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NOVEL POSSIBILITY PYTHAGOREAN INTERVAL VALUED FUZZY SOFT SET METHOD FOR A DECISION MAKING

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ABSTRACT. We discuss the theory of possibility Pythagorean interval valued fuzzy soft set, possibility interval valued fuzzy soft set and define some related the operations namely complement, union, intersection, AND and OR. The possibility Pythagorean interval valued fuzzy soft sets are a generalization of soft sets. Notably, we showed De-Morgan's laws that are valid in possibility Pythagorean interval valued fuzzy soft set theory. Also, we propose an algorithm to solve the decision making problem based on soft set method. To compare two possibilities Pythagorean interval valued fuzzy soft sets for dealing with decision making problems and find a similarity measure is obtained. Finally, an illustrative example is discussed to prove that they can be effectively used to solve problems with uncertainties.

Keywords: Interval valued fuzzy soft set, Pythagorean interval valued fuzzy soft set, Possibility Pythagorean interval valued fuzzy soft set.

AMS Subject Classification: 03E72.

1. INTRODUCTION

Pythagorean fuzzy set has attracted great attentions of many researchers and subsequently, the concept has been applied to many application areas such as decision-making, aggregation operators, and information measures. Rahman et al. [15] worked on some geometric aggregation operators on interval valued Pythagorean fuzzy sets and applied same to group decision-making problem. Perez Dominguez presented a multi objective optimization on the basis of ratio analysis (MOORA) under Pythagorean fuzzy set setting and applied it to MCDM problem [14]. Liang and Xu proposed the idea of Pythagorean fuzzy sets [8] in hesitant environment and its MCDM ability by employing TOPSIS using energy project selection model. Rahman et al. [16] proposed some approaches to multi attribute group decision making based on induced interval valued Pythagorean fuzzy Einstein aggregation operator. The theory of Pythagorean fuzzy soft set to solve the real

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world problems that intuitionistic fuzzy soft set can not deal with the situation that the sum of membership degree and non membership degree of the parameter is larger than 1. It makes the (MADM) limited, and affects the optimum decision. The Pythagorean fuzzy soft set provides a large number of applications to the (MADM) for such real world problems.

In most real problems, uncertainty can be seen everywhere. In order to cope with the uncertainties, many uncertain theories such as fuzzy set [21], intuitionistic fuzzy set [3], Xiao et al initiated the concept of interval valued fuzzy soft sets [18] and Pythagorean fuzzy set [20] are put forwarded. Zadeh was introduced by Fuzzy set suggests that decision makers are to be solving uncertain problems by considering membership degree. After, the concept of intuitionistic fuzzy set is introduced by Atanassov and is characterized by a degree of membership and non-membership satisfying the condition that sum of its membership degree and non membership degree is not exceeding 1 [3]. However, we may interact with a problem in decision making events where the sum of the degree of membership and non-membership of a particular attribute is exceeding 1. So Yager was introduced the concept of Pythagorean fuzzy sets. It has been to extend the intuitionistic fuzzy sets and characterized by the condition that the square sum of its degree of membership and non membership is not exceeding 1.

The theory of soft sets proposed by Molodtsov [11]. It is a tool of parameterization for coping with the uncertainties. In comparison with other uncertain theories, soft sets more accurately reflects the objectivity and complexity of decision making during actual situations. It has been a great achievements both in theories and applications. Moreover, the combination of soft sets with other mathematical models is also a critical research area. For example, Maji et al. proposed by the concept of fuzzy soft set [9] and intuitionistic fuzzy soft set [10]. These two theories are applied to solve various decision making problems. Alkhazaleh et al [1] defined the concept of possibility fuzzy soft sets.

In recent years, Peng et al [12] has extended fuzzy soft set to Pythagorean fuzzy soft set. This model solved a class of multi attribute decision making consists sum of the degree of membership and non membership value is exceeding 1 but the sum of the squares is equal or not exceeding 1. In general, the possibility degree of belongingness of the elements should be considered in multi attribute decision making problems. However, Peng et al [12] failed to do it. As for the problem, the purpose of this paper is to extend the concept of possibility Pythagorean fuzzy soft set to parameterization of possibility Pythagorean interval valued fuzzy set. We obtain a possibility Pythagorean interval valued soft set model. We shall further establish a similarity measure method based on this model and apply it to decision making problems by a suitable examples.

2. Preliminaries

Definition 2.1. [19, 20] *Let X be a non-empty set of the universe, Pythagorean fuzzy set (PFS) A in X is an object having the following form : $A = \{x, \mu_A(x), \nu_A(x) | x \in X\}$, where $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership of A respectively. Consider the mapping $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ and $0 \leq (\mu_A(x))^2 + (\nu_A(x))^2 \leq 1$. The degree of indeterminacy is determined as $\pi_A(x) = \left[\sqrt{1 - (\mu_A(x))^2 - (\nu_A(x))^2}\right]$. Since $A = \langle \mu_A, \nu_A \rangle$ is called a Pythagorean fuzzy number(PFN).*

Definition 2.2. [6, 22, 13] *Let X be a non-empty set of the universe, Pythagorean interval valued fuzzy set (PIVFS) A in X is an object having the following form : $\widetilde{A} =$*

$\{x, \widetilde{\mu}_A(x), \widetilde{\nu}_A(x) | x \in X\}$, *where* $\widetilde{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)]$ *and* $\widetilde{\nu}_A(x) = [\nu_A^L(x), \nu_A^U(x)]$ *represent the degree of membership and degree of non-membership of A respectively. Consider the mapping* $\widetilde{\mu}_A : X \to [0,1]$, $\widetilde{\nu}_A : X \to [0,1]$ *and* $0 \leq (\widetilde{\mu}_A(x))^2 + (\widetilde{\nu}_A(x))^2 \leq 1$ *means that* $0 \leq (\mu_A^U(x))^2 + (\nu_A^U(x))^2 \leq 1$. *The degree of indeterminacy is determined as* $\widetilde{\pi}_A(x) = [\pi_A^L(x), \pi_A^U(x)] = \left[\sqrt{1 - (\mu_A^U(x))^2 - (\nu_A^U(x))^2},\ \sqrt{1 - (\mu_A^L(x))^2 - (\nu_A^L(x))^2}\,\right]$. *Since* $A = \langle [\mu_A^L, \mu_A^U], [\nu_A^L, \nu_A^U] \rangle$ *is called a Pythagorean interval valued fuzzy number(PIVFN).*

Definition 2.3. [6, 22, 13] *Given that* $\widetilde{\beta_1} = A(\widetilde{\mu}_{\beta_1}, \widetilde{\nu}_{\beta_1})$, $\widetilde{\beta_2} = A(\widetilde{\mu}_{\beta_2}, \widetilde{\nu}_{\beta_2})$ *and* $\widetilde{\beta_3} = A(\widetilde{\mu}_{\beta_3}, \widetilde{\nu}_{\beta_3})$ *are any three Pythagorean interval valued fuzzy numbers(PIVFNs) over* (X, E), *then the following properties are holds:*
(i) $\widetilde{\beta_1}^c = (\widetilde{\nu}_{\beta_1}, \widetilde{\mu}_{\beta_1})$
(ii) $\widetilde{\beta_2} \cup \widetilde{\beta_3} = (\max(\widetilde{\mu}_{\beta_2}, \widetilde{\mu}_{\beta_3}), \min(\widetilde{\nu}_{\beta_2}, \widetilde{\nu}_{\beta_3}))$
(iii) $\widetilde{\beta_2} \cap \widetilde{\beta_3} = (\min(\widetilde{\mu}_{\beta_2}, \widetilde{\mu}_{\beta_3}), \max(\widetilde{\nu}_{\beta_2}, \widetilde{\nu}_{\beta_3}))$
(iv) $\widetilde{\beta_2} \geq \widetilde{\beta_3}$ *iff* $\widetilde{\mu}_{\beta_2} \geq \widetilde{\mu}_{\beta_3}$ *and* $\widetilde{\nu}_{\beta_2} \leq \widetilde{\nu}_{\beta_3}$
(v) $\widetilde{\beta_2} = \widetilde{\beta_3}$ *iff* $\widetilde{\mu}_{\beta_2} = \widetilde{\mu}_{\beta_3}$ *and* $\widetilde{\nu}_{\beta_2} = \widetilde{\nu}_{\beta_3}$.

Definition 2.4. [2] *Let* X *be a non-empty set of the universe and* E *be a set of parameter. The pair* $(\widetilde{\mathscr{F}}, A)$ *is called an interval valued fuzzy soft set(IVFSS) on* X *if* $A \subseteq E$ *and* $\widetilde{\mathscr{F}} : A \to \widetilde{\mathscr{F}(X)}$, *where* $\widetilde{\mathscr{F}(X)}$ *is the set of all interval valued fuzzy subsets of* X.

Definition 2.5. [12] *Let* X *be a non-empty set of the universe and* E *be a set of parameter. The pair* $(\mathscr{F}, A)$ *is called a Pythagorean fuzzy soft set(PFSS) on* X *if* $A \subseteq E$ *and* $\mathscr{F} : A \to P\mathscr{F}(X)$, *where* $P\mathscr{F}(X)$ *is the set of all Pythagorean fuzzy subsets of* X.

