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Paul Copp

Ashley Whitney-Rawls

Jace Carter

Tarun Goswami

RESEARCH ARTICLE

# Application of probabilistic methods to turbine engine disk life prediction and risk assessment

**Paul Copp**[1] **Ashley Whitney-Rawls**[1] **Jace Carter**[1] **Tarun Goswami**[1*]

[1] Department of Biomedical, Industrial and Human Factors Engineering, Wright State University, Dayton, OH 45435, USA



**Correspondence to:** Tarun Goswami, Department of Biomedical, Industrial and Human Factors Engineering, Wright State University, Dayton, OH 45435, USA; Email: tarun.goswami@wright.edu

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**Abstract:** Turbine engine disk life prediction and understanding the associated risk remains a significant challenge for today's designer. Despite advances made in materials testing and characterization, as well as, the application of damage tolerance and linear elastic fracture mechanics modeling, there remains a void in properly assessing loading, geometry, and material design property variability. Add to this the application of advanced hybrid and composite material systems and the need to accurately deal with material variability is even greater. There still remain incidents of failure of critical components which were not properly accounted for by the existing analytical methods, testing, and inspections employed today. Application of probabilistic methods offers an effective and useful approach to modeling this variability while also providing a means by which to assess random variable sensitivity and risk assessment. Current research, as well as, applicable industry and government regulatory guidelines and publications were examined and will be presented. An assessment of the most effective tools, modeling methods, and predictive risk of failure assessments together with recommendations for future work will be discussed. The potential for probabilistic methods to provide a cost-effective way to manage fleet engine and component usage is presented, as well as, its ability to enhance the safe implementation of Retirement for Cause concepts to fleet management.

**Keywords:** damage tolerant life, life extension, disk components, probability of failure, non-destructive testing, risk assessment

## 1 Introduction

The application of damage tolerance and retirement for cause concepts is not new. Since 1975, the United States Air Force (USAF) has been trying to effectively deal with the problem of inherent (embedded) flaws from material processing and those resulting from machining and maintenance issues, such as, surface flaws which arise during service usage. USAF requirements and procedures are documented in the Military Handbook 1738B, Engine Structural Integrity Program (ENSIP) [1] and the Military Standard 3024, Propulsion System Integrity Program (PSIP) [2]. The use of deterministic analyses to predict safe life is a wide spread practice used by most government and military airworthiness certification organizations. The Federal Aviation Administration (FAA) uses it with industry for determining the design target rate (DTR), but has recently added damage tolerance requirements for critical components to address failures due to inherent defects. The FAA governing regulations and circulars [3–6] detail the requirements. The European Aviation Safety Agency (EASA) uses deterministic methods in accordance with their regulatory guidelines Certification Specification for Engines [7]. The United Kingdom's Ministry of Defence (MoD), uses both EASA specifications plus damage tolerance methods [8] similar to that of the USAF. The U.S. Navy employs deterministic methods via the Department of Defense (DOD) Joint Service Specification Guide (JSSG-2006) [9]. The deterministic method assumes the largest, single undetectable flaw exists in the most critical location of a structural element and uses the largest principal stresses at that location during the expected operational use of the engine/aircraft. Safe life then determined using minimum material properties comparable to the -3$\sigma$ limitation and design allowables selected to achieve an overall failure probability of 1in 1000, Figure 1.

In addition, Figure 1 shows how damage tolerance is used to determine a safe inspection interval for engine component maintenance schedules as derived from this deterministic analysis of the predicted time for the flaw to grow to a critical size. A key problem with this conservative approach is that widespread fatigue damage for an aging fleet of aircraft/engines is not amenable to analyses based on the growth of a monolithic crack. One ends up with an unacceptable

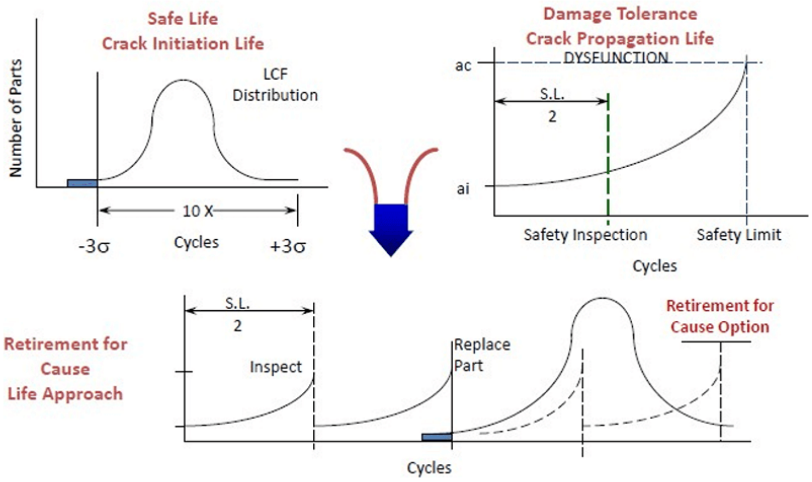



**Figure 1** ENSIP & PSIP design philosophy incorporates safe life & damage tolerance

inspection interval requirement which becomes too costly and forces premature retirement of useful engine components. Koul et al [10] and Goswami [11, 12], as well as, Vukelich [13] discuss the short comings of the safe life approach and introduce damage tolerance methodology and retirement for cause (RFC) as a means to overcome this conservatism. Here, it is assumed that fracture critical areas contain manufacturing or service-induced defects that give rise to cracks that grow during service. By monitoring the crack growth and using fracture mechanics principals a safe inspection interval ($S_{II}$) is determined and the part is not retired until a crack is initiated. This process is also shown in the latter part of Figure 1.

Similarly, Bere and Koul [14] noted using a safe life approach for turbine discs, with a goal to assure only 1 in 1000 components is likely to develop a small fatigue crack at the end of safe life is limiting. Examining Figure 2, shows that one could achieve twice the component life with an 88% probability of no failed parts before crossing the peak of the failure curve by employing an alternative method. This represents a considerable extension of life and resulting cost savings.

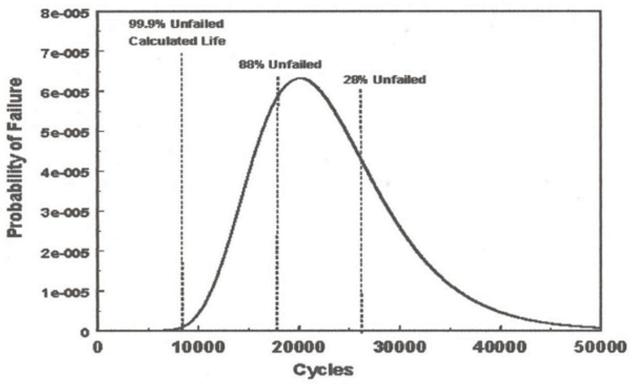



**Figure 2** Probability of failure in safe life approach

As such, damage tolerance methods can be used to assure the continued safe use of components via an inspection life cycle management approach which relies on nondestructive inspection (NDI) of components at overhaul. In essence it is a RFC approach which uses probability of detection curves and inspection data. At the end of one $S_{II}$ all components are inspected and crack-free components are returned to service for another $S_{II}$ cycle. Using this procedure a component can be kept in service until a crack is found, thus allowing it to be retired on an individual basis when conditions warrant it. Deterministic fracture mechanics calculations are used to predict $S_{II}$ and probabilistic fracture mechanics methodologies are used to quantify risk.

