

Interval Type-2 Fuzzy Control Using Distending Function

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Abstract. In this paper, we present a novel interval type-2 fuzzy controller. Its unique features are: 1) A new interval type-2 membership function called type-2 Distending Function (T2DF) is used. It can represent and handle different types of uncertainties using a few parameters; 2) The proposed control design is based on fuzzy arithmetic operations. As compared to existing methods there is no type reduction step; 3) The expressions used are in closed form which makes it suitable for on-line implementation; 4) The proposed design is simple, intuitive, computationally fast and handles uncertainties. The effectiveness of the proposed design is shown by an altitude control of a quadcopter.

Keywords. Interval type-2 fuzzy control, Distending function, Fuzzy arithmetic, Quadcopter altitude control.

Introduction

Type-2 fuzzy sets were introduced by Zadeh to handle the uncertainty in type-1 fuzzy sets [1]. Later, this concept was extended by Karnik and Mendel [2]. The different sources of uncertainties associated with type-I fuzzy systems are: 1) The words used in the fuzzy rules have uncertain meanings (words may convey different meaning to people); 2) The experts do not agree on the values in the consequents; 3) Noise appears in the measured signal and the sensing devices are imprecise. Because of these uncertainties, the membership functions are no longer certain i.e. the grade of membership functions is not a crisp value. To solve this problem, type-2 membership functions were introduced.

Compared to type-1, the type-2 fuzzy systems are better at handling uncertainties, produces smoother control response, are more adaptive and uses smaller rule base [3]. From practical application and computational point of view, interval type-2 fuzzy systems have been introduced [4]. These systems have been successfully used in control systems [5], data mining [6], cost and risk assessment [7], time series predictions [8], urban planning [9] and human resource management [10]. The design of interval type-2 fuzzy system consists of five steps: 1) Fuzzification of the inputs using type-2 MFs; 2) Calculation of rules firing strengths. The firing strength is now an interval; 3) Implication and aggregation is used to produce the outputs. These operations produces also a type-2 fuzzy set; 4) Type reduction is applied to convert type-2 fuzzy set into type-I fuzzy sets; 5) Defuzzification is performed to get the crisp output value. This process is similar to design

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of type-I fuzzy system but here we have type reduction as an additional step. This step convert type-2 fuzzy sets to type-I fuzzy sets. The type reduction is achieved using the so called Karnik Mendel (KM) iterative algorithm [11]. This algorithm defines the switching points of the lower and upper firing strengths. Using these points, the KM algorithms generates two type-I fuzzy sets. These sets are defuzzified to get crisp output. There are some drawbacks in the above mentioned approach:

1. The choice of type-2 membership function and its systematic connection with the uncertainty are not clear. Different type-I membership functions can be combined to generate type-2 membership function. However, it is not clear which type of membership functions should be used for particular uncertainty case.
2. The type reduction step is based on KM algorithm, which is computational expensive [12]. Due to iterative nature, it is not suited for on-line applications. There are some alternative solutions which reduces the computation burden but these are approximations [13].
3. Although type-2 fuzzy logic system (FLS) requires less rules compared to type-I fuzzy systems but the number of parameters is comparatively large. So the optimization is not easy in this case.
4. The implication and aggregation steps also increases the computation complexity of the type-2 FLS.

Here, we solve some of these issues by proposing a new type of interval type-2 FLS. It solves the issues using the following unique features:

1. A new type of parametric membership function called Distending Function (DF) is used. Different types of uncertainties can be expressed by associating it with the parameters of DF. It can effectively represent most of the forms of uncertainties being used in type-2 fuzzy systems.
2. Fuzzy arithmetics approach is utilized here for designing type-2 fuzzy logic controller. So it has no type reduction step and it does not require the iterative algorithms. It is simple, computationally fast and suitable for on-line implementations.
3. Most of the parameters of the T2DF are fixed. Usually the parameter associated with the uncertainty is varied only. We can say that the number of parameters are same as in type-I FLS. The optimization process is easy and fast.
4. There is no implication and aggregation and it is computationally fast.

Because of these features, the proposed approach provides a complete framework for handling the uncertainty using type-2 fuzzy systems.

The rest of the paper is structured as follows. In Section 1, we briefly introduce the interval type-2 distending function and representation of uncertainties using its parameters. In Section 2, we present linear combination of T2DFs. In Section 3, we explain the proposed fuzzy controller design approach using fuzzy arithmetics. In Section 4, we outline the benchmark system, simulations and discuss the results. Lastly, in Section 5, we present our conclusions and directions for future work.

