto:ciasco@unimc.it)  
[pretaroli@unimc.it](mailto:pretaroli@unimc.it)  
[francesca.severini@unimc.it](mailto:francesca.severini@unimc.it)

**Paper:**

**Title of the Article :** Policies for electricity production from renewable sources: the Italian case  
**Hard Copy Submitted :** yes  
**Hard Copy # of Pages :** 17  
**Hard Copy # of Graphs :** 22  
**Hard Copy # of Tables :** 24  
**Digital Copy as Attachment :** no  
**Digital Copy on Diskette :** no  
**Categories** Economist Note  
**Keywords :** Environmental policy, Hybrid I-O model, Macro multiplier approach

**Comments :**

**Abstract :** The Italian Government introduced the Green Certificate market in order to stimulate the production of electricity form

Renewable Energy Sources (RES). The suppliers are obliged to produce a share of renewable electricity otherwise they must buy a number of certificates corresponding to the quota. The paper aims to quantify the economic impact of a reform on Green Certificate market through the Hybrid Input-Output. Moreover, through the singular value decomposition of the inverse matrix of the model, we identify the appropriate key structure able to obtain both the expected positive total output change and the increase of electricity production from RES.

**File Uploaded: :** [JPMarticle rev.pdf](#)

© 2013 - Society for Policy Modeling. All rights reserved.  
Published by [Elsevier Science Inc.](#)

*Policies for electricity production from renewable sources:  
the Italian case*

Maurizio Ciaschini, Rosita Pretaroli, Francesca Severini, Claudio Socci\*

*Department of Economics and Law*  
University of Macerata, ITALY

The Italian Government introduced the Green Certificate market in order to stimulate the production of electricity from Renewable Energy Sources (RES). The suppliers are obliged to produce a share of renewable electricity otherwise they must buy a number of certificates corresponding to the quota. The paper aims to quantify the economic impact of a reform on Green Certificate market through the Hybrid Input-Output. Moreover, through the singular value decomposition of the inverse matrix of the model, we identify the appropriate key structure able to obtain both the expected positive total output change and the increase of electricity production from RES.

**Keywords:** Environmental policy, Hybrid I-O model, Macro multiplier approach.

**JEL classification:** C67, E23, Q43, Q48.

---

\*Corresponding author e-mail: [socci.claudio@unimc.it](mailto:socci.claudio@unimc.it), Dipartimento di Economia e Diritto, P.zza Oberdan 9, 62100 Macerata-Italy; Phone +39 0733 258 2560, Fax +39 0733 258 3960. The authors gratefully thank the referees for their helpful comments.

## 1 Introduction

The Italian Green Certificates scheme (GC) represents one of the four Italian basic mechanisms implemented in 2002 after the liberalization of the electricity market, according to the energy market reform (legislative decree 79/99)<sup>1</sup>. In accordance with the Italian GC system, all suppliers or distributors of electricity - that lay on the network more than 100 GWh year - are compelled to produce a quota of renewable electricity in proportion to their extra sales: the quota is represented by the 2% of the excess in total production of electricity<sup>2</sup>. In other words, producers are obliged to produce or purchase a share of renewable electricity in proportion to their extra sales when they exceed the annual quota<sup>3</sup>. The production of green electricity is certified by the Italian Authority for the Energy Services (ESM) that emits the certificates, which represent the green quality of each unit of renewable electricity generation<sup>4</sup>.

A green certificates (GC) market can be organized following two different schemes depending on the identity of the agent that purchases the certificate property right. It might correspond to the energy producer and/or distributor as well as the final consumer<sup>5</sup>. In both cases, since every unit of renewable electricity generation is represented both by its physical output and its associated green value, a new market will be established alongside the traditional physical electricity market. A market where green certificates can be accumulated and eventually sold<sup>6</sup>.

The GC scheme aims to create a market where electricity from renewable sources can be sold with high margins of profit so that traditional electricity producing industries are stimulated to change their processes towards ways of production characterized by lower costs of production as well as lower emissions of  $CO_2$ <sup>7</sup>.

As an incentive for the use of renewable energy sources (RES) in electricity production processes, the GC scheme can be conceptually referred to the general issue concerning policy instruments in markets affected by externalities. In presence of negative externalities, such as costs of pollution, Government can restore economic efficiency using command-and-control regulations,

<sup>1</sup>The other mechanisms introduced after the liberalization of the market are respectively: energy account both for solar photovoltaic and thermodynamic; grants from EU, National and Regional Governments; voluntary certification of quality.

<sup>2</sup>The quota has been incremented: +0.35% from 2004 to 2006 and +0.75% from 2007 to 2012.

<sup>3</sup>Many other reforms modified the Italian GC system during the last decade: financial law 2008, D.M. 18/12/2008 and law 99/09.

<sup>4</sup>At present, the market of GC and its development represents a crucial tool in the recent European energy policy, which fixed an ambitious goal: the increase of 20% in the energy production by renewable sources for the year 2020 (Telli et al. 2008).

<sup>5</sup>This mechanism supposes that energy consumers (households and firms) are responsible for environmental damage and gives the possibility to consider the generation of electricity from renewable sources. This setting is adopted in Denmark but it is also characterised by lofty transaction costs that make it unpopular within consumers judgment. According to the first scheme energy producers and/or distributors receive green certificates equivalent to the amount of renewable electricity produced. The policy maker imposes a quota of renewable electricity to suppliers in proportion to their sales. The operators that are subjected to the quotas have two possibilities to respect their quota: producing themselves the quantity of renewable electricity buying new technologies or, in alternative, buying each year the certificates corresponding to the quotas. The choice between this two arrangements depends on the opportunities to get a revenue from the certificate trading.

<sup>6</sup>In this respect the GC mechanism facilitates trade of green electricity since the obligation may be fulfilled by buying GC either together with electricity from renewable sources or separately.

<sup>7</sup>It is commonly known that the potential of renewable supply energy is very high. No resource constraints exist for solar, wind, geothermal and wave, but the expansion of the hydro energy production is limited and there is no consensus as regards the limits for sustainable bio energy (Stoutenborough and Beverlin 2008, Haug 2007).

or in alternative, market-based policies (Parry 2002). These approaches include taxes on Greenhouse Gas emissions by firms and subsidy programs that are known as policy instruments for dealing with externalities (Baumol and Oates 1988). As an alternative to taxes and subsidies, which usually are discouraged because of their potential consequences on income distribution between Household groups, there exist many other market-based instruments such as GC system that avoid the direct Government involvement (Goulder et al. 1999). Most European countries adopted a set of economic instruments based on price regulation mechanisms (feed-in tariffs)<sup>8</sup> or quantity regulation mechanisms (tradable energy quotas or green certificate)<sup>9</sup> to encourage the production of RES electricity (Salerian et al. 2000, Carter et al. 2012). Nevertheless neither the economic theory nor the practical experience in either the practice of green certificates and that of feed-in tariffs can suggest a clear advantage of one instrument over the other even though both are distinct in terms of cost-efficacy<sup>10</sup>.

Under this aspect, the element that becomes more prominent is represented by the interaction between policy on RES and the climate change policy. It has to be stressed that the mechanisms of GC do not directly determine an environmental benefit in terms of reducing  $CO_2$  emissions (Amundsen and Nese 2009, Vlachou et al. 1996). However, the promotion of RES can be justified by the environmental improvement obtained each time the production process of energy will replace fossil fuels with renewable sources. Moreover the need of promoting renewable energy sources in electricity generation allows considering the development of the market for GC as an opportunity to achieve economic objectives as the positive change in total output (or GDP). From that point of view, the policies designed to encourage RES usage through the green certificates system, might have major economic relevance in terms of positive impact on industrial production because of the existence of multisectoral interdependency between all components of total output. Since the level of demand for green certificates is imposed by Government through the definition of a predetermined target, a policy establishing a higher target may lead both to a positive change in industrial output and a better balance between renewable and non-renewable energy.

In other words the use of the policy instrument GC with the aim of reaching a predetermined environmental policy target, as a rise in output of energy from RES, poses a problem of economic sustainability, as the evaluation of the impact on GDP, as well as a problem of neutrality with respect to other policy instruments as for example the exogenous component of final demand.

Stimulating renewable energy output through the introduction of regulations establishing shares for energy from RES on total energy output, without reference to the general economic framework, i.e. without restructuring exogenous final demand, can compromise the expected

---

<sup>8</sup>Used in Germany, Spain, France and Portugal.

<sup>9</sup>United Kingdom, Italy, Belgium, Sweden, Netherlands and Denmark.

<sup>10</sup>Exchangeable quotas of green certificates were introduced in Netherlands, United Kingdom, Belgium, Italy, Denmark and Sweden only in 2001 for the electricity market (McKibbin and Wilcoxon 2009). For an extended analysis focused on institutional setting for green certificate in these countries see Schaeffer et al. (2000), Van Dijk (2003), Jensen and Skytte (2002). Recently the European Commission has strongly encouraged the adoption of these instruments in an harmonised way with the aim of limiting the cost of European policy by allowing the development of the renewable energy sources (EC 2004).

result both from the environmental and the general economic policy viewpoint.

