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Average tail risk and aggregate stock returns

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ABSTRACT

We investigate the role of the average risk across stocks in predicting subsequent market returns using measures of risk that capture the higher moments of the return distribution including variance, skewness and kurtosis, as well as measures of tail risk that combine these. We find that average tail risk has statistically and economically significant predictive ability for market returns, even after controlling for market tail risk, suggesting that average *idiosyncratic* tail risk contains information about future returns. Average tail risk dominates other measures of average risk that have been documented in the literature, such as variance and skewness. Our results are robust to the inclusion of control variables that capture business cycle effects, and to the use of different measures of tail risk.

1. Introduction

This paper investigates the ability of the average tail risk across individual stocks to predict subsequent aggregate market returns. Asset pricing theory posits that expected returns should be a function of systematic risk alone, since idiosyncratic risk is diversified when stocks are combined into a portfolio and hence investors should not be compensated for bearing this risk. At the aggregate level, therefore, there should be a positive relation between the return of the market portfolio and the (weighted) average systematic risk across stocks, i.e., the total risk of the market portfolio. A number of studies have investigated the explanatory power of average *total* risk across stocks for future market returns. Since average total risk reflects both average systematic risk and average idiosyncratic risk, it should not have predictive ability for returns beyond that of the total risk of the market portfolio. However, empirical evidence suggests that this is not the case. For example, Goyal and Santa-Clara (2003) find a significant positive relation between the average variance across individual stocks and subsequent market returns, even after controlling for the market variance. Similarly, Jondeau et al. (2019) show that the average skewness across individual stocks has statistically significant predictive power for market returns after controlling for market skewness.

Building on these studies, we investigate more broadly the relation between average risk and subsequent aggregate returns, where risk is captured by variance, skewness and kurtosis, as well as composite measures that combine these, such as tail risk. As systematic risk measures, these have all been found to be relevant from an asset pricing perspective both theoretically and empirically. While the Intertemporal CAPM of Merton (1973) implies that the expected excess return of the market portfolio is proportional to the variance of returns, this rests on the assumption that either investors have quadratic utility or returns are drawn from a multivariate elliptical distribution. However, neither of these assumptions holds in practice. First, financial asset returns are characterised by both skewness and leptokurtosis, neither of which is preserved when assets are combined into portfolios, which is inconsistent with returns being elliptically distributed (see Tang and Choi, 1998, for example). Second, studies have shown that investors have preferences for higher skewness and lower kurtosis, implying that they do not have quadratic utility. For example, Scott and Horvath (1980) show that investors should have preferences for odd moments of the return distribution and preferences against even moments (see

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also Arditti, 1967; Kane, 1982). Consistent with these findings, empirical evidence suggests that skewness and kurtosis are relevant for explaining the cross-section of stock returns. In particular, asset-pricing models that explicitly allow for investors' preferences over the higher moments of returns are generally supported empirically (see, for example, Kraus and Litzenberger, 1976; Fang and Lai, 1997; Dittmar, 2002). At the aggregate level, therefore, we would expect market returns to be negatively related to the skewness of market portfolio returns and positively related to their kurtosis. However, the evidence for this is weak: previous studies have reached mixed conclusions concerning the time series relation between market returns and aggregate skewness (see, for example, Chang et al., 2011; Garcia et al., 2014; Jondeau et al., 2019), and there is no evidence to date on the relation between market returns and aggregate kurtosis. More generally, we would expect market returns to be positively related to the downside risk of the market portfolio, and in particular, to its left tail risk. Indeed, a number of papers have shown that, empirically, tail risk has explanatory power for the cross-section of stock returns (see, for example, Huang et al., 2012; Chabi-Yo et al., 2018; Harris et al., 2019a). At the aggregate level, Bali et al. (2009) show that there is a significant positive relation between market returns and tail risk, which they measure by the previous month's expectation of the value at risk (VaR) of the market portfolio. Moreover, they show that the relation between returns and tail risk is stronger than between returns and variance.[1] Kelly and Jiang (2014) propose a measure of the common fluctuation of individual stock tail risk based on the Hill (1975) power law estimator by pooling all individual returns within each month and using these to estimate a common tail index. They show that this market tail risk measure has predictive power for returns over horizons of one month to five years.

In this paper, we study of the role of average tail risk in predicting subsequent market returns for the U.S. equity market over the period August 1963 to January 2020, and its marginal predictive ability beyond that of the tail risk of the market portfolio. We measure tail risk using value at risk at various confidence levels, which we estimate semi-parametrically using the Cornish–Fisher expansion, and construct both the value weighted average value at risk across individual stocks and the value at risk of the market portfolio. We compare the predictive ability of average and market tail risk with that of corresponding risk measures based on the individual moments of returns. We undertake an out-of-sample analysis to establish the economic significance of the predictive ability of the average and market risk variables. In our robustness analysis, we also consider alternative measures of risk that capture the shape of the return distribution, including downside variance, a non-parametric asymmetry measure and expected shortfall, as well as alternative estimates of value at risk.

Our findings can be summarised as follows. First, we find that the role of average skewness in predicting returns is fragile, and depends on the period considered. In particular, the significant relation between market returns and average skewness identified by Jondeau et al. (2019) disappears in the second half of our sample. Second, neither the kurtosis of the market portfolio nor the average kurtosis across individual stocks is able to predict aggregate stock returns, either individually or in combination. This is true both in the full sample and in both pre- and post-1991 sub-samples. Third, the significant positive relation between market tail risk and market returns documented by Bali et al. (2009) disappears in our extended sample. Finally, we find that average tail risk has a significantly positive relation with subsequent market returns in the full sample, even after controlling for both market tail risk and macroeconomic factors. We show that the role of average tail risk is even more pronounced in the post-1991 sub-sample, which is when average skewness becomes insignificant.