1. Distending Function and Uncertainty Representations

Distending function is a parametric membership function [14]. It has two forms: 1) Symmetric; 2) Asymmetric. The symmetric form is described by

$$\delta_{\epsilon, \nu}^{(\lambda)}(x - c) = \frac{1}{1 + \frac{1-\nu}{\nu} \left| \frac{x-c}{\epsilon} \right|^\lambda}, \tag{1}$$

where $\nu \in (0, 1)$, $\epsilon > 0$, $\lambda \in (1, +\infty)$ and $c \in \mathbb{R}$. $\delta_{\epsilon, \nu}^{(\lambda)}(x - c)$ will be denoted by $\delta_s(x)$. The parameters are the threshold (ν), tolerance/error (ϵ) and sharpness (λ). DF has a peak value of 1 at $x - c = 0$. If the input is in the interval $[-\epsilon, \epsilon]$, then the value of the DF is greater than ν and also $\delta_s(x - c = \pm\epsilon) = \nu$. The parameter λ controls the sharpness of the DF. If $\lambda \rightarrow \infty$ then the DF approaches the characteristic function. With an appropriate value of ν, ϵ and λ , all the existing membership functions (Trapezoidal, Gaussian, Sigmoidal, etc) can be approximated. The membership function can be shifted by a parameter c . As symmetric DF is an even function, the coordinate of the Centre of Gravity (COG) of is c .

The DF function consists of right hand side and left hand side functions (shown in Fig. 1) given by

$$\delta_L(x - c) = \frac{1}{1 + \frac{1-\nu}{\nu} \left| \frac{x-c}{\epsilon} \right|^\lambda \frac{1}{1+e^{(\lambda^*(x-c))}}}, \tag{2}$$

$$\delta_R(x - c) = \frac{1}{1 + \frac{1-\nu}{\nu} \left| \frac{x-c}{\epsilon} \right|^\lambda \frac{1}{1+e^{(-\lambda^*(x-c))}}}. \tag{3}$$

Here, λ^* is a free parameter and $\lambda^* \gg \lambda$. These LHS and RHS functions can be combined using Dombi conjunctive operator and it results in a DF as shown in Fig. 1 and defined by Eq. (1).

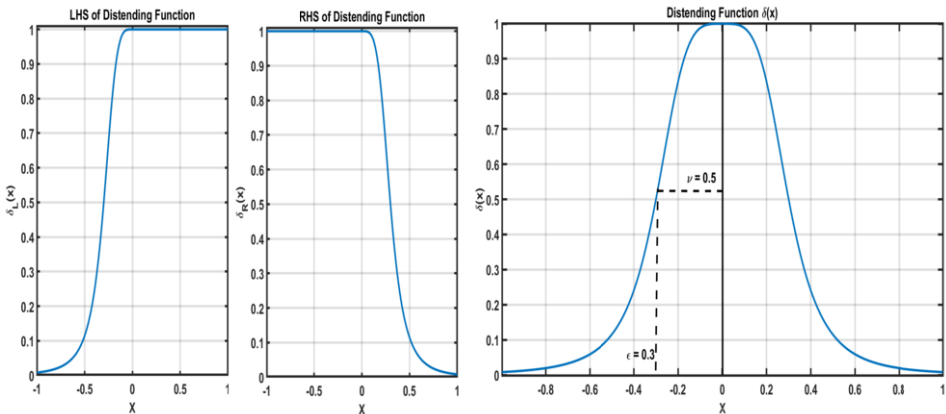


Figure 1. LHS and RHS of DF (Left and Middle). Distending Function (Right)

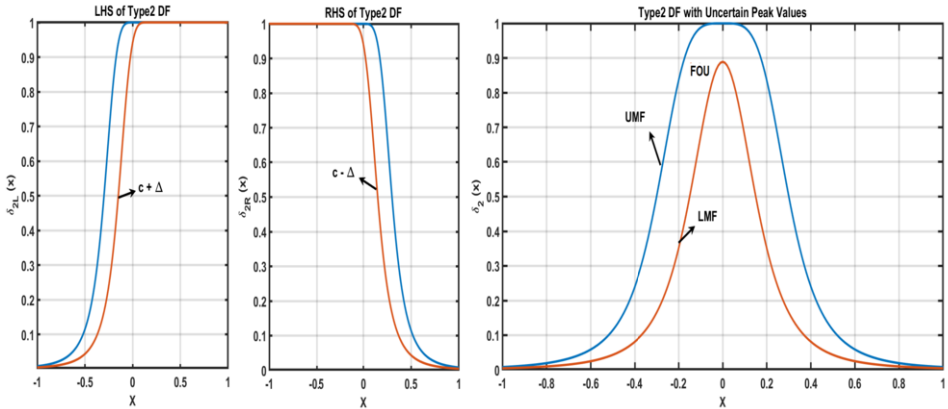


Figure 2. LHS and RHS of uncertain peak values T2DF (Left and Middle). Uncertain peak values T2DF (Right)