Definition 2.6. [1] *Let* X *be a non-empty set of the universe and* E *be a set of parameter. The pair* (X, E) *is a soft universe. Consider the mapping* $\mathscr{F} : E \to \mathscr{F}(X)$ *and* μ *be a fuzzy subset of* E, *ie.* $\mu : E \to \mathscr{F}(X)$. *Let* $\mathscr{F}_\mu : E \to \mathscr{F}(X) \times \mathscr{F}(X)$ *be a function defined as* $\mathscr{F}_\mu(e) = (\mathscr{F}(e)(x), \mu(e)(x)), \forall x \in X$. *Then* $\mathscr{F}_\mu$ *is called a possibility fuzzy soft set(PFSS) on* (X, E).

3. Possibility Pythagorean interval valued fuzzy soft set

We beginning the concept of possibility Pythagorean interval valued fuzzy soft set to generalize possibility fuzzy soft set is connected with the parameterization of Pythagorean fuzzy set and Pythagorean interval valued fuzzy set.

Definition 3.1. *Let* X *be a non-empty set of the universe and* E *be a set of parameter. The pair* (X, E) *is a soft universe. Let* $\widetilde{\mathscr{F}} : E \to \widetilde{\mathscr{F}(X)}$, *and* μ *be a interval valued fuzzy subset of* E, *ie.* $\mu : E \to \widetilde{\mathscr{F}(X)}$. *Let* $\widetilde{\mathscr{F}_\mu} : E \to \widetilde{\mathscr{F}(X)} \times \widetilde{\mathscr{F}(X)}$ *be a function defined as follows* $\widetilde{\mathscr{F}_\mu}(e) = (\widetilde{\mathscr{F}}(e)(x), \widetilde{\mu}(e)(x)), \forall x \in X$. *Then* $\widetilde{\mathscr{F}_\mu}$ *is a possibility interval valued fuzzy soft set(PIVFSS) over* (X, E).

Definition 3.2. *Let* X *be a non-empty set of the universe and* E *be a set of parameter. The pair* $(\widetilde{\mathscr{F}}, A)$ *is a Pythagorean interval valued fuzzy soft set(PIVFSS) on* X *if* $A \subseteq E$ *and* $\widetilde{\mathscr{F}} : A \to \widetilde{P\mathscr{F}(X)}$, *where* $\widetilde{P\mathscr{F}(X)}$ *is the set of all Pythagorean interval valued fuzzy subsets of* X.

Example 3.1. *A set of three children's* $X = \{x_1, x_2, x_3\}$ *and a set of parameter* $E = \{e_1 =$ *running nose,* $e_2 =$ *throat infection,* $e_3 =$ *cough*$\}$.*Suppose that* $\widetilde{\mathscr{F}} : E \to IV\widetilde{P\mathscr{F}(X)}$ *is* *given by*

$$\widetilde{\mathscr{F}}_p(e_1)=\left\{\frac{x_1}{\langle[0.6\ \ 0.7],[0.4\ \ 0.6]\rangle}\ \frac{x_2}{\langle[0.4\ \ 0.6],[0.5\ \ 0.7]\rangle}\ \frac{x_3}{\langle[0.3\ \ 0.5],[0.2\ \ 0.4]\rangle}\right\};\ \widetilde{\mathscr{F}}_p\langle e_2\rangle=\left\{\frac{x_1}{\langle[0.5\ \ 0.7],[0.3\ \ 0.6]\rangle}\ \frac{x_2}{\langle[0.6\ \ 0.7],[0.4\ \ 0.6]\rangle}\ \frac{x_3}{\langle[0.3\ \ 0.5],[0.4\ \ 0.5]\rangle}\right\};\ \widetilde{\mathscr{F}}_p\langle e_3\rangle=\left\{\frac{x_1}{\langle[0.4\ \ 0.6],[0.5\ \ 0.6]\rangle}\ \frac{x_2}{\langle[0.3\ \ 0.5],[0.4\ \ 0.5]\rangle}\ \frac{x_3}{\langle[0.4\ \ 0.6],[0.5\ \ 0.7]\rangle}\right\};$$

Matrix form: $\begin{pmatrix}\langle[0.6\ \ 0.7],[0.4\ \ 0.6]\rangle & \langle[0.4\ \ 0.6],[0.5\ \ 0.7]\rangle & \langle[0.3\ \ 0.5],[0.2\ \ 0.4]\rangle\\ \langle[0.5\ \ 0.7],[0.3\ \ 0.6]\rangle & \langle[0.6\ \ 0.7],[0.4\ \ 0.6]\rangle & \langle[0.3\ \ 0.5],[0.4\ \ 0.5]\rangle\\ \langle[0.4\ \ 0.6],[0.5\ \ 0.6]\rangle & \langle[0.3\ \ 0.5],[0.4\ \ 0.5]\rangle & \langle[0.4\ \ 0.6],[0.5\ \ 0.7]\rangle\end{pmatrix}$

Definition 3.3. *Let X be a non-empty set of the universe and E be a set of parameter. The pair (X,E) is called a soft universe. Suppose that $\widetilde{\mathscr{F}}: E \to \widetilde{P\mathscr{F}(X)}$, and $\widetilde{p}$ is a Pythagorean interval valued fuzzy subset of E. That is $\widetilde{p}: E \to \widetilde{P\mathscr{F}(X)}$, where $\widetilde{P\mathscr{F}(X)}$ denotes the collection of all Pythagorean interval valued fuzzy subsets of X. If $\widetilde{\mathscr{F}_p}: E \to \widetilde{P\mathscr{F}(X)} \times \widetilde{P\mathscr{F}(X)}$ is a function defined as $\widetilde{\mathscr{F}_p}(e) = (\widetilde{\mathscr{F}}(e)(x), \widetilde{p}(e)(x)), x \in X$, then $\widetilde{\mathscr{F}_p}$ is a Possibility Pythagorean interval valued fuzzy soft set (PPIVFSS) on (X,E). For each parameter e, $\widetilde{\mathscr{F}_p}(e) = \{\langle x, (\mu_{\widetilde{\mathscr{F}}(e)}(x), \nu_{\widetilde{\mathscr{F}}(e)}(x)), (\mu_{\widetilde{p}(e)}(x), \nu_{\widetilde{p}(e)}(x))\rangle,\ x \in X\}$*

Example 3.2. *Let $X = \{x_1, x_2, x_3\}$ be a set of three Tuberculosis patient's under treatment of a decision maker to heaviest Tuberculosis effect, $E = \{e_1 =$ high fever, $e_2 =$ high weight loss, $e_3 =$ organs effect$\}$ is a set of parameters. Suppose that $\widetilde{\mathscr{F}_p}: E \to \widetilde{P\mathscr{F}(X)} \times \widetilde{P\mathscr{F}(X)}$ is given by*

$$\widetilde{\mathscr{F}}_p(e_1)=\left\{\frac{x_1}{\langle([0.6\ \ 0.7],[0.4\ \ 0.6]),\ ([0.7\ \ 0.8],[0.2\ \ 0.5])\rangle}\ \frac{x_2}{\langle([0.4\ \ 0.6],[0.5\ \ 0.7]),\ ([0.4\ \ 0.7],[0.1\ \ 0.3])\rangle}\ \frac{x_3}{\langle([0.3\ \ 0.5],[0.2\ \ 0.4]),([0.4\ \ 0.6],[0.1\ \ 0.2])\rangle}\right\};\ \widetilde{\mathscr{F}}_p(e_2)=\left\{\frac{x_1}{\langle([0.5\ \ 0.7],[0.3\ \ 0.6]),\ ([0.6\ \ 0.7],[0.3\ \ 0.4])\rangle}\ \frac{x_2}{\langle([0.6\ \ 0.7],[0.4\ \ 0.6]),([0.5\ \ 0.6],[0.2\ \ 0.3])\rangle}\ \frac{x_3}{\langle([0.3\ \ 0.5],[0.4\ \ 0.5]),([0.4\ \ 0.5],[0.1\ \ 0.2])\rangle}\right\}$$

$$\widetilde{\mathscr{F}}_p(e_3)=\left\{\frac{x_1}{\langle([0.4\ \ 0.6],[0.5\ \ 0.6]),\ ([0.5\ \ 0.7],[0.2\ \ 0.4])\rangle}\ \frac{x_2}{\langle([0.3\ \ 0.5],[0.4\ \ 0.5]),([0.6\ \ 0.8],[0.3\ \ 0.4])\rangle}\ \frac{x_3}{\langle([0.4\ \ 0.6],[0.5\ \ 0.7]),([0.5\ \ 0.6],[0.2\ \ 0.3])\rangle}\right\}$$

Definition 3.4. *Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ are two PPIVFSSs on (X,E). Now $\widetilde{\mathscr{G}_q}$ is a possibility Pythagorean interval valued fuzzy soft subset of $\widetilde{\mathscr{F}_p}$ (denoted by $\widetilde{\mathscr{G}_q} \subseteq \widetilde{\mathscr{F}_p}$) if and only if*
(i) $\widetilde{\mathscr{G}}(e)(x) \subseteq \widetilde{\mathscr{F}}(e)(x)$ if $\mu_{\widetilde{\mathscr{F}}(e)}(x) \geq \mu_{\widetilde{\mathscr{G}}(e)}(x)$, $\nu_{\widetilde{\mathscr{F}}(e)}(x) \leq \nu_{\widetilde{\mathscr{G}}(e)}(x)$,
(ii) $\widetilde{q}(e)(x) \subseteq \widetilde{p}(e)(x)$ if $\mu_{\widetilde{p}(e)}(x) \geq \mu_{\widetilde{q}(e)}(x)$, $\nu_{\widetilde{p}(e)}(x) \leq \nu_{\widetilde{q}(e)}(x)$, $\forall e \in E$.