We make a number of contributions to the literature. First, we contribute to the literature on the role of higher moments in asset pricing and the implications for market returns. We show that the relation between average skewness and subsequent market returns documented by Jondeau et al. (2019) is dependent on the period considered and, in particular, disappears in more recent data. This mirrors the findings of Bali et al. (2005) concerning the role of average variance, who show that the results of Goyal and Santa-Clara (2003) regarding variance do not hold for their extended sample. Our finding that there is no relation between market returns and either market kurtosis or average stock kurtosis in either the full sample or the two sub-samples stands in contrast with the evidence of a significantly positive cross-sectional relation between idiosyncratic kurtosis and expected returns reported by Bali et al. (2019). Second, we contribute to the literature that documents the role of idiosyncratic risk in asset pricing. Measures of average total risk, such as those considered in this paper, capture both systematic (i.e., market) risk and idiosyncratic risk. Controlling for market risk thus reveals the marginal role of idiosyncratic risk in predictive regressions of market returns. Consequently, our finding that average total tail risk contains information that is useful for predicting market returns supports the idea that investors have preferences over idiosyncratic risk, and represents further evidence against standard asset pricing models such as the Intertemporal CAPM. This is consistent with evidence reported for the cross-sectional relation between idiosyncratic volatility and returns by, for example, Ang et al. (2006), who empirically show that stocks with high idiosyncratic volatility earn low average returns. Third, we contribute to the recent literature on the existence of common factors in idiosyncratic risk. Herskovic et al. (2016) show that there is a very distinct common factor in idiosyncratic volatility, and that exposure to this factor is priced in the cross-section of stock returns. While the present paper is not concerned with cross-sectional asset pricing, our results reveal common factors in measures of total risk that are distinct from market risk and hence, by implication, common factors in idiosyncratic risk, not just for variance but for other measures of risk also, most notably tail risk.

The outline of the remainder of this paper is as follows. Section 2 describes the data that we use in the empirical analysis and presents some summary statistics. Section 3 outlines the econometric methodology and reports the results of the in-sample analysis. Section 4 reports the out-of-sample analysis. In Section 5 we investigate the robustness of our results. Section 6 offers some concluding remarks and suggestions for further research.

[1] Harris et al. (2019b) note that the relation between tail risk and market returns breaks down in high volatility periods owing to volatility leverage and feedback effects, but they show this can be mitigated by using longer horizon forecasts of VaR formed at longer lags.

2. Data

2.1. Returns

In the empirical analysis, we use monthly excess market returns and a number of monthly measures computed from daily returns for both the market and for individual stocks. The market excess return is defined as the aggregate stock return minus the short-term interest rate, where the aggregate stock return is the simple return on the value-weighted CRSP-index including dividends, and the short-term interest rate is the one-month Treasury bill rate.[2] For the measures based on individual stock returns, we use daily firm-level returns for all common stocks with share codes 10 or 11 from the CRSP dataset. Each month, we use all stocks with at least 10 daily non-missing returns, excluding the least liquid stocks with an Amihud illiquidity measure in the highest 0.1% percentile and the lowest-priced stocks with a price less than \$1.[3] The sample period is from August 1963 to January 2020 (678 observations), with the start date determined by the cross-section expansion of CRSP beginning in 1962. In the robustness analysis, in addition to the full sample, we also report results for two sub-samples, August 1963 to October 1991 (339 observations) and November 1991 to January 2020 (339 observations).

2.2. Individual stock risk measures

For each stock, we compute the following risk measures. Following Jondeau et al. (2019), the variance of stock i in month t is defined as

$$V_{i,t} = \sum_{d=1}^{D_t}(r_{i,d} - \bar{r}_{i,t})^2 + 2\sum_{d=2}^{D_t}(r_{i,d} - \bar{r}_{i,t})(r_{i,d-1} - \bar{r}_{i,t}) \tag{1}$$

where $r_{i,d}$ is the return of stock i on day d, $\bar{r}_{i,t} = \frac{1}{D_t-2}\sum_{d=1}^{D_t-2} r_{i,d}$ and D_t is the number of days in month t. This definition excludes the last two days of the month when calculating the mean, which avoids the turn-of-the-month effect in stock returns identified by Lakonishok and Smidt (1988). Again following Jondeau et al. (2019), we define the skewness of stock i in month t as:

$$S_{i,t} = \sum_{d=1}^{D_t}\left(\frac{r_{i,d} - \bar{r}_{i,t}}{\sqrt{\sum_{d=1}^{D_t}(r_{i,d} - \bar{r}_{i,t})^2}}\right)^3 \tag{2}$$

We define the kurtosis of stock i in month t analogously as[4]:

$$K_{i,t} = \sum_{d=1}^{D_t}\left(\frac{r_{i,d} - \bar{r}_{i,t}}{\sqrt{\sum_{d=1}^{D_t}(r_{i,d} - \bar{r}_{i,t})^2}}\right)^4 \tag{3}$$

