Adaptive Savitzky-Golay filtering and its applications

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Abstract: Noise reduction is a central issue of the theory and practice of signal processing. The Savitzky-Golay (SG) smoothing and differentiation filter is widely acknowledged as a simple and efficient method for denoising. However only few book on signal processing contain this method. As is well known, the performance of the classical SG-filter depends on the appropriate setting of the window length and the polynomial degree, which should match the scale of the signal since, in the case of signals with high rate of change, the performance of the filter may be limited. This paper presents a new adaptive strategy to smooth irregular signals based on the Savitzky-Golay algorithm. The proposed technique ensures high precision noise reduction by iterative multi-round smoothing and correction. In each round the parameters dynamically change due to the results of the previous smoothing. Our study provides additional support for data compression based on optimal resolution of the signal with linear approximation. Here, simulation results validate the applicability of the novel method.

Keywords: Savitzky-Golay filter; adaptive multi-round smoothing; iterative smoothing and correction; noise removal; data compression; digital smoothing polynomial filter; noise removal; data compression; linear approximation; intelligence paradigm; fuzzy set theory; EMG signal; fast-varying signal.

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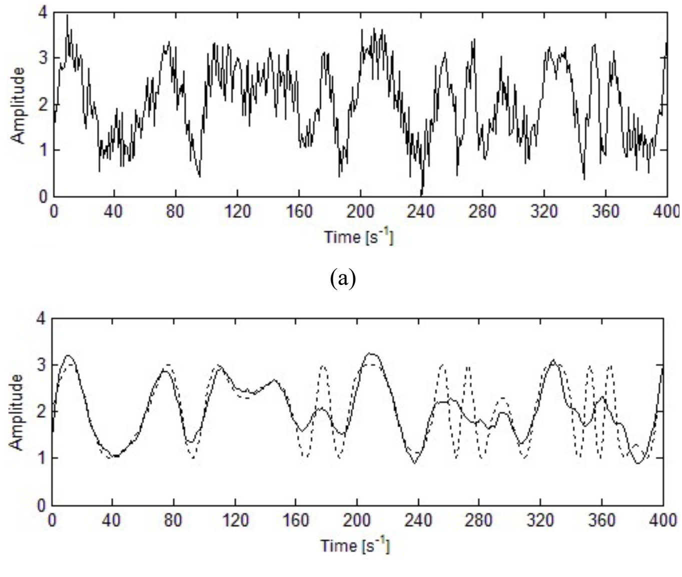
1 Introduction