Example 3.3. *Consider the PPIVFSS $\widetilde{\mathscr{F}_p}$ over (X,E) in Example 3.2. Let $\widetilde{\mathscr{G}_q}$ be another PPIVFSS over (X,E) defined as:*

$$\widetilde{\mathscr{G}}_q(e_1)=\left\{\frac{x_1}{\langle([0.3\ \ 0.6],[0.5\ \ 0.7]),\ ([0.4\ \ 0.7],[0.3\ \ 0.6])\rangle}\ \frac{x_2}{\langle([0.2\ \ 0.5],[0.6\ \ 0.8]),\ ([0.3\ \ 0.6],[0.2\ \ 0.5])\rangle}\ \frac{x_3}{\langle([0.2\ \ 0.4],[0.5\ \ 0.7]),\ ([0.3\ \ 0.4],[0.2\ \ 0.3])\rangle}\right\};\ \widetilde{\mathscr{G}}_q(e_2)=\left\{\frac{x_1}{\langle([0.4\ \ 0.6],[0.5\ \ 0.7]),\ ([0.5\ \ 0.6],[0.4\ \ 0.5])\rangle}\ \frac{x_2}{\langle([0.3\ \ 0.5],[0.6\ \ 0.8]),\ ([0.4\ \ 0.5],[0.3\ \ 0.4])\rangle}\ \frac{x_3}{\langle([0.2\ \ 0.3],[0.6\ \ 0.7]),\ ([0.3\ \ 0.4],[0.2\ \ 0.3])\rangle}\right\}$$

$$\widetilde{\mathscr{G}}_q(e_3)=\left\{\frac{x_1}{\langle([0.3\ \ 0.4],[0.7\ \ 0.8]),\ ([0.4\ \ 0.6],[0.3\ \ 0.5])\rangle}\ \frac{x_2}{\langle([0.2\ \ 0.4],[0.5\ \ 0.7]),\ ([0.5\ \ 0.7],[0.4\ \ 0.5])\rangle}\ \frac{x_3}{\langle([0.3\ \ 0.5],[0.6\ \ 0.8]),\ ([0.4\ \ 0.5],[0.3\ \ 0.4])\rangle}\right\}$$

Definition 3.5. *Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ are two PPIVFSSs on (X,E). Now $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ are possibility Pythagorean interval valued fuzzy soft sets equal(denoted by $\widetilde{\mathscr{F}_p} = \widetilde{\mathscr{G}_q}$) if and only if*
(i) $\widetilde{\mathscr{F}_p} \subseteq \widetilde{\mathscr{G}_q}$
(ii) $\widetilde{\mathscr{F}_p} \supseteq \widetilde{\mathscr{G}_q}$.

Definition 3.6. *Let X be a non-empty set of the universe and E be a set of parameter. Let $\widetilde{\mathscr{F}_p}$ be a PPIVFSS on (X, E). The complement of $\widetilde{\mathscr{F}_p}$ is denoted by $\widetilde{\mathscr{F}_p^c}$ and is defined by $\widetilde{\mathscr{F}_p^c} = \langle \widetilde{\mathscr{F}^c}(e)(x), \widetilde{p}^c(e)(x) \rangle$, where $\widetilde{\mathscr{F}^c}(e)(x) = \langle \nu_{\widetilde{\mathscr{F}}(e)}(x), \mu_{\widetilde{\mathscr{F}}(e)}(x) \rangle$, $\widetilde{p}^c(e)(x) = \langle \nu_{\widetilde{p}(e)}(x), \mu_{\widetilde{p}(e)}(x) \rangle$. It is true that $(\widetilde{\mathscr{F}_p^c})^c = \widetilde{\mathscr{F}_p}$*

Definition 3.7. *Let X be a non-empty set of the universe and E be a set of parameter. Let $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ be two PPIVFSSs on (X, E). Let $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ be two PPIVFSSs on (X, E). The union and intersection of $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ over (X, E) are denoted by $\widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{G}_q}$ and $\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{G}_q}$ respectively and is defined by $\widetilde{J}_j : E \to \widetilde{P\mathscr{F}(X)} \times \widetilde{P\mathscr{F}(X)}$, $\widetilde{I}_i : E \to \widetilde{P\mathscr{F}(X)} \times \widetilde{P\mathscr{F}(X)}$ such that $\widetilde{J}_j(e)(x) = (\widetilde{J}(e)(x), \widetilde{j}(e)(x))$, $\widetilde{I}_i(e)(x) = (\widetilde{I}(e)(x), \widetilde{i}(e)(x))$, where $\widetilde{J}(e)(x) = \widetilde{\mathscr{F}}(e)(x) \cup \widetilde{\mathscr{G}}(e)(x)$, $\widetilde{j}(e)(x) = \widetilde{p}(e)(x) \cup \widetilde{q}(e)(x)$, $\widetilde{I}(e)(x) = \widetilde{\mathscr{F}}(e)(x) \cap \widetilde{\mathscr{G}}(e)(x)$ and $\widetilde{i}(e)(x) = \widetilde{p}(e)(x) \cap \widetilde{q}(e)(x)$, for all $x \in X$.*

Example 3.4. *Let $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ be the two PPIVFSSs on (X, E) is defined by*

$$\widetilde{\mathscr{F}}_p(e_1) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.5\ \ 0.7],[0.3\ \ 0.6]),\ ([0.6\ \ 0.7],[0.4\ \ 0.5])\rangle} \\ \frac{x_2}{\langle([0.6\ \ 0.7],[0.3\ \ 0.5]),\ ([0.5\ \ 0.6],[0.3\ \ 0.5])\rangle} \\ \frac{x_3}{\langle([0.2\ \ 0.3],[0.4\ \ 0.5]),\ ([0.4\ \ 0.6],[0.2\ \ 0.3])\rangle} \end{array} \right\} ; \widetilde{\mathscr{F}}_p(e_2) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.3\ \ 0.6],[0.5\ \ 0.7]),\ ([0.7\ \ 0.8],[0.4\ \ 0.5])\rangle} \\ \frac{x_2}{\langle([0.4\ \ 0.6],[0.5\ \ 0.7]),\ ([0.4\ \ 0.5],[0.3\ \ 0.4])\rangle} \\ \frac{x_3}{\langle([0.2\ \ 0.4],[0.2\ \ 0.4]),\ ([0.2\ \ 0.6],[0.2\ \ 0.5])\rangle} \end{array} \right\}$$

$$\widetilde{\mathscr{F}}_p(e_3) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.3\ \ 0.4],[0.7\ \ 0.8]),\ ([0.4\ \ 0.7],[0.3\ \ 0.6])\rangle}, \\ \frac{x_2}{\langle([0.3\ \ 0.5],[0.5\ \ 0.7]),([0.6\ \ 0.8],[0.4\ \ 0.6])\rangle}, \\ \frac{x_3}{\langle([0.4\ \ 0.6],[0.5\ \ 0.7]),([0.5\ \ 0.6],[0.2\ \ 0.5])\rangle} \end{array} \right\}$$

$$\widetilde{\mathscr{G}}_q(e_1) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.4\ \ 0.6],[0.5\ \ 0.7]),\ ([0.5\ \ 0.8],[0.4\ \ 0.6])\rangle} \\ \frac{x_2}{\langle([0.3\ \ 0.5],[0.4\ \ 0.6]),\ ([0.4\ \ 0.7],[0.3\ \ 0.4])\rangle} \\ \frac{x_3}{\langle([0.3\ \ 0.5],[0.6\ \ 0.7]),([0.3\ \ 0.4],[0.2\ \ 0.4])\rangle} \end{array} \right\} ; \widetilde{\mathscr{G}}_q(e_2) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.6\ \ 0.7],[0.4\ \ 0.6]),\ ([0.4\ \ 0.7],[0.3\ \ 0.6])\rangle} \\ \frac{x_2}{\langle([0.2\ \ 0.5],[0.6\ \ 0.8]),([0.3\ \ 0.6],[0.2\ \ 0.5])\rangle} \\ \frac{x_3}{\langle([0.3\ \ 0.5],[0.5\ \ 0.7]),([0.3\ \ 0.4],[0.2\ \ 0.6])\rangle} \end{array} \right\}$$

$$\widetilde{\mathscr{G}}_q(e_3) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.4\ \ 0.6],[0.5\ \ 0.6]),\ ([0.4\ \ 0.6],[0.4\ \ 0.5])\rangle}, \\ \frac{x_2}{\langle([0.2\ \ 0.4],[0.4\ \ 0.5]),([0.5\ \ 0.7],[0.4\ \ 0.5])\rangle}, \\ \frac{x_3}{\langle([0.3\ \ 0.5],[0.6\ \ 0.8]),([0.4\ \ 0.7],[0.3\ \ 0.4])\rangle} \end{array} \right\}$$

$$(\widetilde{\mathscr{F}}_p \cup \widetilde{\mathscr{G}}_q)(e_1) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.5\ \ 0.7],[0.3\ \ 0.6]),\ ([0.6\ \ 0.8],[0.4\ \ 0.5])\rangle}, \\ \frac{x_2}{\langle([0.6\ \ 0.7],[0.3\ \ 0.5]),\ ([0.5\ \ 0.7],[0.3\ \ 0.4])\rangle}, \\ \frac{x_3}{\langle([0.3\ \ 0.5],[0.4\ \ 0.5]),([0.4\ \ 0.6],[0.2\ \ 0.3])\rangle} \end{array} \right\} ; (\widetilde{\mathscr{F}}_p \cup \widetilde{\mathscr{G}}_q)(e_2) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.6\ \ 0.7],[0.4\ \ 0.6]),\ ([0.7\ \ 0.8],[0.3\ \ 0.5])\rangle}, \\ \frac{x_2}{\langle([0.4\ \ 0.6],[0.5\ \ 0.7]),([0.4\ \ 0.6],[0.2\ \ 0.4])\rangle}, \\ \frac{x_3}{\langle([0.3\ \ 0.5],[0.2\ \ 0.4]),([0.3\ \ 0.6],[0.2\ \ 0.5])\rangle} \end{array} \right\}$$

$$(\widetilde{\mathscr{F}}_p \cup \widetilde{\mathscr{G}}_q)(e_3) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.4\ \ 0.6],[0.5\ \ 0.6]),\ ([0.4\ \ 0.7],[0.3\ \ 0.5])\rangle}, \\ \frac{x_2}{\langle([0.3\ \ 0.5],[0.4\ \ 0.5]),([0.6\ \ 0.8],[0.4\ \ 0.5])\rangle}, \\ \frac{x_3}{\langle([0.4\ \ 0.6],[0.5\ \ 0.7]),([0.5\ \ 0.7],[0.2\ \ 0.4])\rangle} \end{array} \right\}$$

