Online Identification of the Open-Circuit Voltage of Lithium-Ion Batteries with the Use of Interval Methods

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Introduction

The charging/discharging dynamics of Lithium-ion batteries can be approximated by using equivalent circuit models. According to [1]–[3], these models consist of a finite number of RC sub-networks as well as series resistances and a state of charge (SOC) dependent voltage source (open-circuit voltage $v_{\mathrm{OC}}(t)$) as shown in Fig. 1.

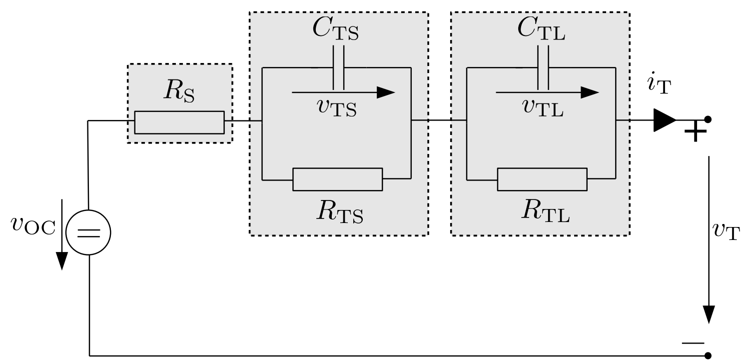

Figure 1: Equivalent circuit model of a Lithium-ion battery (cf. [1]).

In this presentation, two RC sub-networks representing processes with short (T_{TS}) and large (T_{TL}) time constants are considered, which result from polarization effects and concentration losses as described in [1], [2].

The SOC $\sigma(t)$ and the voltages across the RC sub-networks $v_{\mathrm{TS}}(t)$ and $v_{\mathrm{TL}}(t)$ are chosen as the state variables. With the state vector

$$\mathbf{x}(t) = \begin{bmatrix}\sigma(t) & v_{\mathrm{TS}}(t) & v_{\mathrm{TL}}(t)\end{bmatrix}^T \quad , \tag{1}$$

the quasi-linear, continuous-time battery model is obtained as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\left(\sigma(t)\right) \cdot \mathbf{x}(t) + \mathbf{b}\left(\sigma(t)\right) \cdot u(t) \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-1}{C_{\mathrm{TS}}(\sigma(t)) \cdot R_{\mathrm{TS}}(\sigma(t))} & 0 \\ 0 & 0 & \frac{-1}{C_{\mathrm{TL}}(\sigma(t)) \cdot R_{\mathrm{TL}}(\sigma(t))} \end{bmatrix} \cdot \mathbf{x}(t) + \begin{bmatrix} \frac{-1}{C_{\mathrm{Bat}}} \\ \frac{1}{C_{\mathrm{TS}}(\sigma(t))} \\ \frac{1}{C_{\mathrm{TL}}(\sigma(t))} \end{bmatrix} \cdot i_{\mathrm{T}}(t)\end{aligned} \tag{2}$$

with the terminal current $i_{\mathrm{T}}(t)$ as the system input.
By applying Kirchhoff's voltage law, the terminal voltage is obtained as

$$v_{\mathrm{T}}(t) = v_{\mathrm{OC}}(\sigma(t)) - v_{\mathrm{TS}}(t) - v_{\mathrm{TL}}(t) - i_{\mathrm{T}}(t) \cdot R_{\mathrm{S}}\left(\sigma(t)\right) \quad , \tag{3}$$

where $v_{\mathrm{OC}}(\sigma(t))$ and $R_{\mathrm{S}}\left(\sigma(t)\right)$ are represented by nonlinear expressions of the SOC. For more details, see [1], [2].

In classical state estimation approaches, the parameters are identified experimentally (cf. [1]–[3]). But they are subject to aging- and temperature-induced variations, which are shown in [2]. The aging of battery cells leads to a loss of the total capacity, an increasing Ohmic cell resistance and changes in the charging/discharging efficiency as well as changes of the time constants. Additionally, there are other influence factors such as the cell temperature. The first-mentioned variations can be estimated with the help of an augmented state vector, but this approach does not allow for estimating nonlinear dependencies of the circuit elements on the SOC or other influence factors.
For the identification of the nonlinear dependency of the open-circuit voltage on the SOC with underlying aging- and temperature-induced

variations, it is assumed that the parameters are known and not yet affected by aging. The aging- and temperature-induced variations can be mapped onto the open-circuit voltage.
This presentation proposes a two-stage identification for nonlinear dependencies with the dependency of the open-circuit voltage on the SOC as an example. The state variables of the dynamic system are estimated in the first stage with an interval observer. In the second stage, the a-priori knowledge is corrected using the estimated state variables.

In this presentation the notations $\underline{\mathbf{M}}$ and $\overline{\mathbf{M}}$ for a matrix $\mathbf{M}$ denote the element-wise lower and upper bounds.

