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Creep Fatigue, Part III: Diercks Equation: Modification and Applicability

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CREEP FATIGUE:
PAPER III
Diercks Equation: Modification and Applicability

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ABSTRACT

Creep-fatigue data of low allow steels were compiled from international sources, and trends in creep-fatigue behavior were identified in Paper I. Methods of life prediction and their trends were examined in Paper II with respect to compiled data. Diercks equation, a multivariate creep-fatigue life extrapolation equation, for SS, 304, in terms of strain range, temperature, hold time and strain rate parameters is modified and extended to the life prediction of low alloy steels in this paper.

KEY WORDS: life prediction, observed life, factor of ± x2, over or under-predicted, standard error, equivalent factor on life, low alloy steels, strain range, material-dependent equivalent strain rate.

INTRODUCTION

Several high temperature, low cycle fatigue (HTLCF) or creep fatigue, life prediction methods /1-7/ have been developing during the last two decades. These methods were reviewed in Paper II /8/. Phenomenological methods of creep-fatigue life prediction require several combinations of laboratory tests for material parameter determination. Strain rate, frequency, continuous fatigue, hold time, balanced and unbalanced dwells, equal and unequal ramp rates and relaxation details are a few requirements. Diercks equation /7/ is a multivariate equation, in terms of test variables, obtained by a least square best fit equation from a bank of creep-fatigue data on SS 304. With modifications /9,10/, prediction by this method was found to be superior to the damage summation technique and comparable to or better than the strain range partitioning technique, when extended to 2.25Cr-Mo and 9Cr-1Mo steels, involving hold times of a few minutes. The details of those modifications are available in publications /8,11/. Diercks equation and its modification are discussed below.

DIERCKS EQUATION

Diercks equation has been discussed in /9,10/ with other methods of life prediction. ASME code case N 47 specifies this equation as a basis for the design of fatigue diagrams /10/ in high temperature low cycle fatigue of SS 304. A least square best fit equation of SS 304, expressed by multivariate regression functions in strain range, strain rate, temperature and hold time parameters, was evolved. The equation with the above variables has the following form:

$$(\log N_f)^{-1/2} = 1.20551064 + 0.66002143\ S + 0.18040042\ S^2 - 0.00814329\ S^4 + 0.00025308\ RS^4 + 0.00021832\ TS^4 - 0.00054660\ RT^2 - 0.005567\ RH^2 - 0.00293919\ HR^2 + 0.0119714\ HT - 0.00051639\ H^2T^2 \quad (1)$$

where S is a strain range parameter ($S=\Delta\varepsilon_t/100$), R a strain rate parameter ($R=\log\dot{\varepsilon}$), T a temperature parameter ($T=T_c/100$), and H is a hold time parameter ($H=\log(1+t_h)$), $\Delta\varepsilon_t$ is the % total strain range, $\dot{\varepsilon}$ is the strain rate, T_c is the test temperature of SS 304 and t_h is the duration of hold time in hours in the above equation.

Kitagawa *et al.* /9,10/ modified this equation which has been the subject of an extensive study /11/. Equation (1) was corrected for fatigue, by life or cycle ratio (α), which is a ratio of cyclic life of SS 304 with the low alloy steel being investigated for the same strain range, temperature and strain rate. Thus, if the right hand side of Equation (1) is denoted by a constant term C for a particular S, T, R and H, Kitagawa's modification takes the following form:

$\alpha =$ N_f (of SS 304) / N_f (of low alloy steel) under the same conditions. (2)

$$[\log (\alpha N_f)]^{-1/2} = C$$

The fatigue correction factor or the cycle ratio was observed to be dependent on temperature, strain range, and strain rate. It varied from 1 to 5, from high to low strain ranges, when other test parameters remained the same. This, in turn, requires material data for both materials under similar conditions, which is difficult where a data bank is not available.

The life prediction by the modified Equation (2) was within a factor of 2 for 2.25Cr-Mo and 9Cr-1Mo low alloy steels /9,10/. However, this modification cannot be applied where data on the pure fatigue behavior of low alloy steel and that of SS 304 at the same temperatures and strain rates are not available. These data are essential to determine the cycle ratio (α). It was assessed with hold periods from 1 to 10 minutes. The above modification was not assessed for several hours of holds, where the predicted life was nearly the same for the total strain ranges from 0.55 to 2%. It predicted a saturation effect of dwells. No account of decreasing strain rate (which is a rate of deformation [$d\varepsilon/d\tau$] or when integrated for a cycle $\Delta\varepsilon/\tau$, where τ is the cycle time) with increasing hold period was made in the model. Instead, the strain rate of the pure fatigue tests was considered.

MODIFICATION OF DIERCKS EQUATION

Replacement of cycle ratio (α) with cycle time (τ) factor:

Owing to the limitations discussed above and the advantage of the equation that it does not require any creep-fatigue tests, a further modification was undertaken to eliminate the requirement of relative material properties of SS 304 and low alloy steels. Equation (2) was modified, replacing α (cycle ratio) with a cycle time parameter (τ), which is a ratio of total strain parameter (S), ($\Delta\varepsilon_t/100$), to strain rate (%/sec). The equation was proposed in a modified form:

$$[\log (\tau N_f)]^{-1/2} = C \quad (3)$$

The life of various low alloy steels under creep-fatigue, as determined from Equation (3) was found to be same, if at the same strain range, temperature and hold time the strain rate parameter was constant. Hence, to apply this equation for a range of low alloy steels, a material-dependent equivalent strain rate ($\dot{\varepsilon}_e$), was introduced.