Definition 3.8. *A PPIVFSS $\widetilde{\emptyset}_\theta(e)(x) = \langle \widetilde{\emptyset}(e)(x), \widetilde{\theta}(e)(x) \rangle$ is said to a possibility null Pythagorean interval valued fuzzy soft set $\widetilde{\emptyset}_\theta : E \to \widetilde{P\mathscr{F}(X)} \times \widetilde{P\mathscr{F}(X)}$, where $\widetilde{\emptyset}(e)(x) = ([0, 0], [1, 1])$ and $\widetilde{\theta}(e)(x) = ([0, 0], [1, 1])$, $\forall x \in X$.*

Definition 3.9. *A PPIVFSS $\widetilde{\Omega}_\Lambda(e)(x) = \langle \widetilde{\Omega}(e)(x), \widetilde{\Lambda}(e)(x) \rangle$ is said to a possibility absolute Pythagorean interval valued fuzzy soft set $\widetilde{\Omega}_\Lambda : E \to \widetilde{P\mathscr{F}(X)} \times \widetilde{P\mathscr{F}(X)}$, where $\widetilde{\Omega}(e)(x) = ([1, 1], [0, 0])$ and $\widetilde{\Lambda}(e)(x) = ([1, 1], [0, 0])$, $\forall x \in X$.*

Theorem 3.1. *Let $\widetilde{\mathscr{F}_p}$ be a PPIVFSS on (X, E). Then the following properties are holds:*
(i) $\widetilde{\mathscr{F}_p} = \widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{F}_p}$, $\widetilde{\mathscr{F}_p} = \widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{F}_p}$
(ii) $\widetilde{\mathscr{F}_p} \subseteq \widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{F}_p}$, $\widetilde{\mathscr{F}_p} \subseteq \widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{F}_p}$
(iii) $\widetilde{\mathscr{F}_p} \cup \widetilde{\emptyset}_\theta = \widetilde{\mathscr{F}_p}$, $\widetilde{\mathscr{F}_p} \cap \widetilde{\emptyset}_\theta = \widetilde{\emptyset}_\theta$
(iv) $\widetilde{\mathscr{F}_p} \cup \widetilde{\Omega}_\Lambda = \widetilde{\Omega}_\Lambda$, $\widetilde{\mathscr{F}_p} \cap \widetilde{\Omega}_\Lambda = \widetilde{\mathscr{F}_p}$.

Remark 3.1. *Let $\widetilde{\mathscr{F}_p}$ be a PPIVFSS on (X,E). If $\widetilde{\mathscr{F}_p} \neq \widetilde{\Omega}_\Lambda$ or $\widetilde{\mathscr{F}_p} \neq \widetilde{\emptyset}_\theta$, then $\widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{F}_p^c} \neq \widetilde{\Omega}_\Lambda$ and $\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{F}_p^c} \neq \widetilde{\emptyset}_\theta$.*

Theorem 3.2. *Let $\widetilde{\mathscr{F}_p}$, $\widetilde{\mathscr{G}_q}$ and $\widetilde{\mathscr{H}_r}$ are three PPIVFSSs over (X,E), then the following properties are hold:*
(1) $\widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{G}_q} = \widetilde{\mathscr{G}_q} \cup \widetilde{\mathscr{F}_p}$,
(2) $\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{G}_q} = \widetilde{\mathscr{G}_q} \cap \widetilde{\mathscr{F}_p}$,
(3) $\widetilde{\mathscr{F}_p} \cup (\widetilde{\mathscr{G}_q} \cup \widetilde{\mathscr{H}_r}) = (\widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{G}_q}) \cup \widetilde{\mathscr{H}_r}$,
(4) $\widetilde{\mathscr{F}_p} \cap (\widetilde{\mathscr{G}_q} \cap \widetilde{\mathscr{H}_r}) = (\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{G}_q}) \cap \widetilde{\mathscr{H}_r}$.
(5) $(\widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{G}_q})^c = \widetilde{\mathscr{F}_p^c} \cap \widetilde{\mathscr{G}_q^c}$,
(6) $(\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{G}_q})^c = \widetilde{\mathscr{F}_p^c} \cup \widetilde{\mathscr{G}_q^c}$,
(7) $(\widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{G}_q}) \cap \widetilde{\mathscr{F}_p} = \widetilde{\mathscr{F}_p}$,
(8) $(\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{G}_q}) \cup \widetilde{\mathscr{F}_p} = \widetilde{\mathscr{F}_p}$,
(9) $\widetilde{\mathscr{F}_p} \cup (\widetilde{\mathscr{G}_q} \cap \widetilde{\mathscr{H}_r}) = (\widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{G}_q}) \cap (\widetilde{\mathscr{F}_p} \cup \widetilde{\mathscr{H}_r})$.
(10) $\widetilde{\mathscr{F}_p} \cap (\widetilde{\mathscr{G}_q} \cup \widetilde{\mathscr{H}_r}) = (\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{G}_q}) \cup (\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{H}_r})$.

Proof. The proof follows from Definition 3.6 and 3.7. □

Definition 3.10. *Let $(\widetilde{\mathscr{F}_p}, A)$ and $(\widetilde{\mathscr{G}_q}, B)$ be two PPIVFSSs on (X,E). Then the operations "$(\widetilde{\mathscr{F}_p}, A)$ AND $(\widetilde{\mathscr{G}_q}, B)$" is denoted by $(\widetilde{\mathscr{F}_p}, A) \wedge (\widetilde{\mathscr{G}_q}, B)$ and is defined by $(\widetilde{\mathscr{F}_p}, A) \wedge (\widetilde{\mathscr{G}_q}, B) = (\widetilde{\mathscr{H}_r}, A \times B)$, where $\widetilde{\mathscr{H}_r}(\alpha,\beta) = (\widetilde{\mathscr{H}}(\alpha,\beta)(x), \widetilde{r}(\alpha,\beta)(x))$ such that $\widetilde{\mathscr{H}}(\alpha,\beta) = \widetilde{\mathscr{F}}(\alpha) \cap \widetilde{\mathscr{G}}(\beta)$ and $\widetilde{r}(\alpha,\beta) = \widetilde{p}(\alpha) \cap \widetilde{q}(\beta)$, for all $(\alpha,\beta) \in A \times B$.*

Definition 3.11. *Let $(\widetilde{\mathscr{F}_p}, A)$ and $(\widetilde{\mathscr{G}_q}, B)$ be two PPIVFSSs on (X,E), then the operations "$(\widetilde{\mathscr{F}_p}, A)$ OR $(\widetilde{\mathscr{G}_q}, B)$" is denoted by $(\widetilde{\mathscr{F}_p}, A) \vee (\widetilde{\mathscr{G}_q}, B)$ and is defined by $(\widetilde{\mathscr{F}_p}, A) \vee (\widetilde{\mathscr{G}_q}, B) = (\widetilde{\mathscr{H}_r}, A \times B)$, where $\widetilde{\mathscr{H}_r}(\alpha,\beta) = (\widetilde{\mathscr{H}}(\alpha,\beta)(x), \widetilde{r}(\alpha,\beta)(x))$ such that $\widetilde{\mathscr{H}}(\alpha,\beta) = \widetilde{\mathscr{F}}(\alpha) \cup \widetilde{\mathscr{G}}(\beta)$ and $\widetilde{r}(\alpha,\beta) = \widetilde{p}(\alpha) \cup \widetilde{q}(\beta)$, for all $(\alpha,\beta) \in A \times B$.*

Remark 3.2. *Let $(\widetilde{\mathscr{F}_p}, A)$ and $(\widetilde{\mathscr{G}_q}, B)$ be two PPIVFSSs on (X,E). For all $(\alpha,\beta) \in A \times B$, if $\alpha \neq \beta$, then $(\widetilde{\mathscr{F}_p}, A) \vee (\widetilde{\mathscr{G}_q}, B) \neq (\widetilde{\mathscr{G}_q}, B) \vee (\widetilde{\mathscr{F}_p}, A)$ and $(\widetilde{\mathscr{F}_p}, A) \wedge (\widetilde{\mathscr{G}_q}, B) \neq (\widetilde{\mathscr{G}_q}, B) \wedge (\widetilde{\mathscr{F}_p}, A)$.*

Theorem 3.3. *Let $(\widetilde{\mathscr{F}_p}, A)$ and $(\widetilde{\mathscr{G}_q}, B)$ be two PPIVFSSs on (X,E), then*
(i) $((\widetilde{\mathscr{F}_p}, A) \wedge (\widetilde{\mathscr{G}_q}, B))^c = (\widetilde{\mathscr{F}_p}, A)^c \vee (\widetilde{\mathscr{G}_q}, B)^c$
(ii) $((\widetilde{\mathscr{F}_p}, A) \vee (\widetilde{\mathscr{G}_q}, B))^c = (\widetilde{\mathscr{F}_p}, A)^c \wedge (\widetilde{\mathscr{G}_q}, B)^c$.