Several areas of signal processing, image enhancement, etc., require the development of highly efficient data processing methods. In a certain class of problems, for instance chemical spectroscopy, smoothing and differentiation is especially important. Much work on a particular family of smoothing filters has been carried out. In their cutting edge paper of 1964, Savitzky and Golay introduced a particular type of low-pass filter, commonly referred as the digital smoothing polynomial filter (DISPO) or Savitzky-Golay (SG) filter (Savitzky and Golay, 1964). Corrections of the original technique can be found, e.g., in Steiner et al. (1972) and Madden (1978). The principle of their approach is fitting a low degree polynomial in least squares sense on the samples within a sliding window, in the time domain. The smoothed value of the centrepoint derived from convolution. There is a considerable amount of literature on the properties and improvements of SG filters (Edwards and Willson, 1974; Ahnert and Abel, 2007; Browne et al., 2007; Wayt and Integrated, 2007; Zhao et al., 2014; Persson and Strang, 2003; Krishnan and Seelamantula, 2013; Engel et al., 2013). The use of SG smoothing in chemometric algorithms, chemical imaging, etc., is well established (Komsta, 2009; Finta et al., 2013; Tong et al., 2015). Few studies have emphasised the frequency domain properties of SG-filters (Schafer, 2011b, 2011a; Bromba and Ziegler, 1981; Hamming, 1989). Schafer proposed a very plausible and practically useful approximate formula for the 3dB cut-off frequency as a function of polynomial order and impulse response half-length. In Luo et al. (2005b), the properties of the SG digital differentiator filters are discussed in detail and recommendations are made for the choice of filter length in order to maintain the resolution of the signal derivative. In Luo et al. (2005a), an extension is described to calculate the filter coefficients for even-numbered data using a matrix form. Some attempts have been made with the purpose of the construction of fractional-order SG differentiators. It has been shown, that by applying the Riemann-Liouevile fractional order definition in the SG-filter, the fractional order derivative can be obtained of corrupted signals (Chen et al., 2011). Moreover recent findings highlight the advantages of fractional order SG filters, as new solutions to smooth noisy data obtained from, for say, near infrared spectral analysis (Tong et al., 2015). It is worth noting that there are several sources of noise, including electronic noise, acoustic noise, electromagnetic and electrostatic noise, that may limit the performance and accuracy in signal processing systems (Vaseghi, 2008). In signal processing it is commonly assumed that the noise is an additive white Gaussian noise (AWGN) process. In practice, often non-stationary, impulsive type disturbances, burst noises, etc., can also occur. Although for several problems mathematically convenient and efficient filtering and smoothing algorithms exist, most of them require the characterisation and model of the noise process. The SG method was originally developed to make discernible the relative widths and heights of spectral lines. It smooth's equally the noise and the signal components, as it leads to bias and reduction in resolution. For denoising signals with a large spectral dynamic or with a high rate of change, the classical SG filtering is an unsuitable method. In addition, the efficiency depends on the appropriate selection of the polynomial order and the window length, which should match the intrinsic scale of the input signal. However the SG-filters give excellent results while preserving simplicity and speed, but most of the applications require the users to arbitrarily select the polynomial order and size of the sliding window. In general, the SG filters perform well when we apply a low order polynomial with long window length or low degree with short window and repeated smoothing. It has also been shown that the smoothing effect decreases by applying low order polynomial on higher frequencies or high order polynomials on lower frequency parts of the signal. With this in mind, in this study we introduce an adaptive smoothing method based on SG filtering, which provides a good performance independent of the type of noise process. This brief communication is organised as follows. In Section 2 we summarise the basic mathematical background of the classical SG smoothing and we discuss its performance. In Section 3 we derive the new adaptive multi-round technique. Then the simulation results are presented in Section 4. After, in Section 5 we draw some pertinent conclusions.

Figure 1 Performance of the SG filter, (a) signal with contaminating noise (b) dotted line – original signal, solid line – smoothed signal, $k = 3$, $m = 35$



2 Theoretical background

In the following, a brief summary of the mathematical background of SG filtering is provided that is based on Flannery et al. (1992). First, consider a sequence of equally spaced input data $n\{x_j, y_j\}$, $j = 1,\ldots,n$. The smoothed values derives from convolution, given by

$$g_i = \sum_{i=-m}^{m} c_i y_{k+1}, \tag{1}$$

where the window length $M = 2m + 1$, $i = -m,\ldots,\lambda,\ldots,m$, and λ denotes the index of the middle point. The k^{th} order polynomial P can be written as

$$P = a_0 + a_1 (x > x_\lambda) + a_2 (x > x_\lambda)^2 + \ldots + a_k (x > x_\lambda)^k \tag{2}$$

The task is to evaluate the coefficients of equation (2) by minimising the fitting error in the least squares sense. The Jacobian matrix is given by

$$J = \frac{\partial P}{\partial a} \tag{3}$$

The polynomial at x = x_ is a0, hence in order to evaluate the polynomial in the window we have to solve a system of M equations which can be written in matrix form

$$J \cdot a = y \tag{4}$$

$$\begin{pmatrix} 1 & (x_{\lambda-m} - x_\lambda) & \cdots & (x_{\lambda-m} - x_\lambda)^k \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & (x_{\lambda+m} - x_\lambda) & \cdots & (x_{\lambda+m} - x_\lambda)^k \end{pmatrix} \times \begin{pmatrix} a_0 \\ \vdots \\ \vdots \\ \vdots \\ a_k \end{pmatrix} = \begin{pmatrix} y_{\lambda-m} \\ \vdots \\ \vdots \\ \vdots \\ y_{\lambda+m} \end{pmatrix}$$