The modification made by Kitagawa *et al.* /10/ also involved a temperature correction factor, which scales the creep rupture properties of SS 304 with respect to the low alloy steel being investigated. The iso-stress creep rupture properties of low alloy steels range from 50 to 100°C lower than the creep rupture properties of SS 304. With the introduction of a temperature correction factor, the creep-fatigue response of low alloy steels does not change appreciably (from 10-15 cycles). Hence, the temperature correction factor has not been changed.

Material-dependent equivalent strain rate ($\dot{\varepsilon}_e$)

The material-dependent equivalent strain was determined by trial and error, as follows:

1. obtain a total strain versus life extrapolation with a few creep-fatigue data points in terms of
   $$\Delta\varepsilon_t = A (N_f)^{-\beta} \quad (4)$$
2. determine the parameters (A and -β) of the best fit

equation discussed in Paper I /12/ (there are 50 combinations of hold times and other test variables; these can be used to generate a response curve or analyzed statistically for a mean or average line),

3. extrapolate this equation at several strain levels,
4. use Equation (3) probabilistically with several assumed values of material-dependent equivalent strain rate (range from 0.05 to 1.0 for most materials),
5. select the value of the material-dependent equivalent strain rate ($\dot{\varepsilon}_e$), when there is a good degree of fit between the extrapolated life and that predicted by Equation (3). Choice can be made between the most conservative and the over-predicted response.

The material-dependent equivalent strain rate varies from data to data as the parameters of the extrapolated equation change with the creep-fatigue test type. A range of constants of Equation (4) has been tabulated in Paper I /12/ for a range of hold times. From a large number of low alloy steels studied, the material-dependent equivalent strain rate was considered "constant" for each low alloy steel grade for simplicity. However, one value of the material-dependent equivalent strain rate will be very conservative for one type of creep-fatigue sequence, for a particular hold direction and may over-predict for the other holds. Hence, the material-dependent strain rate should be determined by appropriate data fitting. A value of 0.1 was obtained for 0.5Cr-Mo-V and 1Cr-Mo-V, 0.25 for 1.25Cr-Mo and 0.5%/sec for 2.25Cr-Mo, 2.25Cr-Mo-V and 9Cr-1Mo steels. These values have been kept constant for all combinations of hold times and strain ranges for the above grades of low alloy steels, assessed in this investigation. Figure 1 describes the fit between the extrapolated behavior (drawn with data, Ref. Table 22, Paper I /12/) and the life predicted by Equation (3), with different values of material-dependent equivalent strain rate ($\dot{\varepsilon}_e$). Further derivatives of the material-dependent equivalent strain rate are left to the individual user and his data by the five-stage method described above.

Equation (3), when assessed, showed life prediction of low alloy steels within a factor of 2 for a short hold of a few minutes to holds of 16 hours for most test data points. The merits and demerits of this method have been described in Paper II /8/. A few useful, relevant points in this paper are that Equations (1-3) only consider hold periods in which they are more damaging, that is, either tension or compression for the balanced cycles. From the existing knowledge on dwell sensitivity, 1Cr-Mo-V is a tensile dwell sensitive material whereas 2.25Cr-Mo is compressive.

With the above modifications, this method has been assessed with the creep-fatigue data compiled in Paper I /12/, as discussed below.

APPLICABILITY OF MODIFIED DIERCKS EQUATION

Life Prediction by Modified Diercks Equation for 0.5Cr-Mo-V Steel

The following are parameters of the modified Diercks equation for 0.5Cr-Mo-V steel:

Temperature parameter = [(test temperature + 100°C)/100]

Material-dependent equivalent strain rate = 0.1

Equation (3) was used to evaluate life.

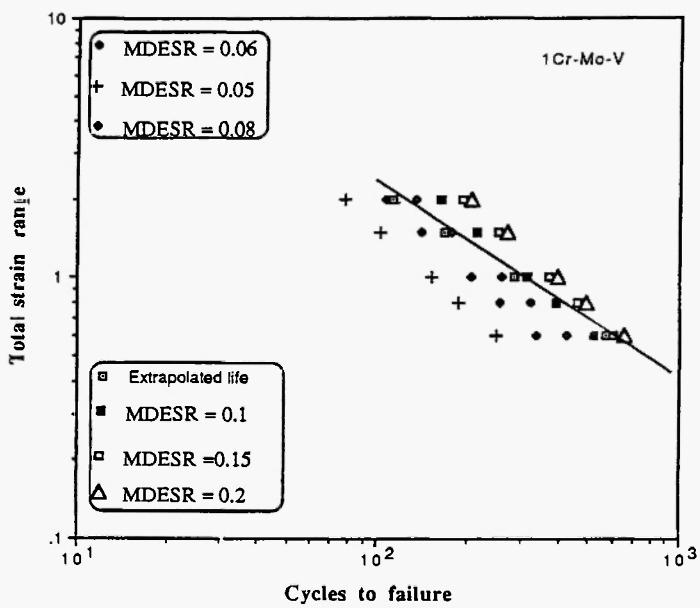

Fig. 1: Determination of the material-dependent equivalent strain rate (MDESR) for 1Cr-Mo-V.

Batch 1: Limited data have been compiled for this low alloy steel type. When assessed with the modified Diercks equation, only 6% of the test data points were outside the factor of ± x 2. 94% of test data points were predicted within a factor of 2. From one-half hour to 16 hours tensile hold times were assessed and plotted in Fig. 2.

Life Prediction by Modified Diercks Equation for 1Cr-Mo-V Steel

The following are the parameters of the modified Diercks equation for 1Cr-Mo-V steel:

Temperature parameter = (cycle temperature + 100°C)/100

Material-dependent equivalent strain rate = 0.1

Equation (3) was used to evaluate life.