Proof. (i) Suppose that $(\widetilde{\mathscr{F}_p}, A) \wedge (\widetilde{\mathscr{G}_q}, B) = (\widetilde{\mathscr{H}_r}, A \times B)$.
Now, $\widetilde{\mathscr{H}_r^c}(\alpha,\beta) = (\widetilde{\mathscr{H}^c}(\alpha,\beta)(x), \widetilde{r}^c(\alpha,\beta)(x))$, for all $(\alpha,\beta) \in A \times B$. By Theorem 3.2 and Definition 3.10, $\widetilde{\mathscr{H}^c}(\alpha,\beta) = (\widetilde{\mathscr{F}}(\alpha) \cap \widetilde{\mathscr{G}}(\beta))^c = \widetilde{\mathscr{F}^c}(\alpha) \cup \widetilde{\mathscr{G}^c}(\beta)$ and $\widetilde{r}^c(\alpha,\beta) = (\widetilde{p}(\alpha) \cap \widetilde{q}(\beta))^c = \widetilde{p}^c(\alpha) \cup \widetilde{q}^c(\beta)$. On the other hand, given that $(\widetilde{\mathscr{F}_p}, A)^c \vee (\widetilde{\mathscr{G}_q}, B)^c = (\widetilde{\Lambda}_o, A \times B)$, where $\widetilde{\Lambda}_o(\alpha,\beta) = (\widetilde{\Lambda}(\alpha,\beta)(x), \widetilde{o}(\alpha,\beta)(x))$ such that $\widetilde{\Lambda}(\alpha,\beta) = \widetilde{\mathscr{F}^c}(\alpha) \cup \widetilde{\mathscr{G}^c}(\beta)$ and $\widetilde{o}(\alpha,\beta) = \widetilde{p}^c(\alpha) \cup \widetilde{q}^c(\beta)$ for all $(\alpha,\beta) \in A \times B$. Thus, $\widetilde{\mathscr{H}_r^c} = \widetilde{\Lambda}_o$. Hence $((\widetilde{\mathscr{F}_p}, A) \wedge (\widetilde{\mathscr{G}_q}, B))^c = (\widetilde{\mathscr{F}_p}, A)^c \vee (\widetilde{\mathscr{G}_q}, B)^c$. Similarly to prove other part. □

4. Similarity measure between two possibilities Pythagorean interval valued fuzzy soft sets

Definition 4.1. *Let X be a non-empty set of the universe and E be a set of parameter. Suppose that $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ are two PPIVFSSs on (X, E). The similarity measure between two PPIVFSSs $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$ is denoted by $Sim(\widetilde{\mathscr{F}_p}, \widetilde{\mathscr{G}_q})$ and is defined as:*

$$Sim(\widetilde{\mathscr{F}_p}, \widetilde{\mathscr{G}_q}) = \left[Sim(\mathscr{F}_p^L, \mathscr{G}_q^L),\ Sim(\mathscr{F}_p^U, \mathscr{G}_q^U)\right]$$
$$= \left[(\varphi(\mathscr{F}^L, \mathscr{G}^L) \cdot \psi(p^L, q^L)),\ (\varphi(\mathscr{F}^U, \mathscr{G}^U) \cdot \psi(p^U, q^U))\right]$$

such that $\varphi(\widetilde{\mathscr{F}}, \widetilde{\mathscr{G}}) = \left[\varphi(\mathscr{F}^L, \mathscr{G}^L),\ \varphi(\mathscr{F}^U, \mathscr{G}^U)\right] =$

$$\left[\frac{T(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)) + S(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x))}{2}, \frac{T(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x)) + S(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x))}{2}\right] \text{ and}$$

$$\psi(\widetilde{p}, \widetilde{q}) = \left[\psi(p^L, q^L),\ \psi(p^U, q^U)\right] = \left[1 - \frac{\sum|\alpha_i^L - \beta_i^L|}{\sum|\alpha_i^L + \beta_i^L|}\ ,\ 1 - \frac{\sum|\alpha_i^U - \beta_i^U|}{\sum|\alpha_i^U + \beta_i^U|}\right],$$

where $\left[T(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)), T(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x))\right] =$

$$\left[\frac{\sum_{i=1}^n(\mu^L_{\mathscr{F}(e_i)}(x) \cdot \mu^L_{\mathscr{G}(e_i)}(x))}{\sum_{i=1}^n(1 - \sqrt{(1-\mu^{2L}_{\mathscr{F}(e_i)}(x)) \cdot (1-\mu^{2L}_{\mathscr{G}(e_i)}(x))})}\ ,\ \frac{\sum_{i=1}^n(\mu^U_{\mathscr{F}(e_i)}(x) \cdot \mu^U_{\mathscr{G}(e_i)}(x))}{\sum_{i=1}^n(1 - \sqrt{(1-\mu^{2U}_{\mathscr{F}(e_i)}(x)) \cdot (1-\mu^{2U}_{\mathscr{G}(e_i)}(x))})}\right] \text{ and}$$

$$\left[(S(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)))\ , (S(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x)))\right] =$$

$$\left[\sqrt{1 - \frac{\sum_{i=1}^n |\nu^{2L}_{\mathscr{F}(e_i)}(x) - \nu^{2L}_{\mathscr{G}(e_i)}(x)|}{\sum_{i=1}^n 1 + ((\nu^{2L}_{\mathscr{F}(e_i)}(x)) \cdot (\nu^{2L}_{\mathscr{G}(e_i)}(x)))}}\ ,\ \sqrt{1 - \frac{\sum_{i=1}^n |\nu^{2U}_{\mathscr{F}(e_i)}(x) - \nu^{2U}_{\mathscr{G}(e_i)}(x)|}{\sum_{i=1}^n 1 + ((\nu^{2U}_{\mathscr{F}(e_i)}(x)) \cdot (\nu^{2U}_{\mathscr{G}(e_i)}(x)))}}\right] \text{ and}$$

$$\alpha_i^L = \frac{\mu^{2L}_{p(e_i)}(x)}{\mu^{2L}_{p(e_i)}(x) + \nu^{2L}_{p(e_i)}(x)}, \quad \beta_i^L = \frac{\mu^{2L}_{q(e_i)}(x)}{\mu^{2L}_{q(e_i)}(x) + \nu^{2L}_{q(e_i)}(x)},$$
$$\alpha_i^U = \frac{\mu^{2U}_{p(e_i)}(x)}{\mu^{2U}_{p(e_i)}(x) + \nu^{2U}_{p(e_i)}(x)}\ , \quad \beta_i^U = \frac{\mu^{2U}_{q(e_i)}(x)}{\mu^{2U}_{q(e_i)}(x) + \nu^{2U}_{q(e_i)}(x)}.$$

Theorem 4.1. *Let $\widetilde{\mathscr{F}_p}$, $\widetilde{\mathscr{G}_q}$ and $\widetilde{\mathscr{H}_r}$ be the any three PPIVFSSs over (X, E). Then the following statements are holds:*
(i) $Sim(\widetilde{\mathscr{F}_p}, \widetilde{\mathscr{G}_q}) = Sim(\widetilde{\mathscr{G}_q}, \widetilde{\mathscr{F}_p})$
(ii) $[0, 0] = 0 \leq Sim(\widetilde{\mathscr{F}_p}, \widetilde{\mathscr{G}_q}) \leq [1, 1] = 1$
(iii) $\widetilde{\mathscr{F}_p} = \widetilde{\mathscr{G}_q} \implies Sim(\widetilde{\mathscr{F}_p}, \widetilde{\mathscr{G}_q}) = 1$
(iv) $\widetilde{\mathscr{F}_p} \subseteq \widetilde{\mathscr{G}_q} \subseteq \widetilde{\mathscr{H}_r} \implies Sim(\widetilde{\mathscr{F}_p}, \widetilde{\mathscr{H}_r}) \leq Sim(\widetilde{\mathscr{G}_q}, \widetilde{\mathscr{H}_r})$
(v) $\widetilde{\mathscr{F}_p} \cap \widetilde{\mathscr{G}_q} = \{\phi\} \Leftrightarrow Sim(\widetilde{\mathscr{F}_p}, \widetilde{\mathscr{G}_q}) = 0.$