The coefficients are found from the normal equation in the following writing

$$J^T (Ja) = (J^T J)\, a \tag{5}$$

so

$$a = (J^T J)^{-1} (J^T y) \tag{6}$$

Since

$$P(x_\lambda) = a_0 = (J^T J)^{-1} (J^T y), \tag{7}$$

by replacing y with a unit vector in equation (6) the c_0 coefficient can be calculated as

$$c_j = \sum_{i=1}^{k+1} \left| (J^T J)^{-1} \right|_{0i,}\ J_{ij} \tag{8}$$

Table 1 Some SG coefficients

SG coefficients		
M	*k*	*Coefficients*
9	2	–0.0909 0.0606 0.1688 0.2338 0.2554 0.2338 0.1688 0.0606 –0.0909
	4	0.0350 –0.1282 0.0699 0.3147 0.4172 0.3147 0.0699 –0.1282 0.0350
11	3	–0.0839 0.0210 0.1026 0.1608 0.1958 0.2075 0.1958 0.1608 0.1026 0.0210 –0.0839
	5	0.0420 –0.1049 –0.0233 0.1399 0.2797 0.3333 0.2797 0.1399 –0.0233 –0.1049 0.0420

Note: $M = 2m + 1$ is the window length and k denotes the polynomial degree.

With a size of $(2m + 1)x(k + 1)$ the G matrix of the convolution coefficients

$$G = J(J^T J) = [g_0, g_1, \ldots, g_j], \tag{9}$$

the coefficients for the γ^{th} order derivative being derived from the formula below (Orfanidis, 1995),

$$c_k^{(\gamma)} = \gamma! \sum_{-m}^{m} g_\gamma(-m) x(k-m) \tag{10}$$

3 Adaptive multi-round smoothing

The proposed method seeks to overcome the above mentioned limitations of SG filtering. This new adaptive concept ensures the application of a suitable polynomial order and window length at the different frequency components of the signal. Thus, it is possible to avoid the undershoots and preserve the peaks that could be important from different data analysis aspects. Since we perform in the time domain, this method provides efficient results independent of the type of contaminating noise.

3.1 Adaptive construction of SG-filter

In order to separate the signal components of different frequency-density, first of all, a conventional SG filtering is performed. Because we assume that only the corrupted signal is available, this step serves to reveal the peaks, hence the window length and degree of the polynomial may be arbitrary. After, the first smoothing the coordinates of the local minimum and maximum points can be obtained in consecutive order:

$$C = \begin{pmatrix} x_1 x_2 \cdots x_u \\ y_1 y_2 \cdots y_u \end{pmatrix} \tag{11}$$

Then the d distance vector is introduced, which contains the number of samples between two neighbouring points of local minima and maxima:

$$d = (\delta_1 \delta_2 \ldots \delta_{u-1}) \tag{12}$$

The separation of $R = [r_1, \ldots r_l]$ number of parts of the signal containing similar frequency components is based on the following measures. The bordering points are marked out for detecting the sections between the δ local extrema one-by-one. If the variance given by $S_{(d)} = 1 \big/ (u-1) \sum_{i=1}^{u-1} \delta_i^2 - \bar{\delta}^2$, of the actual section based on the previous values is >> ϵ_1, then this will be the first section of the next part. In each part of the signal which contains similar frequency components, the applied window length (M) and polynomial degree (k) is determined as follows. The window should match the scale of the signal and the polynomial degree should vary by depending on the framesize and frequency. Since the next fuzzy relation can be defined between the section length;

$$F\left(d_{\max} >> \bar{d}_R\right) = \frac{1}{1 + e^{-\left(\delta_{\max} - \bar{d}_R\right)}} \in [0, 1] \tag{13}$$

where $\bar{\delta}_R$ stands for the average length of the sections in the current R parts of the signal, while $\delta_{\max}$ = max(d) in the observed signal. If $g(d_{\max}, \bar{d}_R) = 1$, the current part of the signal contains high frequency components. Hence, the following rules are applied:

$$\text{if } 1 > g\left(d_{\max}, \bar{d}_R\right) > 0.9 \text{ then } k = 5, M = nint\left(0.3\bar{\delta}_R\right) \tag{14}$$

$$\text{if } 0.89 > g = \left(d_{\max}, \bar{d}_R\right) > 0.75 \text{ then } k = 4, M = nint\left(0.5\bar{\delta}_R\right) \tag{15}$$

$$\text{if } 0.75 > g\left(d_{\max}, \bar{d}_R\right) > 0.45 \text{ then } k = 3, M = nint\left(\bar{\delta}_R\right) \tag{16}$$

$$\text{if } 0.44 > g\left(d_{\max}, \bar{d}_R\right) > 0.2 \text{ then } k = 2, M = nint\left(0.5R_n\right) \tag{17}$$

