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Marketing Mix and New Product Diffusion Models

Luís G. Pinto[a], Luís Cavique[b], Jorge M.A. Santos[c,*]

[a] *Institute of Advanced Training and Research of the University of Évora, Largo dos Colegiais 2, 7000 Évora, Portugal.*
[b] *Department of Science and Technology of the Universidade Aberta, R. Escola Politécnica 147, 1269-001 Lisboa, Portugal.*
[c] *Department of Mathematics of the University of Évora, Largo dos Colegiais 2, 7000 Évora, Portugal.*

Abstract

In this paper we analyze the relationship between the marketing mix and new product diffusion models. The goal is to obtain a general new product diffusion model that incorporates the classic 4Ps model of the Marketing Mix: Product, Price, Place, Promotion. An empirical study was conducted using mobile broadband adoption data in Japan.

Keywords: new product diffusion; innovation diffusion; marketing science; forecasting

1. Introduction

Business firms operate in a highly competitive environment and require decision support systems to increase market share and profitability. Thus, providing decision support systems for specific managerial functions (i.e. marketing management) is a vital development. Over the last few decades, new product diffusion models have become widely employed in the industry as a way of supporting marketing decisions related with the launch of new innovative products. The most prominent of these is the Bass diffusion model [3]. Several extensions have been proposed that incorporate marketing mix variables [9][12][13]. However, existing innovation diffusion models do not include all four variables of the classic marketing mix concept: Product, Price, Place, Promotion.

In this article, we introduce a general innovation diffusion model which integrates the four marketing mix variables. The resulting model can be used as a decision support system for marketers. An empirical application with real-world data is also presented, based on the diffusion of mobile broadband in Japan.

* Corresponding author. Tel.: +351-266-740-800.
Email address: jmas@uevora.pt (Jorge M.A. Santos)

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2. Preliminaries

2.1. Marketing Mix

In the marketing literature, *Marketing Mix* typically refers to the *4Ps* model first introduced by McCarthy [11]. According to this model, marketing can be seen as a process where marketing managers allocate investment across four independent variables: Product, Price, Place, and Promotion. Product refers to investment decisions regarding product attributes and their quality. Price refers to the price level and structure. Place refers to the distribution efforts. Promotion refers to the investment in advertising and publicity, as well as other communication media. Other marketing mix theories have been proposed, with specific variables for services, retail, digital marketing, or other application contexts, however, in most cases we can typically map the additional variables to the classic four dimensions.

2.2. New Product Diffusion Models

The new product diffusion or innovation diffusion theory was extensively debated by Rogers in [17]. This theory attempts to explain and predict how new products and innovations spread in a population. Bass [2] placed the theory in a formal mathematical setting, incorporating some influence from the epidemiological literature, such as the SIR model [8].

2.2.1. Bass Diffusion

The *Bass model of innovation diffusion* [2] is an ordinary differential equation with the following form [9]:

$$\dot{f}(t) = p\bar{m} + (q-p)f(t) - \frac{q}{\bar{m}}f^2(t). \tag{1}$$

Where $f(t)$ is the total sales up to t, $\dot{f}(t)$ its derivative (the sales at t), $p > 0, q > 0$ are the coefficients of innovation and imitation, and m is the fixed number of potential adopters (market potential). This model was initially derived by Bass, assuming that a new product is being adopted by a market with an unobserved social network structure (homogeneous network assumption). However several extensions of this model have been proposed which integrate the knowledge of the network topology [5][7][16].

A Generalized version of the Bass model was then proposed which included decision variables for price and advertising [4][9]:

$$f(X_i, t) = [p\bar{m} + (q-p)f(X_i, t) - \frac{q}{\bar{m}}f^2(X_i, t)]h(X_i, t) \tag{2}$$

$$\begin{aligned} h(X_i, t) = 1 + \alpha\frac{P_i(t) - P_i(t-1)}{P_i(t-1)} \\ +\beta Max\{0, \frac{[A_i(t) - A_i(t-1)]}{A_i(t-1)}\}, \end{aligned} \tag{3}$$

$$X_i = (P_i(t), A_i(t)) \tag{4}$$

Where X_i is the *marketing mix vector* or *marketing tactic*, which in this case is made up of two time dependent functions, respectively representing the *price* $P(t)$ and *advertising expenditure* $A(t)$ over time, and α being the price impact parameter, and β the advertising efficiency parameter.