1.1. Uncertainty and Type-2 Distending Function (T2DF)

DF has four parameters i.e. ν , ε , λ and c . Uncertainty can appear in any of these parameters and as a result various DFs are obtained. The DF with highest grade values is called upper membership function (UMF) and that with lowest values is called lower membership function (LMF). The UMF, LMF and various DFs in between, can be combined to form a interval T2DF. Here we consider only the uncertainty in the c parameter. It will result in generation of interval T2DF with uncertain peak values.

Uncertainty in the peak values

If the peak value of DF becomes uncertain, then it can be represented using interval T2DF having uncertain ' c ' value. Consider the right and left hand side of DF as shown in Fig. 1. Now if the peak value becomes uncertain by a magnitude of Δ , then this uncertainty can be added to the ' c ' value of the LHS and RHS of DF as shown in Fig. 2. These LHS and RHS can be combined using Dombi conjunctive operator. It results in T2DF with uncertain peak values as shown in Fig. 2.

Next we show that various T2DF can be combined using fuzzy arithmetics and the results is also a T2DF.

2. Linear Combination of T2DFs

Zadeh suggested that that fuzzy quantities can be combined arithmetically [1]. Later, many researchers explored this direction [15, 16]. Using fuzzy arithmetic operations, we show here that T2DFs are closed under linear combination (i.e. the linear combination of T2DFs is also a T2DF). Here, we take the case of T2DFs with uncertain peak values. Consider n T2DFs $\delta_2^1, \delta_2^2, \dots, \delta_2^n$ with the coordinates of the peak values c_1, c_2, \dots, c_n , the uncertainties in the coordinates of peak values $\Delta_1, \Delta_2, \dots, \Delta_n$ and tolerance values $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ respectively. Let

$$\begin{aligned}
 c_a &= \sum_{i=1}^n w_i c_i, \quad \varepsilon_a = \sum_{i=1}^n w_i \varepsilon_i, \quad \Delta_a = \sum_{i=1}^n w_i \Delta_i, \\
 \bar{c}_a &= \sum_{i=1}^n \bar{w}_i \bar{c}_i, \quad \bar{\varepsilon}_a = \sum_{i=1}^n \bar{w}_i \bar{\varepsilon}_i.
 \end{aligned}
 \tag{4}$$

where w_i and \bar{w}_i are the weights of the LMF and UMF of i th T2DF respectively. It can be shown that

$$\bar{\delta}_2^a(x) = \frac{1}{1 + \frac{1-\nu}{\nu} \left| \frac{x - \bar{c}_a}{\bar{\varepsilon}_a} \right|^\lambda},
 \tag{5}$$

$$\delta_2^a(x) = \frac{1}{1 + \frac{1-\nu}{\nu} \left| \frac{x - (c_a + \Delta_a)}{\varepsilon_a} \right|^\lambda + \left| \frac{x - (c_a - \Delta_a)}{\varepsilon_a} \right|^\lambda}.
 \tag{6}$$

Where $\bar{\delta}_2^a(x)$ is UMF and $\delta_2^a(x)$ is LMF of the aggregated membership function. As $\bar{\delta}_2^a(x)$ and $\delta_2^a(x)$ are T2DF, so the linear combination of n T2DFs is also a T2DF.

3. Control design approach

Our design methodology is motivated by our previous study where a fuzzy control was designed using fuzzy arithmetic operations and DFs [14].

Multi input single output (MISO) system is described by the following rules

$$\text{If } x_1 \text{ is } A_1^i \text{ and } \dots \text{ and } x_n \text{ is } A_n^i \text{ then } y \text{ is } B^i.
 \tag{7}$$

where x_i is the i th input linguistic variable, A_i is the i th input fuzzy subset, B_i is the i th output fuzzy subset and y is the output of the system. $i = 1, \dots, l$ are the number of fuzzy rules. The part of the fuzzy rule before *then* is called antecedent part and the part after it is called consequent part. Uncertainty in the system is handled (modeled and minimized) by defining interval type-2 fuzzy sets for A_1, A_2, \dots, A_n and B . These type-2 sets will be represented using T2DFs. The inference mechanism will minimize this uncertainty and map the input-output space to generate a crisp output. In our approach, the fuzzy rules are evaluated by dealing with the antecedent and consequent parts separately.