Our primary measure of tail risk is value at risk (VaR), which we estimate semi-parametrically with the Cornish–Fisher (CF) expansion, which approximates the quantile of the return distribution using the skewness and kurtosis of returns.[5] The standardised Cornish–Fisher VaR at the p% confidence level is given by:

$$VaR_{i,t}^{p} = -\{\Phi_{1-p}^{-1} + \frac{\tilde{S}_{i,t}}{6}[(\Phi_{1-p}^{-1})^2 - 1] + \frac{\tilde{K}_{i,t} - 3}{24}[(\Phi_{1-p}^{-1})^3 - 3\Phi_{1-p}^{-1}] - \frac{\tilde{S}_{i,t}^2}{36}[2(\Phi_{1-p}^{-1})^3 - 5\Phi_{1-p}^{-1}]\} \tag{4}$$

where $\tilde{S}_{i,t} = \sqrt{D_t}S_{i,t}$ and $\tilde{K}_{i,t} = D_t K_{i,t}$ are the standardised skewness and kurtosis coefficients, respectively, and Φ_{1-p}^{-1} is the $1-p\%$ quantile of the standard normal distribution (Christoffersen, 2011). We compute VaR at the 90% and 95% confidence levels. In the robustness section, we consider alternative ways of estimating value at risk, as well as other measures of tail risk.

2.3. Aggregate risk measures

For each of the individual stock risk measures described above, we consider two measures of aggregate risk. The first is the value-weighted average of the individual stock risk measure, i.e., a measure of average total risk. The market capitalisation for each

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2 The monthly market excess return and the one-month Treasury bill rate are obtained from Kenneth French's website.

3 The Amihud (2002) illiquidity of stock i in month t is defined as $Illiq_{i,t} = \frac{1}{D_t}\sum_{d=1}^{D_t}\frac{|r_{i,t}|}{Vol_{i,d}}$, where $Vol_{i,d}$ is the dollar trading volume of the stock on day d and D_t is the number of days in month t.

4 Note that the measure of skewness given by Eq. (2) used by Jondeau et al. (2019) and the corresponding measure of kurtosis given by Eq. (3) differ from the standardised skewness and kurtosis coefficients by a factor of $\frac{1}{\sqrt{D_t}}$ and $\frac{1}{D_t}$, respectively, and are therefore not invariant to the number of days in the month. Consequently, there is a degree of variation in these measures over time arising solely from differences in the number of working days each month. However, in the robustness section, we show that use of the standardised skewness and kurtosis coefficients leads to very similar results.

5 A potential concern of the Cornish–Fisher approach is possible non-monotonicity of the estimated quantiles, especially at very low probabilities (see, for example, Amédée-Manesme and Barthélémy (2020)). However, this does not appear to be an issue in our empirical analysis at the VaR confidence levels that we consider.

Table 1
Summary statistics. The table presents summary statistics for the following variables: value-weighted average variance ($V_{vw,t}$), skewness ($S_{vw,t}$), kurtosis ($K_{vw,t}$), 90% VaR ($90\% VaR_{vw,t}$), 95% VaR ($95\% VaR_{vw,t}$) (Columns 1–6), market variance ($V_{m,t}$), skewness ($S_{m,t}$), kurtosis ($K_{m,t}$), 90% VaR ($90\% VaR_{m,t}$), 95% VaR ($95\% VaR_{m,t}$) and the market return ($r_{m,t}$) (Columns 7–12). p-values to test the equality of the mean and variance between the average and market values are reported in columns 13 and 14. The sample period is August 1963 to January 2020.

	(1)	(2)	(3)	(4)	(5)	(6)		(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	Value-weighted average across stocks							Value-weighted market portfolio						Equality tests (p values)	
	Mean	Q1	Med	Q3	Std Dev	N		Mean	Q1	Med	Q3	Std Dev	N	T-test	F-test
							$R_{m,t}$	0.005	−0.079	0.009	0.077	0.044	678		
$V_{vw,t} \times 100$	0.844	0.287	0.640	2.506	0.008	678	$V_{m,t} \times 100$	0.194	0.017	0.109	0.713	0.004	678	0.000%	0.000%
$S_{vw,t}$	0.032	−0.050	0.035	0.104	0.042	678	$S_{m,t}$	−0.011	−0.267	−0.005	0.213	0.136	678	0.000%	0.000%
$K_{vw,t}$	0.164	0.139	0.160	0.202	0.018	678	$K_{m,t}$	0.147	0.092	0.135	0.264	0.056	678	0.000%	0.000%
$90\% VaR_{vw,t}$	1.273	1.231	1.273	1.314	0.023	678	$90\% VaR_{m,t}$	1.304	1.130	1.313	1.434	0.081	678	0.000%	0.000%
$95\% VaR_{vw,t}$	1.583	1.480	1.581	1.688	0.055	678	$95\% VaR_{m,t}$	1.650	1.320	1.658	1.936	0.172	678	0.000%	0.000%

firm in each month is computed as the product of the month-end number of shares outstanding and the month-end stock price, both of which are obtained from CRSP. The value-weighted average risk measures take the form

$$X_{vw,t} = \sum_{i=1}^{N_t} w_{i,t} X_{i,t} \tag{5}$$

where $X \in \{V, S, K, VaR\}$ is the risk measure, $w_{i,t}$ is the market capitalisation weight of firm i in month t, and N_t is the number of firms in month t.