2.2.2. Mesak Diffusion

The generalized Bass model served as the foundation of the extension by Mesak [1] [12] which incorporated the price, place and promotion variables of the marketing mix:

$$f_t(X_i, t) = (pP_i(t) + qf(X_i, t))[\bar{m}D_i(t) - f(X_i, t)]A_i(t) \tag{5}$$

[1] We use M7 from the original work given that it was the model with both the best empirical performance, as well as the most theoretically robust.

$$X_i = (P_i(t), D_i(t), A_i(t)). \tag{6}$$

Where $f_t(X_i, t) = \frac{\partial f(X_i,t)}{\partial t}$ is the sales rate on moment t given a *marketing tactic* X_i.

3. Proposed Model

3.1. Marketing Mix Diffusion (MMD)

Mesak in [12] suggets incorporating the product quality as modeled by the Narasimhan-Ghosh-Mendez (NGM) Diffusion model [13]. Therefore we will now introduce a general model for the marketing mix function which includes the Product variable using the NGM model. According to the NGM model we have a function $U_t(U_i, t) = \frac{\partial U}{\partial t}$ which gives us the rate at which the quality weighted quantity of goods in the market cease to influence consumer's behavior:

$$U_t(U_i, t) = \frac{1}{\eta}(U_i(t) f(X_i, t) - U(U_i, t))) \tag{7}$$

In order for this model to be estimated, we require a solution for this PDE.

Theorem 1. *The quality weighted quantity of goods in the market that influence consumers' behavior, $u(Q_i, t)$, is given by a linear Volterra integral equation of the first kind:*

$$U(U_i, t) = e^{-\frac{t}{\eta}} \int_1^t \frac{e^{\xi/\eta} f(\xi) U_i(\xi)}{\eta} d\xi + \gamma e^{-\frac{t}{\eta}} \tag{8}$$

Proof. We can rewrite (7) as $U_t(U_i, t) + \frac{1}{\eta} U(U_i, t) = \frac{1}{\eta} f(X_i, t) U_i(t)$. By multiplying both sides by $e^{\frac{t}{\eta}}$ and substituting $\frac{e^{t/\eta}}{\eta} = \frac{\partial e^t \eta}{\partial t}$ we get $e^{t/\eta} \frac{\partial U(U_i,t)}{\partial t} + \frac{\partial e^{t/\eta} U(X_i,t)}{\partial t} = \frac{e^{t/\eta} f(X_i,t) U_i(t)}{\eta}$. Applying the reverse product rule to the left-hand side we get $\frac{\partial e^{t/\eta} U(X_i,t)}{\partial t} = \frac{e^{t/\eta} f(X_i,t) U_i(t)}{\eta}$. Integrating both sides with respect to t we get $\int_0^t \frac{\partial}{\partial t} e^{t/\eta} U(U_i, t) dt = \int_0^t \frac{1}{\eta} e^{t/\eta} f(X_i, t) U_i(t) dt$. Evaluating the integrals we get $U(U_i, t) e^{\frac{t}{\eta}} = \int_1^t \frac{e^{\xi/\eta} f(\xi) U_i(\xi)}{\eta} d\xi + \gamma$, where γ is an arbitrary constant, which can be rearranged to get $U(U_i, t) = e^{-\frac{t}{\eta}} \int_1^t \frac{e^{\xi/\eta} f(\xi) U_i(\xi)}{\eta} d\xi + \gamma e^{-\frac{t}{\eta}}$. □