3.1. The antecedent part

The antecedent part of the i th fuzzy rule is

$$\mathcal{L}(\delta_1(x_1)^i, \delta_2(x_2)^i, \dots, \delta_n(x_n)^i) = \hat{w}_i(x),
 \tag{8}$$

where \mathcal{L} is the fuzzy logical expression and $\hat{w}_i(x)$ is the rule applicability interval. $\hat{w}_i(x)$ contains two values corresponding to upper and lower membership grades in T2DFs. For a specific input values \underline{x}^* , Eq. (8) can be evaluated using a general parametric operator [17] and this results in an interval $[\hat{w}_i(\underline{x}^*) \quad \bar{\hat{w}}_i(\underline{x}^*)]$

$$\mathcal{L}(\underline{\delta}_1^i(x_1^*), \underline{\delta}_2^i(x_2^*), \dots, \underline{\delta}_n^i(x_n^*)) = \hat{w}_i(\underline{x}^*),$$

$$\mathcal{L}(\bar{\delta}_1^i(x_1^*), \bar{\delta}_2^i(x_2^*), \dots, \bar{\delta}_n^i(x_n^*)) = \hat{w}_i(\underline{x}^*),$$

where $\underline{\delta}_n^i(x)$ is the LMF of the n th T2DF and $\bar{\delta}_n^i(x)$ is the UMF of the n th T2DF. $\hat{w}_i(\underline{x}^*)$ is called the upper strength and $\bar{w}_i(\underline{x}^*)$ is called the lower strength of the i th rule. We normalize these strengths (to compare the rules) to get the upper firing strengths $\bar{w}_i(x^*)$ and lower firing strengths $w_i(x^*)$. The firing strengths gives the probability of the rule. The lower and upper firing strengths of the i th rule are

$$w_i(\underline{x}^*) = \frac{\hat{w}_i(\underline{x}^*)}{\sum_{i=1}^l \hat{w}_i(\underline{x}^*)}, \quad \bar{w}_i(\underline{x}^*) = \frac{\hat{w}_i(\underline{x}^*)}{\sum_{i=1}^l \hat{w}_i(\underline{x}^*)}, \quad (9)$$

where $\sum_{i=1}^l w_i(\underline{x}^*) = 1, \quad \sum_{i=1}^l \bar{w}_i(\underline{x}^*) = 1.$

3.2. The consequent part

This part of the rule is a type-2 fuzzy set represented by a single T2DF. The upper and lower firing strengths of each rule (calculated from the antecedent part) are multiplied by the UMF and LMF of the consequent T2DF. As a result the fuzzy output obtained from each rule valuation is also a T2DF. By combining all the rules, we can generate an LMF and UMF of the aggregated T2DF.

If $w_1(\underline{x}^*), w_2(\underline{x}^*), \dots, w_l(\underline{x}^*)$ are the lower firing strengths and $\underline{\delta}_{1o}(x), \underline{\delta}_{2o}(x), \dots, \underline{\delta}_{lo}(x)$ are the l LMFs of the type-2 consequents, then the LMF of the aggregated output ($\underline{\delta}_a(x)$) of the l fuzzy rule is given by Eq. (6), where

$$\underline{c}_a = \sum_{i=1}^l w_i(\underline{x}^*) \underline{c}_i, \quad \underline{\varepsilon}_a = \sum_{i=1}^l w_i(\underline{x}^*) \underline{\varepsilon}_i. \quad (10)$$

\underline{c}_i and $\underline{\varepsilon}_i$ are the parameters of the LMF of i th consequent T2DF. Similarly, the UMF of the aggregated output T2DF ($\bar{\delta}_a(x)$) has the following form given by Eq. (5), where

$$\bar{c}_a = \sum_{i=1}^l \bar{w}_i(\underline{x}^*) \bar{c}_i, \quad \bar{\varepsilon}_a = \sum_{i=1}^l \bar{w}_i(\underline{x}^*) \bar{\varepsilon}_i. \quad (11)$$

Here \bar{c}_i and $\bar{\varepsilon}_i$ are the parameters of the UMF of i th T2DF consequent. Now the crisp output can be generated as

$$c_{crisp} = \frac{\underline{c}_a + \bar{c}_a}{2}. \quad (12)$$

The whole procedure is summarized in the Algorithm.1.