The second is the market risk measure corresponding to each of the individual stock risk measures, which we compute by applying the risk measure to the value-weighted portfolio of all stocks with share codes 10 or 11 from the CRSP database. This is equivalent to the value-weighted average systematic risk across the individual stocks. Therefore, we define the market variance as

$$V_{m,t} = \sum_{d=1}^{D_t} (r_{m,d} - \bar{r}_{m,t})^2 + 2\sum_{d=2}^{D_t} (r_{m,d} - \bar{r}_{m,t})(r_{m,d-1} - \bar{r}_{m,t}) \tag{6}$$

where $r_{m,d}$ is the daily return from the value-weighted CRSP portfolio. The market skewness is defined as

$$S_{m,t} = \sum_{d=1}^{D_t} \left(\frac{r_{m,d} - \bar{r}_{m,t}}{\sqrt{\sum_{d=1}^{D_t} (r_{m,d} - \bar{r}_{m,t})^2}}\right)^3 \tag{7}$$

The market kurtosis is defined as

$$K_{m,t} = \sum_{d=1}^{D_t} \left(\frac{r_{m,d} - \bar{r}_{m,t}}{\sqrt{\sum_{d=1}^{D_t} (r_{m,d} - \bar{r}_{m,t})^2}}\right)^4 \tag{8}$$

Similarly, the standardised market Cornish–Fisher VaR at the $p\%$ confidence level is given by

$$VaR^p_{m,t} = -\{\Phi^{-1}_{1-p} + \frac{\tilde{S}_{m,t}}{6}[(\Phi^{-1}_{1-p})^2 - 1] + \frac{\tilde{K}_{m,t} - 3}{24}[(\Phi^{-1}_{1-p})^3 - 3\Phi^{-1}_{1-p}] - \frac{\tilde{S}^2_{m,t}}{36}[2(\Phi^{-1}_{1-p})^3 - 5\Phi^{-1}_{1-p}]\} \tag{9}$$

where $\tilde{S}_{m,t} = \sqrt{D_t} S_{m,t}$ and $\tilde{K}_{m,t} = D_t K_{m,t}$ are the standardised skewness and kurtosis coefficients, respectively.

2.4. Summary statistics

Table 1 reports summary statistics for the market excess return and the average and market risk variables for the full sample from August 1963 to January 2020. The last two columns report the p-values of tests of equality of the mean (Column 13) and variance (Column 14) between the average and market values of each risk measures. All of the differences are statistically significant. Table 2 reports the correlation matrix for the average and market risk variables, as well as for the future and current market excess return. The average value and market value for each risk measure are, in most cases, highly correlated, with correlations between 0.561 and 0.851. The market return has a positive correlation of 0.061 with its first lag, although this is not statistically significant at the 5% level. The average and market risk measures are highly correlated with the contemporaneous market return, as would be expected given that they are constructed from returns, but they are also correlated, to a lesser extent, with the following month's market return. The risk measures themselves are correlated with each other to varying degrees. Average skewness is highly correlated with average 95% value at risk, but less so with average 90% value at risk. A similar result holds for market skewness and market value at risk. The relation between kurtosis and value at risk is less clear, reflecting the fact that an increase in kurtosis is associated with higher value at risk at high confidence levels but lower value at risk at low confidence levels. Both average and market variance have only low correlations with the other risk variables.

Table 2

Correlation matrix. The table reports correlations for the following variables: value-weighted average variance ($V_{vw,t}$), skewness ($S_{vw,t}$), kurtosis ($K_{vw,t}$), 90% VaR ($90\%VaR_{vw,t}$), 95% VaR ($95\%VaR_{vw,t}$), market variance ($V_{m,t}$), skewness ($S_{m,t}$), kurtosis ($K_{m,t}$), 90% VaR ($90\%VaR_{m,t}$), 95% VaR ($95\%VaR_{m,t}$) the current market return ($r_{m,t}$) and the following month's market return ($r_{m,t+1}$). The sample period is August 1963 to January 2020.

	(1) $r_{m,t}$	(2) $r_{m,t+1}$	(3) $V_{vw,t}$	(4) $S_{vw,t}$	(5) $K_{vw,t}$	(6) $90\% VaR_{vw,t}$	(7) $95\% VaR_{vw,t}$	(8) $V_{m,t}$	(9) $S_{m,t}$	(10) $K_{m,t}$	(11) $90\% VaR_{m,t}$	(12) $95\% VaR_{m,t}$
$r_{m,t}$	1.000											
$r_{m,t+1}$	0.061	1.000										
$V_{vw,t}$	−0.226	−0.090	1.000									
$S_{vw,t}$	0.217	−0.092	0.083	1.000								
$K_{vw,t}$	−0.012	−0.022	−0.086	−0.123	1.000							
$90\%VaR_{vw,t}$	−0.167	0.100	−0.083	−0.769	−0.312	1.000						
$95\%VaR_{vw,t}$	−0.225	0.096	−0.074	−0.979	−0.036	0.856	1.000					
$V_{m,t}$	−0.345	−0.071	0.851	0.011	0.006	−0.099	−0.016	1.000				
$S_{m,t}$	0.041	−0.062	0.081	0.660	−0.117	−0.497	−0.650	0.048	1.000			
$K_{m,t}$	−0.037	0.008	0.000	−0.182	0.561	−0.181	0.092	0.049	−0.217	1.000		
$90\%VaR_{m,t}$	−0.060	0.052	−0.079	−0.499	−0.172	0.617	0.542	−0.073	−0.749	−0.383	1.000	
$95\%VaR_{m,t}$	−0.031	0.063	−0.087	−0.633	0.016	0.542	0.644	−0.063	−0.979	0.023	0.829	1.000

3. Models and empirical results

In this section, we evaluate the ability of the risk measures defined in the previous section to predict the subsequent market excess return, both individually and in different combinations.