Numerous researchers [15–24] have shown how using probabilistic methods designers can safely use components beyond conservative deterministic levels. These methods apply advanced probabilistic failure assessment techniques, such as, the estimated mean value method (MVM), a first order reliability method (FORM), and /or a Monte Carlo simulation method with importance sampling (MCSIS). However, these approaches cannot be blindly used without risk. Annis and Vukelich [25] studied several probabilistic approaches for risk assessment of structural components. They considered the Normal Distribution, the Log-Normal Distribution, the Weibull Distribution, and the Beta Distribution for fatigue type data. What they found was the underlying assumption of a specific distribution could generate errors in risk assessment analysis. Figure 3 shows the differing results of stress versus number of cycles to failure using Log-Normal, Normal, and 2 Parameter Weibull Distributions. There is good agreement in behavior in the center region, but a divergence of behavior occurs in the tail regions. This divergence is related to the initial distribution assumption of each technique.While it is true that probabilistic analysis methods are a major improvement to the previously used empirical techniques, there is still room for improvement in consistently applying these tools and developing a physics' based probabilistic solution for risk assessment.

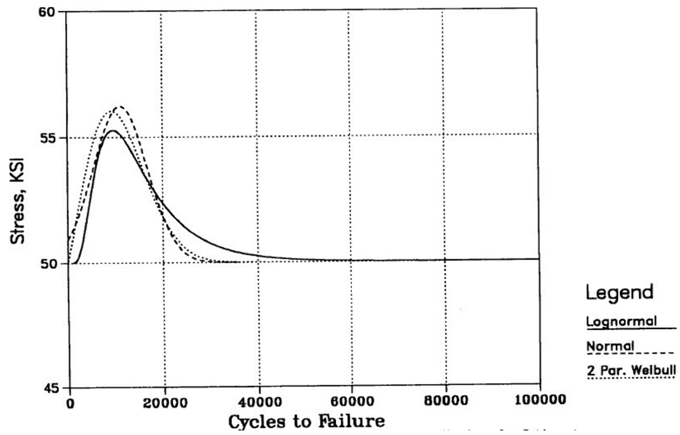



**Figure 3** Log-Normal, Normal, and 2 Parameter Weibull distributions for fatigue-type data show similar behavior near center and different tail behaviors

Much of the success seen to date is indebted to a ground breaking 1983 effort initiated by the North Atlantic Treaty organization (NATO) Advisory Group for Aerospace Research and Development (AGARD) which established the Engine Disk Cooperative Test Program. The AGARD Structures and Materials Panel was charged with executing this effort involving military and industry partners from the participating NATO nations. The study focused on damage tolerance in titanium alloys used in turbine engine disk components. It grew beyond building a comprehensive damage tolerance database for Ti-6Al-4V titanium alloy using 13 international laboratories. This core work is documented in AGARD-R-766 [26]. The intent of the "Core Program" was to familiarize participants with state-of-the-art test techniques using a well behaved titanium alloy, Ti-6Al-4V. Detailed test procedures were written. Four specimen geometries were selected; two investigating low cycle fatigue properties and two damage tolerance properties. The effort was expanded to include the study of other titanium alloys (IMI 685 and Ti-17) behavior under simplified test and actual flight loading conditions, as well as, a comparative study of five different crack growth models and their verification used by the participating team members.

Based on his core work in AGARD-R-766, Raizenne [17], noted in deterministic cases, structural risk analyses are used to assess the structural integrity of the load path, "structural integrity is characterized in terms of the single flight probability of failure of the load path." Probabilistic evaluation of strength versus stress is dynamic since strength degrades as fatigue cracks in the load path grow and conditions change during maintenance actions. The structural condition is analyzed as a distribution of damage at the critical locations and fracture mechanics tools are used to predict the growth of the damage distributions as a function of flight hours. The probability of failure is calculated from the strength and stress distributions at time "T." The engine component maintenance actions are therefore scheduled at intervals that provide an acceptably small probability of failure. Our interest lies in the comparison of the probabilistic predictive tools versus the test data. Each model used the established material database and a series of simplified loadings (Figure 4 and 5), as well as, some actual flight test loading profiles called TURBISTAN (Figure 6). The simple sequences SS1, SS2, and SS3 were selected to study the effect of minor cycles on a single major cycle. An overload ratio of 1.7 was also chosen and the wave shape used for this sequence was a simple triangular shape, SS4 [26].

"TURBISTAN is a spectrum load history for fighter aircraft engine disks. The TURBISTAN sequence contains 15,452 load reversals in a block of 100 different flights, whose average length is about 80 cycles [26]. A sample of TURBISTAN, comprising flights 66-69, is shown in Figure 6. It must be noted that the frequent load excursions for actual flights can affect the fatigue

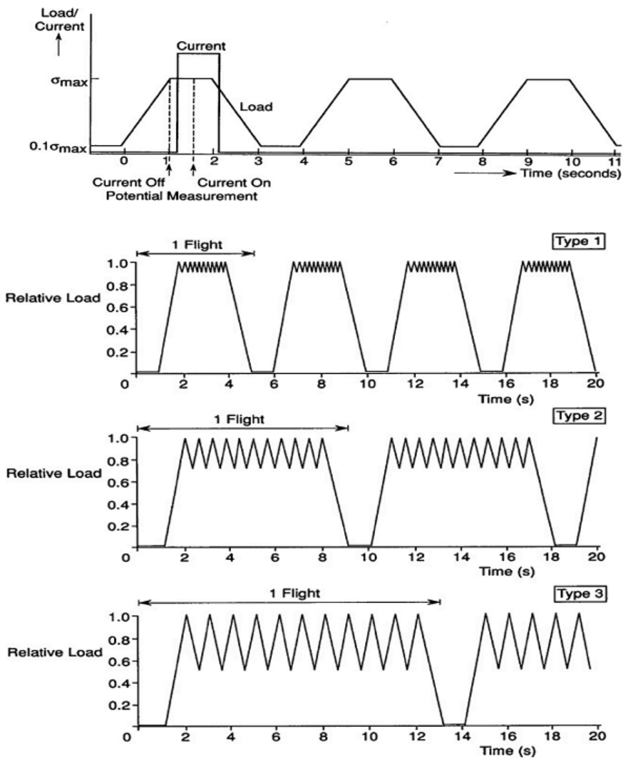



**Figure 4** Trapezoidal wave form used for constant amplitude test and graphical presentation of simple load sequences SS1, SS2, and SS3