4. Simulations, Results and Discussion

The effectiveness of the proposed technique is shown by designing an altitude control system for a quadcopter (Parrot mini-drone).

Algorithm 1. Algorithm for type-2 fuzzy control using T2DF

- Step 1:** Transform the inputs into $[0, 1]$ interval.
- Step 2:** Define the T2DFs for the input and output linguistic variables.
- Step 3:** Fuzzify the crisp inputs using Eq. (1).
- Step 4:** Construct the rule base from the knowledge using Eq. (7).
- Step 5:** Calculate the upper and lower strength of each rule using Eq. (8) by choosing the appropriate fuzzy conjunctive/disjunctive operators.
- Step 6:** Calculate the l upper and l lower firing strengths using Eq. (9).
- Step 7:** Calculate the parameters $(\bar{c}_a, \bar{\varepsilon}_a, \underline{c}_a, \underline{\varepsilon}_a)$ of the aggregated output T2DF by using Eq. (10) and Eq. (11).
- Step 8:** Generate the UMF of the aggregated output T2DF using Eq. (5) and LMF of the aggregated output T2DF using Eq. (6).
- Step 9:** Get the crisp output control signal u using Eq. (12).

4.1. Parrot Mini-Drone Mambo

The flight simulation model of the Mambo quadcopter is available in Matlab Simulink [18]. The model consists of: 1) Environment model; 2) Airframe model; 3) Sensors; 4) Flight control system. The Airframe model describes the 6 DOF dynamical model of the quadcopter structure. It includes the angular speeds ω , motor torques τ , upward forces f , three translational components (x, y, z) and three rotational components (ϕ, θ, ψ) . The system model is described as

$$\dot{X} = f(X, U) + W,$$

where

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ x \\ y \\ z \end{bmatrix}; \quad U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ \Omega_r \end{bmatrix} = \begin{bmatrix} b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ b(-\omega_2^2 + \omega_4^2) \\ b(\omega_1^2 - \omega_3^2) \\ d(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2) \\ -\omega_1 + \omega_2 - \omega_3 + \omega_4 \end{bmatrix}; \quad W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}.$$

Where X is the state vector consisting of three translational and three rotational components, W is the additive noise affecting all these states of the quadcopter and U is the input to the system. U_1 is the total thrust and it governs the altitude z of the quadcopter. U_2, U_3, U_4 controls the roll, pitch and yaw rotations and ω_r represents the overall residual angular speed. Here we have designed a type-2 fuzzy controller which regulates the altitude z of the quadcopter by generating an appropriate total thrust U_1 .

4.2. Control scenario: Altitude Control

A type-2 fuzzy controller based on the proposed technique is designed to control the altitude of the quadcopter in a situation where quadcopter initiates the takeoff operation and reaches an altitude of 1m. It then increases its altitude to 2m and maintain this altitude for sometime. The quadcopter then comes back to an altitude of 1m. The type-2

controller generates the required total thrust U_1 for all these events. The altitude z and rate of change of altitude \dot{z} are the two inputs of the type-2 controller. The T2DFs of z input and control output U_1 are shown in Fig. 5. The upper and lower control surfaces of the controller are shown in Fig. 3. The controller output surface, used to control the quadcopter is also shown in Fig. 3. The reference signal commands the quadcopter to increase the altitude to 2m at 10 sec and come back to an altitude of 1m at 20 sec. The reference signal and the altitude of the quadcopter are plotted in Fig. 4. For comparison purpose, the altitude of the quadcopter is also controlled using a tuned PD controller. The response of the PD controller is also shown in Fig. 4.

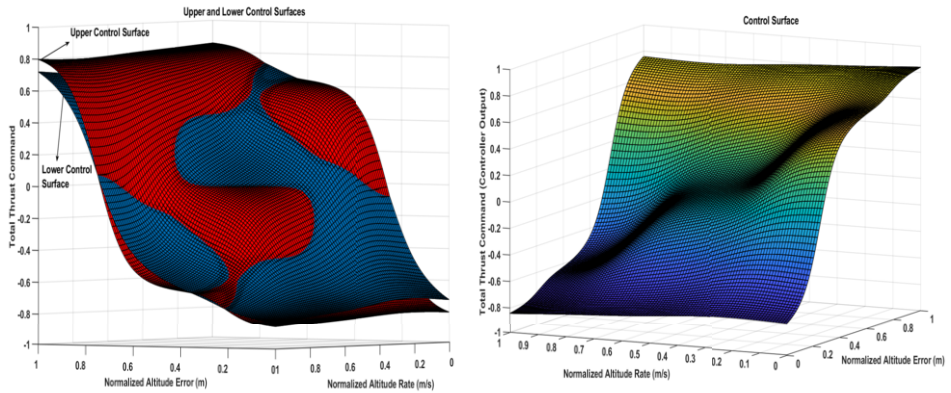


Figure 3. Upper (red) and lower (blue) control surfaces (Left). Output Control surface (Right)