3.1. One-factor regressions

We first consider each risk variable separately and estimate the following regression:

$$r_{m,t+1} = a + b_0 r_{m,t} + bX_t + \epsilon_{t+1} \tag{10}$$

where $r_{m,t+1}$ is the market excess return at time $t+1$, $r_{m,t}$ is the market excess return at time t, X_t is either the value-weighted average measure of risk or the market measure of risk at time t. We include the current market return in all of the regressions that we estimate and, in the robustness section, also consider control variables to capture business cycle effects.[6] Table 3 reports the results of estimating this regression. Returns are negatively related to average variance and average skewness and positively related to the two average VaR measures. The coefficients on average skewness, average 90% VaR and average 95% VaR are significant at the 1% level, and the coefficient on average variance is significant at the 5% level. The coefficient on average kurtosis is negative but not statistically significant. Of the average risk measures, the strongest predictor of market returns is skewness and 95% VaR, followed by 90% VaR and then average variance. Of the market risk measures, only variance and skewness are significant, and then only at the 10% level. The p-value of 95% market VaR is 11.8%, which is broadly consistent with the findings reported by Bali et al. (2009) and Harris et al. (2019b), both of which find a significantly positive relation between market returns and the 1-month ahead expectation of VaR.[7] The coefficient on the current market return is positive, reflecting the positive serial correlation in returns reported in Table 1, and is significant at the 10% level in the regressions for average 90% VaR, and at the 5% level for average skewness and average 95% VaR measures.

3.2. Two-factor regressions

To establish the marginal explanatory power of the average risk measures, we first estimate two-factor regressions that include, for each risk measure, both the value-weighted average variable and the market variable:

$$r_{m,t+1} = a + b_0 r_{m,t} + b_1 X_{vw,t} + b_2 X_{m,t} + \epsilon_{t+1} \tag{11}$$

where $X_{vw,t}$ is the value-weighted average risk measure and $X_{m,t}$ is the corresponding market risk measure, at time t. This regression allows us to discern the marginal predictive ability of the idiosyncratic component of average total risk across firms, since the inclusion of $X_{m,t}$ controls for the systematic component of total risk. Table 4 reports the results for these two-factor regressions. The average variance is now not significant at all. The strongest predictor of market returns among the average risk variables is 95% VaR, which remains significant at the 1% level, while 90% VaR and skewness are now significant at the 5% level. Notably, none of the market risk variables is now significant, suggesting that idiosyncratic tail risk contains more information about future market returns than does systematic tail risk. This implies that the role of systematic tail risk in the results reported by Bali et al. (2009) and Harris et al. (2019b) is primarily as a proxy for idiosyncratic risk, with which it is correlated.

[6] The inclusion of the current market return is motivated by the well-documented existence of time series momentum in stock returns (see, for example, Moskowitz et al., 2012). Excluding the current market return does not materially affect our results, and the conclusions drawn are the same.

[7] Results for the one-factor regression using the same sample as Harris et al. (2019b) (not reported) show that 95% market VaR has a p-value of 4.1%.

Table 3
One-factor regressions. The table reports the estimated parameters from the one-month-ahead one-factor predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$ on each average or market risk measure separately and the current market return $r_{m,t}$. The two-sided p-values based on Newey–West adjusted t-statistics with six lags are reported in parentheses. The last two rows present the R^2 and adjusted R^2 values. The sample period is from August 1963 to January 2020.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Constant	0.009	0.009	0.013	−0.270	−0.142	0.006	0.005	0.004	−0.034	−0.022
	(0.000)	(0.000)	(0.348)	(0.011)	(0.005)	(0.000)	(0.007)	(0.365)	(0.248)	(0.211)
r_m	0.042	0.085	0.060	0.080	0.087	0.041	0.063	0.061	0.064	0.063
	(0.284)	(0.049)	(0.155)	(0.059)	(0.043)	(0.311)	(0.133)	(0.152)	(0.131)	(0.137)
$V_{vw,t}$	−0.450									
	(0.043)									
$S_{vw,t}$		−0.115								
		(0.004)								
$K_{vw,t}$			−0.051							
			(0.545)							
$90\%VaR_{vw,t}$				0.216						
				(0.009)						
$95\%VaR_{vw,t}$					0.093					
					(0.004)					
$V_{m,t}$						−0.654				
						(0.099)				
$S_{m,t}$							−0.021			
							(0.095)			
$K_{m,t}$								0.008		
								(0.771)		
$90\%VaR_{m,t}$									0.030	
									(0.178)	
$95\%VaR_{m,t}$										0.017
										(0.118)
R^2	0.984%	1.537%	0.413%	1.626%	1.640%	0.650%	0.790%	0.379%	0.674%	0.790%
$Adj R^2$	0.691%	1.245%	0.118%	1.334%	1.348%	0.355%	0.496%	0.083%	0.379%	0.496%

Table 4
Two-factor regressions. The table reports the estimated parameters from the one-month-ahead two-factor predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$ on the average and market value of each risk measure and the current market return $r_{m,t}$. The two-sided p-values based on Newey–West adjusted t-statistics with six lags are reported in parentheses. The last two rows report the R^2 and adjusted R^2 values. The sample period is from August 1963 to January 2020.