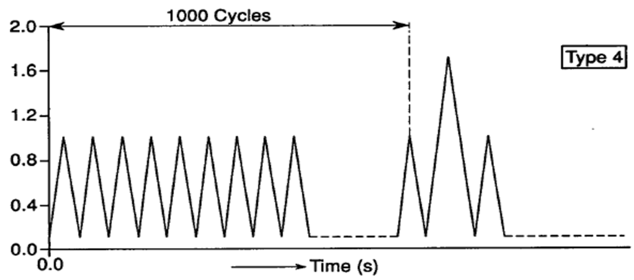



**Figure 5** Graphical presentation of overload load sequence SS4

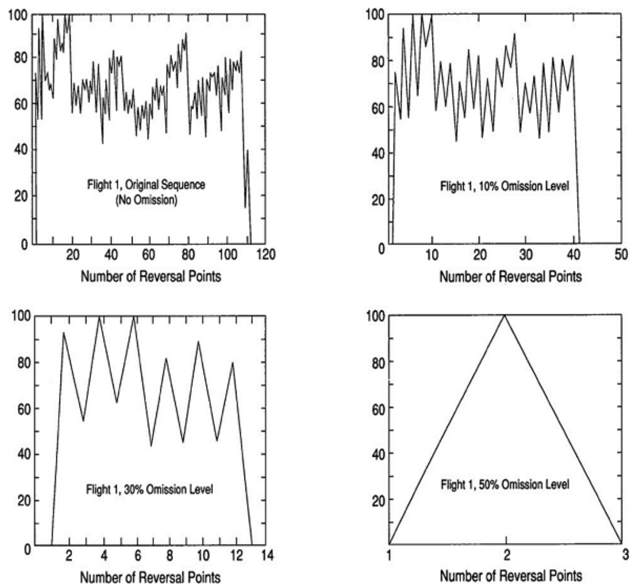



**Figure 6** Flight #1 of TURBISTAN load sequence with various omittance levels

crack growth behavior. "The TURBISTAN fatigue crack growth tests were done in laboratory air at room temperature (293-295$^0$K). A constant loading rate was maintained, resulting in cycle frequencies ranging from 1 Hertz for the largest load excursions to 20 Hertz for the smallest load excursions. Automated crack growth measurements were made using the direct current (DC) potential drop technique."

A closer examination of sequence SS1 shows a small alternating stress component and a large mean stress level applied for a longer period than any of the other cycles. In fact the test data we will discuss later, shows these cycles being applied for significant periods which resembles a HCF load case. Parker [27] showed this same phenomena for cyclic torsion loads applied to the TF41 low pressure compressor shaft. (See Figure 7)

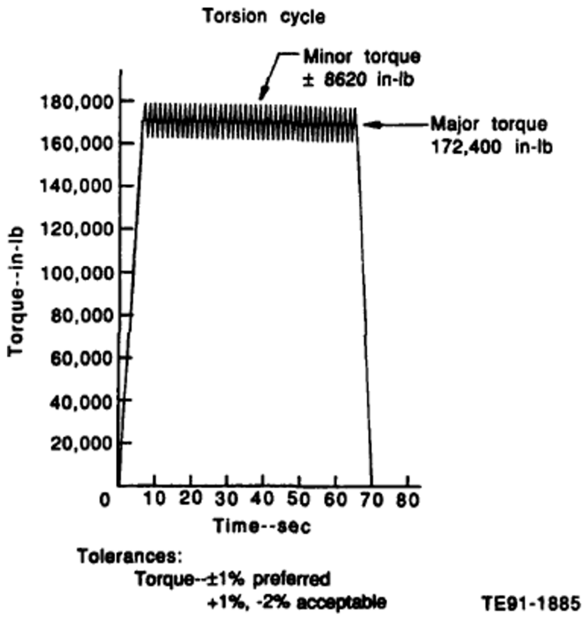



**Figure 7** Torsion load cycle used for fatigue tests

This was a test case used in the AGARD Engine Life Assessment study [27]. It showed how HCF and LCF loadings interact in real applications and can change engine component lifing measurements. This could have happened in some of the SS1 case studies. In such an instance, the low cycle load is applied for a matter of seconds while the high cycle load is applied for hundreds or thousands of hertzs. The cumulative damage that would result from a combined loading case like this would not be determined using a simple liner Miner's rule. The AGARD TF41 case [27] did not include high temperature effects, which is an additional requirement for future analyses and tests to further develop accurate tools to predicted component life. Petrovich and Zeigler [28] studied HCF and LCF crack growth interactions in Inconel 718 a nickel based super alloy. Their work showed distinct regions where the combined loads dominated behavior at 649$^o$C. Crack retardation was observed when high frequency loading was applied in the low cycle dominated regime. A linear summation technique would have produced erroneous results. Crack growth prediction was complicated by the transient effects associated with high cycle growth rate retardation [27]. To date, the FAA does not address this issue in their regulations, but USAF does so in ENSIP, however uses an unrealistic alternating stress level of 30 percent.

Following the material characterization studies, five NATO participants agreed to evaluate their respective crack growth models and verify their results against experimental data. The five participants were The Aeronautical Testing Center (CEAT) in Toulouse, France, Swedish Defense Research Agency (FAA), National Aerospace Laboratory (NLR), Amsterdam, The Netherlands, National Aeronautics and Space Administration (NASA), Langley, and Rolls Royce (RR), Derby, United Kingdom. A total of 60 loading cases for three titanium alloys were analyzed under this study. We selected two cases to examine as they represent two significant limiting cases. First, consider Case 02 for a Ti-6Al-4V Corner Crack (CC) specimen with a constant amplitude load ratio (R) equal to 0.1. Here every prediction falls below the experimental data and thus yields a conservative solution. (see Figure 8)

The Rolls Royce model was closest to matching the actual test data. This was likely due to their having specifically designed specimens for CC testing and analysis. Over all, here we see good model behavior for the first 40,000 cycles. However, after that it is clear the models no longer yield the same results. For our purposes we are treating the test results as the "correct"

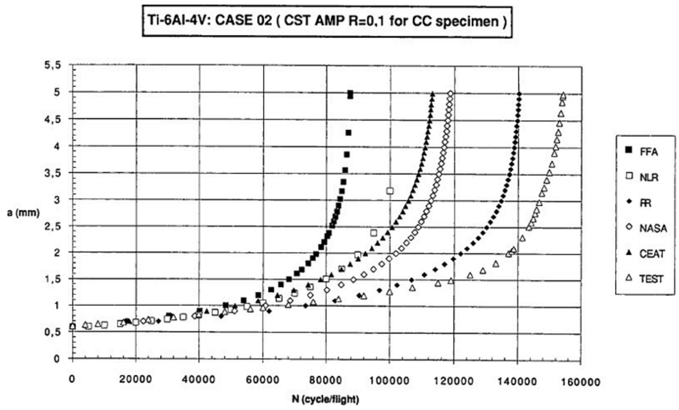



**Figure 8** Ti-6Al-4V case 02

solution and judging model performance against that standard. If you are attempting to extend the useful life of a component these divergences in results becomes a serious concern. (see Figure 9)

Next, consider Case 55 for Ti 17 Compact Tension (CT) specimen at 30% of the TURBISTAN load conditions.

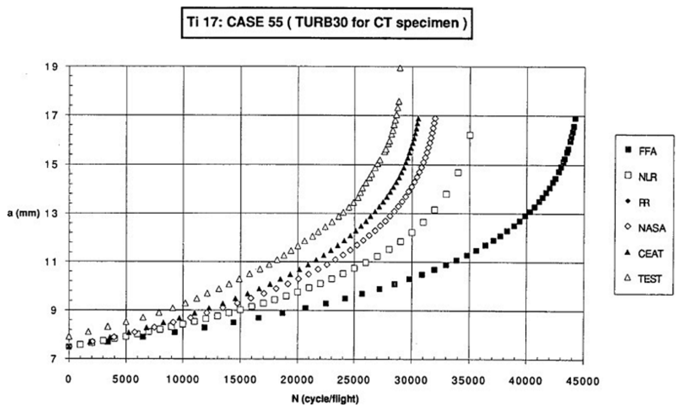



**Figure 9** Ti 17 case 55

This time the actual test results for Ti 17 are not accurately predicted by any of the models. All do reflect the correct behavior (i.e., curvature), but all over estimate the true life. The results are not conservative and divergence begins at 5,000 cycles. This implies the initial input conditions may have been inaccurate. As such, none of the approaches is particularly accurate, they all have short comings. The important point here is that there remains a great deal of research needed to develop sound physics based tools to consistently predict engine disk component life.