Batches 1 and 2 of the 1Cr-Mo-V steels comprise interspersion fatigue-creep tests with 23 and 47 hours tensile hold times. The effect of combined cycles on the creep-fatigue behavior has been noted in Paper I /12/. There are beneficial as well as damaging effects of combined cycles that cannot be accounted for in the model. Hence, these sequences have not been assessed with the method to comment on the applicability of the modified Diercks equation.

Batch 3: When assessed, 70% of test data points were predicted within a factor of ± x2, and the remaining 30% of the data within a factor from 3 to 11, as shown in Fig. 3. The discrepancy exists /13/ in the very nature of this data. Transformation of inelastic to total strain range has been made within a temperature difference of 50°C in the table (Paper I, Table 8) /12/.

Batch 4: A large number of test conditions exist for this batch. The prediction by the modified method was found to be conservative within a factor of ± x2, as shown in Fig. 4. The beneficial effect of an unbalanced hold (16/0.003 hrs.) enhances life by a factor of 3 as compared to only 16 hours tensile hold cycle. Such effects cannot be described by this method. However, life prediction of a 16 hr. tensile dwell cycle was within a factor of ± x2.

Batch 5: Hold time sequences of one-half hour to 16 hours have been assessed with this model. At lower strain ranges (0.5 to 0.6%), with hold times of one-half and 16 hours, life prediction by the modified Diercks equation was very conservative. However, 75% of the test data points were predicted within a factor of ± x2 in Fig. 5.

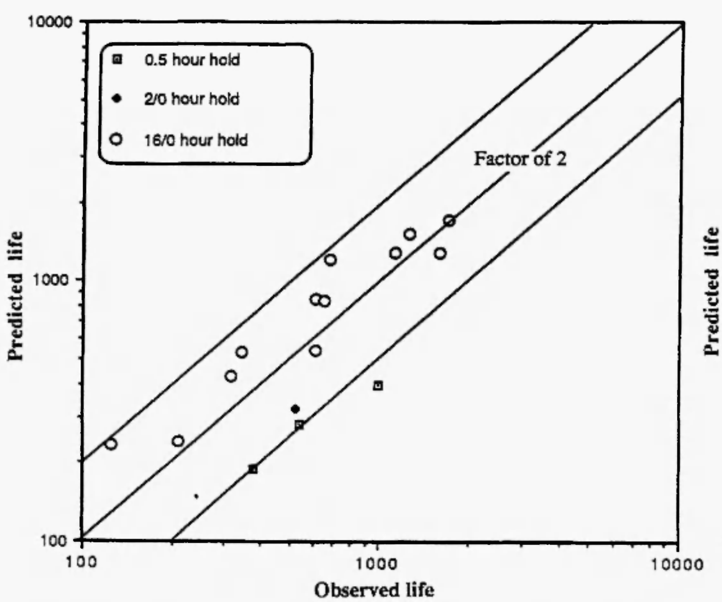

Fig. 2: Life prediction of 0.5Cr-Mo-V Batch 1 by the modified Diercks equation.

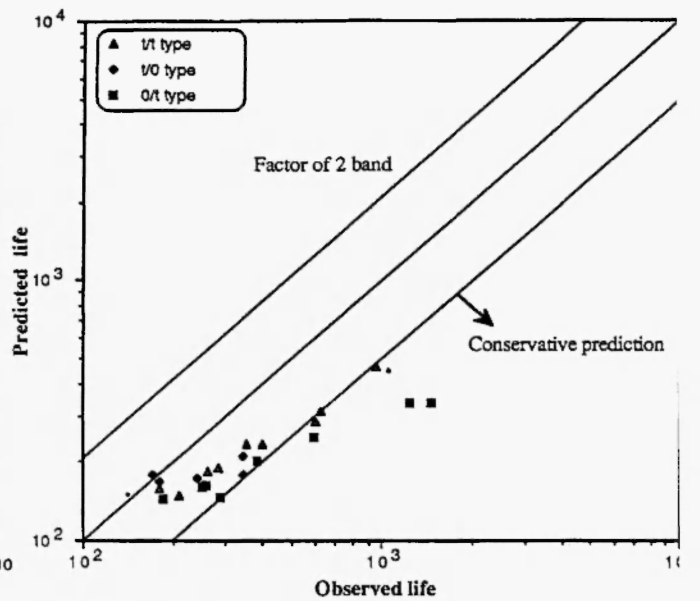

Fig. 3: Life prediction of 1Cr-Mo-V Batch 3 by the modified Diercks equation.

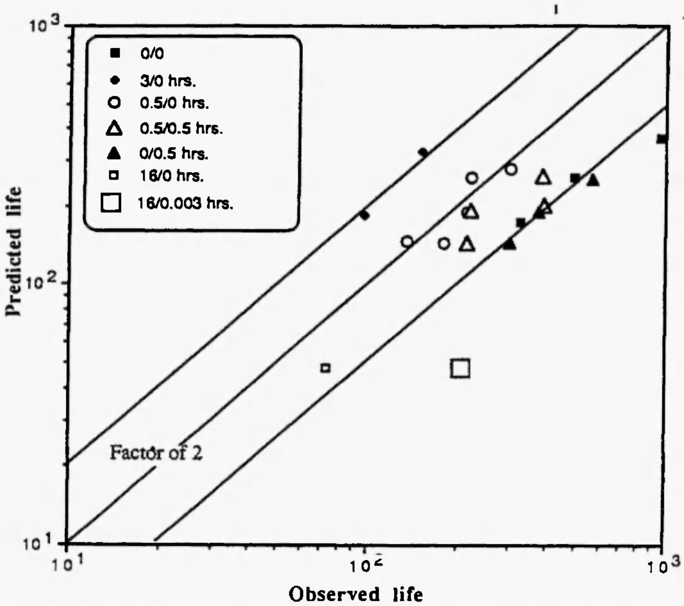

Fig. 4: Life prediction of 1Cr-Mo-V Batch 4 by the modified Diercks equation.