Proof. The proof (i), (ii) and (v) are trivial. Suppose that $\widetilde{\mathscr{F}_p} = \widetilde{\mathscr{G}_q}$ implies that $\mu_{\widetilde{\mathscr{F}}(e)}(x) = \mu_{\widetilde{\mathscr{G}}(e)}(x)$, $\nu_{\widetilde{\mathscr{F}}(e)}(x) = \nu_{\widetilde{\mathscr{G}}(e)}(x)$, $\mu_{\widetilde{p}(e)}(x) = \mu_{\widetilde{q}(e)}(x)$ and $\nu_{\widetilde{p}(e)}(x) = \nu_{\widetilde{q}(e)}(x)$.
Now, $\left[T(\mathscr{F}^L(e)(x), \mathscr{F}^L(e)(x)),\ T(\mathscr{F}^U(e)(x), \mathscr{F}^U(e)(x))\right]$

$$\begin{aligned} &= \left[\frac{\sum_{i=1}^n(\mu^L_{\mathscr{F}(e_i)}(x))^2}{\sum_{i=1}^n(1 - 1 + (\mu^L_{\mathscr{F}(e_i)}(x))^2)}, \frac{\sum_{i=1}^n(\mu^U_{\mathscr{F}(e_i)}(x))^2}{\sum_{i=1}^n(1 - 1 + (\mu^U_{\mathscr{F}(e_i)}(x))^2)}\right] \\ &= \left[\frac{\sum_{i=1}^n(\mu^L_{\mathscr{F}(e_i)}(x))^2}{\sum_{i=1}^n(\mu^L_{\mathscr{F}(e_i)}(x))^2}, \frac{\sum_{i=1}^n(\mu^U_{\mathscr{F}(e_i)}(x))^2}{\sum_{i=1}^n(\mu^U_{\mathscr{F}(e_i)}(x))^2}\right] \\ &= 1 \end{aligned}$$

and $\Big[S(\mathscr{F}^L(e)(x),\mathscr{F}^L(e)(x)), S(\mathscr{F}^U(e)(x),\mathscr{F}^U(e)(x))\Big] = \Big[\sqrt{(1-0)}, \sqrt{(1-0)}\Big] = 1$.
Thus, $\varphi(\widetilde{\mathscr{F}},\widetilde{\mathscr{F}}) = \Big[\varphi(\mathscr{F}^L,\mathscr{F}^L),\ \varphi(\mathscr{F}^U,\mathscr{F}^U)\Big] = \Big[\frac{1+1}{2}, \frac{1+1}{2}\Big] = 1$ and
$\psi(\widetilde{p},\widetilde{p}) = \Big[\psi(p^L,p^L),\ \psi(p^U,p^U)\Big] = 1$.
Hence $Sim(\widetilde{\mathscr{F}}_p,\widetilde{\mathscr{F}}_q) = \Big[Sim(\mathscr{F}^L_p,\mathscr{F}^L_q),\ Sim(\mathscr{F}^U_p,\mathscr{F}^U_q)\Big] = 1$. This proves (iii).
(iv) Given that

$$\left\{\begin{array}{l} \widetilde{\mathscr{F}}_p \subseteq \widetilde{\mathscr{G}}_q \implies \mu_{\widetilde{\mathscr{F}}(e)}(x) \le \mu_{\widetilde{\mathscr{G}}(e)}(x),\ \ \nu_{\widetilde{\mathscr{F}}(e)}(x) \ge \nu_{\widetilde{\mathscr{G}}(e)}(x) \\ \mu_{\widetilde{p}(e)}(x) \le \mu_{\widetilde{q}(e)}(x),\ \ \nu_{\widetilde{p}(e)}(x) \ge \nu_{\widetilde{q}(e)}(x) \\ \widetilde{\mathscr{F}}_p \subseteq \widetilde{\mathscr{H}}_r \implies \mu_{\widetilde{\mathscr{F}}(e)}(x) \le \mu_{\widetilde{\mathscr{H}}(e)}(x),\ \ \nu_{\widetilde{\mathscr{F}}(e)}(x) \ge \nu_{\widetilde{\mathscr{H}}(e)}(x) \\ \mu_{\widetilde{p}(e)}(x) \le \mu_{\widetilde{r}(e)}(x),\ \ \nu_{\widetilde{p}(e)}(x) \ge \nu_{\widetilde{r}(e)}(x) \\ \widetilde{\mathscr{G}}_q \subseteq \widetilde{\mathscr{H}}_r \implies \mu_{\widetilde{\mathscr{G}}(e)}(x) \le \mu_{\widetilde{\mathscr{H}}(e)}(x),\ \ \nu_{\widetilde{\mathscr{G}}(e)}(x) \ge \nu_{\widetilde{\mathscr{H}}(e)}(x) \\ \mu_{\widetilde{q}(e)}(x) \le \mu_{\widetilde{r}(e)}(x),\ \ \nu_{\widetilde{q}(e)}(x) \ge \nu_{\widetilde{r}(e)}(x) \end{array}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(*)$$

Clearly, $\mu_{\widetilde{\mathscr{F}}(e)}(x)\cdot\mu_{\widetilde{\mathscr{H}}(e)}(x) \le \mu_{\widetilde{\mathscr{G}}(e)}(x)\cdot\mu_{\widetilde{\mathscr{H}}(e)}(x)$ implies that
$\sum_{i=1}^n(\mu_{\widetilde{\mathscr{F}}(e_i)}(x)\cdot\mu_{\widetilde{\mathscr{H}}(e_i)}(x)) \le \sum_{i=1}^n(\mu_{\widetilde{\mathscr{G}}(e_i)}(x)\cdot\mu_{\widetilde{\mathscr{H}}(e_i)}(x))$ ....(1)
Clearly, $(\mu_{\widetilde{\mathscr{F}}(e)}(x))^2 \le (\mu_{\widetilde{\mathscr{G}}(e)}(x))^2$ implies that
$(1-(\mu_{\widetilde{\mathscr{F}}(e)}(x))^2)\cdot(1-(\mu_{\widetilde{\mathscr{H}}(e)}(x))^2) \ge (1-(\mu_{\widetilde{\mathscr{G}}(e)}(x))^2)\cdot(1-(\mu_{\widetilde{\mathscr{H}}(e)}(x))^2)$ and
$1-\sqrt{(1-(\mu_{\widetilde{\mathscr{F}}(e)}(x))^2)\cdot(1-(\mu_{\widetilde{\mathscr{H}}(e)}(x))^2)} \ \le\ 1-\sqrt{(1-(\mu_{\widetilde{\mathscr{G}}(e)}(x))^2)\cdot(1-(\mu_{\widetilde{\mathscr{H}}(e)}(x))^2)}$
and $\sum_{i=1}^n 1-\sqrt{(1-(\mu_{\widetilde{\mathscr{F}}(e_i)}(x))^2)\cdot(1-(\mu_{\widetilde{\mathscr{H}}(e_i)}(x))^2)} \le \sum_{i=1}^n 1-\sqrt{(1-(\mu_{\widetilde{\mathscr{G}}(e_i)}(x))^2)\cdot(1-(\mu_{\widetilde{\mathscr{H}}(e_i)}(x))^2)}$.......(2)
Equation (1) is divided by (2),

$$\frac{\sum_{i=1}^n(\mu_{\widetilde{\mathscr{F}}(e_i)}(x)\cdot\mu_{\widetilde{\mathscr{H}}(e_i)}(x))}{\sum_{i=1}^n 1-\sqrt{(1-(\mu_{\widetilde{\mathscr{F}}(e_i)}(x))^2)\cdot(1-(\mu_{\widetilde{\mathscr{H}}(e_i)}(x))^2)}} \le \frac{\sum_{i=1}^n(\mu_{\widetilde{\mathscr{G}}(e_i)}(x)\cdot\mu_{\widetilde{\mathscr{H}}(e_i)}(x))}{\sum_{i=1}^n 1-\sqrt{(1-(\mu_{\widetilde{\mathscr{G}}(e_i)}(x))^2)\cdot(1-(\mu_{\widetilde{\mathscr{H}}(e_i)}(x))^2)}} \ \ldots\ldots..(3)$$

Clearly, $\nu^2_{\widetilde{\mathscr{F}}(e)}(x) \ge \nu^2_{\widetilde{\mathscr{G}}(e)}(x)$ and $\nu^2_{\widetilde{\mathscr{F}}(e)}(x) - \nu^2_{\widetilde{\mathscr{H}}(e)}(x) \ge \nu^2_{\widetilde{\mathscr{G}}(e)}(x) - \nu^2_{\widetilde{\mathscr{H}}(e)}(x)$.
Hence $\sum_{i=1}^n \Big|\nu^2_{\widetilde{\mathscr{F}}(e_i)}(x) - \nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)\Big| \ge \sum_{i=1}^n \Big|\nu^2_{\widetilde{\mathscr{G}}(e_i)}(x) - \nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)\Big|$ ....(4)
Also, $(\nu^2_{\widetilde{\mathscr{F}}(e)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e)}(x)) \ge (\nu^2_{\widetilde{\mathscr{G}}(e)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e)}(x))$ implies that
$\sum_{i=1}^n 1+(\nu^2_{\widetilde{\mathscr{F}}(e_i)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)) \ge \sum_{i=1}^n 1+(\nu^2_{\widetilde{\mathscr{G}}(e_i)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x))$ ....(5)
Equation (4) is divided by (5), we get

$$\frac{\sum_{i=1}^n|\nu^2_{\widetilde{\mathscr{F}}(e_i)}(x)-\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)|}{\sum_{i=1}^n 1+(\nu^2_{\widetilde{\mathscr{F}}(e_i)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x))} \ge \frac{\sum_{i=1}^n|\nu^2_{\widetilde{\mathscr{G}}(e_i)}(x)-\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)|}{\sum_{i=1}^n 1+(\nu^2_{\widetilde{\mathscr{G}}(e_i)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x))} \text{ and}$$

$$1-\frac{\sum_{i=1}^n|\nu^2_{\widetilde{\mathscr{F}}(e_i)}(x)-\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)|}{\sum_{i=1}^n 1+(\nu^2_{\widetilde{\mathscr{F}}(e_i)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x))} \le 1-\frac{\sum_{i=1}^n|\nu^2_{\widetilde{\mathscr{G}}(e_i)}(x)-\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)|}{\sum_{i=1}^n 1+(\nu^2_{\widetilde{\mathscr{G}}(e_i)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x))} \text{ and}$$

$$\sqrt{1-\frac{\sum_{i=1}^n|\nu^2_{\widetilde{\mathscr{F}}(e_i)}(x)-\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)|}{\sum_{i=1}^n 1+(\nu^2_{\widetilde{\mathscr{F}}(e_i)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x))}} \le \sqrt{1-\frac{\sum_{i=1}^n|\nu^2_{\widetilde{\mathscr{G}}(e_i)}(x)-\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x)|}{\sum_{i=1}^n 1+(\nu^2_{\widetilde{\mathscr{G}}(e_i)}(x)\cdot\nu^2_{\widetilde{\mathscr{H}}(e_i)}(x))}} \ \ldots\ldots\ldots\ldots(6)$$