Wanhill [29] found similar behavior, i.e., limited initial agreement of analytical tools with test data but the models all diverge with higher cycle levels, when he investigated Inconel 718 (IN718). Connolley, Reed, and Starink [30] found even greater variability for short crack growth, interaction and coalescence in IN718. Konig and Bergmann [31] conducted probabilistic damage comparison studies of powder metallurgy Udimet 700 for cracks emanating from various defects. They used TURBISTAN loading cycles and found there was again reasonable agreement between the various predictive tools at low cycle numbers, but a diverge as the number of cycles increased, their focus was primarily on hot disk applications.

The level of variability has led many researchers to pursue probabilistic methods to model such behavior. Kappas [32] provides a fairly complete summary of the risk and reliability associated with the various probabilistic methods in use in 2000 for aircraft gas turbine engines.

# 2 Application of probabilistic methods

Many advances have been made since the groundbreaking work done under the AGARD subcommittee. Cesare and Sues [35] of Applied Research Associates, Inc., Raleigh, N.C. used probabilistic methods to account for design uncertainties, manufacturing tolerances, as well as, to make product reliability and risk assessments. They used probabilistic distributions for design tolerances, loads, material properties, and boundary conditions which were treated as random variables. The main tools they employed were ProFES with ANSYS and MSC/NASTRAN. ProFES is a probabilistic finite element analysis system that allows designers to perform analysis in a 3D graphical environment. Essentially, the probabilistic analysis tool has been integrated with the Finite Element tool in order to match the required inputs and outputs and minimize the designer's effort. This work was sponsored under a government Small Business Independent Research (SBIR) project with Applied Research Associates and the Air Force Research Laboratory (AFRL). Other participants included General Electric, Pratt & Whitney, Allison Engine, General Motors and commercial finite element vendors ANSYS and MSC/NASTRAN.

Cesare and Sues analyzed a simplified model of a disk section using a rectangular bar with a hole under a uniform load. The input random variables were the elastic modulus, Poisson's Ratio, loading, and the radius (see Table 1). The user enters this data via dialogue boxes; it should also be noted, 9 possible probabilistic distributions are available to describe a variables behavior. The finite element model was directly imported into ProFES as a data input file.

**Table 1** Input random variables

| Parameter | Distribution | Mean | SD | Units |
|---|---|---|---|---|
| Modulus | Lognormal | $2.9\text{x}10^7$ | $2.9\text{x}10^6$ | psi |
| Poisson's Ratio | Truncated Lognormal | 0.25 | 0.025 | |
| Load | Lognormal | 1.0 | 0.1 | Pounds (lb) |
| Radius | Lognormal | 0.5 | 0.05 | inches |

Next, ProFES probabilistic analysis was used to determine the failure modes, called limit-states, which were modeled using the equation, G = R – S, R is resistance (i,e., yield strength or cycles to failure), S is load effect (i.e., max stress or desired life) and failure is defined when G = 0.

All probabilistic methods involve repeated evaluation of a limit-state function. Monte Carlo Simulation (MSC) generates samples of each random variable, and runs deterministic model predictions at each combination. Statistics and probabilities are determined by a simple statistical analysis. First-Order Second-Moment (FOSM) method finds the gradients of the limit state function at the mean values of the random variables, fits a linear response surface at this point, and estimates mean and standard deviation of the response. First-Order Reliability Method (FORM) searches input variables for the combination that is most likely to cause failure (*most probable point* (MPP)). Then fits a linear surface at the MPP and uses this surface to compute probabilities. ProFES output response variables options allow the user to define desired response variables to be collected for use in statistical analysis or the limit-state function via dialogue boxes. ProFES presents the user with a list of options, e.g. displacement, stress, strain, etc. For our case, they examined the equivalent stress versus the classical stress concentration factor. The MCS was used and a cumulative distribution function (CDF) was calculated. The probability results showed, for the uniform far field stress of 1.0, the resulting stress concentration factor (SCF) was 3.45 with a standard deviation of 0.23, since the classical deterministic solution is SFC $< 4.0$ there would be no failure. Next, they introduced a geometry factor as a random variable, i.e., the radius was input as a normal distribution with a mean of 0.5" and a standard deviation of 0.05" and the solution was recomputed using the FORM method. This time the results yielded a probability of failure of 0.131, so the reliability becomes 1 – 0.131 = 0.869, which means there is a 13.1% chance of failure! Using the ProFES sensitivity analysis option the authors showed the impact of the random variable probabilistic distributions on the solution. (see Figure 10). The sensitivity factors are used to tell us which variables contribute the most to uncertainty, which in our case were Load P with a 75.51% and Radius R with a 24.48%. It is also clear that standard material properties like modulus and Poisson's Ratio have virtually no impact when modeled probabilistically. However design variables such as load and geometry show a dramatic impact on uncertainty and one which is not readily apparent to the designer without conducting a probabilistic analysis. In contrast, using a classic deterministic solution, i.e. $3.5 < 4.0$ with a safety margin of 4.0/3.5 = 1.14 one would have been led us to believe the plate could never fail. i.e. have an SFC greater than 4.0.

Jameel [36] from Honeywell, tackled a more complex disk model in support of the FAA sponsored Rotor Integrity SubCommittee (RISC). The goal was to take a generic disk ring which met the FAA AC 33.70-2 [26] bolt hole test case defined in the appendix. Jameel used the FAA probabilistic analysis tool DARWIN to assess the probability of fracture (POF) due to surface damage in a highly stressed bolt hole of a nickel component. He then compared the DARWIN results with the Honeywell in-house turbine engine proprietary design tools.

The RISC*-TEC bolt hole test case is a titanium ring disk with 40, inch, bolt holes. The blade

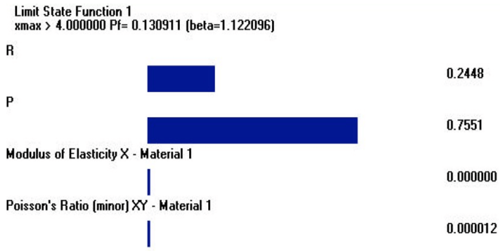



**Figure 10** Results of ProFES sensitivity analysis

load on the ring disk is simulated through an external pressure load of 4.786 KSI. The loading cycle consists of a centrifugal load at room temperature due to cycling between a maximum speed of 5,700 RPM and 0 RPM. An ANSYS 3D elastic-plastic model (Figure 11) of a wedge section of the disk was created and a contour plot of the maximum principal stress S1 is shown here corresponding to the point of maximum loading in the cycle. In this problem the material properties were not treated probabilistically. Location 4 (indicated on the figure) was used for all sensitivity studies.

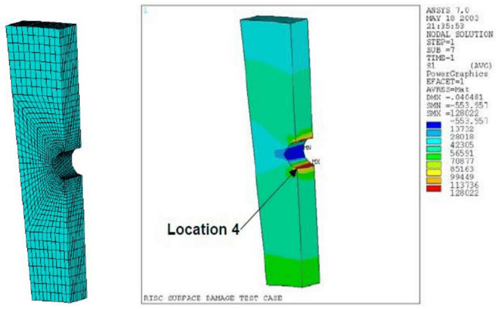



**Figure 11** ANSYS FE model and RISC-TEC bolt hole principal stress solution

The Ti–6Al–4V Alloy input data used in the analyses is shown below in Table 2.