Using this solution, and the usual operationalization functional forms for P_i,D_i and A_i (according to [12]), we can now get the following representation of the general model[2]:

$$R(P_i, D_i, A_i, t) = p P_i(t) [\bar{m} D_i(t) - f(X_i, t)] A_i(t) \tag{9}$$

$$S(X_i, t) = q U(U_i, t) \tag{10}$$

$$f_t(X_i, t) = S(U_i, t) + R(P_i, D_i, A_i, t) \tag{11}$$

$$U(U_i, t) = e^{-\frac{t}{\eta}} \int_1^t \frac{e^{\xi/\eta} f(\xi) U_i(\xi)}{\eta} d\xi + \gamma e^{-\frac{t}{\eta}} \tag{12}$$

$$P(P_i, t) = P_i(t) \tag{13}$$

$$D(D_i, t) = D_i(t) \tag{14}$$

$$A(A_i, t) = \sqrt{A_i(t)} \tag{15}$$

[2] The power pricing formula suggested by Mesak was not employed given its poor empirical fit.

$$X_i = (U_i(t), P_i(t), D_i(t), A_i(t)). \tag{16}$$

Where $f_t(X_i, t)$ is the sales rate on moment t given a *marketing tactic* X_i, p is the coefficient of innovation (or coefficient of external influence), q is the coefficient of imitation (or coefficient of internal influence), η is a quality influence decay coefficient, γ the product launch quality coefficient, and $\bar{m}$ is the market potential.

This model was inspired by the work of Shinohara [18], which, by using a physical analogy, introduced a novel taxonomy of diffusion types. In this model we separate the effects of the marketing mix variables between the *radiation* and *diffusion* components, which can be roughly interpreted as *global* and *local* (i.e. individual level) effects. Therefore we can define the *Radiation* (9) and *Diffusion* (10) functions, which contain the separated effects of product (diffusion) as well as price, place and promotion (radiation), as well as a combined *Radiation-Diffusion* function (11).

4. Empirical Results

An empirical application was conducted using OECD mobile broadband adoption data from Japan (2014-2019) [14]. A marketing mix dataset was constructed combining data from several sources. To approximate Product quality-related investments, we used mobile broadband speed in Japan information from Akamai's State of Internet Reports 2014-2019 [1][3]. Pricing data was approximated using the Consumer Price Index for Mobile Charges in Japan [15]. Advertising investment data for the entire Information/Communications industry sector was used as a proxy for the communications related to mobile broadband adoption [6]. While point-of-sale investment data was not available at the industry sector level, *digital distribution efforts* were approximated using the Alexa website rank for the three major telco players in Japan (nttdocomo.co.jp, KDDI.com and softbank.jp) [19]. The latter times series data was subjected to a Dynamic Factor Analysis procedure to extract a latent "digital distribution" factor that summarizes the entire Telco sector. Polynomial interpolation (order 4) was applied to all series to obtain a continuous time series approximation and to adjust the data with different granularity levels (month, quarter or year). The final dataset included 72 periods (months) for adoption, product, price, place, and promotion investment data between January 2014 and December 2019. We considered all of the previously seen models: Bass diffusion, Mesak diffusion, Marketing Mix diffusion (MMD). However, the empirical operationalization of these models required an addition of a number of statistical adjustment variables for better fit. Therefore we've considered the following modifications to the models:

$$f_{empirical}(X_i, t) = w_0 + f(X_i, t) \tag{17}$$

$$P_{empirical}(P_i, t) = w_1 P(P_i, t) \tag{18}$$

$$D_{empirical}(D_i, t) = w_2 D(D_i, t) \tag{19}$$

$$A_{empirical}(A_i, t) = w_3 A(A_i, t) \tag{20}$$

$$U_{empirical}(U_i, t) = w_4 U(U_i, t) \tag{21}$$

For the MMD model, the parameters η and γ were manually set to $\eta = \gamma = 1000$.