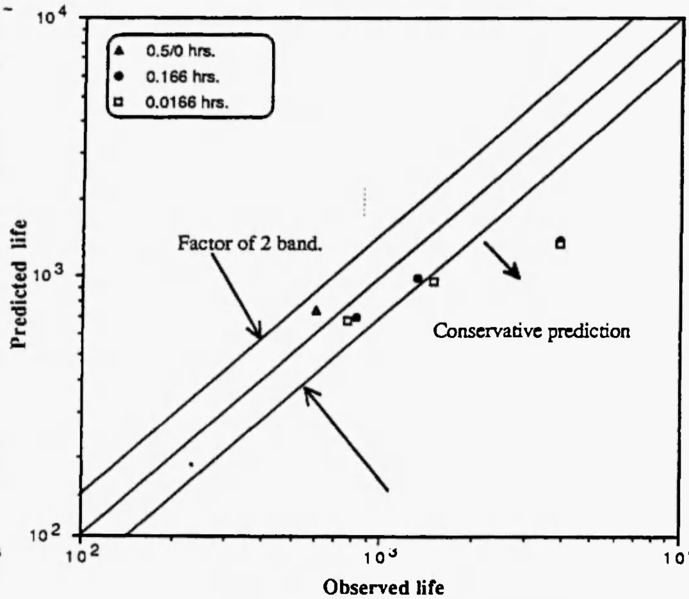

Fig. 6: Life prediction of 1.25 Cr-Mo Batch 1 by the modified Diercks equation.

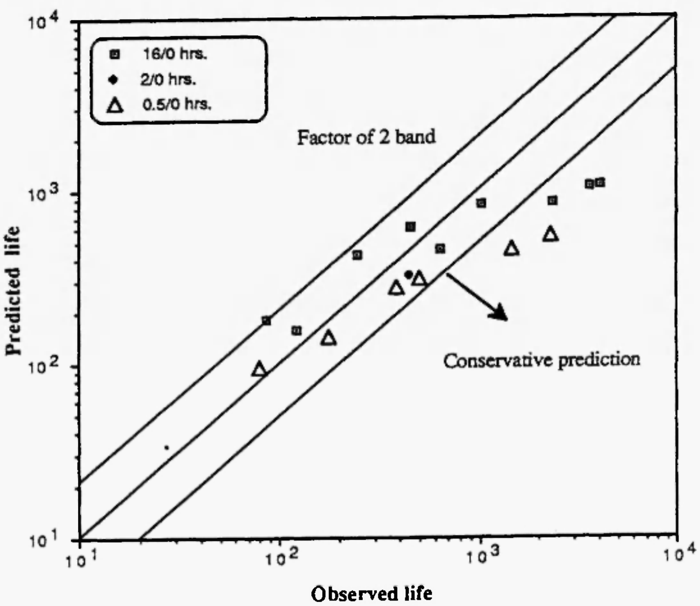

Fig. 5: Life prediction of 1Cr-Mo-V Batch 5 by the modified Diercks equation.

Life Prediction by Modified Diercks Equation for 1.25Cr-Mo Steel:

The following are the parameters of the modified Diercks equation for 1.25Cr-Mo steel:

Temperature parameter = (cycle temperature + 100°C)/100

Material-dependent equivalent strain rate = 0.25

Equation (3) was used to evaluate life.

Batch 1: Pure fatigue and creep-fatigue combinations were assessed. 66% of the test data points were predicted within a factor of ± x2 and the remaining 33% were within a factor of 4 as shown in Fig. 6. The discrepancy in the predicted life may be due to the definition of the failure criterion, where cycles to failure were expressed in terms of 20, 33 and 40% in load drop. It was different for various test conditions. For this reason alone, the prediction band was within a factor of 3 for the combination of strain range and hold time (0.5% and 0.166hr hold), respectively, when 40% decrease in the peak tensile load was the failure criterion.

Batch 2: Prediction in the case of creep-fatigue cycles was very good. 90% of the test data points were predicted within a factor of ± x2. In the case of a one-hour hold cycle, at 2.03% strain range, representing 10% of the test data, prediction by this method was within a factor of 2.06, which may be considered as 2. Hence all the creep-fatigue data were predicted within a factor of ± x2, as shown in Fig. 7.

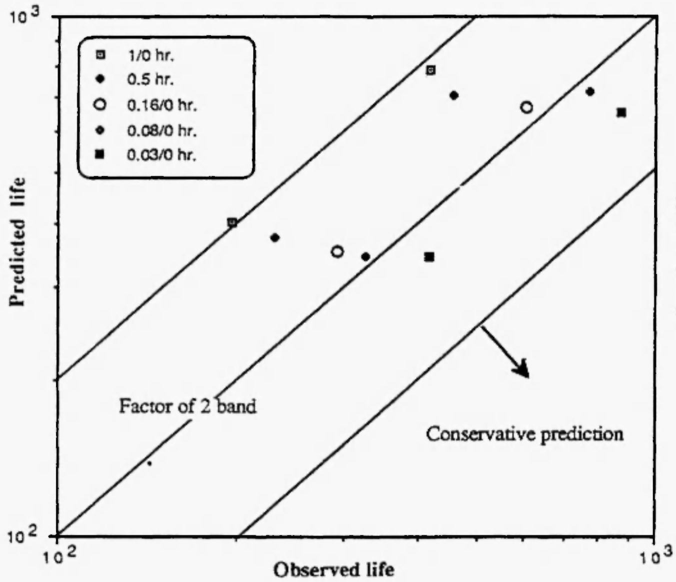

Fig. 7: Life prediction of 1.25 Cr-Mo Batch 2 by the modified Diercks equation.