**Table 2** Physical propeties for Ti – 6Al – 4V

| Variable | Value |
|---|---|
| Density lb/in$^3$ | 0.161 |
| E (KSI) | 1.74E+04 |
| YS (KSI) | 121 |
| UTS (KSI) | 132 |
| e (%) | 10 |
| RA (%) | 20 |
| Poisson's ratio | 0.361 |
| Hardening Exponent | 20 |

Crack growth properties for two load (R) ratios in the form of Paris equations were used:

$$\mathrm{R} = 0 : \mathrm{da/dN} = 5.248\text{E-}11(\Delta\mathrm{K})^{3.87} \tag{1}$$

$$\mathrm{R} = -1 : \mathrm{da/dN} = 7.2684\text{E-}12(\Delta\mathrm{K})^{3.87} \tag{2}$$

DARWIN methodology for surface damage risk analysis using 3D finite element models consists of determining the principal stress plane and extracting the bivariant stresses at the point of interest (Figure 12a). The univariant stress gradient is extracted from the 3D model using ANSYS and compared with the DARWIN univariant stress gradient (Figure 12b). DARWIN and internal Honeywell code results show good agreement.

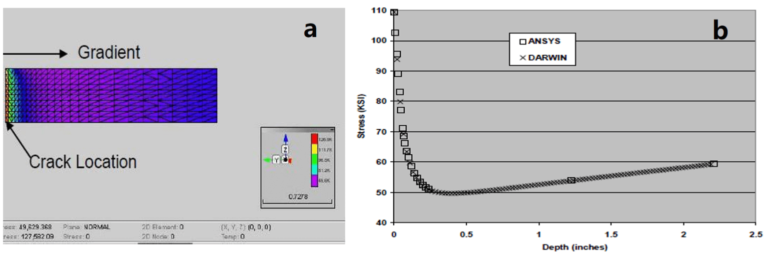



**Figure 12** a: Principal Stress Plane; b: Univariant Stress Gradients.

Next he compared DARWIN deterministic calculated crack growth lives versus the Honeywell in-house code. The stresses in the maximum principal stress plane were used to deterministically compute the crack growth life of an initial flaw (0.001 x 0.001 inches). (see Figure 13).

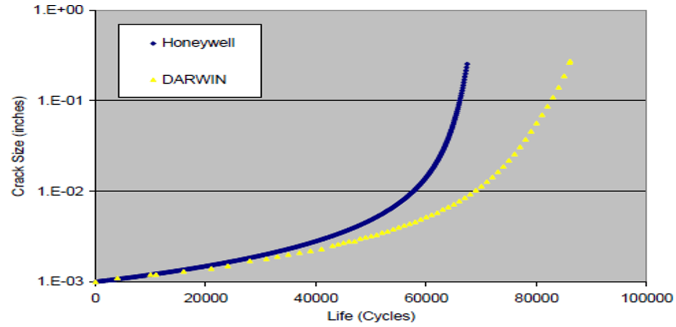



**Figure 13** Comparison of DARWIN vs Honeywell Crack Growth Life Predictions

Figure 13, Comparison of DARWIN vs Honeywell Crack Growth Life Predictions

Notice that the crack growth life results at the end of life are diverging. Such differences can arise due to variations in stress intensity factor solutions used by the two different programs.

However, notice that the crack growth predictions at 20,000 cycles are very similar between the two solutions. We observed the same behavior in the earlier AGARD studies. (see Figure 14)

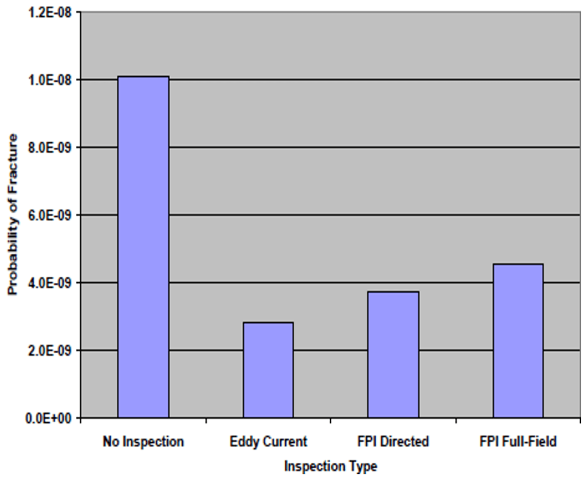



**Figure 14** Probability of fracture of location 4 on Bolt hole test case for three different inspection techniques applied at 10,000 cycles

Risk of fracture of the RISC-TEC bolt hole test case due to surface damage was evaluated as shown in Figure 15. DARWIN contains internal probability of detection (POD) curves for the various inspection techniques: Eddy current (EC),Ultrasonic (UT), and fluorescent penetrant inspection (FPI), as well as, an anomaly exceedeance curve. Impact of various inspections on risk of fracture shows the relative efficacy of the inspections with **eddy current** inspections at half-life giving the greatest benefit. The goals were two-fold, demonstration of the methodology, and comparisons with independent solutions as a benchmark, both of which were achieved.

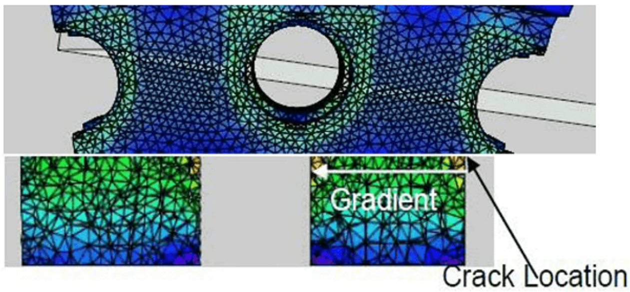



**Figure 15** Principal Stress Solution for High Energy Disk Component

The next challenge was to perform a risk of fracture analysis of a Honeywell engine high energy rotating component due to surface damage. Often in design a new material is needed to overcome field problems, etc. This requires a new risk analysis of fracture, as well as other analysis to meet engine requirements. The finite element model for a bolt hole is shown in the Figure 15. The maximum principal stress plane is shown in the lower portion at the right.

The results for the maximum principal stresses at the critical location due to changing from Material A to Material B are shown in Figure 16. Overall, it appears Material B can meet the design requirements.

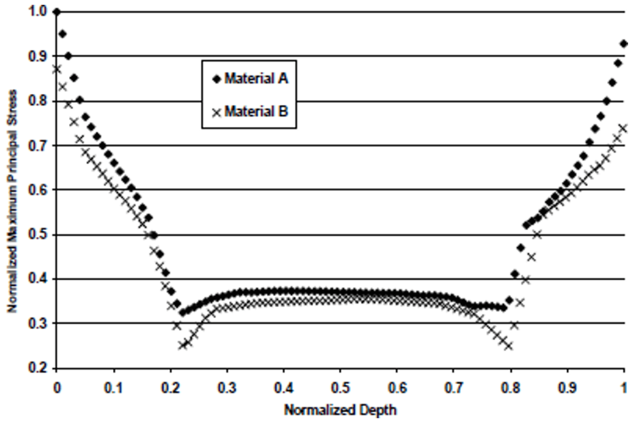



**Figure 16** Comparison Of Principal Stress Fields for the two Materials

However, if one completes the analysis for residual life and compares the relative magnitudes of the risk of fracture one finds for the various inspection techniques that Material B has a much greater probability of fracture without inspection as shown in Figure 17. The crucial issue is that probabilistic analysis has helped to identify this design limitation which would not have been realized using deterministic calculations alone.