Adding Equation (3), (6) and divided by 2, $\varphi(\widetilde{\mathscr{F}},\widetilde{\mathscr{H}}) \le \varphi(\widetilde{\mathscr{G}},\widetilde{\mathscr{H}})$ ....(7)
By Equation (*), Clearly $\Big[\alpha_i^L,\alpha_i^U\Big] \le \Big[\beta_i^L,\beta_i^U\Big] \le \Big[\gamma_i^L,\gamma_i^U\Big]$, where

$$\Big[\alpha_i^L,\alpha_i^U\Big] = \left[\frac{\mu^{2L}_{p(e_i)}(x)}{\mu^{2L}_{p(e_i)}(x)+\nu^{2L}_{p(e_i)}(x)}, \frac{\mu^{2U}_{p(e_i)}(x)}{\mu^{2U}_{p(e_i)}(x)+\nu^{2U}_{p(e_i)}(x)}\right],$$

$$\Big[\beta_i^L,\beta_i^U\Big] = \left[\frac{\mu^{2L}_{q(e_i)}(x)}{\mu^{2L}_{q(e_i)}(x)+\nu^{2L}_{q(e_i)}(x)}, \frac{\mu^{2U}_{q(e_i)}(x)}{\mu^{2U}_{q(e_i)}(x)+\nu^{2U}_{q(e_i)}(x)}\right] \text{ and}$$

$$\Big[\gamma_i^L,\gamma_i^U\Big] = \left[\frac{\mu^{2L}_{r(e_i)}(x)}{\mu^{2L}_{r(e_i)}(x)+\nu^{2L}_{r(e_i)}(x)}, \frac{\mu^{2U}_{r(e_i)}(x)}{\mu^{2U}_{r(e_i)}(x)+\nu^{2U}_{r(e_i)}(x)}\right].$$

Now, $\left[\alpha_i^L, \alpha_i^U\right] - \left[\gamma_i^L, \gamma_i^U\right] \leq \left[\beta_i^L, \beta_i^U\right] - \left[\gamma_i^L, \gamma_i^U\right]$.
Since $\left[\alpha_i^L, \alpha_i^U\right], \left[\beta_i^L, \beta_i^U\right], \left[\gamma_i^L, \gamma_i^U\right]$ are numerical values.
Hence $\left|\left[\beta_i^L, \beta_i^U\right] - \left[\gamma_i^L, \gamma_i^U\right]\right| \leq \left|\left[\alpha_i^L, \alpha_i^U\right] - \left[\gamma_i^L, \gamma_i^U\right]\right|$ and
$-\left|\left[\alpha_i^L, \alpha_i^U\right] - \left[\gamma_i^L, \gamma_i^U\right]\right| \leq -\left|\left[\beta_i^L, \beta_i^U\right] - \left[\gamma_i^L, \gamma_i^U\right]\right|$ ....(8) and
$\left|\left[\alpha_i^L, \alpha_i^U\right] + \left[\gamma_i^L, \gamma_i^U\right]\right| \leq \left|\left[\beta_i^L, \beta_i^U\right] + \left[\gamma_i^L, \gamma_i^U\right]\right|$ ....(9)
Equation (8) is divided by (9), we get

$$\frac{-\left|\left[\alpha_i^L,\alpha_i^U\right]-\left[\gamma_i^L,\gamma_i^U\right]\right|}{\left|\left[\alpha_i^L,\alpha_i^U\right]+\left[\gamma_i^L,\gamma_i^U\right]\right|} \leq \frac{-\left|\left[\beta_i^L,\beta_i^U\right]-\left[\gamma_i^L,\gamma_i^U\right]\right|}{\left|\left[\beta_i^L,\beta_i^U\right]+\left[\gamma_i^L,\gamma_i^U\right]\right|} \text{ and}$$

$$1 - \frac{\left|\left[\alpha_i^L,\alpha_i^U\right]-\left[\gamma_i^L,\gamma_i^U\right]\right|}{\left|\left[\alpha_i^L,\alpha_i^U\right]+\left[\gamma_i^L,\gamma_i^U\right]\right|} \leq 1 - \frac{\left|\left[\beta_i^L,\beta_i^U\right]-\left[\gamma_i^L,\gamma_i^U\right]\right|}{\left|\left[\beta_i^L,\beta_i^U\right]+\left[\gamma_i^L,\gamma_i^U\right]\right|}.$$

Hence $\psi(\widetilde{p}, \widetilde{r}) \leq \psi(\widetilde{q}, \widetilde{r})$ ....(10)
Multiply by Equation (7) and (10), $\varphi(\widetilde{\mathscr{F}}, \widetilde{\mathscr{H}}) \cdot \psi(\widetilde{p}, \widetilde{r}) \leq \varphi(\widetilde{\mathscr{G}}, \widetilde{\mathscr{H}}) \cdot \psi(\widetilde{q}, \widetilde{r})$.
Hence $Sim(\widetilde{\mathscr{F}_p}, \widetilde{\mathscr{H}_r}) \leq Sim(\widetilde{\mathscr{G}_q}, \widetilde{\mathscr{H}_r})$. This proves (iv). □

Example 4.1. *Calculate the similarity measure between the two PPIVFSSs, $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$. We choose the first sample of $\widetilde{\mathscr{F}_p}$ and $\widetilde{\mathscr{G}_q}$, $E = \{e_1, e_2, e_3, e_4\}$ can be defined as below:*

$\widetilde{\mathscr{F}_p}(e)$	e_1	e_2	e_3	e_4
$\widetilde{\mathscr{F}}(e)$	([0.6 0.7], [0.4 0.6])	([0.5 0.7], [0.3 0.6])	([0.4 0.6], [0.5 0.7])	([0.3 0.5], [0.6 0.8])
$\widetilde{p}(e)$	([0.7 0.8], [0.2 0.5])	([0.6 0.7], [0.3 0.4])	([0.5 0.7], [0.6 0.7])	([0.4 0.6], [0.3 0.7])

$\widetilde{\mathscr{G}_q}(e)$	e_1	e_2	e_3	e_4
$\widetilde{\mathscr{G}}(e)$	([0.3 0.5], [0.5 0.7])	([0.4 0.5], [0.6 0.7])	([0.3 0.4], [0.7 0.8])	([0.2 0.6], [0.4 0.5])
$\widetilde{q}(e)$	([0.4 0.5], [0.6 0.8])	([0.5 0.6], [0.4 0.5])	([0.4 0.6], [0.6 0.7])	([0.6 0.7], [0.5 0.6])

Using Definition 4.1 and routine calculation, we get

$$\begin{aligned}
T(\widetilde{\mathscr{F}}(e)(x), \widetilde{\mathscr{G}}(e)(x)) &= \left[T(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)),\ T(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x))\right] \\
&= \left[0.883061,\ 0.927425\right]. \\
S(\widetilde{\mathscr{F}}(e)(x), \widetilde{\mathscr{G}}(e)(x)) &= \left[S(\mathscr{F}^L(e)(x), \mathscr{G}^L(e)(x)),\ S(\mathscr{F}^U(e)(x), \mathscr{G}^U(e)(x))\right] \\
&= \left[0.901041,\ 0.913370\right]. \\
\varphi(\widetilde{\mathscr{F}}, \widetilde{\mathscr{G}}) &= \left[\varphi(\mathscr{F}^L, \mathscr{G}^L),\ \varphi(\mathscr{F}^U, \mathscr{G}^U)\right] = \left[0.892051, 0.920397\right]. \\
\psi(\widetilde{p}, \widetilde{q}) &= \left[\psi(p^L, q^L),\ \psi(p^U, q^U)\right] = \left[0.791039, 0.805205\right].
\end{aligned}$$

$$\begin{aligned} Sim(\widetilde{\mathscr{F}_p},\widetilde{\mathscr{G}_q}) &= \Big[Sim(\mathscr{F}_p^L,\mathscr{G}_q^L),\, Sim(\mathscr{F}_p^U,\mathscr{G}_q^U)\Big] \\ &= \Big[0.892051, 0.920397\Big] \times \Big[0.791039, 0.805205\Big] \\ &= \Big[0.705648, 0.741108\Big]. \end{aligned}$$

5. Application of similarity measure in decision making of Parental Choice of School

Decisions in most real life problems, such as education, economy, management, politics and technology in daily life. In economy, we know that decisions have a major impact on customer cost, manufacturing, service and efficiency etc. The same is true for school education. It is the best results for education to choose the best school education property. In the selection of school teaching education, the evaluation of teaching education is carried out according to various standards of experts. There are various studies, primarily conducted that have investigate the reasons why parents select a school, which they perceive best meets their childrens needs and parental aspirations for their children. We identify a factor regarded as parental decision making: Academic Factor - divided into five identified elements namely Class size, Fees structure, Quality, Location and Student/Teacher relationship.