With the bounding system $\mathbf{x} \in [\underline{\mathbf{x}}\ ;\ \overline{\mathbf{x}}]$ and $\hat{\mathbf{x}} \in \left[\underline{\hat{\mathbf{x}}}\ ;\ \overline{\hat{\mathbf{x}}}\right]$, $\mathbf{x}$ is given as $\mathbf{x} \in \left[\underline{\hat{\mathbf{x}}}\ ;\ \overline{\hat{\mathbf{x}}}\right]$ and the interval observer is obtained according to ([3], [4])

$$\underline{\mathbf{A}}_{\mathrm{O}}\underline{\hat{\mathbf{x}}} + \underline{\mathbf{B}}\mathbf{u} + \mathbf{H}\underline{\mathbf{y}}_{\mathrm{m}} \leq \dot{\hat{\mathbf{x}}} \leq \overline{\mathbf{A}}_{\mathrm{O}}\overline{\hat{\mathbf{x}}} + \overline{\mathbf{B}}\mathbf{u} + \mathbf{H}\overline{\mathbf{y}}_{\mathrm{m}} \tag{4}$$

with the observer system matrices

$$\underline{\mathbf{A}}_{\mathrm{O}} = \underline{\mathbf{A}} - \mathbf{HC} \quad \text{and} \quad \overline{\mathbf{A}}_{\mathrm{O}} = \overline{\mathbf{A}} - \mathbf{HC} \tag{5}$$

and the uncertain measurements

$$[\mathbf{y}_{\mathrm{m}}] := \left[\underline{\mathbf{y}}_{\mathrm{m}}\ ;\ \overline{\mathbf{y}}_{\mathrm{m}}\right] = \mathbf{y}_{\mathrm{m}} + [-\Delta\mathbf{y}_{\mathrm{m}}\ ;\ \Delta\mathbf{y}_{\mathrm{m}}]\ . \tag{6}$$

Here, the system matrix $\mathbf{A}\left(\sigma(t)\right)$ has the following sign pattern

$$\mathbf{A}\left(\sigma(t)\right) = \begin{bmatrix} \leq 0 & \geq 0 & \geq 0 \\ \geq 0 & \leq 0 & \geq 0 \\ \geq 0 & \geq 0 & \leq 0 \end{bmatrix} \in \left[\ \underline{\mathbf{A}}\ \ ;\ \ \overline{\mathbf{A}}\ \right]\ . \tag{7}$$

The output equation is given as

$$\mathbf{y}(t) = \tilde{v}_{\mathrm{T}}(t) = \left[\tilde{v}_{\mathrm{OC}}(t) - v_{\mathrm{TS}}(t) - v_{\mathrm{TL}}(t) - i_{\mathrm{T}}(t) \cdot R_{\mathrm{S}}(t)\right] \tag{8}$$

with the associated quasi-linear representation

$$\begin{aligned}\mathbf{y}^*(t) &= \mathbf{y}(t) - \mathbf{D}\left(\sigma(t)\right) \cdot i_{\mathrm{T}}(t) = \mathbf{C}\left(\sigma(t)\right) \cdot \mathbf{x}(t) \\ &= \begin{bmatrix} \eta_{\mathrm{OC}}\left(\sigma(t)\right) & -1 & -1 \end{bmatrix} \cdot \mathbf{x}(t) \in [\mathbf{y}_{\mathrm{m}}] \quad ;\end{aligned} \tag{9}$$

$\tilde{v}_{\mathrm{OC}}(t)$ is obtained by subtracting the constant, state independent terms from the expression for the open-circuit voltage $v_{\mathrm{OC}}(t)$ to turn this expression into a quasi-linear form, see [1].

Based on the design of a robust interval observer shown in [3], the observer matrix $\mathbf{H}$ is hereby assigned as

$$\mathbf{H} = \begin{bmatrix} h_1 & 0 & 0 \end{bmatrix}^T \quad , \quad h_1 > 0 \ . \tag{10}$$

With the help of interval methods, the nonlinear dependency of the open-circuit voltage on the SOC is identified as shown in Fig. 2.

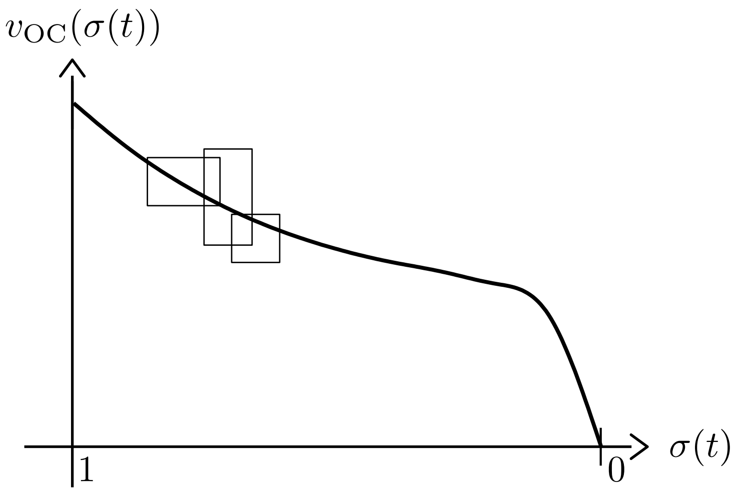

Figure 2: Identification of nonlinear dependencies using interval methods.

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