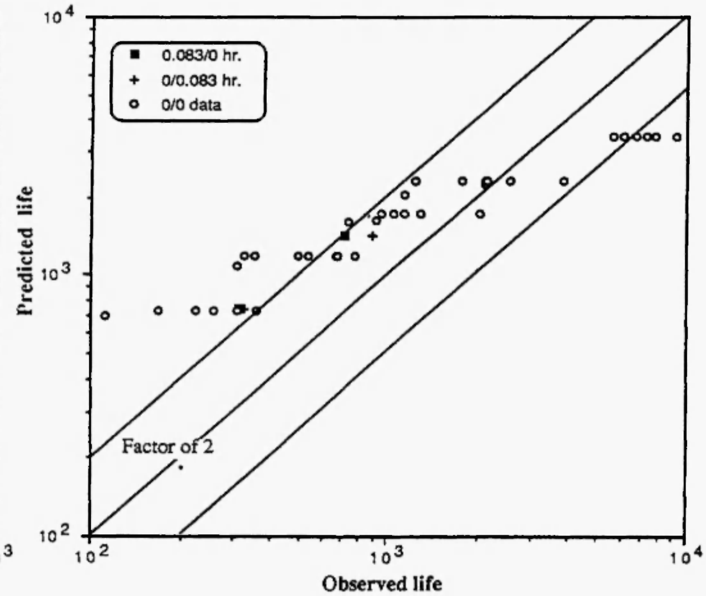

Fig. 8: Life prediction of 2.25 Cr-Mo Batch 3 by the modified Diercks equation.

Life Prediction by Modified Diercks Equation for 2.25Cr-Mo Steel:

The following are the parameters of the modified Diercks equation for 2.25Cr-Mo steel:

Temperature parameter = (cycle temperature + 100°C)/100

Material-dependent equivalent strain rate = 0.5

Equation (3) was used to evaluate life.

Batches 1 and 2: Data compiled in Batches 1 and 2 contain the interspersion type of tests involving combined cycles. It has been pointed out also in the case of 1Cr-Mo-V steel Batches 1 and 2 that combined cycles are associated with a healing and/or detrimental effect with respect to the number of such fatigue cycles that cannot be accounted for in the model. Instead, they are not assessed to comment on the applicability of the modified Diercks equation.

Batch 3: When assessed, 67.5% of the test data points were predicted within a factor of ± x2 and the remaining points were within a factor of ± x3, as shown in Fig. 8. It may be seen from the data that 5 minutes tensile and comprehensive hold times are very small and may not cause any damage at all. Batch 3 is mostly a continuous fatigue, with two tests having a hold of 5 minutes in either direction, to cause interaction effects of creep with fatigue.

Batch 4: When assessed below a cyclic life range of 10^4 cycles, balanced, compression and tension only dwells, life was predicted within a factor of ± x2, as shown in Fig. 9.

Batch 5: Data analyzed by the modified equation gave a prediction within a factor of ± x2 for 100% of the test data points. This method was found better than

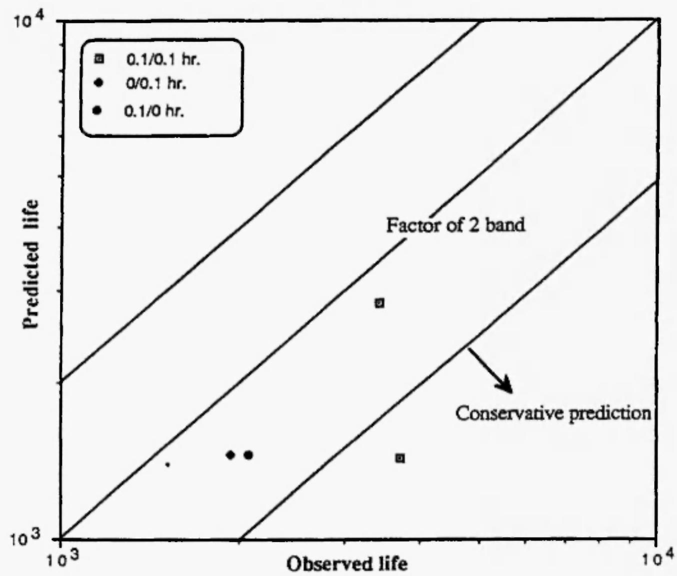

Fig. 9: Life prediction of 2.25 Cr-Mo Batch 4 by the modified Diercks equation.

the damage summation and comparable to the strain range partitioning techniques, as shown in Fig. 10.

Batch 6: Data were analyzed by the modified Diercks equation, where 88% of the test data points were predicted within a factor of ± x2. The remaining 12% of the test data points represented three test conditions, of which one involved error in part of the testing, where the life of 1.2% total strain range and the same hold time was 3/4 or 75% of the life at 4.30% total strain range. The remaining two cases, representing 8% of test data points, were within a factor of 2.2, as shown in Fig. 11.

Batch 7: Data were assessed with the modified Diercks equation, where 91% of the test data points were prdicted within a factor of ± x2. The remaining 9% of the test data points were within a factor of 3. The capability of this model was verified at a high strain rate of 1.48% /sec, when prediction was found within a factor of 2, as shown in Fig. 12.

Batch 8: A comparatively new alloy, information regarding the material behavior and life prediction by other models is not available. When assessed with the modified equation, 64% of the test data points were within a factor of ± x2 and the remaining 36% of the test data points were within a factor of 4 to 5, as shown in Fig. 13. The above discrepancies, where 36% of test data points were within a much larger error band of 4/5, were due to the assumption that this material is similar to 2.25Cr-Mo steel, and the tests involved only continuous fatigue cycling. The modified Diercks equation is applicable better under creep-fatigue conditions with hold times.