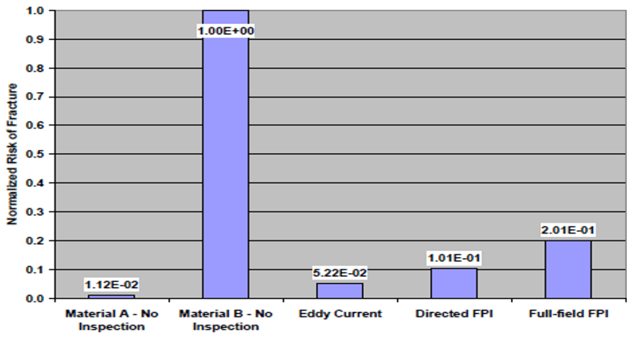



**Figure 17** Impact of Inspections on extending Residual life of the component

Since the FAA has certified DARWIN as an acceptable tool for calculating rotating component probability of failure anyone attempting to analyze real turbine engine components has resorted to using it as the preferred tool or at least as a comparison tool. As such, we need to discuss the programs features to better understand how to accurately employ it. Under FAA sponsorship, Enright et al [37] of Southwest Research Institute developed a new method for defining surface damage-related crack growth. It uses a 3D finite element solution as input to define principal stress fracture planes. It uses basic fracture mechanics principles integrated with probabilistic analysis tools, and allows for aerospace component performance and risk assessment. DARWIN uses a zone-based approach, i.e., the component is divided into volumes of approximately equal risk, and the total risk is the sum of all zones. A probabilistic methodology has been developed to predict the risk of fracture associated with, (a) inherent (embedded) material defects, and (b) surface damage-based defects. Figure 18 shows both defect types as modeled in DARWIN.

The strategy for handling inherent (embedded) material defects revolves around the zone-based approach and subdividing a component into volumes of "equal risk." (see Figure 19) A region of approximately equal risk is defined to have the same uniform stress state ($\sigma$**),** same fatigue crack growth properties, inspection schedules, POD curves, and anomaly distributions. In contrast, surface damage-based defects are assumed to be concentrated in specific regions (e.g., bolt holes, or surfaces subjected to abusive machining).

A new method of modeling surface damage-related crack growth uses a 3D finite element analysis and assumes Mode I cracks propagate in a plane normal to the maximum principal

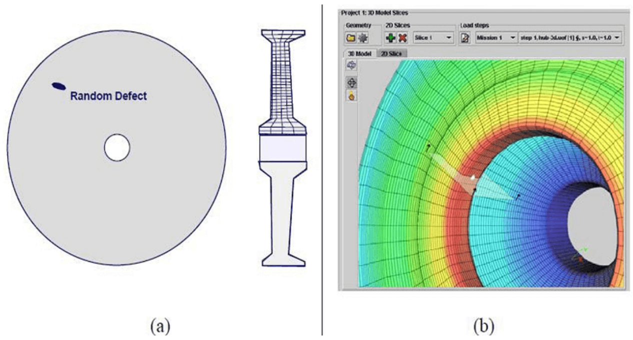



**Figure 18** DARWIN modeling of inherent and surface damage defects

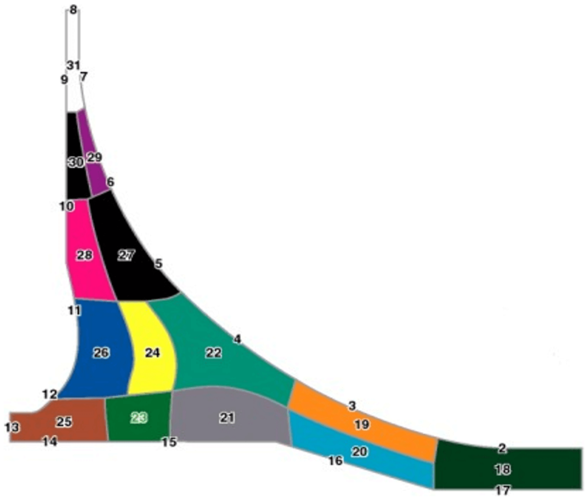


**Figure 19** Example of Zone-Based development in DARWIN

stress ($\sigma_{max}$) at the critical location. This is depicted graphically in Figure 20. DARWIN has a graphical interface (GUI) to automatically identify the principal stress plane and then slices the model along the principal stress plane to reveal the crack propagation plane and uses the associated rectangular plate for making a fracture mechanics assessment. MSC can require a large number of samples for accuracy thus leading to a lengthy computation. The GUI is one approach to reduce the required computation time while retaining solution accuracy.

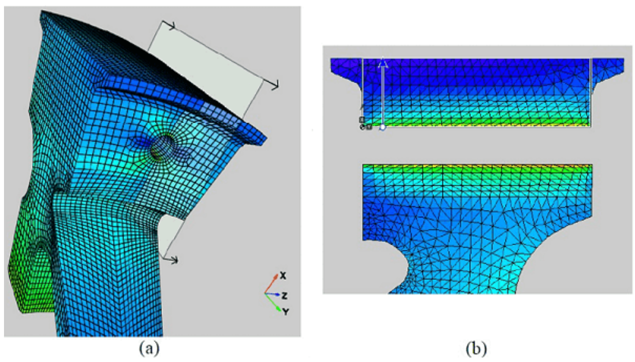



**Figure 20** Example of mode I crack plane modeling in DARWIN

Another area of considerable interest has been trying to understand the impact of residual stress on crack growth propagation. Millwater, Larsen, and John [38], attacked this problem using probabilistic methods to assess the impact on crack growth fatigue life due to residual stress (RS) effects on disk surfaces by treating it as a random variable. RS was varied over the surface and depth using a probabilistic distribution plus scaling factor. The simplified approach adopted here treats the entire RS profile as a function of a single scaling parameter which is then modeled as a random variable as shown below.

$$\sigma_{\mathrm{RS}}(s) = R\sigma_{\mathrm{RS-Reference}}(s) \tag{3}$$

where, s - distance from the surface, $\sigma_{RS}$(s) is applied probabilistic RS profile, R is random variable and $\sigma_{RS-Reference}$(s) is a reference RS profile that is scaled by R. (see Figure 21)

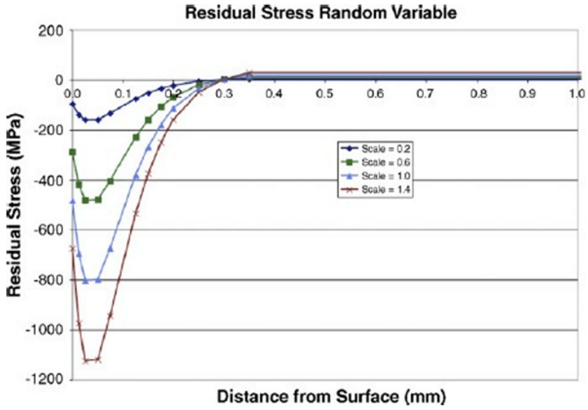



**Figure 21** RS material data obtained from X-Ray diffraction and polishing techniques

Probabilistic Sensitivity is defined by the partial derivatives of the POF following the work of Karamchandani [44] and Wu [45].

$$\frac{\partial P}{\partial \theta_j} = E\left[I(\mathbf{x})\frac{\partial f_{x_j}(x_j)}{\partial \theta_j}\frac{1}{f_{x_j}(x_j)}\right] + \text{ boundary term} \tag{4}$$

where *P* is the probability-of-fracture, $\theta_j$ denotes a parameter of random variable *j*, **x** represents a vector of random variables, $f_x(\mathbf{x})$ is the joint density function of **x** and the indicator function $I(\mathbf{x})$ is defined as equal to one if failure occurs and zero otherwise.