The consequences might in fact imply that environmental policies do not generate a satisfactory result with respect to the resources involved and, furthermore, negative externalities could be induced whose sterilization could require further policy effort.

In terms of policy we then deal with two relevant targets:

- i) rise in energy output from renewable sources, that can be measured by the rise of the ratio between RES and total energy, both evaluated in physical terms, or the rise in the number of GC and of the exchange volume on the corresponding market.
- ii) compatibility of environmental policy with the industry framework in the attempt of associating the environmental policy with a policy of final demand designed to dampen the possible trade off between environmental and economic targets.

In this respect, the paper aims to quantify the economic impact of the GC market and the change in the renewable and non-renewable energy balance (Gallagher et al. 2003, Chontanawat et al. 2008). The object is to verify the effects of policies designed to promote energy from RES by means of the Hybrid multisectoral approach, which evaluates both the interdependence between all production processes and the relevance of each commodity in the whole system. The Hybrid Input-Output (I-O) model is the suitable tool in order to analyze the energy commodity that is characterized by non unitary pricings, which are ruled by regulation in primary and final markets. This feature is inconsistent with traditional Input-Output approach which assumes unitary pricing across all commodities (Dietzenbacher and Stage 2006)<sup>11</sup>. In this case, since the flows of energy commodity would be assessed in monetary terms the presence of administered pricings would lead to ambiguous results (Lahr 1993). Furthermore, the hybrid I-O is particularly useful in order to evaluate effects of policies designed for the GC market where the governmental quota is expressed in physical terms (GWh year).

This approach allows expressing the flows in physical and monetary terms where the rows include flows measured in energy units (GWh) corresponding to energy deliveries. Thus by means of the hybrid I-O model it is possible to find the Leontief inverse, which can be used to compare the results between the innovative approach of the Macro Multipliers (MM) and the traditional analysis of multipliers (Ciaschini and Socci 2007). Through the MM approach that is based on the decomposition of the inverse matrix of the model, the key structure of the exogenous variable (final demand change) can be identified in order to obtain the expected total output change or the expected renewable and non-renewable energy balance (Ciaschini and Socci 2006). In fact, since the results of the traditional multipliers analysis are affected by the unrealistic structure of the exogenous shock (Ciaschini et al. 2009), the Macro Multipliers analysis overcomes this limit by the singular value decomposition (SVD) of the Leontief inverse. Furthermore the MM approach allows for the identification and quantitative determination of the aggregated Macro Multipliers (MM), which lead the economic interactions, and the key structures of macroeconomic variables that either hide or activate these forces.

---

<sup>11</sup>The hybrid Input-Output model is commonly applied to analyse the impact of environmental and energy policies because it usually overcomes the limits of a monetary evaluation of the commodity flows (Miller and Blair 2009).

For this purpose, section two of this paper illustrates the hybrid I-O model based on the Input-Output table for Italian economy for the 2005 (EUROSTAT 2008, 2009) which is integrated with data on energy demand from the RES demand in physical terms (GWh) (ISTAT 2007). The third section describes the innovative MM approach based on the Singular Value Decomposition of the inverse matrix of the Hybrid I-O model. In the fourth section the results of the policies are shown. In particular we will implement the empirical simulation focusing on three different scenarios all of them oriented towards the attainment of a complex target in which both the environmental and the economic impact are jointly evaluated. In all scenarios the commitment to produce energy from renewable sources is put together with the rearrangement of the policy control - the exogenous final demand - to be evaluated on the target vector. This is given by total industry outputs, which are expressed in value terms, and energy from renewable sources, expressed in physical terms. The first scenario is based on an exogenous shock on final demand that has the same structure of the demand vector observed in the IO table. The second scenario reproduces an exogenous shock on final demand according to the dominating key structure suggested by the MM approach. This type of policy, that is oriented to achieve the maximum output change, might realize a more satisfactory result with reference to the balance between energy production through fossil fuel and renewable electricity. The third one aims to quantify the impact on both the balance between energy from renewable and non-renewable sources and output change when the exogenous shock is modelled according to a policy control structure oriented to reach the maximum change of RES production.

## 2 Hybrid Input-Output model

The Hybrid I-O model as well as the traditional I-O approach allows for the evaluation of the effects of a final demand change on the economy as a whole, given the structural interrelations among industries (Polenske 1976). The hybrid approach also allows evaluating the effects of a policy of reform, but modelled in physical and monetary terms (Miller and Blair 2009).

The hybrid model is built putting together the economic variables describing the industry production process, whose variables are traditionally quantified in terms of current values, with those variables, that can be considered environmental, relating to the obtainment of energy, evaluated in physical quantities. Correspondingly the model will simultaneously generate an articulated policy that involves both an "economic" outcome, whose variables are determined in value units, and an environmental outcome, whose results are determined in physical units.

The Hybrid I-O model then refers to  $n$  commodities where  $n = m + k$ . The first  $m$  commodity flows are evaluated in current prices euros, according the traditional I-O practice, while the remaining  $k$  commodities are expressed in gigawatt hours (GWh). Thus, the total requirement of energy by commodity, the "energy intensity" in GWh, can be easily determined, simultaneously with the industry outputs, by solving the Hybrid I-O model.



The fundamental equation of the model is given by:

$$\mathbf{x}^* = \mathbf{A}^* \mathbf{x}^* + \mathbf{f}^* \quad (1)$$

vector  $\mathbf{x}^*$  is the output vector and its elements are all expressed in monetary terms (€) with the exception of energy commodity, for which we get also an evaluation in physical terms (GWh). The same detail is adopted for the elements of vector  $\mathbf{f}^*$  that is the vector of the hybrid final demand. Moreover,  $\mathbf{A}^*$  is the matrix of the hybrid technical coefficients that can be defined as:

$$\mathbf{A}^* = \mathbf{B}^* (\hat{\mathbf{x}}^*)^{-1} \quad (2)$$

where matrix  $\mathbf{B}^*$  is the hybrid matrix of I-O intermediate flows. Matrix  $\mathbf{B}^*$  is  $n \times n$  matrix and can be defined as the following:

$$\mathbf{B}^* = \begin{cases} \mathbf{b}_m^* \\ \mathbf{b}_k^* \end{cases}$$

where  $\mathbf{b}_m^*$  is a block of commodity flows which is expressed in monetary values and block  $\mathbf{b}_k^*$  is the block of energy commodities which is expressed in physical quantities. Analogously the  $n \times 1$  vectors of total output  $\mathbf{x}^*$  and final demand  $\mathbf{f}^*$  can be partitioned as:

$$f^* = \begin{cases} f_m^* \\ f_k^* \end{cases}, \quad x^* = \begin{cases} x_m^* \\ x_k^* \end{cases}$$

where blocks  $\mathbf{f}_m^*$  and  $\mathbf{x}_m^*$  are also evaluated in monetary values and blocks  $\mathbf{f}_k^*$  and  $\mathbf{x}_k^*$  in physical terms. The matrices blocks whose elements are expressed according their own measurement unit, can be represented as follows:

$$\mathbf{B}^* = \begin{bmatrix} \text{€} & \text{€} \\ GWh & GWh \end{bmatrix}, \quad \mathbf{f}^* = \begin{bmatrix} \text{€} \\ GWh \end{bmatrix}, \quad \mathbf{x}^* = \begin{bmatrix} \text{€} \\ GWh \end{bmatrix}$$

In this respect, according equation 2, blocks of matrix  $\mathbf{A}^*$  will result as ratios of flows quantified as follows:

$$\mathbf{A}^* = \begin{bmatrix} \text{€}/\text{€} & \text{€}/GWh \\ GWh/\text{€} & GWh/GWh \end{bmatrix} \quad (3)$$

Therefore, the solution of the hybrid model is expressed by the equation:

$$\Delta \mathbf{x}^* = [\mathbf{I} - \mathbf{A}^*]^{-1} \Delta \mathbf{f}^* \quad (4)$$

that describes the relation between the change on policy control (final demand change,  $\Delta \mathbf{f}^*$ ) and the resulting change in the objective variable (total output change,  $\Delta \mathbf{x}^*$ ). The inverse matrix

can be defined as:

$$\mathbf{R}^* = [\mathbf{I} - \mathbf{A}^*]^{-1} \quad (5)$$

and represents the Leontief inverse of the hybrid model, which quantifies the direct and indirect effects of final demand on total output.

### 3 Macro Multiplier approach

Traditional analysis, based on the inverse matrix  $\mathbf{R}^*$ , gives a complete picture of the economic connections, both direct and indirect, between the variables represented in the model (Round 2003). However, the predetermined structure of the exogenous shock, which must be adopted when the traditional multipliers and linkage analysis is performed, represents an important shortcoming that has led a major part of the literature to advise against this approach (Skolka 1986).