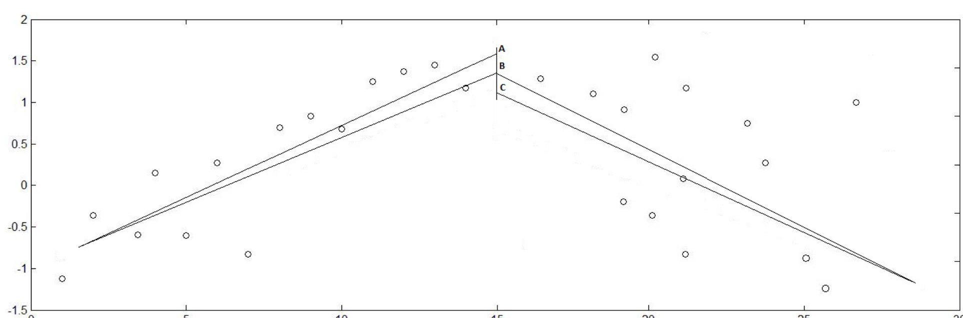
$$\text{else } k = 1, M = nint\left(0.8R_n\right) \tag{18}$$

where R_n is the total number of samples of the R part, and we can assign the k and M values to each R part of the signal. The values for the bounds have been determined according to the formula $2k$ modified by experimental results.

3.2 Multi-round correction with linear approximation

As we have the coordinates of the local minimum and maximum points and the vector d, we can easily construct the linear approximation $ax + b = y$ of the sections. It serves for performing two important tasks. First, after the first adaptive SG-smoothing subtracting it from the smoothed signal the imprecision or inflexion points can be revealed. Therefore it makes the correction process possible by introducing new cutting points for the next adaptive smoothing. However, we can compress the data in such a way that it could be useful for analysing economic trends. Alternatively, a linear regression line could be also fitted on the smoothed data between two local extrema. In this case, the ending and starting points of two consecutive lines do not necessary follow so as to match at the same value. Therefore further corrections can be introduced by marking out the middle value between the ending point of the first line and the starting point of the second line. After, the new lines are defined at these new points. For details, see Figure 2. The first regression line ends on point A and the next regression line starts in point C. The new lines are defined at point B. After the correction of the smoothed signal, an adaptive smoothing is once again performed. If the difference between the linear approximate and the smoothed function is greater then ϵ_2, then adaptive smoothing is necessary again. This iterative method of smoothing and corrections assists the detection and preservation of the soft shapes of the signal.

Figure 2 Illustration of joining the regression lines



We list below the steps of the adaptive multi-round smoothing procedure: let y_0 denote the corrupted input signal and let y_i denote the corresponding output data.

1 perform SG-filtering on y_0 and get y_1

2 detect local extrema [C] and compute the distances [d]

3 determine [R]

4 perform adaptive SG-smoothing on y_1

5 repeat step 2 on y_1

6 fit linear lines and get y_{lin}

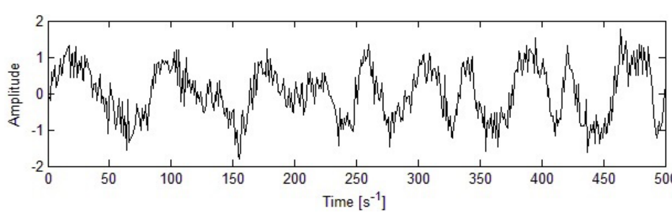
7 calculate $\epsilon = y_1 - y_{lin}$

8 if $\epsilon > v$ then repeat steps 3 through 7.