Probabilistic sensitivities of parameters that define external random variables is represented by $\theta_j$ with a tilde and can be obtained as follows.

$$S_{\tilde{\theta}j} = \frac{\partial P_{\mathrm{f}}}{\partial \tilde{\theta}_j} = E\left[P_{\mathrm{fc}}(\tilde{\mathbf{x}}, \hat{\mathbf{x}})\kappa\left(x_j, f_{Xj}, \theta\right)\right] \tag{5}$$

where the expected value operator is taken with respect to external variables only and the boundary term is omitted for conciseness, but should be included if necessary. Also $\kappa$ is related to the distribution and parameters of the external random variables.

The sensitivity of the probability-of-failure with respect to the parameters of the internal variables, represented by $\theta_j$ with a hat is determined by taking the expected value of the sensitivity of the conditional probability-of-failure.

$$S_{\hat{\theta}_j} = \frac{\partial P_{\mathrm{f}}}{\partial \hat{\theta}_j} = E\left[\frac{\partial P_{\mathrm{fc}}(\tilde{\mathbf{x}}, \hat{\mathbf{x}})}{\partial \hat{\theta}_j}\right] \tag{6}$$

The test case used was a compressor disk. The primary tool for analysis was DARWIN with a finite element code. The code can accommodate variable crack driving stress fields so RS and centrifugal fields are easily combined. Residual stress (RS) was treated as a random external variable; the initial crack size, life scatter; and stress scatter were treated as internal variables. The disk model was subjected to an air to ground loading profiles are shown below (Figure 22).

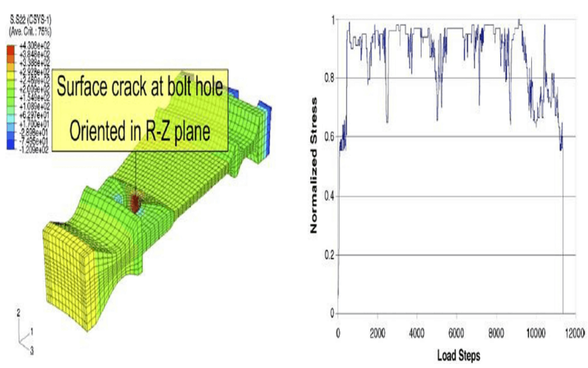



**Figure 22** Disk finite element solution and flight loading profile

The test case input data can be seen in Table 3.

**Table 3** Test case input data

| Property | Description | Comment |
|---|---|---|
| Loading | Spectrum air to ground mission and reference stress | 11360 time points, 70 load pairs<br>Reference stress from Brokman et al. |
| da/dN - DK | C = 4.009E-9, n = 2.443, Kth = 0, Kc = 100.0, a = 2.0 | Paris law with no threshold<br>Newman closure model, SI unit conversion |
| Initial crack size | Tabular probability distribution | Representative of initial cracks at bolt holes in disks |
| Propagation scatter | Median = 1, COV = 0.3 | Based on AGARD data analysis by McClung |
| Stress scatter | Median = 1, COV = 0.05 | log-normal distribution log-normal distribution |
| Failure definition | Nf < 20,000 cycles | |

The results for normalized POF versus flights is shown below in Figure 23 and the sensitivity of the solution to the various random variables is shown in Figure 24.

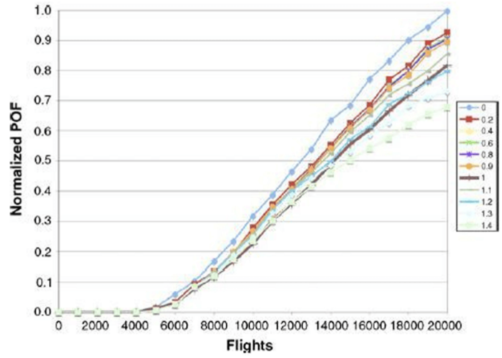



**Figure 23** Normalized POF *vs* Flights Cycles and RS Scaling Factor

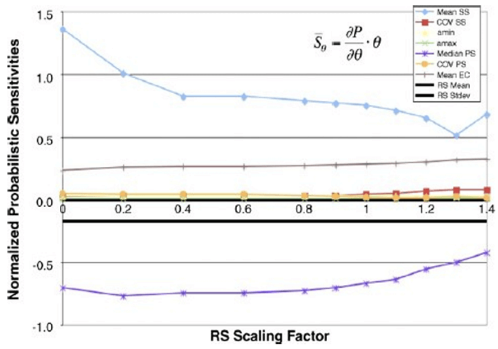



**Figure 24** Variation in normalized sensitivities with respect to RS scaling factor

Surprisingly the results showed RS has only a minor effect of just 15% reduction on the POF. The authors had expected a stronger effect. The sensitivity factor calculations did provide a $1^{st}$-order of magnitude indication of each parameter's importance. Only the mean Stress Scatter and median Propagation Scatter had any significant effect. The other variables were essentially negligible. It is believed that the limited depth of penetration (200-250 $\mu$m) by the shot peening was insufficient to overcome cracks caused by maintenance. The nominal size of a crack that will cause failure is approximately 300$\mu$m in depth initially which grows to 6mm at failure.

There have been a significant number of efforts to improve the mesh zone refinement process used by DARWIN, Enright et al. [39] developed a new method to refine the zone mesh by initially discretizing the component in a course mesh to define critical areas. Then execute the DARWIN risk assessment code and evaluate the results. If total risk is below the Design Target Risk (DTR) you are done. If not, you iterate until the total risk does fall below the DTR level.

In an effort to assess the accuracy of the DARWIN code, several test cases were run versus the proprietary codes of the contractors. The DARWIN results for cycles to failure vs. initial flaw size for the Flight_Life fracture mechanics module in DARWIN are compared to OEM fracture mechanics codes in Figure 25. The results to date have been favorable.

Millwater et al. [40–43] developed a convergent zone-refinement method for risk assessment of gas turbine disks which enhances accuracy. A local metric is used to identify zones needing further refinement leading to more accurate solutions. Knowledgeable users divide a component into zones of equal risk and use the life-limiting location for conservative calculations. The probability of disk fracture is found from system reliability equation.

$$P[\text{ disk }] = P[\text{ fracture in any zone }] = P\left[F_1 \cup F_2 \cup \cdots \cup F_N\right] \\ = 1 - \prod_{i=1}^{n}\left(1 - P\left[F_i\right]\right) \approx \sum_{i=1}^{n} P\left[F_i\right] \tag{7}$$

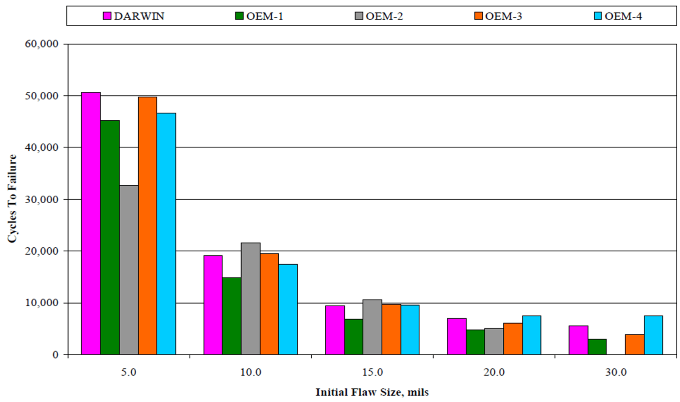



**Figure 25** DARWIN results for cycle to failure for initial flaw size *vs* OEM results

Probability of fracture of an anomaly P[$F_i$] in a zone "i" is found using the adjacent equation where P[$A_i$] is probability of anomaly existing in the zone and P[$B_i$I$A_i$] is the conditional probability of fracture and is computed using probabilistic fracture mechanics based life assessment (Monte Carlo sampling) for low cycle fatigue. Stress Intensity Factors (SIF) are derived using weight functions for rectangular plates which are a good approx since SIF is weakly dependent on the boundary. Variations in FE results are simulated using a multiplier "S" as a random variable. Life scatter factor "B" is used to model the variations in predicted cycles to failure as shown below.