In order to avoid the main criticisms associated to traditional analysis, in this paper we use the Macro Multiplier (MM) for identifying the most convenient structure of the policy control (final demand for renewable energy) through which the actual policy shock on the economy can be modeled. The innovative MM approach that is based on the Singular Value Decomposition of the Leontief inverse, can identify the most efficient structure (or a desired structure) of the control variable that generates the highest effect (or the desired one) in the policy variable (Ciaschini et al. 2009). All policies designed starting from matrix  $\mathbf{R}$  are heavily characterized by the structure of both the exogenous policy control vector, whose role is that of a policy instrument, and the policy target vector, on which we observe the effects. In this respect, the possibility of considering the scale effect of the whole policy control in conjunction with the its composition effect becomes crucial in designing the actual policy variable (Ciaschini 1989).

Matrix  $\mathbf{R}^*$  can be decomposed through the Singular Value Decomposition (Lancaster and Tiesmenetsky 1985) and rewritten as the product of three different matrices:

$$\mathbf{R}^* = \mathbf{ZMP}^T \quad (6)$$

Matrix  $\mathbf{Z} = [\mathbf{z}_1 \dots \mathbf{z}_m]$  is a unitary matrix of dimension  $m \times m$  whose columns represent the structures of the policy-target variable (i.e. total output) through which all the results are observed and evaluated. For this reason we define these as the key-structures of the policy-target. Matrix  $\mathbf{P} = [\mathbf{p}_1 \dots \mathbf{p}_n]$  is a unitary matrix of dimension  $n \times n$  whose rows represent the structures of the policy-control variable. Since these structures represent the composition of the policy-controls we define them as key-structures of the policy-control. Finally matrix  $\mathbf{M}$  is an  $m \times n$  diagonal matrix with all elements equal to zero outside the diagonal. The elements along the diagonal represent aggregate multipliers, which are all real, positive and ordered according their magnitude as:  $m_1 \geq m_2 \geq \dots \geq m_p \geq 0$ .

The structures identified play a fundamental role in determining the potential behaviour of the

economic system: we can in fact evaluate the different impacts on total output (target) of a final demand vector (control) of predetermined unit scale, which however changes its inner structure. This is done taking into account that matrix  $\mathbf{R}$  hides the fundamental combinations of the policy variables, the policy key structures. The unit control impact will be then determined multiplying the corresponding combination of final demand, control key structure, by a predetermined scalar, which plays in fact the role of aggregated multiplier (Ciaschini et al. 2009, 2010).

The decomposition of the inverse matrix of the Hybrid I-O model can be compacted as:

$$\mathbf{R} = [\mathbf{Z}_1 \mathbf{Z}_2] \begin{bmatrix} M_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^T \\ \mathbf{P}_2^T \end{bmatrix} \quad (7)$$

that is

$$\mathbf{R} = \mathbf{Z}_1 \mathbf{M}_1 \mathbf{P}_1^T \quad (8)$$

where  $\mathbf{M}_1$  is a  $rxr$  diagonal matrix where  $m$  are the non-zero Macro Multipliers.  $\mathbf{Z}_1$   $m \times r$  represents the first  $r$  columns of matrix  $\mathbf{Z}$  and is the orthonormal base in the objective space  $\mathbf{Z}(\mathbf{R})$ . In the same way  $\mathbf{P}_1$  ( $n \times r$ ) represents the first  $r$  columns of matrix  $\mathbf{P}$  and corresponds to the orthonormal base in the policy control space  $\vartheta(\mathbf{R})$ .

From this considerations it is possible to emphasize some interesting features of the decomposition proposed. If  $\mathbf{R}^T \mathbf{R} = (\mathbf{ZMP}^T)^T (\mathbf{ZMP}^T) = \mathbf{PM}^2 \mathbf{P}^T$  Macro Multipliers are the square root of  $\mathbf{R}^T \mathbf{R}$  eigenvalues, that is  $m_i = \sqrt{\lambda_i(\mathbf{R}^T \mathbf{R})}$ . Moreover the policy controls key-structures  $\mathbf{p}_i$  are obtained as eigenvectors of  $\mathbf{R}^T \mathbf{R}$ .

Similarly, if we consider  $\mathbf{RR}^T = (\mathbf{ZMP}^T)(\mathbf{Z} \cdot \mathbf{MP}^T)^T = \mathbf{ZM}^2 \mathbf{Z}^T$  Macro Multipliers can also be calculated as square root of  $\mathbf{RR}^T$  eigenvalues, that is  $m_i = \sqrt{\lambda_i(\mathbf{RR}^T)}$ . Moreover the vectors that represent the key structures of policy objective  $\mathbf{z}_i$  correspond to the eigenvectors of  $\mathbf{RR}^T$ .

It is worthwhile to mention that the key structures of policy objective are different from the key structures of policy control since the matrix  $\mathbf{R}$  is not symmetrical.

We can write:

$$\mathbf{R} \mathbf{p}_1 = m_1 \mathbf{z}_1 \quad (9)$$

where  $\mathbf{p}_1$  corresponds to the dominating key structure of policy control and  $\mathbf{z}_1$  is the corresponding key structure of the policy target<sup>12</sup>.

Once determined the set of key structures both for the policy control variable and the policy target variable, it is necessary to focus on some methodological aspects concerning the definition of a suitable measure of the aggregate value of each policy variable. This is done for a correct evaluation of the changes in the scale of these variables.

Given a vector that shows the value of the sectoral components of a macro variable, defining both the scale and the structure of the sectoral components of this macro variable, a delicate

---

<sup>12</sup>All methodological details about MM approach are defined in appendix B (Ciaschini and Socci 2007, Ciaschini et al. 2009, 2010, 2011).

question arises of defining the scale of the policy variable as a whole.

The matrices of the key-structures  $\mathbf{P}$  and  $\mathbf{Z}$ , that operate on the policy control to transform it into the policy target, have the ability to compress and expand vectors and this will result in a change in the vector's scale. The vector's scale can be determined according various aggregation criteria. Economists usually refer to the sum of the sectoral final demands in order to determine the total final demand which represents the scale of the final demand. However the axis rotation alters the vectors coordinates and the transformation is not uniform. Two vectors whose elements' sums are equal with reference to a system of coordinates (basis) will result into two vectors with different sums in a new system of coordinates. This is the reason why we need to refer also to a further aggregation criteria able to generate a set of vectors whose characteristics are neutral with the respect to axes rotation. In this case all changes in the vectors' scale can be correctly attributed to changes in the vectors structure. An aggregation criterion that overcomes these drawbacks is that of assigning to the vectors scale the value of its modulus:  $modulus(\mathbf{p}) = \sqrt{\sum_i p_i^2}$ . All the policy vectors that have the same modulus, by describing a circle whose radius corresponds to the modulus, are invariant with respect to rotations of the axis.

As we mentioned in multisectoral economics the most immediate aggregation criterion is represented by the sum of sectoral elements. If we consider that every single component can assume both positive or negative value - because they may represent the activity balances of some variables (foreign debt) or the modification of a pre-existing situation - we define this procedure synthetically as  $sum(\mathbf{p}) = \sum_i p_i$ . This aggregation procedure can be meaningful in simulation to determine the net balance within the policy variable as, for example, the net final demand change. One special case is the zero-sum policy where the aggregate change of the macro variable, final demand in our case, remains unchanged since all sectoral demand changes compensate within the same control-variable.

If the sum identifies the net balance it is however apparent that this criterion is unable to capture the amount of change detected within the policy variable since two vectors of equal sum may hide changes of relevant magnitude. An aggregation criterion that quantifies conveniently the real amount of resources that have been activated is represented by the sum of the absolute values of the vector components:  $abs\ change(\mathbf{p}) = \sum |p_i|$ . The absolute change of vector  $\mathbf{p}$  quantifies the amount of the policy maneuver in terms both of expansion and restraints imposed to sectors. In the income redistribution process for example, this measure indicates the total effort of higher revenues to maintain a certain level and the expansion of lower revenues. In the illustration of the results we refer to the absolute change as the suitable and convenient measure of the scale of a vector that shows the changes in sectoral components of either the output and the final demand both in monetary and physical terms.

#### 4 Policies for electricity production from renewable energy sources: Italian case

The application that we propose aims to evaluate the impact of a policy that stimulates the production of energy by means the production of energy from renewable sources. The analysis is based on the Italian I-O table for the year 2005 (ISTAT 2007) that has a disaggregation of 59x59 commodities. We integrated the I-O flows with data regarding the requirement of renewable energy per commodity in physical terms. Our manipulation on the Italian I-O table results in a 60x60 hybrid I-O table with 59 commodities and one commodity that represents the renewable energy source good.