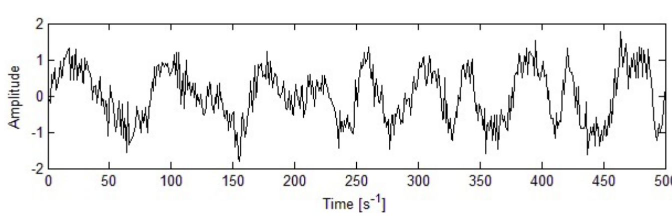
4 Simulation results

The performance of the proposed method has been tested on a one-dimensional signal corrupted with additive noise (Figure 3). The simulation was carried out by using Matlab7. After the first smoothing (Figure 3), the cutting points were determined in $r = [150, 421]$. Then the adaptive smoothing was performed (Figure 4). Comparing Figure 3 and Figure 4, it can be seen that the undershoot have been corrected by applying the appropriate polynomial degree and window length. With the linear approximation (Figure 5) of the signal obtained, a further correction for the new repartioning is got, where the cutting points are defined in $r = [150, 387, 450]$. The result of the next adaptive SG-smoothing is depicted in Figure 6. It is apparent, that the soft cambers are tracked and the shape of the signal preserved with correct elimination of noise components. This iterative method of smoothing and correction provides a good performance in the case of irregular signals. For fast and simple performance the coefficients are obtained from tables. In case of high sampling rate the signal is resampled with $s(x) = \frac{x}{100}$ in order to match for shorter running window. Then, the missing values are replaced with simple nearest neighbour interpolation.

Figure 3 (a) Original signal corrupted with noise (b) smoothing with the classical SG filter, $k = 3$, $m = 35$, dotted line – original signal, solid line – smoothed signal, stars – local minima and maxima

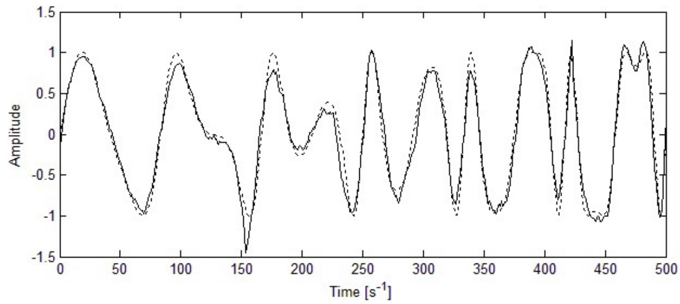


(a)



(b)

Figure 4 Smoothing with adaptive SG filtering



Note: Solid line – smoothed signal, dotted line – original signal,
$k_1 = 3$, $m_1 = 39$, $k_2 = 3$, $m_2 = 21$, $k_3 = 4$, $m_3 = 31$.

Figure 5 Linear approximation of the signal

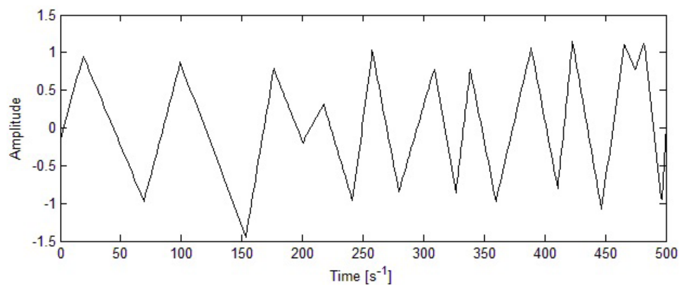
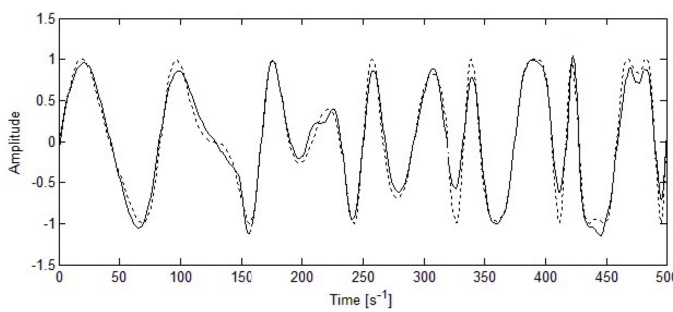


Figure 6 Smoothing with adaptive SG filtering after correction



Note: Solid line – smoothed signal, dotted line – original signal,
$k_1 = 2$, $m_1 = 45$, $k_2 = 3$, $m_2 = 25$, $k_3 = 4$, $m_3 = 15$, $k_4 = 5$, $m_4 = 9$.