$$P\left[F_i\right] = P\left[A_i\right] P\left[B_i \mid A_i\right] \tag{8}$$

$$\sigma = \sigma(\mathrm{FE}) S \tag{9}$$

$$N = N(FM) B \tag{10}$$

The risk contribution factors (RCF) are found using

$$\mathrm{RCF}_i = \frac{P\left[F_i\right]}{\sum_{j=1}^{n} P\left[F_j\right]} = \frac{P\left[A_i\right]^* P\left[B_i \mid A_i\right]}{\sum_{j=1}^{n} P\left[A_j\right]^* P\left[B_j \mid A_j\right]} \tag{11}$$

RCF's are defined such that:

$$\sum_{i=1}^{n} \mathrm{RCF}_i = 1 \tag{12}$$

Applying this methodology to an impeller the authors readily converged to a solution where the mesh elements were all below the DTR (Figure 26).

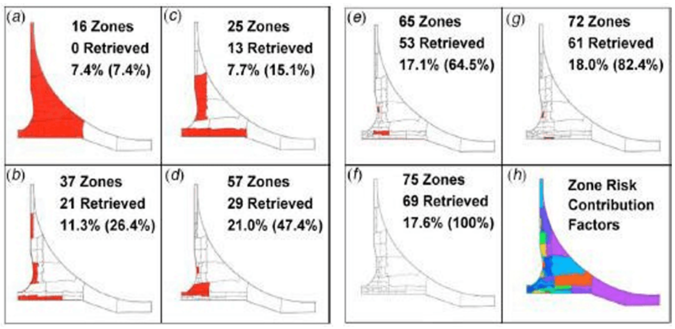



**Figure 26** DARWIN mesh refinement process to meet DTR requirements

For more on mesh refinement see Moody, Millwater, and Enright [42] "Adaptive Risk Refinement Methodology for Gas Turbine Engine Rotor Disks" AIAA 2008-2224. It provides a detailed example of using DARWIN's zone-based risk analysis and defines probability functions to analyze an impeller disk component. They employ a probability of failure surface to help guide the mesh refinement process. Also, Kappas [32] provides a comprehensive review of current (2002) probabilistic approaches. He discusses the Monte Carlo, $1^{st}$-order and $2^{nd}$ order Reliability Methods and reviews probabilistic equations used to model material, loading, life, risk, etc. He discusses finite element analysis and using tools like DARWIN for calculating POF.

As part of our code verification effort Wright State University (WSU) performed a number of analyses to demonstrate various code capabilities. In particular, WSU solved the FAA Advisory Circular test case using ABAQUS as a finite element tool for the disk ring and DARWIN for the probability of failure risk assessment. WSU worked with Southwest Research Institute (SwRI) to integrate ABAQUS output results with DARWIN. Thus far DARWIN had only been designed to accept input from ANSYS since the initial developers and

industry partners use it as their principal finite element analysis tool. WSU used ABAQUS 6.8 and 6.9 and a DARWIN 7.0 beta version for this analysis. The results from ABACUS using AC33.70-2 [6] input data and material properties are shown in Figure 27. The maximum principal stress is in excellent agreement with those from Jameel [36]. This information was then feed into the DARWIN probabilistic code and the probability of failure was computed. The results (Figure 28) are quite good and fall within the FAA AC's acceptable range.

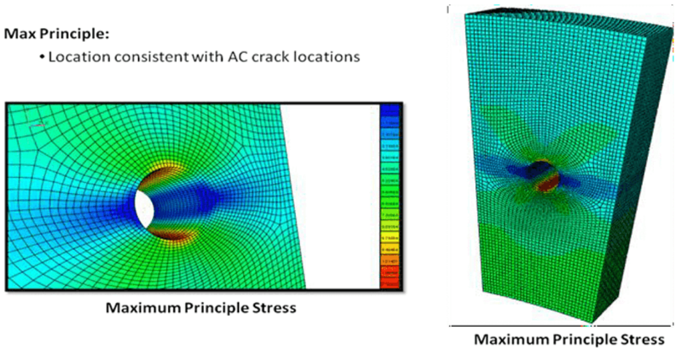
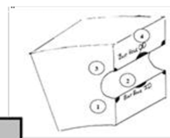



**Figure 27** ABAQUS calibration test stress results for AC 33.70-2

**POF of Rotating Titanium Ring:**
- Crack location 2 has the highest sensitivity
- Monte Carlo method with LAF (100,000 samples)
- Total feature area taken into account in zone definition



| AC 33.70-2 Probability of Facture at 20,000 Cycles | |
|---|---|
| Without Inspection | With Inspection |
| 2.886E-4 | 1.81E-5 |
| Percent Deviation from Mean Given | |
| +0.2% | +41.4% |

**Results Range:**
- Without inspection as a percent of the mean value: +11.8% to -10.4%
- With inspection as a percent of the mean value: +75.8% to -43.0%.

**Results ranges point to more uncertainty associated with how the impacts of inspections on the POF are calculated by the various manufacturers**

**Figure 28** DARWIN probability of failure results for AC 33.70-2

# 3 Conclusions

It has been shown that damage tolerance and probabilistic methods when used in combination can effectively deal with the random variability that exists in real engine components. Research into sensitivity behavior has shown us that classic material properties such as Young's Modulus and Poisson's ratio do not benefit from a probabilistic treatment where as design properties, such as crack growth, component life, residual stress, etc., as well as, geometry and loading do benefit from using probabilistic distributions. It is also clear that deterministic analysis alone is often too conservative and at times misleading as evidenced by Cesare and Sues where the actual failure risks were not properly accounted for in the plate with a hole problem. It has been shown that in order to tackle real turbine engine disk components with complex geometries the designer must have a robust finite element code, such as, ANSYS or ABAQUS and a robust probabilistic analysis code with fracture mechanics modules. Current investigators and authors have tended to use some existing Government or Industry developed probabilistic tool that is interfaced with a finite element program. Although most industry codes are proprietary, WSU was able to closely approximate such results using ABAQUS and DARWIN. This is a promising development and will allow us to move on to a "realistic" disk geometry subjected to "realistic" operational flight load profiles. In addition, WSU will develop an anomaly distribution representative of the results observed in the USAF logistics depots during engine overhauls in order to enhance the accuracy of our calculations and analysis.

Combined damage tolerance and probabilistic methods offer great promise to be effective tools to deal with engine fleet management issues in a cost effective way. In addition, they are crucial to permit the safe application of retirement for cause concepts. However, much remains to be done before such methods can be implemented into regulatory guidelines. We believe a new AGARD "like" study is needed which would focus on use of probabilistic methods much as was done in the late 70's and early 80's. This would require an industry, government, and academia team to tackle such a demanding problem.

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