[3] Since we are interested in the perceived quality, we've applied a Weber-Fechner Law transformation ($U_i = \kappa \, ln \frac{U_i^*}{U_0}$) with parameters $\kappa = 1$ and $U_0 = 0.01$ Mbps, with U_0 being the minimum perceived mobile internet speed, beyond which consumers will consider quality to be zero, and where U_i^* is the observed average mobile internet speed.

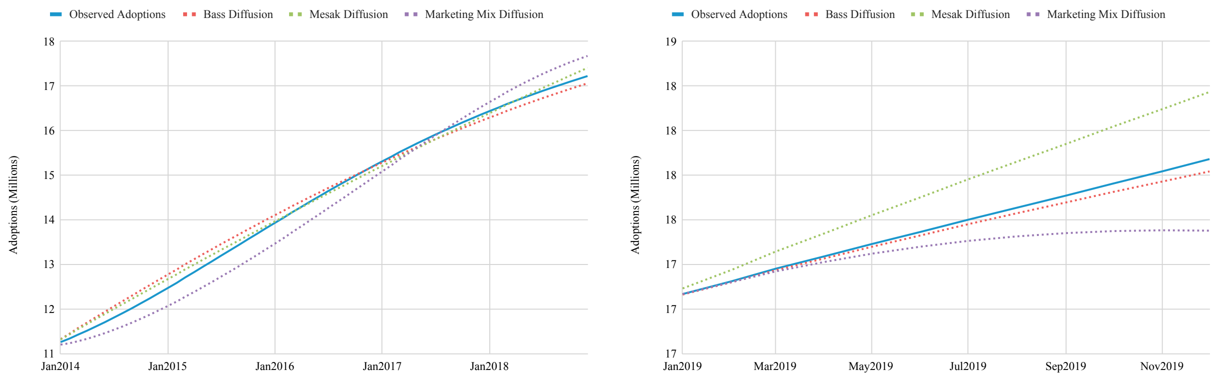

Fig. 1: In-sample (Left) and Out-of-sample (Right) results.

Table 2: Out-of-sample Results

	Bass Diffusion	Mesak Diffusion	MMD
MSE	**8.71E08**	4.05E10	2.67E10
RMSE	**29,521**	201,311	163,550
NRMSE	**0.16%**	1.12%	0.91%
MAPE	**0.13%**	1.01%	0.51%

Table 1: Estimated Parameters

		Bass	Mesak	MMD
Numb. of Params.		2	6	7
Coeff. of Innovation	p	0.000	0.000	0.003
Coeff. of Imitation	q	0.025	0.000	0.000
Baseline Adopt.	w_0		117,368	1.299
Price Elast.	w_1		-11.441	-1.825
Distrib. Intensity	w_2		2.405	0.007
Advertising Invest.	w_3		12.778	3.363
Prod. Quality Impact	w_4			0.084
Market Constant	$\bar{m}$	20,000,000	253,521	-1,278,165

The four models were estimated using a least-squares approach, by minimizing the sum of square errors (SSE). While improved estimation procedures based on Maximum Likelihood Estimation (MLE) as well as the Nonlinear Least Squares method, are available for the classic Bass model [20], none of such methods are known for the remaining ones. Therefore we've opted for a simplified general process of adjusting the models using the L-BFGS-B method [10]. The resulting parameter estimates are displayed in Table I. All models were implemented using Python and the Scipy library and estimated in a local Intel Core i7 CPU machine running Windows 10. Table II and Figure 1 summarize the results. To compare the different models we used the Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Normalized Root Mean Squared Error (NRMSE) and the Mean Absolute Percentage Error (MAPE). The results suggest that the Bass model without decision variables has an overall better fit, consistent with previous known results [4]. However, the proposed model (MMD) outperforms the Mesak model with three decision variables.