	(1)	(2)	(3)	(4)	(5)
Constant	0.010	0.009	0.017	−0.287	−0.149
	(0.000)	(0.000)	(0.293)	(0.009)	(0.004)
r_m	0.050	0.086	0.061	0.081	0.088
	(0.215)	(0.047)	(0.146)	(0.056)	(0.040)
$V_{vw,t}$	−0.676				
	(0.114)				
$S_{vw,t}$		−0.124			
		(0.012)			
$K_{vw,t}$			−0.094		
			(0.407)		
$90\%VaR_{vw,t}$				0.242	
				(0.013)	
$95\%VaR_{vw,t}$					0.101
					(0.009)
$V_{m,t}$	0.573				
	(0.492)				
$S_{m,t}$		0.004			
		(0.798)			
$K_{m,t}$			0.025		
			(0.494)		
$90\%VaR_{m,t}$				−0.012	
				(0.654)	
$95\%VaR_{m,t}$					−0.004
					(0.767)
R^2	1.046%	1.546%	0.483%	1.656%	1.653%
$Adj R^2$	0.605%	1.107%	0.040%	1.217%	1.215%

Table 5
Multi-factor regressions. The table reports the estimated parameters from the one-month-ahead multi-factor predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$ on combinations of risk measures and the current market return $r_{m,t}$. The two-sided p-values based on Newey–West adjusted t-statistics with six lags are reported in parentheses. The last two rows report the R^2 and adjusted R^2 values. The sample period is from August 1963 to January 2020.

	(1)	(2)	(3)	(4)	(5)
Constant	0.012	0.027	0.032	−0.277	−0.133
	(0.000)	(0.092)	(0.053)	(0.015)	(0.019)
r_m	0.075	0.052	0.077	0.077	0.079
	(0.077)	(0.201)	(0.070)	(0.077)	(0.065)
$V_{vw,t}$	−0.559	−0.745	−0.641	−0.725	−0.583
	(0.229)	(0.081)	(0.161)	(0.086)	(0.203)
$S_{vw,t}$	−0.114		−0.116		
	(0.034)		(0.032)		
$K_{vw,t}$		−0.130	−0.132		
		(0.243)	(0.233)		
$90\%VaR_{vw,t}$				0.239	
				(0.018)	
$95\%VaR_{vw,t}$					0.094
					(0.023)
$V_{m,t}$	0.473	0.683	0.615	0.890	0.532
	(0.617)	(0.399)	(0.494)	(0.267)	(0.566)
$S_{m,t}$	0.004		0.004		
	(0.802)		(0.803)		
$K_{m,t}$		0.029	0.017		
		(0.427)	(0.659)		
$90\%VaR_{m,t}$				−0.014	
				(0.585)	
$95\%VaR_{m,t}$					−0.004
					(0.752)
R^2	2.001%	1.244%	2.210%	2.230%	2.122%
$Adj R^2$	1.271%	0.508%	1.186%	1.501%	1.393%

3.3. Multi-factor regressions

We now consider different combinations of the risk measures, and report results for the following multi-factor regression:

$$r_{m,t+1} = a + b_0 r_{m,t} + \mathbf{b_1} \mathbf{X}_{vw,t} + \mathbf{b_2} \mathbf{X}_{m,t} + \epsilon_{t+1} \tag{12}$$

where $\mathbf{X}_{vw,t}$ is a vector of the value-weighted average risk factors and $\mathbf{X}_{m,t}$ is the vector of the corresponding market risk factors. Column 1 of Table 5 reports results for the combination of variance and skewness. In this regression, average variance is insignificant, as are the market risk variables, but skewness is significant at the 5% level. This is consistent with the results reported by Jondeau et al. (2019). Column 2 reports results for variance and kurtosis, which, as before, has a coefficient that is negative but insignificant, while column 3 reports results for the combination of variance, skewness and kurtosis. The inclusion of kurtosis marginally increases the significance of skewness. In Columns 4 and 5, we report results for the combination of variance with each of the two VaR measures. Both average VaR measures are significant at the 5% level. However, average variance is only significant at a 10% level in the regression which combines average variance with average 90% VaR. For the 90% VaR and 95% VaR measures, the explanatory power of the regression, as measured by the adjusted R-squared, are 1.501% and 1.393%, compared to 1.271% for skewness. Taken together, the results reported in Tables 3–5 therefore suggest that average tail risk, as measured by either 90% VaR or 95% VaR, represents a better predictor of future market returns than does average skewness.

4. Out-of-sample evaluation

The in-sample analysis of the previous section suggests that there is a statistically significant relation between average tail risk and subsequent market returns, and that the strength of this relation is stronger than it is between average skewness and returns. In this section, we investigate the out-of-sample predictive ability of the average risk measures. We then evaluate the performance of trading strategies based on each of the risk measures and compare it with that of a naive strategy based on the historical rolling-window mean return.

Following DeMiguel et al. (2009), we predict the 1-month ahead excess market return using a rolling window approach, with an estimation window of length $M = 240$ months.[8] For the sake of parsimony, we focus on the one-factor regressions and exclude the current market return. For each month $t = M, \ldots, T-1$, we use the sample from $t - M + 1$ to t to estimate the following regression:

$$r_{m,t+1} = \alpha + bX_{vw,t} + \eta_{t+1} \tag{13}$$

[8] We also experimented with an expanding window approach, as in Jondeau et al. (2019), and the conclusions are broadly similar.