The first block of the data base represents the flows of intersectoral commodity flows expressed in monetary terms apart from the flows of "renewable energy sources" that are expressed in euro and GWh<sup>13</sup>. The second block refers to final demand and the last row is headed to renewable energy sources flows. The I-O table is closed by the value added block and the row of imports which guarantee the correspondence between row and column totals. This new Hybrid I-O table represents the consistent data set to implement the Hybrid I-O model.

The original problem of the I-O model lies in the search of the output vector consistent with the final demand vector for I-O sectors, given the structural interrelation among commodities. Such a vector faces both the predetermined final demand vector ( $\mathbf{f}^*$ ) by commodity and the induced commodity demand. From the I-O matrix it is possible to derive the constant technical coefficients matrix ( $\mathbf{A}^*$ ) and the inverse of the model, obtained according equation 4, which shows the total requirements of commodity output per unit of final demand (exogenous variable).

In our case the policy target appears to be complex since it aims at the search of the compatibility between the environmental and the economic aspects. In particular it is necessary to verify the effect in physical terms on the output of energy from renewable sources, considering the corresponding trend of GC issued, and the induced changes in value added and GDP. This target is realized not only through the introduction of an output share of energy from renewable source imposed to the energy producers, but also through the compatibility check with the actual production structure. The incentive to be introduced has to be evaluated in a general economic framework and then it has to be coordinated with the other policy instruments with the aim of avoiding the possible negative externalities that may emerge. In the scenarios that will be presented the environmental policy will always be associated with a policy based on the restructuring of exogenous final demand, with exception of the first scenario. In this case we will show an environmental policy associated not with a restructuring of final demand but with the actually observed change in final demand.

In particular, the simulations experiments proposed are designed on three different scenarios: i) in the first a shock in final demand of 0.10% of the observed final demand value in 2005 is assumed in order to determine its impact on the balance between renewable and non-renewable

---

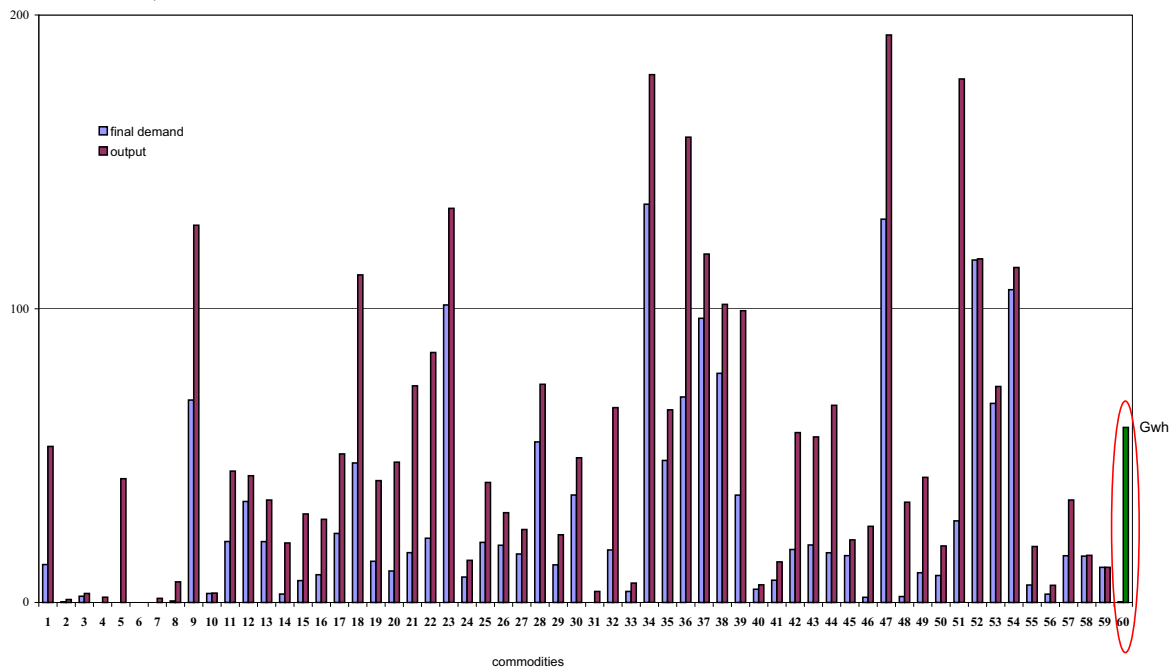
<sup>13</sup>The official statistics distinguish the total demand of energy from renewable energy sources expressed in GWh (ISTAT 2007), in intermediate consumption by commodity and final consumption. The total renewable energy sources production is 59,600 GWh.

electricity and total output. ii) in the second scenario the shock in final demand is kept at the same scale of the first scenario but its structure is put equal to that implied by the key structure in the dominating policy, put in evidence by the MM analysis. This scenario has the aim of achieving an increase on the balance between renewable and non-renewable energy with a maximum total output change. iii) in the third scenario the scale of the change in final demand is kept at the level of the previous scenarios while its structure is assumed equal to the key structure most favourable to renewable energy source production. This scenario has the aim of maximizing the ratio between renewable and non-renewable energy. This ratio is evaluated by the quotient between renewable energy output and total energy output (RNR ratio).

i) *First scenario: observed structures policy control*

The scale of the change in final demand for the first simulation amounts to of 1,683 million of Euros<sup>14</sup>, corresponding to the 1‰ of the "observed" final demand i.e. total final demand in the I-O table. From the point of view of its sectoral composition, the observed structure has been imposed to a change of such a scale. This structure, even if the observed, is not the "best" in terms of its economic performance as we will see further on in this paper. However the sectoral outcomes of the simulation experiment under the first scenario, displayed in figure 1, show a general positive impact on all output sectors with most relevant effects on the service sectors (sectors from 35 "Trade, maintenance and repair services of motor vehicles and motorcycles; retail sale of automotive fuel" to 59 "Private households with employed persons")<sup>15</sup>.

Figure 1. Sectoral impacts of a final demand change (scale: 1‰ of the observed scale; structure: observed structure  $f_{I-O}$ )



<sup>14</sup>The total change in final demand is determined as the sum of the absolute values of the vector elements. This figure represents the aggregate value of the policy.

<sup>15</sup>The classification of commodities is shown in appendix A, table A1.

In aggregate figures the results of this simulation are summarized in Table 1. The main economic and energetic indicators are shown in the second column of the table. The shock in final demand generates an increase in total output and a consequent change in value added which however remains still at 1,683 million of Euros, the same amount of the final demand policy shock. Energy output from renewable sources raises in absolute terms of 12 GWh<sup>16</sup>, but the ratio between renewable and non-renewable energy does not change in percentage terms due to a corresponding increase in non-renewable energy output. It remains in fact fixed at the level 16.89% before and after the policy shock. The final demand structure designed in this first simulation generates an increase in green certificates supply of 1.35%, and this can be interpreted as a positive result even though, as we have pointed out, the policy on the whole is "neutral" in terms of the change in both value added (0.10%) and renewable and non-renewable energy ratio.

The analysis performed under this first scenario provides policy recommendations for the design of an environmental policy constrained by the observed structure of final demand. In this case the impact on the policy target proves to be rather limited both from the environmental and economic standpoint. The change in final demand designed according the observed structure, i.e. according the structure of the observed data on final demand, as reported in the I-O table, puts in evidence that the environmental target has no tight connection with a final demand characterized by high demands towards the outputs of the services sectors. If relevant effects in environmental terms have to be realized a policy has to be designed towards those commodities that can generate relevant direct and indirect effects on the output of energy from renewable sources.

We need then to investigate which sectoral rearrangement of final demand, subject to the resources constraint of 1,683 million of euros, is able to generate the greatest effect on the environmental target without neglecting that on Value Added (GDP). In this sense such an effort is directed to the definition of a complex target where the traditional economic target (sectoral output and GDP) is determined and evaluated simultaneously with the environmental target (RNR ratio GC). In order to evaluate the relevance of the other policies that will be designed we can assume this first scenario as a benchmark.

Other two simulation scenarios have been identified in which the policy variable structures are oriented to a complex objective, as for example, the increase in total output or/and a better ratio between renewable and non-renewable energy.

ii) *Second scenario: policy control for economic target*

For this purpose in the second application we used the Macro Multiplier approach in order to design a more effective composition of the policy control according to the objective of the policy maker. The singular value decomposition of the inverse matrix of the model reveals and quantifies the key structures ( $m_i \cdot \mathbf{z}_i$ ) among which choose the most favourable for the selected output. Figure A1, in appendix A, illustrates all the 60 key structure. Only the first one, the dominating

---

<sup>16</sup>In figure 1 the change in energy production is circled.