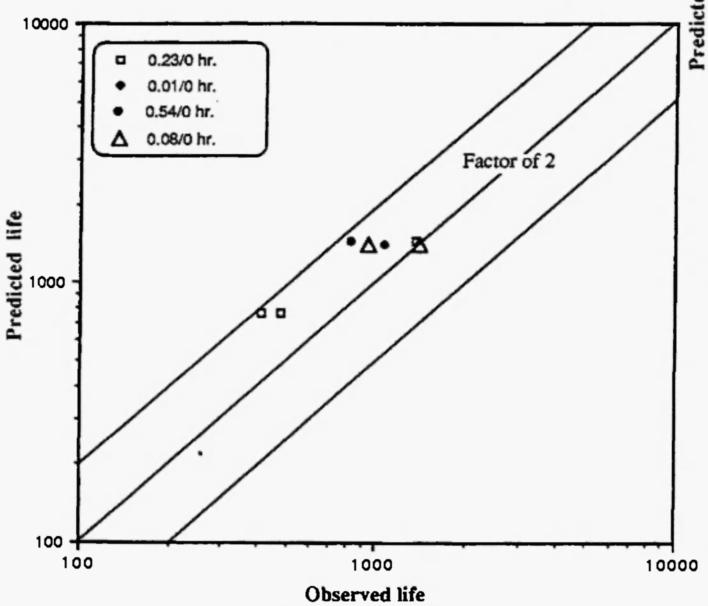

Fig. 10: Life prediction of 2.25 Cr-Mo Batch 5 by the modified Diercks equation.

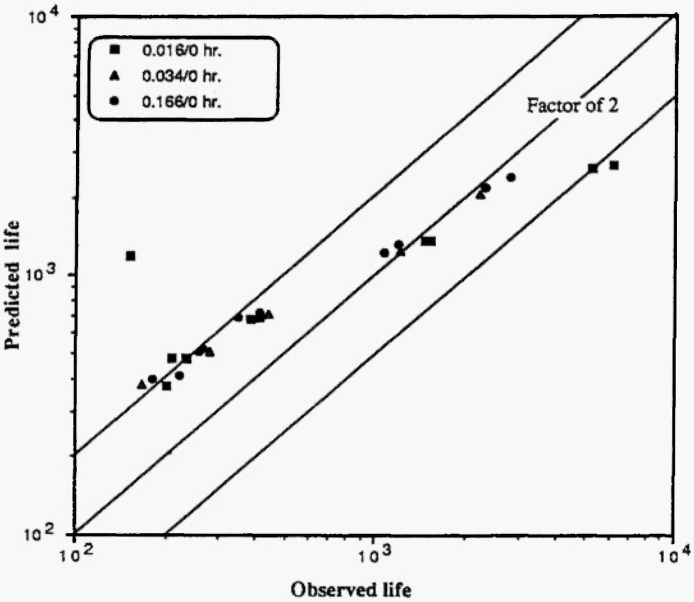

Fig. 11: Life prediction of 2.25 Cr-Mo Batch 6 by the modified Diercks equation.

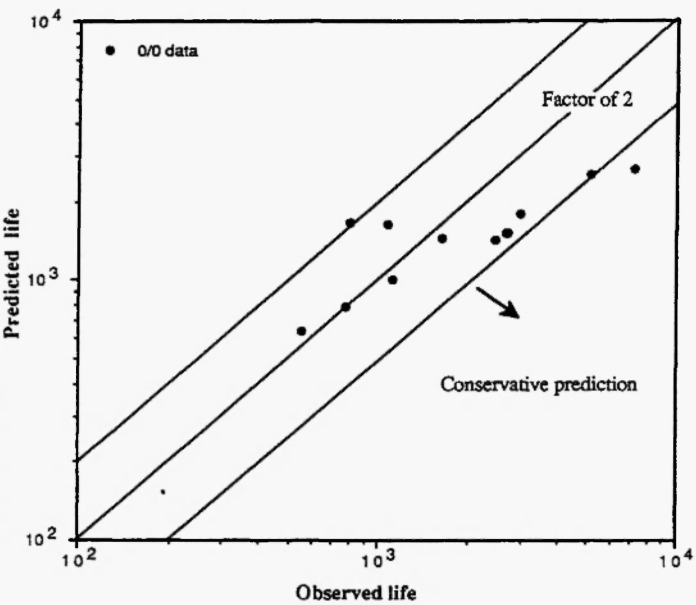

Fig. 12: Life prediction of 2.25 Cr-Mo Batch 7 by the modified Diercks equation.

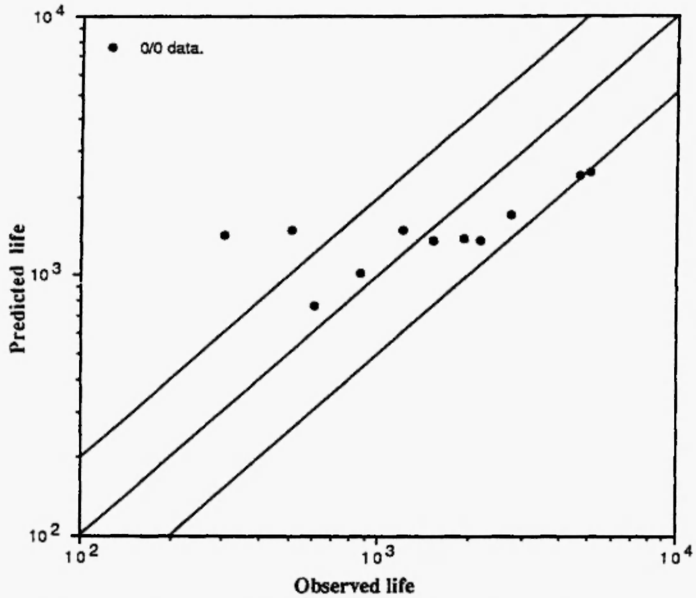

Fig. 13: Life prediction of 2.25 Cr-Mo Batch 8 by the modified Diercks equation.