5 Application example: EMG signal denoising

The processing of electromyography (EMG) signals are highly important in several biomedical applications. Recently, many researches focus on the noise-removal issues on EMG signals which is very desirable in order to developed precisely standardised biomedical technologies (Chowdhury et al., 2013; De Luca et al., 2010). While, for instance, study (Clancy et al., 2002) concerns the sources and processing of surface EMG measurement noises and discusses amplitude estimating and noise-cancellating techniques of the denoised signal.

For an application example, our technique was tested on a noisy EMG signal. The original data was taken from Goldberger, et al. (2000) which was corrupted with AWGN. Performance of the adaptive multi-round filter can be seen on Figures 7 and 8. The signal-to-noise ratio was SNR = 0.3744 [dB] before and SNR = 7.3421 [dB] after the smoothing. These results validate that this technique can be valuable for several applications such as EMG processing applications.

Figure 7 EMG signal corrupted with noise (see online version for colours)

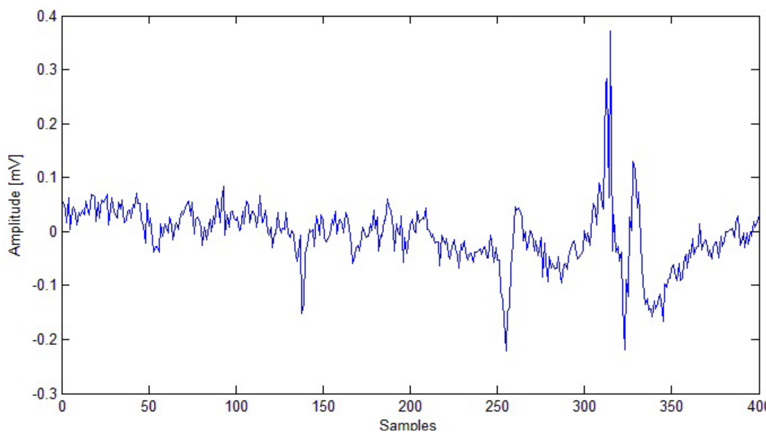
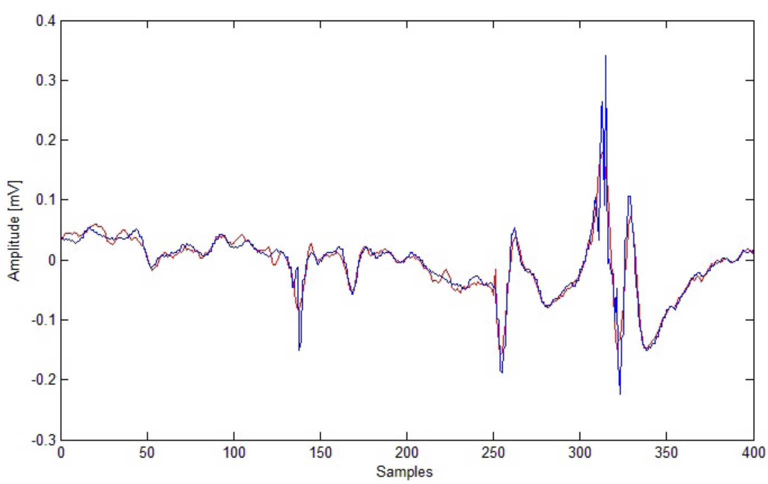


Figure 8 Denoised EMG signal with adaptive multi-round SG filter (see online version for colours)



Notes: Blue line-original signal, red line – denoised signal.

6 Conclusions

In this paper the adaptive multi-round smoothing based on the SG algorithm was introduced. An important premise of the classical SG filtering is that the signal should be slowly-varying. The proposed method automatically selects the polynomial order and window length according to the signal form, thus signals with high rate of change are also can be smoothed correctly. For precise smoothing the algorithm applies linear approximation of the signal. The optimal resolution of the signal is based on the local extrema points. The method iteratively performs the adaptive smoothing and correction, hence the shape of also fast-varying signals can be precisely detected. Thus the important details of the signal are preserved besides full elimination of the contaminating components independent of the character of noise process. Further, such a decomposition of the signal with linear approximation allows convenient data compression. Simulation results have shown, that the proposed technique allows excellent performance. Our procedure is a clear improvement on current methods.

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