Table 1. Aggregate results: comparison among the results of three simulations under different scenarios

POLICY CONTROL			
<i>Scale of the final demand change</i>	1‰ of the "observed" vector's scale	1‰ of the "observed" vector's scale	1‰ of the "observed" vector's scale
<i>Structure of the final demand change</i>	Observed Structure	Computed: Key Structure 1	Computed: Key Structure 51
Multiplier change <sup>(a)</sup>	1.874	2.005	1.514
POLICY TARGET			
RNR ratio <sup>(b)</sup>	16.89%	16.92%	17.05%
Green certificates variation	1.35%	3.32%	11.72%
Value added variation	0.10%	0.63%	-0.03%

<sup>(a)</sup>Determined as the ratio abs change(x) / abs change(f) value.

<sup>(b)</sup>Ratio between renewable energy output and total energy output.

key structure, allows to achieve a policy objective as that of maximizing the change on total output. This is the reason why the simulation scenario considers this structure while assuming the same scale of simulation (i) for the policy shock on final demand (0.10% corresponding to 1.683 million of euro). Figure 2 puts in evidence a comparison between the final demand structure adopted in simulation (i), the observed structure, and the structure chosen in simulation (ii), key structure 1 of policy control.

Figure 3 shows the sectoral results for simulation (ii) whereas the relevant results in aggregate terms are described by the third column of Table 1. According to key structure 1 of policy control, the increase of final demand of 0.10% generates a multiple effect higher than the previous scenario. Value added in fact rises of 0.63%. At the same time the ratio between renewable and non-renewable energy registers a slight improvement and the percentage reaches the 16.92%. This result depends on the increase in energy production from renewable sources that is equal to 147 GWh and generates an increase in green certificates emission (+3.32%). As it can be seen from table 1 key structure 1 on final demand generates the most favourable impact in terms of total output and value added increases. Moreover this policy generates a more suitable ratio between renewable and non-renewable energy.

The sectoral rearrangement of final demand suggested by this policy, involving the same amount of resources, is able to induce an improvement both in the economic and in the environmental performance with respect to the benchmark result. Here a first policy recommendation emerges for which we need to associate to the introduction of an amount of GC to associate a balanced arrangement of final demand, i.e. an arrangement that involves all types of commodities. This feature could emerge from the weak linkage of service outputs with energy output especially energy output from renewable sources. Moreover the adoption of the new arrangement of final demand allows to exclude the presence of trade offs between environmental and economic policy. In this case we can say that externalities are positive.



Figure 2. Comparison between two final demand changes with same scale (scale: 1‰ of the observed scale; structure: observed,  $\mathbf{f}_{I-O}$ , and computed,  $\mathbf{p}_1$ )

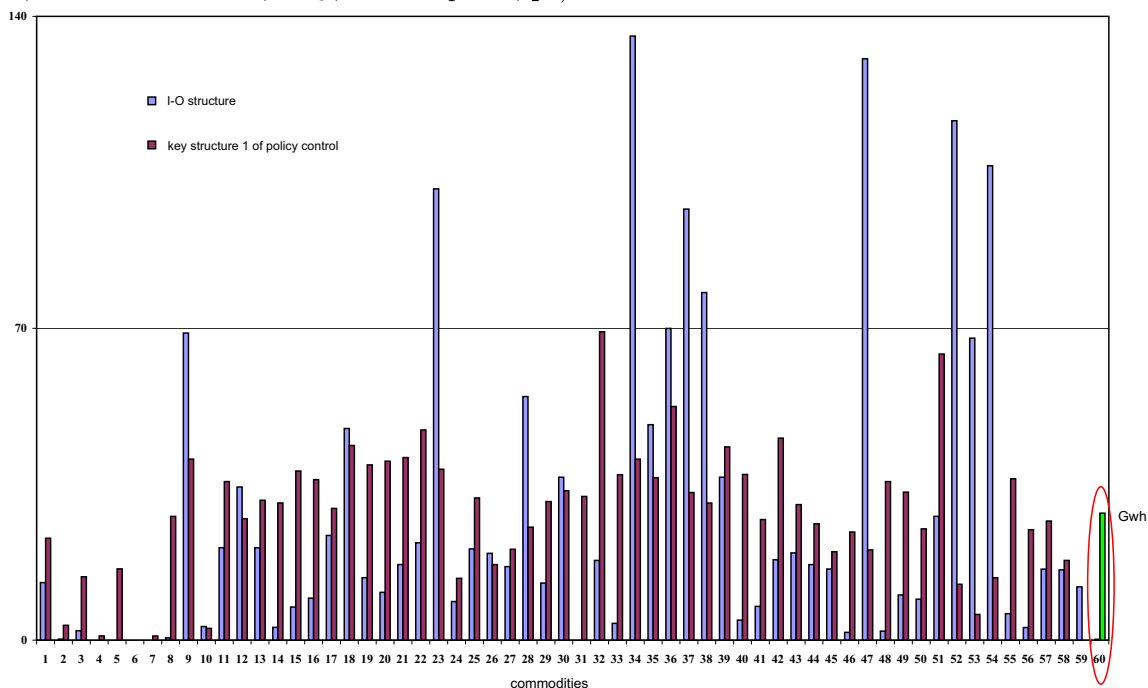
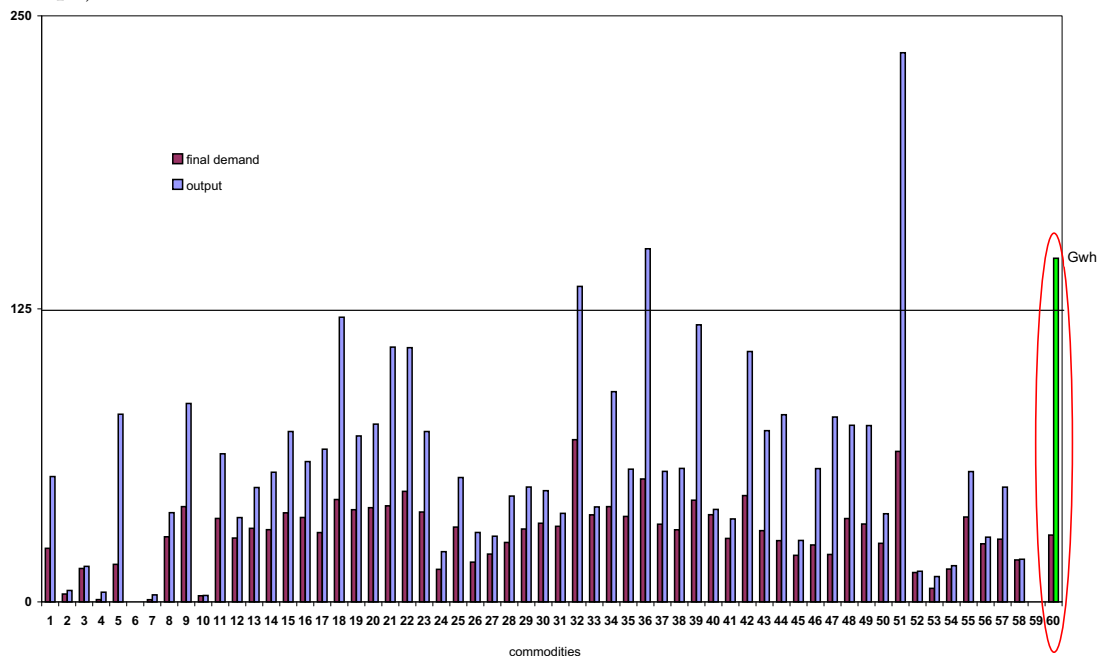


Figure 3. Sectoral impacts of a final demand change (scale: 1‰ of the observed scale; structure: computed structure  $\mathbf{p}_1$ )



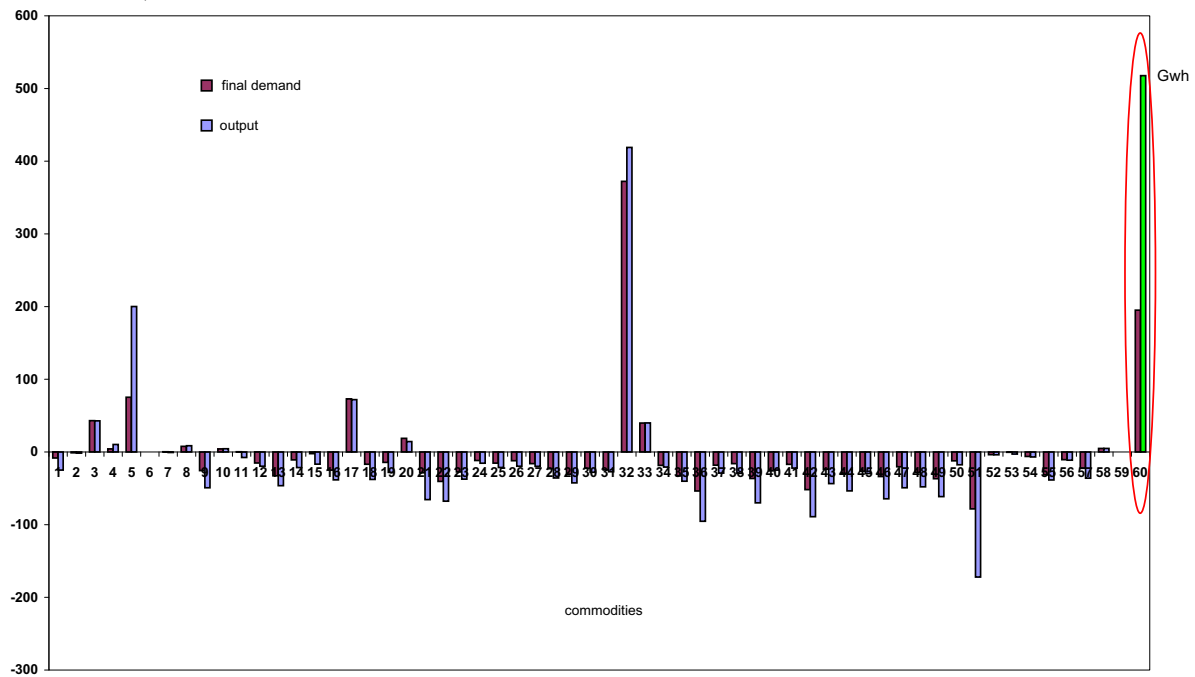
iii) *Third scenario: policy control for environmental target*