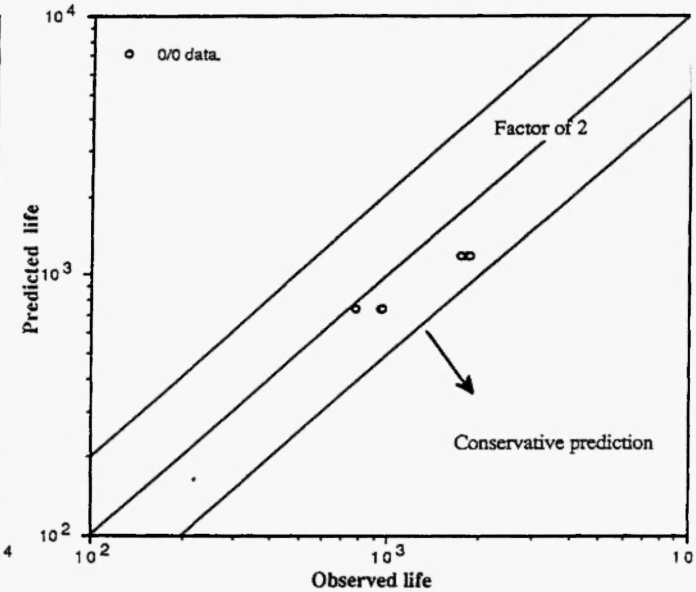

Fig. 14: Life prediction of 9Cr-1Mo Batch 1 by the modified Diercks equation.

Life Prediction by Modified Diercks Equation for 9Cr-1Mo Steel:

The following are the parameters of the modified Diercks equation in the case of 9Cr-1Mo steel:

Material-dependent equivalent strain rate = 0.5

Temperature parameter = [(test temperature + 50) / 100]

Batch 1: The modified Diercks equation was applied to the HTLCF data of 9Cr-1Mo steel. Below a life range of 10^4 cycles, 90% of the test data points were predicted within a factor of ± x2, as shown in Fig. 14.

Batch 2: The modified Diercks equation was assessed with the creep-fatigue data of 9Cr-1Mo steel. At lower strain ranges (0.5%), with 15 minutes tensile hold, prediction by the modified Diercks equation was very poor. However, as the hold time and temperature increased, the prediction was found to be very good. 70% of the test data points were predicted within a factor of ± x2. These data have been plotted in Fig. 15.

Prediction Capability and Limitations of the Modified Equation:

The success of a life prediction method depends on the spread of the error band in which the observed and predicted data fall. In creep-fatigue life prediction, the acceptable band is ±2, since it maintains a very high statistical confidence (95%). All methods do not predict 100% of the test data points within a factor of ± x2 in the case of low alloy steels, as reviewed in Paper II /8/. Table 1 compares the predictabilities of the damage summation technique and the strain range partitioning

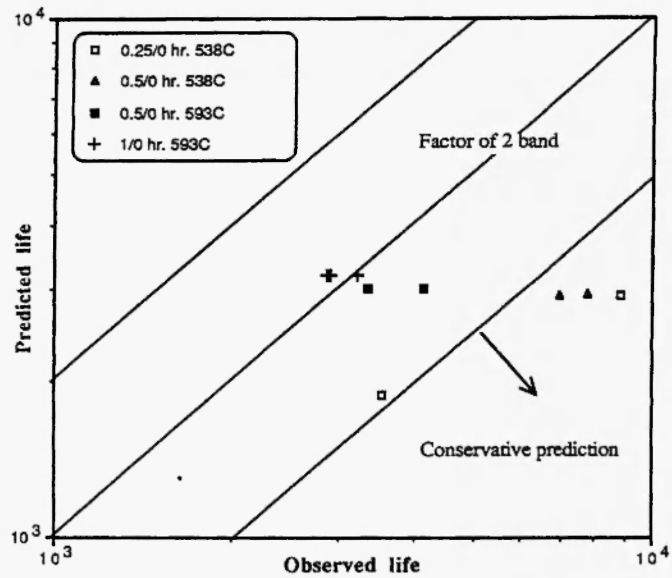

Fig. 15: Life prediction of 9Cr-1Mo Batch 2 by the modified Diercks equation.

Table 1
Comparison of the modified method with DSA and SRP

Material	Batch No.	Temperature	Heat Treatment	Prediction by DSA	Prediction by SRP	Prediction by MDE
1Cr-Mo-V	1	540°C	N&T	69%	75%	
	1	485°C	N&T	100%	100%	
	4	565°C	N&T	57%	85%	100%
2.25Cr-Mo	1	540°C	A	29%	100%	
	1	540°C	N&T	82%	96%.	
	1	485°C	Q&T	100%	58%	
	3	600°C	N&T	70%	100%	67.5%
	5	600°C	N&T	0%	100%	100%

technique under various conditions and the percentage of test data points predicted within a factor of ± x2.

At least 67.5% of the test data points were predicted within a factor of ± x2 by the modified Diercks equation. This is a simpler method, does not require material tests and predicts lives comparable to those of SRP and DSA.

Only Batch 4 of 1Cr-Mo-V was characterized for different hold times in tension and compression of the types t /0, t/t, 0/t and xt/yt /13/. This data set is ideal to assess the capability of a method. This batch has been assessed with all the popular methods of life prediction /13/, where it was concluded that no method is adequate. All the test data points were predicted within a factor of ± x2 by the modified Diercks equation.