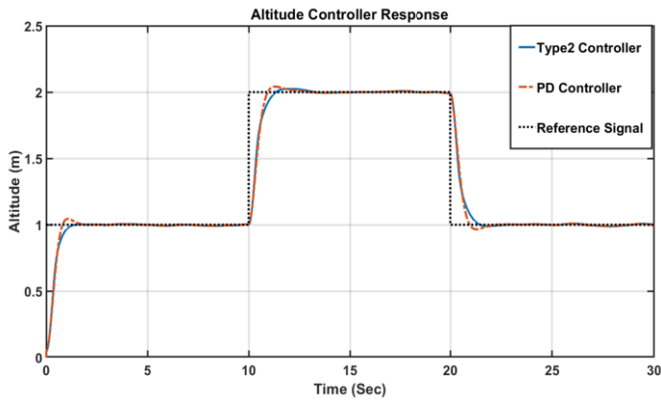


Figure 4. Altitude Response of proposed type-2 controller and PD controller

4.3. Discussion

T2DFs with uncertain peak values are used in this simulation study to generate a control signal for altitude control. Appropriate values of $\lambda, \varepsilon, \nu$, and Δ have been selected

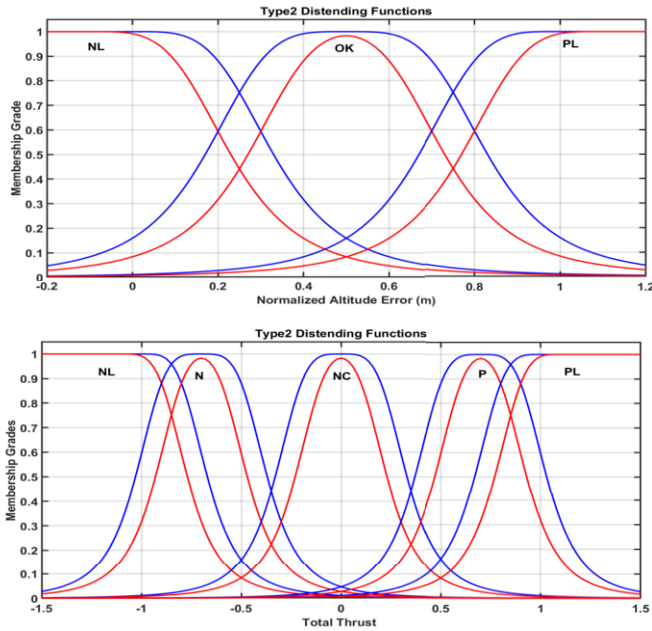


Figure 5. T2DFs for Altitude Error and Total Thrust

and kept fixed for the T2DFs of antecedents and consequents. With a few rules, a very smooth control surface is generated. A few closed form expressions are used (mentioned in Algorithm 1), the computation cost is low as compared to conventional iterative procedures for interval type-2 fuzzy controllers. The results are comparable to a PD controller as shown in Fig. 4. In addition, the proposed type-2 controller handles the uncertainties using T2DFs.

5. Conclusion and future work

A new interval type-2 fuzzy controller is proposed. The controller uses a new type-2 membership function i.e. T2DF. Using T2DFs, uncertainties can be handled easily. The proposed inference mechanism is based on the fuzzy arithmetic and uncertainty calculations. There are no type reduction, implication and aggregation steps. Therefore, the computational complexity is low and the proposed controller can be implemented on-line. The limitation of the design is the assumption that the experts knowledge is available in the form of if-else rules which is not true in some cases. In such cases, the data-driven design of the proposed controller can be developed in future. Also, the proposed techniques can be extended to design an adaptive type-2 fuzzy controller using optimization techniques.

Acknowledgements

The project was supported by the European Union and co-funded by the European Social Fund (no. EFOP-3.6.3-VEKOP-16-2017-0002).

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