Finally, the third scenario aims to identify the final demand composition suitable for the best result in terms of ratio between renewable and non-renewable energy: in this case the policy maker aims to reach the maximum level of the environmental indicator. For this purpose the proper key structure of the policy control variable (final demand) among the 60 key structures described in figure A1, is the one that is activated by the key structure of the policy variable

that presents the highest effect on the production of renewable energy sources.

The structure consistent with this objective is the structure number 51<sup>17</sup>. Once the key structure has been identified ( $\mathbf{p}_{51}$ ) the final demand shock of 1,683 million of euro, the 1‰ of the "observed" final demand, will be formatted accordingly and the results are shown in figure 4. The environmental objective can be achieved only implementing a policy based on quite complex changes in final demand.

Figure 4. Sectoral impacts of a final demand change (scale: 1‰ of the observed scale; structure: computed structure  $\mathbf{p}_{51}$ )



The aggregate results of this application are summarised in the fourth column of table 1. Even if the total value added decreases of -0.03%, the ratio between renewable and non-renewable energy reaches the highest value: 17.05%. The production of renewable energy sources increases of 518 GWh and the supply of green certificates raises of 11.72%. This policy is then an environmental oriented policy that requires a predetermined composition of the final demand change in order to promote the production of renewable energy intensive commodities. This policy, in fact, generates the best results in terms of balance between renewable and non-renewable energy with a slight negative effect on the change of value added.

In this last case the sectoral rearrangement of final demand is designed in such a way to obtain the greatest possible effect from environmental policy with the minimal effect on output change. the rearrangement of final demand shows changes of alternating signs in order to stimulate those outputs that show an intensive use of energy in particular energy from renewable sources: 3, 4, 5, 8, 10, 17, 32, 33 and 59. Compared with the first policy we observe that the environmental policy is highly oriented towards traditional outputs unlike the first policy which is unbalanced towards services.

<sup>17</sup>In figure A1, appendix A, the structure 51 is different coloured with respect to the others.

## 5 Conclusion

The promotion of renewable energy sources in electricity production have increased in the last 20 years following the recent consideration of the environmental question. The concern for climate changes in fact led many countries to concentrate in designing optimal instruments to reduce Greenhouse Gas Emissions and face the environmental damage and depleting. Among all environmental policy instruments the promotion of renewable energy sources has received increasing favor from the public authorities and a special suggestion from the European Union.

A set of measures focused on encouraging energy efficiency and promoting renewable energy sources in electricity generation has been activated by Governments from the ending of Nineties. The liberalization of electricity market and the introduction of economic incentives when renewable energy technology is employed, are some examples of these measures. Germany, France, Spain and Portugal adopted policies based on feed-in tariffs while Italy, Belgium, Sweden, Netherlands, Denmark and United Kingdom implemented a system based on exchangeable quotas and tradable green certificates.

Economic theory and practical experience do not confirm the advantage of one instrument on the other, nevertheless the European Union has strongly promoted the adoption of exchangeable quotas in order to harmonize all national support scheme. In Italy the green certificate market has been introduced in recent times and there is no agreement on his effectiveness in terms of environmental and economic benefits. The renewable energy technologies in fact have not reached an adequate level of economic performance even though the production of energy from wind, solar and geothermal sources has been growing according to the emission of green certificates.

In this paper an effort was made both to analyse the relevance of renewable energy sources in electricity production and to find the convenient policy structure able to achieve different targets of the policy maker both environmental and economic. For this purpose we integrate the I-O data for the Italian economy with the statistics on renewable energy sources requirements by goods in physical terms and we implemented a Hybrid I-O model which was used to simulate three scenarios comparing the effects of a final demand change of the same amount using three different structures of the exogenous shock.

The attainment of a complex target where trade-off between the target variables can be put in evidence is a crucial feature for the policy maker. However the adoption of such an environmental policy presents two orders of difficulties: i) it is not indifferent with respect to economic structure on which it is activated, and ii) it can generate externalities that in some cases require sterilization actions.

In the analysis we have performed emerges that the most relevant effect from the environmental standpoint can be attained only through the search of a convenient restructuring of final demand that must be oriented to traditional outputs, outputs that can be considered intensive in the use of energy from renewable sources. This type of policy generates a weak negative externality in terms of a negative, however negligible, change in GDP. Moreover this negative externality disappears when in the evaluation we refer to GDP environment corrected.

If the actual level of GDP or total value added is taken as a target then a good balance between

energy from renewable and non renewable sources can be attained with only a negligible loss in GDP.

In particular, when supposing a change in final demand according the observed I-O structure (first scenario) the increase in final demand generates an increase in energy production from renewable sources and a consequent raise in green certificate emission. From environmental point of view this policy can be considered neutral but on the economic side, there is a small increase in value added.

A better economic and environmental performance takes place in the second scenario where the macro multiplier approach is used. Focusing on the identification of the policy structure able to reach the best results in terms of total output, the shock in final demand can be distributed according the first key structure showing a positive impact on aggregate value added and on balance between renewable and non-renewable energy. This is confirmed by an increase in green certificate exchange.

When the policy maker focuses on the environmental target, the key structure 51 is the most suitable policy for the production of renewable energy commodities. In this case (third scenario) the final demand shock creates an improvement in environmental performance and an increase in the supply of green certificates. This result is extremely significant if the aim of the policymaker is to encourage the production of renewable energy through the green certificate market. Nevertheless in this case a small negative impact on value added is detected which is of limited size, compared with the increase in the renewable and non-renewable energy ratio.

## References

- Amundsen, E.S., and Nese, G. (2009), "Integration of tradable green certificate markets: What can be expected?," *Journal of Policy Modeling*, 31(6), 903–922.
- Baumol, W., and Oates, W., *The Theory of Environmental Policy*, Cambridge: Cambridge University Press (1988).
- Carter, A., Craigwell, R., and Moore, W. (2012), "Price reform and household demand for electricity," *Journal of Policy Modeling*, 34(2), 242–252.
- Chontanawat, J., Hunt, L.C., and Pierse, R. (2008), "Does energy consumption cause economic growth?: Evidence from a systematic study of over 100 countries," *Journal of Policy Modeling*, 30(2), 209–220.
- Ciaschini, M., and Socci, C. (2006), "Income distribution and output change: Macro Multiplier approach," in *Economic Growth and Distribution: On the Nature and Cause of the Wealth of Nations* ed. N. Salvadori, Edward Elgar, chap. 10, pp. 247–270.
- Ciaschini, M., and Socci, C. (2007), "Final demand impact on output: a Macro Multiplier approach," *Journal of Policy Modeling*, 29(1), 115–132.
- Ciaschini, M. (1989), "Scale and structure in economic modelling," *Economic Modelling*, 6, 355–373.
- Ciaschini, M., Pretaroli, R., and Socci, C. (2009), "A convenient multi sectoral policy control for