The modified Diercks equation also has some of the limitations of the other methods of life prediction discussed in /8/, since it considers both the tension and compression holds equally damaging and does not account for dwell sensitivity. In equation (1), a hold time parameter accounts for the time of hold which is log(1+hold time). It is evident that only tensile dwell time can be assessed with this method as log of a negative quantity (more than an hour compressive hold) becomes infinite.

Also accounting for hold times is possible only once in the expression for the hold time parameter. Hence balanced dwell cycles will not be analyzed; instead, occurrence of hold where they are more damaging (longer times and direction) will be considered. Healing effects are more complex when combined cycles or unbalanced holds are applied. Such situations are underpredicted by all the methods of life prediction discussed in /8/.

Amidst these demerits, it predicts life in a reasonable range, which merits its applicability. It does not require the constants determined from creep-fatigue test histories or continuous fatigue data. This alone qualifies applicability of this approach, when other methods fail to derive initial material response without conducting any tests.

The life prediction by the modified Diercks equation was statistically analyzed for standard error and equivalent factor on life /14/, and the results are tabulated in Table 2.

Table 2
Error band in the proposed life prediction equation

Material	Temp.	Batch	% Tests within a factor of			SE	EF
	C°		2	3	4		
1Cr-Mo-V (N&T)	540	1	85	15		0.055	1.135
1Cr-Mo-V (N&T)	485	1	75	25		0.065	1.16
1Cr-Mo-V (N&T)	538	2	50	50		0.1457	1.39
1Cr-Mo-V (N&T)	483	2	50		50	0.081	1.205
1Cr-Mo-V (N&T)	550	3	94	6		0.012	1.02
1Cr-Mo-V (N&T)	565	4	100			0.008	1.01
1.25Cr-Mo (A/R)	550	1	67	33		0.102	1.26
1.25Cr-Mo (N&T)	600	2	100			0.021	1.05
2.25Cr-Mo (A)	540	1	60	20	20/5	0.234	1.71
2.25Cr-Mo (N&T)	540	1	67		33/5	0.058	1.14
2.25Cr-Mo (Q&T)	485	1	67		33	0.0256	1.06
2.25Cr-Mo (A)	538	2	100			0.0435	1.1
2.25Cr-Mo (N&T)	538	2	67	33		0.0136	1.03
2.25Cr-Mo (Q&T)	483	2	33	67		0.1692	1.47
2.25Cr-Mo (N&T)	600	3	67	33		0.005	1.01
2.25Cr-Mo (N&T)	502	4	100			0.0636	1.157
2.25Cr-Mo (N&T)	600	5	100			0.064	1.15
2.25Cr-Mo (N&T)	550	6	88	12		0.0059	1.013
2.25Cr-Mo (N&T)	593	7	91	9		0.055	1.135
2.25Cr-Mo-V	593	8	63.6	18	18	0.0282	1.06
9Cr-1Mo	550	1	90	10		0.0589	1.145
9Cr-1Mo	593	2	70	30		0.0625	1.154

(*not specified in this abstract, SE, standard error and EF, equivalent factor on life determined statistically, when the factor exceeds 4 it is expressed by % test data points / range of factor.)

CONCLUSIONS

1. The modification of the Diercks equation proposed in this paper does not require any relative material properties as required by Kitagawa's modification. Fatigue correction in terms of a cycle ratio requires the ratio of cyclic life of SS 304 and low alloy steel being investigated at the same strain range, strain rate and test temperature. A cycle time parameter was introduced into this modification which is the ratio of the strain range parameter to the material-dependent equivalent strain rate. The temperature parameter has been kept the same as that of Kitagawa *et al.*
2. The material-dependent equivalent strain rate has been determined from data fitting. For the sake of simplicity, its value has been kept constant for each low alloy steel grade.
3. This equation applies to any length of hold times and cyclic lives below 10^4 cycles in the low cycle fatigue regime. However, compressive dwells together with balanced and unbalanced cycles assume only one segment of hold time where it is more damaging.
4. The standard error and equivalent factors between the predicted and observed creep-fatigue lives of low alloy steels were below 2 for all cases, which is better than any other method of life prediction analyzed statistically.
5. An approximate creep-fatigue response of a low alloy steel can be derived without conducting any laboratory tests.

ACKNOWLEDGEMENTS

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REFERENCES

1. Code Case N-47, ASME Boiler and Pressure Vessel Code, ASME NY, 1974.
2. L.F. Coffin, in: MPS Symposium on Creep-Fatigue Interactions, ASME, 1976; p. 346.
3. S.S. Manson, G.R. Halford and M.H. Hirschberg, in: First Symposium on Design for Elevated Temperature Environment, ASME, 1971; p. 12.
4. S. Majumdar and P.S. Maiaya, Argonne National Laboratory Report ANL-76/57, 1976.
5. M. Chrzanowski, *Int. J. Mech. Sci.*, **18**, 69 (1976).
6. W.J. Ostergren, *ASTM J. Testing & Evaluation*, 4, 327 (1976).
7. D. Diercks and D.T. Raske, ANL Report No. 5150.
8. T. Goswami, Paper II, *International J. High Temp. Mater. & Processes*, _________ (1994).
9. K. Sonoya, I. Nonaka and M. Kitagawa, *ISIJ International*, **31**(12), 1424 (1991).
10. M. Kitagawa, I. Nonaka and K. Sonoya, *Fatigue*, **3**, 1537 (1990).
11. T. Goswami, unpublished work (1992).
12. T. Goswami, Paper I, *International J. High Temp. Mater. & Processes*, __________ (1994).
13. R.H. Priest and E.G. Ellison, *Resc. Mechanica*, 4, 127 (1982).
14. M. Spiegel, in: *Schaum's Series*, McGraw-Hill Book Co., N.Y., 1961.