5. Conclusions

In this work we have introduced a novel new product diffusion model which can be used to simulate real world market dynamics taking into account the effects of all marketing mix variables. We provided some evidence that the empirical fit of this model is comparable to the classic Bass diffusion model, while outperforming the Mesak model, the current state of the art diffusion model with marketing mix decision variables. This proposed model might be useful as a decision support system for marketing managers, allocating resources and setting policies related with the 4Ps of the marketing mix.

5.1. Limitations

The main limitation of this study is related with the marketing mix data. While for advertising investment, product quality and price level we have good approximations, the data available for distribution is quite limited. Ideally, to characterize the Place variable of the marketing mix, which is related with distribution intensity, we should consider both offline (point-of-sale) as well as online (e-commerce) efforts. Unfortunately, our only option was to approximate the online distribution component using Alexa website rank data, since the point-of-sales data was not available.

5.2. Future Work

In future work we can extend the MMD model to complex networks, and study the innovation diffusion process in a real world graph network. Another research path includes the numerical approximation of the optimal marketing mix paths using an optimal control approach, while also exploring multi-objective optimization (for instance, maximizing profit as well as market share or total sales).

References

[1] Akamai, *State of Internet Reports*, 2014.

[2] F. M. Bass, *A New Product Growth for Model Consumer Durables*, Management Science **50** (2004), no. 12_Supplement, 1825–1832.

[3] Frank M. Bass, *A New Product Growth for Model Consumer Durables*, Management Science **15** (1969jan), no. 5, 215–227.

[4] Frank M. Bass, Trichy V. Krishnan, and Dipak C. Jain, *Why the Bass Model Fits without Decision Variables*, Marketing Science **13** (1994), no. 3, 203–223.

[5] M. L. Bertotti, J. Brunner, and G. Modanese, *The Bass diffusion model on networks with correlations and inhomogeneous advertising*, Chaos, Solitons and Fractals **90** (2016), 55–63, available at 1605.06308.

[6] Dentsu, *Advertising Expenditures in Japan*, 2020.

[7] Georg Holtz, *An individual level diffusion model, carefully derived from the Bass-model*, 2004.

[8] William Ogilvy Kermack and A. G. McKendrick, *A contribution to the mathematical theory of epidemics*, Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character **115** (1927aug), no. 772, 700–721.

[9] Gary L. Lilien, Arvind Rangaswamy, and Arnaud De Bruyn, *The Bass Model: Marketing Engineering Technical Note*, Principles of marketing engineering, 2007, pp. 1–8.

[10] Dong C. Liu and Jorge Nocedal, *On the limited memory BFGS method for large scale optimization*, Mathematical Programming **45** (1989), no. 1-3, 503–528.

[11] E Jerome McCarthy, *Basic marketing, a managerial approach*, R.D. Irwin, Homewood, Ill., 1964 (English).

[12] Hani I Mesak, *Incorporating price, advertising and distribution in diffusion models of innovation: Some theoretical and empirical results*, Computers and Operations Research **23** (1996), no. 10, 1007–1023.

[13] Ram Narasimhan, Soumen Ghosh, and David Mendez, *A Dynamic Model of Product Quality and Pricing Decisions on Sales Response*, Decision Sciences (1993).

[14] OECD, *Mobile broadband subscriptions* (2018).

[15] Portal Site of Official Statistics of Japan website (https://www.e-stat.go.jp/), *Consumer Price Index 2015 - Base Consumer Price Index*, 2015.

[16] Tae-Hyung Pyo, Thomas S Gruca, and Gary J Russell, *A New Bass Model Utilizing Social Network Data* (2017).

[17] Everett M. Rogers, *Diffusion of Innovation*, Free Press, 2003.

[18] Kazunori Shinohara, *Space-time innovation diffusion based on physical analogy*, Applied Mathematical Sciences **6** (2012), no. 49-52, 2527–2558.

[19] Site Rank Data, *https://siterankdata.com/*, 2021.

[20] V. Srinivasan and Charlotte H. Mason, *Technical Note—Nonlinear Least Squares Estimation of New Product Diffusion Models*, Marketing Science **5** (1986), no. 2, 169–178.