- the ICT in the USA economy,” *Metroeconomica*, 60, 660–685.
- Ciaschini, M., Pretaroli, R., and Socci, C. (2010), “Multisectoral structures and policy design,” *International Journal of Control*, 83, 281–296.
- Ciaschini, M., Pretaroli, R., and Socci, C. (2011), “Balance, Manhattan norm and Euclidean distance of industrial policies for the US,” *Structural Change and Economic Dynamics*, 22(3), 204–206, doi:10.1016/j.strueco.2011.03.002.
- Dietzenbacher, E., and Stage, J. (2006), “Mixing Oil and Water? Using Hybrid Input-Output Tables in a Structural Decomposition Analysis,” *Economic Systems Research*, 18(1), 85–95.
- EC, (2004), “The share of renewable energy in the EU Country profiles. Overview of RES in the enlarged European Union,” *Commission staff working document SEC (2004)*, 547, 26.5.2004.
- EUROSTAT, (2008), “Energy - Yearly statistics 2006,” *EUROSTAT*.
- EUROSTAT, (2009), “Panorama of Energy - Energy statistics to support EU policies and solutions,” *EUROSTAT*.
- Gallagher, P.W., Shapouri, H., Price, J., Schamel, G., and Brubaker, H. (2003), “Some long-run effects of growing markets and renewable fuel standards on additives markets and the US ethanol industry,” *Journal of Policy Modeling*, 25(6-7), 585–608.
- Goulder, L., Parry, I., Williams III, R., and Burtraw, D. (1999), “The Cost-Effectiveness of Alternative Instruments for Environmental Protection in a Second Best Setting,” *Journal of Public Economics*, 72, 523–554.
- Haug, M. (2007), “Renewable Energy in Future Energy Supply: A Renaissance in Waiting,” *Quarterly Journal of International Agriculture*, 46(4), 304–324.
- ISTAT, *I bilanci energetici*, Roma: ISTAT (2007).
- Jensen, S., and Skytte, K. (2002), “Interactions between the power and green certificate markets,” *Energy Policy*, 30, 425–435.
- Lahr, M.L. (1993), “A Review of the Literature Supporting the Hybrid Approach to Constructing REgional Input-Output Models,” *Economic Systems Research*, 5(3), 277–293.
- Lancaster, P., and Tiesmenetsky, M. (1985) *The Theory of Matrices*, second ed., New York: Academic Press.
- McKibbin, W.J., and Wilcoxon, P.J. (2009), “Uncertainty and climate change policy design,” *Journal of Policy Modeling*, 31(3), 463–477.
- Miller, R.E., and Blair, P.D., *Input-Output Analysis: Foundations and Extensions*, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., (2009).
- Parry, I. (2002), “Are Tradable Emissions Permits a Good Idea?,” *Resources for the future*, 02, Washington, D.C.
- Polenske, K.R. (1976), “Multiregional interactions between energy and transportation,” in *Advances in Input-Output Analysis* eds. K.R. Polenske and J.V. Skolka, Cambridge, Mass: Ballinger.
- Round, J. (2003), “Social Accounting Matrices and SAM-based Multiplier Analysis,” in *Techniques for Evaluating the Poverty Impact of Economic Policies* eds. L.A.P. da Silva and

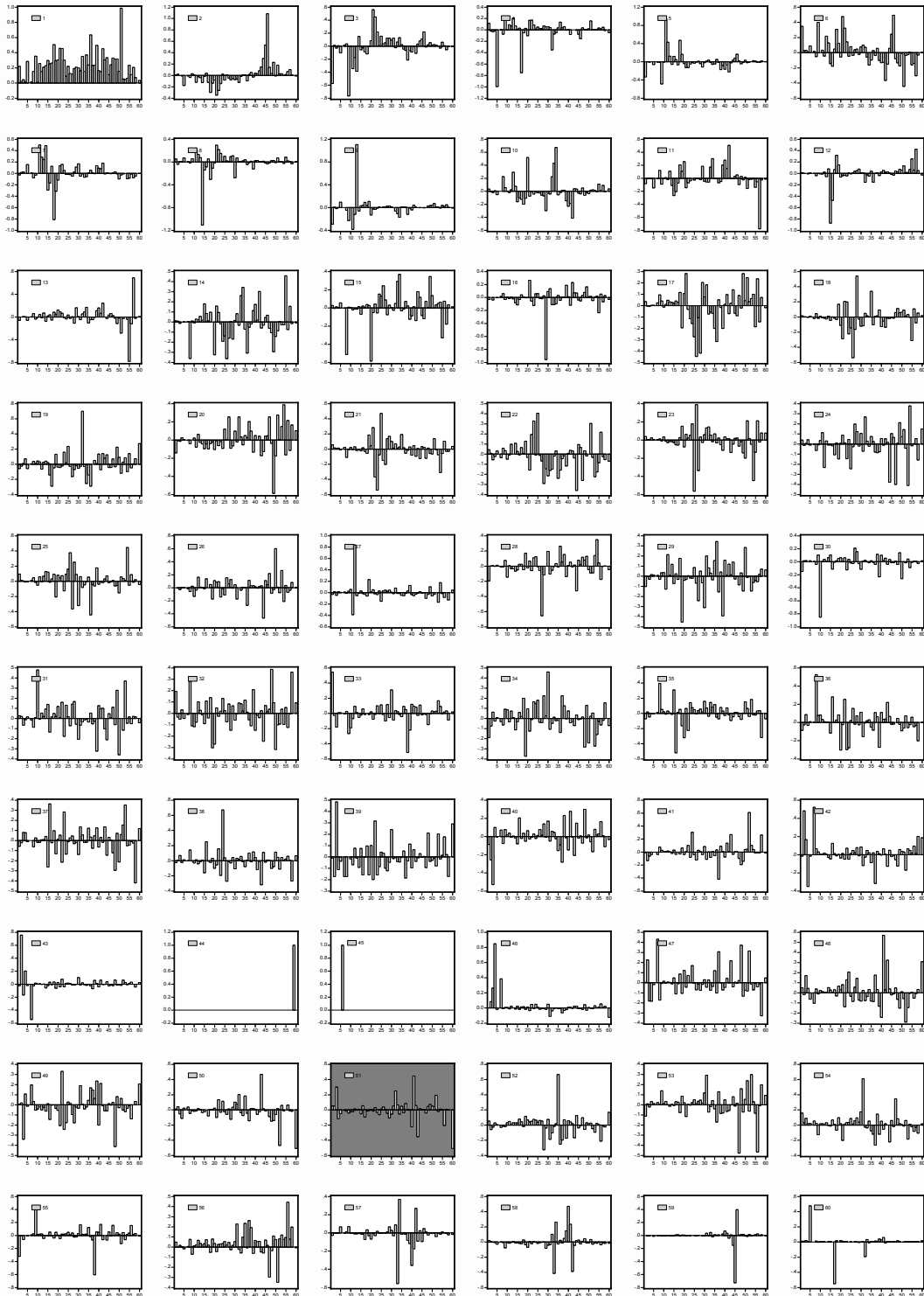
- F. Bourguignon, World Bank and Oxford University Press, chap. 14.
- Salerian, J., Gregan, T., and Stevens, A.H. (2000), "Pricing in Electricity Markets," *Journal of Policy Modeling*, 22(1), 859–893.
- Schaeffer, G., Boots, M., Mitchell, C., Anedersono, T., Timpe, C., and Cames, M. (2000), "Option for design of tradable green certificate systems," *Report ECN-C-00-032*, Petten.
- Skolka, J., "Input Output Multipliers and Linkages." paper presented at the 8th International Conference on Input-Output Techniques - Sapporo (1986).
- Stoutenborough, J.W., and Beverlin, M. (2008), "Encouraging Pollution-Free Energy: The Diffusion of State Net Metering Policies," *Social Science Quarterly*, 89, 1230–1251.
- Telli, a., Voyvoda, E., and Yeldan, E. (2008), "12. Economics of environmental policy in Turkey: A general equilibrium investigation of the economic evaluation of sectoral emission reduction policies for climate change," *Journal of Policy Modeling*, 30(2), 321–340.
- Van Dijk, M.e.a. (2003), "Renewable Energy Policies and Market Developments," *Report ECN-C-99-072*, Petten.
- Vlachou, A., Vassos, S., and Andrikopoulos, A. (1996), "Energy and environment: Reducing CO2 emissions from the electric power industry," *Journal of Policy Modeling*, 18(4), 343–376.