We then use the estimated parameters from this regression and the value of $X_{vw,t}$ in month t to forecast the market excess return in month $t+1$. Rolling the estimation window through the full sample yields $T-M$ out-of-sample forecasts of the excess market return from August 1983 to January 2020:

$$\hat{\mu}_{m,t} = \mathbf{E}[r_{m,t+1}] = \hat{\alpha} + \hat{\beta} X_{vw,t} \tag{14}$$

We evaluate the forecast performance of each of the risk measures using several statistics. The out-of-sample R-squared coefficient compares the predictive power of the regression with the estimation window mean $\bar{r}_{m,t} = \frac{1}{M}\sum_{s=t-M+1}^{t} r_{m,s}$, and is defined as $R^2_{OOS} = 1 - \frac{MSE_P}{MSE_N}$, where $MSE_P = \frac{1}{T-M}\sum_{t=M}^{T-1}(r_{m,t+1} - \hat{\mu}_{m,t})$ is the mean square error of the out-of-sample forecasts based on the model and $MSE_N = \frac{1}{T-M}\sum_{t=M}^{T-1}(r_{m,t+1} - \bar{r}_{m,t})$ is the mean square error based on the estimation window mean. The adjusted out-of-sample R-squared coefficient is defined as

$$adjR^2_{OOS} = R^2_{OOS} - \frac{k}{T-M-k-1}(1 - R^2_{OOS}) \tag{15}$$

where k is the number of predictors. The higher the adjusted out-of-sample R-squared, the better the predictive ability.

We also use the encompassing test statistic proposed by (Clark and McCracken, 2001):

$$ENC = \frac{T-M-k-1}{T-M} \times \frac{\sum_{t=M}^{T-1}((r_{m,t+1} - \bar{r}_{m,t})^2 - (r_{m,t+1} - \bar{r}_{m,t})(r_{m,t+1} - \hat{\mu}_{m,t}))}{MSE_P} \tag{16}$$

The null hypothesis is that the forecasts based on the estimation sample mean encompass the forecasts derived from the model, that is, the variables in the model offer no marginal predictive ability over the sample mean. The ENC statistic follows a non-standard distribution when testing nested models and so we use the critical values provided in Table 1 of Clark and McCracken (2001).

Following Jondeau et al. (2019), we also examine the performance of a trading strategy based on the forecasts of each model, and compare it with that of strategy based on the estimation window mean. The strategy comprises a combination of the market portfolio and the risk-free asset, represented by the one-month treasury bill. Each month, the weight of the market portfolio is determined as

$$w_{m,t} = \frac{\hat{\mu}_{m,t}}{\lambda \hat{V}_{m,t}} \tag{17}$$

where λ is the risk aversion coefficient and $\hat{V}_{m,t}$ is the sample variance of market returns computed over the previous five years. Following Jondeau et al. (2019), we assume the risk-aversion parameter λ is equal to 2. We constrain $w_{m,t}$ to lie within the interval of 0 and 2, which allows a maximum leverage of 100% (Campbell and Thompson, 2008). The annualised average portfolio return is computed as:

$$\bar{r}_p = \frac{12}{T-M}\sum_{t=M}^{T-1} r_{p,t+1} \tag{18}$$

where $r_{p,t+1} = w_{m,t} r_{m,t+1} + (1 - w_{m,t}) r_{f,t}$ is the portfolio return in month t. The annualised portfolio return volatility is calculated as

$$\sigma_p = \sqrt{12} \times std(r_{p,t+1}) \qquad t = M, \ldots, T-1 \tag{19}$$

The Sharpe ratio is defined as

$$SR_p = \frac{\bar{r}_p}{\sigma_p} \tag{20}$$

which measures the risk-adjusted return of the portfolio. We examine whether the trading strategy based on the predictors yields a higher Sharpe ratio than a strategy based on the estimation window mean by applying the test statistic used by DeMiguel et al. (2009). Under the null hypothesis, $\frac{\bar{r}_p}{\sigma_p} - \frac{\bar{r}_0}{\sigma_0} = 0$, the test statistic $z_{SR} = \frac{\sigma_0 \bar{r}_p - \sigma_p \bar{r}_0}{\sqrt{\theta}}$ is asymptotically distributed as a standard normal.[9][10] The final statistic considered in this paper is the annual transaction fee generated by each strategy, which is defined as

$$Fee_p = \frac{12f}{T-M}\sum_{t=M}^{T-1} |w_{m,t+1} - w_{m,t+}| \tag{21}$$

where f is the fee per dollar and $w_{m,t+}$ represents the market weight just before rebalancing at $t+1$. $w_{m,t+}$ is different from $w_{m,t}$ and $w_{m,t+1}$ due to changes in the value of the market portfolio and the risk-free asset. Following Jondeau et al. (2019), we assume a fee of $f = 10$ basis points.

Table 6 reports the results of the out-of-sample analysis, with the adjusted out-of-sample R-squared and ENC test statistic reported in Columns 1–2 and the out-of-sample trading strategy results reported in Columns 3–8. The highest adjusted out-of-sample R-squared is for 90% VaR (0.262%), followed by 95% VaR (0.029%). For the remaining measures—including skewness—the adjusted

[9] θ is computed as $\theta = \frac{1}{T_{OOS}}(2\sigma_p^2\sigma_0^2 - \sigma_p\sigma_0\sigma_{p,0} + \frac{1}{2}\bar{r}_p^2\sigma_0^2 + \frac{1}{2}\bar{r}_0^2\sigma_p^2 - \frac{\bar{r}_p\bar{r}_0}{\sigma_p\sigma_0}\sigma_{p,0}^2)$.

[10] We also computed the certainty equivalent return using a mean–variance utility function with a risk aversion coefficient of 2. This leads to the same ranking of portfolios as the Sharpe ratio and so these results are not reported.