Class size is a very important element in parental considerations when deciding upon which private or public school to choose. Furthermore such parental decisions are based on the assumption that a smaller class equates to a more suitable and better quality learning environment in which the students achievements and development will be enhanced through a constructive relationship between teachers and learners in which teachers have more time to devote to supporting each individual learner. In relation to the importance of academic programmes related to students achievement a lot of the research into public or private schools in terms of student attainment has shown that a high quality academic programme leads to high student achievement. Good teaching is charged with positive emotion, good teachers are not just well oiled machines. They are emotional, passionate beings who connect with their students and fill their work and classes with pleasure, creativity, challenge and joy. Our goal is to select the optimal one out of a great number of alternatives based on the assessment of experts against the criteria.

5.1. **Algorithm for PPIVFSS Model.**
The algorithm for the selection of the best choice is given as:

(1) Input the PPIVFSS in tabular form.
(2) Input the set of choice parameters $A \subseteq E$.
(3) Compute the values of T and S.
(4) Calculate the φ value by taking $\frac{T+S}{2}$.
(5) Determine the value $\psi = 1 - \frac{\sum|\alpha_i-\beta_i|}{\sum|\alpha_i+\beta_i|}$ and $1 \leq i \leq 5$.
(6) Compute the similarity measure by taking the product of φ and ψ.
(7) Determine maximum similarity, where maximum similarity= $Max\{similarity^i\}$ and $1 \leq i \leq 5$.
(8) Finally, decision is to choose as the best solution to the problem.

5.2. **Survey study.**
A parent intends to choose the popular school education property. Here we intends to choose five schools are nominated. The score of the school education

property evaluated by the experts is represented by $E = \{e_1$: Class size, e_2: Fees Structure, e_3 : Quality, e_4 : Location, e_5: Student/Teacher relationship$\}$.
Suppose that decision makers in the school education can provide the PIVFN values for the ideal school education property, which reflect the pursuit of the ideal qualities of the school education property. The evaluations of the school education property as per PPIVFSS are shown as Tables 2-6. The PIVFNs values in Tables 2-6 are provided by the experts, depending on their assessment of the alternatives against the criteria under consideration. In this example, in order to find the school education property which is closest to the ideal school education property, we should calculate the similarity measure of PPIVFSSs in Tables 2-6 with the one in Table 1 based on Definition 4.1. The threshold of the similarity should rely on the school property.

Table 1
PPIVFSS for the ideal school education property

$\widetilde{\mathscr{L}_p}(e)$	e_1	e_2	e_3	e_4	e_5
$\widetilde{\mathscr{L}}(e)$	([0.9 0.95], [0.15 0.2])	([0.8 0.9], [0.2 0.25])	([0.9 0.95], [0.15 0.2])	([0.85 0.9], [0.2 0.3])	([0.8 0.85], [0.3 0.4])
$\widetilde{p}(e)$	([1 1], [0 0])	([1 1], [0 0])	([1 1], [0 0])	([1 1], [0 0])	([1 1], [0 0])

Table 2
PPIVFSS for the first school education property

$\widetilde{\mathscr{A}_{p_1}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widetilde{\mathscr{A}}(e)$	([0.7 0.75], [0.2 0.25])	([0.5 0.6], [0.25 0.3])	([0.6 0.7], [0.23 0.3])	([0.5 0.6], [0.3 0.35])	([0.4 0.6], [0.4 0.45])
$\widetilde{p}_1(e)$	([0.3 0.45], [0.6 0.65])	([0.4 0.6], [0.3 0.45])	([0.5 0.7], [0.2 0.35])	([0.6 0.75], [0.3 0.45])	([0.6 0.8], [0.4 0.5])

Table 3
PPIVFSS for the second school education property

$\widetilde{\mathscr{B}_{p_2}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widetilde{\mathscr{B}}(e)$	([0.7, 0.75], [0.18, 0.22])	([0.7, 0.8], [0.22, 0.28])	([0.75, 0.85], [0.25, 0.3])	([0.8, 0.85], [0.3, 0.35])	([0.6, 0.8], [0.4, 0.45])
$\widetilde{p}_2(e)$	([0.5, 0.6], [0.7, 0.75])	([0.4, 0.5], [0.6, 0.65])	([0.5, 0.6], [0.6, 0.65])	([0.5, 0.6], [0.4, 0.5])	([0.7, 0.8], [0.4, 0.6])

Table 4
PPIVFSS for the third school education property

$\widetilde{\mathscr{C}_{p_3}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widetilde{\mathscr{C}}(e)$	([0.85 0.9], [0.25 0.28])	([0.75 0.8], [0.3 0.35])	([0.7 0.75], [0.2 0.25])	([0.6 0.7], [0.35 0.4])	([0.5 0.6], [0.4 0.45])
$\widetilde{p}_3(e)$	([0.5 0.6], [0.7 0.8])	([0.3 0.4], [0.4 0.5])	([0.4 0.5], [0.3 0.4])	([0.6 0.7], [0.5 0.6])	([0.2 0.3], [0.8 0.9])

Table 5
PPIVFSS for the fourth school education property

$\widetilde{\mathscr{D}_{p_4}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widetilde{\mathscr{D}}(e)$	([0.85 0.9], [0.18 0.22])	([0.8 0.85], [0.25 0.33])	([0.85 0.9], [0.25 0.3])	([0.7 0.8], [0.35 0.4])	([0.7 0.85], [0.45 0.48])
$\widetilde{p}_4(e)$	([0.5 0.6], [0.7 0.75])	([0.6 0.8], [0.5 0.55])	([0.8 0.85], [0.4 0.45])	([0.6 0.7], [0.5 0.6])	([0.7 0.8], [0.5 0.6])

Table 6
PPIVFSS for the fifth school education property

$\widetilde{\mathscr{E}_{p_5}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widetilde{\mathscr{E}}(e)$	([0.8 0.85], [0.2 0.2])	([0.7 0.9], [0.2 0.3])	([0.75 0.9], [0.2 0.3])	([0.5 0.7], [0.3 0.35])	([0.6 0.7], [0.4 0.45])
$\widetilde{p}_5(e)$	([0.3 0.6], [0.4 0.7])	([0.5 0.6], [0.6 0.7])	([0.4 0.5], [0.7 0.8])	([0.2 0.4], [0.5 0.6])	([0.6 0.7], [0.3 0.5])

Calculating the similarity measure for the 1-5 schools education property is given below the table.

	T	S	φ	ψ	$Similarity$
$(\widetilde{\mathscr{L},\mathscr{A}})$	[0.822594 0.859740]	[0.980863 0.982538]	[0.901728 0.921139]	[0.779651 0.783217]	[0.703033 0.721452]
$(\widetilde{\mathscr{L},\mathscr{B}})$	[0.952503 0.964217]	[0.982088 0.985118]	[0.967295 0.974667]	[0.652104 0.658118]	[0.630777 0.641446]
$(\widetilde{\mathscr{L},\mathscr{C}})$	[0.930735 0.935680]	[0.973787 0.976665]	[0.952261 0.956172]	[0.568735 0.578833]	[0.541584 0.553464]
$(\widetilde{\mathscr{L},\mathscr{D}})$	[0.984871 0.989484]	[0.973044 0.975488]	[0.978958 0.982486]	[0.746919 0.760357]	[**0.731202 0.747040**]
$(\widetilde{\mathscr{L},\mathscr{E}})$	[0.926929 0.967446]	[0.984445 0.984798]	[0.955687 0.976122]	[0.561962 0.591115]	[0.537060 0.577000]

From the above results, we find that the fourth school education property is closest to the ideal school education property with the highest value of the similarity measure as [0.731202 0.747040].

6. PIVFSS approach without the generalization parameter

We investigate the above mentioned survey study using the PIVFSS approach to consider the effect of the possibility parameter. Calculating the similarity measure for the mention above 1-5 schools education property as follows. We have

	T	S	$Similarity$
$(\widetilde{\mathscr{L},\mathscr{A}})$	[0.822594 0.859740]	[0.980863 0.982538]	[0.901728 0.921139]
$(\widetilde{\mathscr{L},\mathscr{B}})$	[0.952503 0.964217]	[0.982088 0.985118]	[0.967295 0.974667]
$(\widetilde{\mathscr{L},\mathscr{C}})$	[0.930735 0.935680]	[0.973787 0.976665]	[0.952261 0.956172]
$(\widetilde{\mathscr{L},\mathscr{D}})$	[0.984871 0.989484]	[0.973044 0.975488]	[**0.978958 0.982486**]
$(\widetilde{\mathscr{L},\mathscr{E}})$	[0.926929 0.967446]	[0.984445 0.984798]	[0.955687 0.976122]

It is observed that the first, second, third and fifth schools education property from the perspective of similarity measure are quite away from the ideal school education property. We find that the fourth school education property is closest to the ideal school education property with the highest value of the similarity measure as [0.978958 0.982486].

7. Comparison of PPIVFSS and PIVFSS

If the school education property unit chooses the threshold $[0.70, 0.85]$, we should choose the fourth school education property as a potential school. On the contrary, when using PIVFSS approach without the generalization parameter, we can not distinguish which the schools education property is the best one. So the possibility parameter has an important influence to the similarity measure of the fourth school education property. Therefore, PPIVFSS approach is more scientific and reasonable than PIVFSS approach without the generalization parameter in the process of decision-making.

8. Conclusion

The main goal of this work is to present a possibility Pythagorean interval valued fuzzy soft set to solve the phenomena related to decision making in which the sum of membership and non-membership is exceed 1. Finally, PPIVFSS approach is more scientific and reasonable than PIVFSS approach without the generalization parameter in the process of decision making. So in future, we should consider the possibility Pythagorean Cubic soft set theory and possibility Spherical soft set theory using soft set model.

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