## Appendix A: Tables and figures

Table A1. Commodity classification

- 1 Products of agriculture, hunting and related services
- 2 Products of forestry, logging and related services
- 3 Fish and other fishing products; services incidental of fishing
- 4 Coal and lignite; peat
- 5 Crude petroleum and natural gas; services incidental to oil and gas extraction excluding surveying
- 6 Uranium and thorium ores
- 7 Metal ores
- 8 Other mining and quarrying products
- 9 Food products and beverages
- 10 Tobacco products
- 11 Textiles
- 12 Wearing apparel; furs
- 13 Leather and leather products
- 14 Wood and products of wood and cork (except furniture); articles of straw and plaiting materials
- 15 Pulp, paper and paper products
- 16 Printed matter and recorded media
- 17 Coke, refined petroleum products and nuclear fuels
- 18 Chemicals, chemical products and man-made fibres
- 19 Rubber and plastic products
- 20 Other non-metallic mineral products
- 21 Basic metals
- 22 Fabricated metal products, except machinery and equipment
- 23 Machinery and equipment n.e.c.
- 24 Office machinery and computers
- 25 Electrical machinery and apparatus n.e.c.
- 26 Radio, television and communication equipment and apparatus
- 27 Medical, precision and optical instruments, watches and clocks
- 28 Motor vehicles, trailers and semi-trailers
- 29 Other transport equipment
- 30 Furniture; other manufactured goods n.e.c.
- 31 Secondary raw materials
- 32 Electrical energy, gas, steam and hot water
- 33 Collected and purified water, distribution services of water
- 34 Construction work
- 35 Trade, maintenance and repair services of motor vehicles and motorcycles; retail sale of automotive fuel
- 36 Wholesale trade and commission trade services, except of motor vehicles and motorcycles
- 37 Retail trade services, except of motor vehicles and motorcycles; repair services of personal and household goods
- 38 Hotel and restaurant services
- 39 Land transport; transport via pipeline services
- 40 Water transport services
- 41 Air transport services
- 42 Supporting and auxiliary transport services; travel agency services
- 43 Post and telecommunication services
- 44 Financial intermediation services, except insurance and pension funding services
- 45 Insurance and pension funding services, except compulsory social security services
- 46 Services auxiliary to financial intermediation
- 47 Real estate services
- 48 Renting services of machinery and equipment without operator and of personal and household goods
- 49 Computer and related services
- 50 Research and development services
- 51 Other business services
- 52 Public administration and defence services; compulsory social security services
- 53 Education services
- 54 Health and social work services
- 55 Sewage and refuse disposal services, sanitation and similar services
- 56 Membership organisation services n.e.c.
- 57 Recreational, cultural and sporting services
- 58 Other services
- 59 Private households with employed persons
- 60 Renewable energy

Figure A1. Key structures for policy objective



**Appendix B: Methodological aspects on the MM approach**

The decomposition proposed in section 3 can be applied both to square and non-square matrices. Here the general case of square matrix  $\mathbf{R}$  will be shown<sup>18</sup>. Taking as example a 2x2 model, we

<sup>18</sup>The non-square matrix case is easily developed along the same lines.



will show a Singular Values Decomposition and provide an interpretation of the results in terms of multisectoral economic analysis for policy. Let us consider matrix  $\mathbf{W}$  [2, 2], for example, the square of matrix  $\mathbf{R}$ :

$$\mathbf{W} = \mathbf{R}^T \mathbf{R}$$

Matrix  $\mathbf{W}$  has a positive definite or semi definite square root. Given that  $\mathbf{W} \geq 0$  by construction, its eigenvalues  $\lambda_i$  for  $i = 1, 2$  shall be all real non negative (Lancaster and Tiesmenetsky 1985). The nonzero eigenvalues of matrices  $\mathbf{W}$  and  $\mathbf{W}^T$  coincide. The system of eigenvectors  $\mathbf{z}_i$   $i = 1, 2$  for  $\mathbf{W}$  and  $\mathbf{p}_i$   $i = 1, 2$  for  $\mathbf{W}^T$  are orthonormal basis. We get then

$$\mathbf{R}^T \mathbf{z}_i = \sqrt{\lambda_i} \mathbf{p}_i \quad i = 1, 2$$

We can construct the two matrices

$$\mathbf{Z} = [\mathbf{z}_1 \mathbf{z}_2] \quad \mathbf{P} = [\mathbf{p}_1 \mathbf{p}_2]$$

As defined above, the eigenvalues of  $\mathbf{W}$  coincide with singular values of  $\mathbf{R}$  hence  $m_i = \sqrt{\lambda_i}$  and we get

$$\mathbf{R}^T \mathbf{Z} = [m_1 \mathbf{p}_1 m_2 \mathbf{p}_2] = \mathbf{P} \mathbf{M}$$

Structural matrix  $\mathbf{R}$  in equation 5 can be then decomposed as

$$\mathbf{x} = \mathbf{Z} \mathbf{M} \mathbf{P}^T \mathbf{f} \tag{B1}$$

where  $\mathbf{P}$  is an [2, 2] unitary matrix whose columns define the 2 reference structures for final demand:

$$\mathbf{p}_1 = \begin{bmatrix} p_{11} & p_{12} \end{bmatrix}$$

$$\mathbf{p}_2 = \begin{bmatrix} p_{21} & p_{22} \end{bmatrix}$$

$\mathbf{Z}$  is a [2, 2] unitary matrix whose columns define 2 reference structures for output:

$$\mathbf{z}_1 = \begin{bmatrix} z_{11} \\ z_{21} \end{bmatrix}, \mathbf{z}_2 = \begin{bmatrix} z_{12} \\ z_{22} \end{bmatrix}$$

and  $\mathbf{M}$  is a  $[2, 2]$  diagonal matrix of the type:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

Scalars  $m_i$  all real and positive and can be ordered as  $m_1 > m_2$ . Now we have all the elements to show that, in this decomposition, the singular values represent the MM that quantify the aggregate scale effects of a shock in final demand on total output. We can also appreciate the peculiar role played by the unit structures associated that we define as key-structures. In fact if we express the actual vector  $\mathbf{f}$  in terms of the key-structures specified by matrix  $\mathbf{P}$ , we obtain the final demand vector  $\mathbf{f}^0$ :

$$\mathbf{f}^0 = \mathbf{P}^T \mathbf{f} \quad (\text{B2})$$

On the other hand we can also express total output in terms of the calculated key-structures of total output:

$$\mathbf{x}^0 = \mathbf{Z} \mathbf{x} \quad (\text{B3})$$

Through equations B2 and B3 equation B1 becomes:

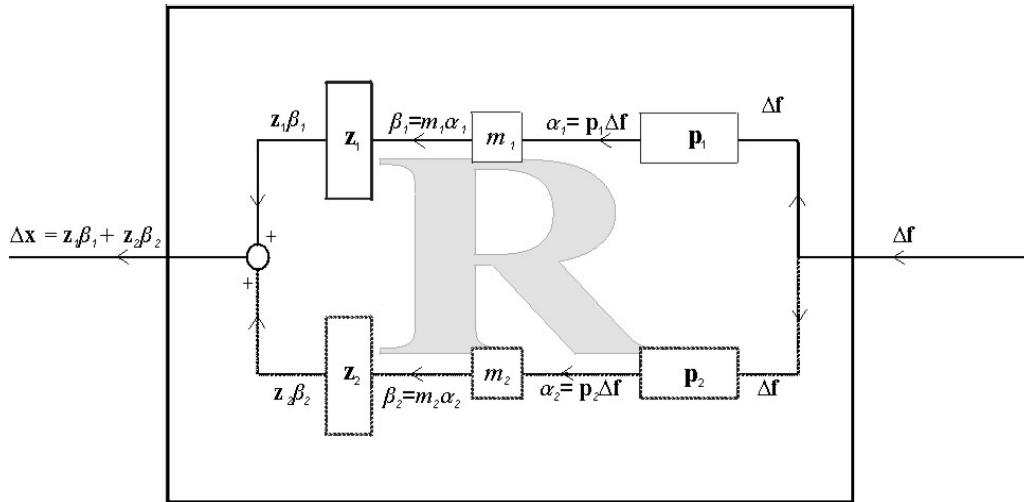
$$\mathbf{x}^0 = \mathbf{M} \mathbf{f}^0 \quad (\text{B4})$$

which implies:

$$x_i^0 = m_i f_i^0 \quad (\text{B5})$$

where  $i = 1, 2$ .

Figure B1. Macro Multipliers  $m_i$  and key structures  $\mathbf{z}_i, \mathbf{p}_i$  in the Leontief inverse



We note that matrix  $\mathbf{R}$ , the Leontief inverse, hides 2 fundamental compositions of the output

vector, given by the unit-modulus output key structures  $\mathbf{z}_1$  and  $\mathbf{z}_2$ , which will shape the structure of the outcome.

As shown in the diagram in figure B1 the inner structure of the resulting output is determined by the combination of the key structures of total output  $\sum \beta_i \mathbf{z}_i$ . The weights of these combinations  $\beta_i$  are determined by the multipliers,  $m_i$ , according the degree of activation of each of them,  $\alpha_i$ .

It is interesting to note that the core of multisectoral interaction operation is determined by multiplications of aggregated scalars  $\beta_i = m_i \alpha_i$ .

The degree of activation,  $\alpha_1$  and  $\alpha_2$  respectively, of each multiplier is determined by the degree at which the structure of final demand change  $\Delta \mathbf{f}$  is equal to the corresponding key structure  $\mathbf{p}_i$ . In the extreme case where the structure of the final demand change is equal to one of the two the key structures, for example  $\mathbf{p}_i$ , then only multiplier  $m_1$  will be activated<sup>19</sup>. And the dotted loop in fig 1B will be deactivated.

No other result can be obtained from matrix  $\mathbf{R}$  (Ciaschini and Socci 2007)<sup>20</sup>.

---

<sup>19</sup>Since  $\mathbf{p}_2 \Delta \mathbf{f} = 0$  due to the requisite of orthogonality of the key structures.

<sup>20</sup>This feature of consistently separating scale effects from structure effects through the MM is also relevant with reference to the difficulties emerging with the aggregation in multisectoral models.