Table 6

Out-of-sample performance. The table reports, for each average risk measure, the adjusted out-of-sample R^2 ($AdjR^2_{OOS}$), the encompassing test statistics (ENC), the average market portfolio weight ($\bar{\omega}_m$), the annualised average return ($\bar{r}_p$), volatility (σ_p), and skewness ($Skew_p$) of the portfolio, the annualised Sharpe ratio (SR), and the annual transaction fee (Fee). The table also reports results for the strategies based on the current return and the historical mean return, and for the buy-and-hold-strategy. The out-of-sample period is from August 1983 to January 2020.

	(1) $Adj\ R^2_{OOS}$ (%)	(2) ENC	(3) $\bar{\omega}$	(4) $\bar{r}$ (%)	(5) σ (%)	(6) $Skew_p$	(7) SR	(8) Fee (%)
$Historical\ mean^t_{t-M+1}$	NaN	NaN	1.392	9.662	21.872	−1.118	0.442	0.075
$Buyandhold$	NaN	NaN	1.000	8.105	15.012	−0.852	0.540	0.000
$r_{vw,t}$	−0.820	−0.211	1.368	8.676	20.642	−0.958	0.420	0.567
$V_{vw,t}$	−0.291	2.588	1.401	11.644	20.144	−0.876	0.578	0.227
$S_{vw,t}$	−0.059	4.793	1.312	13.726	22.977	−0.747	0.597*	0.734
$K_{vw,t}$	−1.223	−1.016	1.309	8.159	21.815	−1.093	0.374	0.470
$90\%VaR_{vw,t}$	0.262	6.941	1.242	13.908	23.043	−0.708	0.604*	0.863
$95\%VaR_{vw,t}$	0.029	5.241	1.277	13.469	23.178	−0.700	0.581	0.789

*Denotes that the Sharpe ratio is higher than that of the historical mean strategy at the 10% significance level.

R-squared is negative. The ENC test shows that we can reject the null hypothesis that the naive forecast encompasses the model forecasts for variance, skewness, 90% VaR and 95% VaR at the 5% significance level. Consistent with the adjusted R-squared results, the strongest rejection is for 90% VaR, followed by 95% VaR and then skewness. Turning to the trading strategy results, the use of 90% VaR as a predictor provides the highest average annualised portfolio return of 13.908%, compared with 13.726% for skewness and 13.469% for 95% VaR. These are substantially higher than the annualised returns of both the naive strategy based on the historical mean return and the buy and hold strategy. The strategies based on the VaR and skewness measures are more volatile than the naive and buy-and-hold strategies, and the 90% VaR-based strategies are marginally more volatile than the skewness-based strategy. All of the strategies have negative return skewness. Overall, the trading strategy based on 90% VaR offers the best out-of-sample risk-adjusted return performance as measured by the Sharpe ratio. The naive strategy has an annualised Sharpe ratio of 0.442, while the buy-and-hold strategy has an annualised Sharpe ratio of 0.540. Only strategies based on 90% VaR and skewness yield a Sharpe ratio that is significantly higher than that of the naive strategy. The strategies based on the lagged return and average kurtosis generate a Sharpe ratio that is lower than that of the naive strategy. Annualised transaction fees generated by each strategy range from 0.227% to 0.863% of portfolio value. The transaction fee for 90% VaR strategy is slightly higher than it is for the skewness strategy. Even after taking transaction fees into account, the 90% VaR strategy still delivers a higher annualised average return than the skewness strategy. Thus, the out-of-sample analysis supports our earlier findings and shows that the superior predictive ability of average tail risk relative to average skewness is both statistically and economically significant.

Fig. 1 provides a graphical representation of the evolution of the out-of-sample performance of each trading strategy. In particular, it reports the difference in the cumulative sum of squared errors (SSE) between the 240-month rolling window sample mean and the out-of-sample forecasts based on each of the risk measures:

$$\Delta CumSSE_{t+1} = \sum_{M+1}^{t+1} (r_{m,t+1} - \bar{r}_{m,t})^2 - \sum_{M+1}^{t+1} (r_{m,t+1} - \hat{\mu}_{m,t})^2 \tag{22}$$

where the mean market return in the estimation window $\bar{r}_{m,t}$ and the prediction $\hat{\mu}_{m,t}$ have the same definition as above. An increase in $CumSSE$ indicates increasing performance relative to the rolling window mean. Both the market return and the average variance outperform the mean return during the subprime crisis, but otherwise generally underperform. Both skewness and VaR systematically outperform up until shortly before the sub-prime crisis. They recover somewhat after the crisis but experience further decline in performance from 2016 onwards.

5. Robustness checks

The empirical evidence reported in the previous sections suggests that average tail risk has predictive ability for future market returns that is both statistically and economically significant, and that it dominates other measures of average risk, including skewness. In this section, we examine the robustness of our results with respect to a number of aspects of the analysis.

5.1. Control variables

First, we control for a number of variables that capture business cycle effects for which the aggregate risk variables potentially act as proxies. In particular, following Jondeau et al. (2019), we include the dividend–price ratio, the default spread, the term spread, the stochastically detrended risk-free rate and the expected component of aggregate illiquidity as control variables. The dividend–price ratio (DP) is defined as the difference between the log of the last 12-month dividends (D) and the log of the current level of the Standard & Poor 500 index.

$$DP_t = \log(\sum_{\tau=t-11}^{t} D_\tau) - \log(P_t) \tag{23}$$

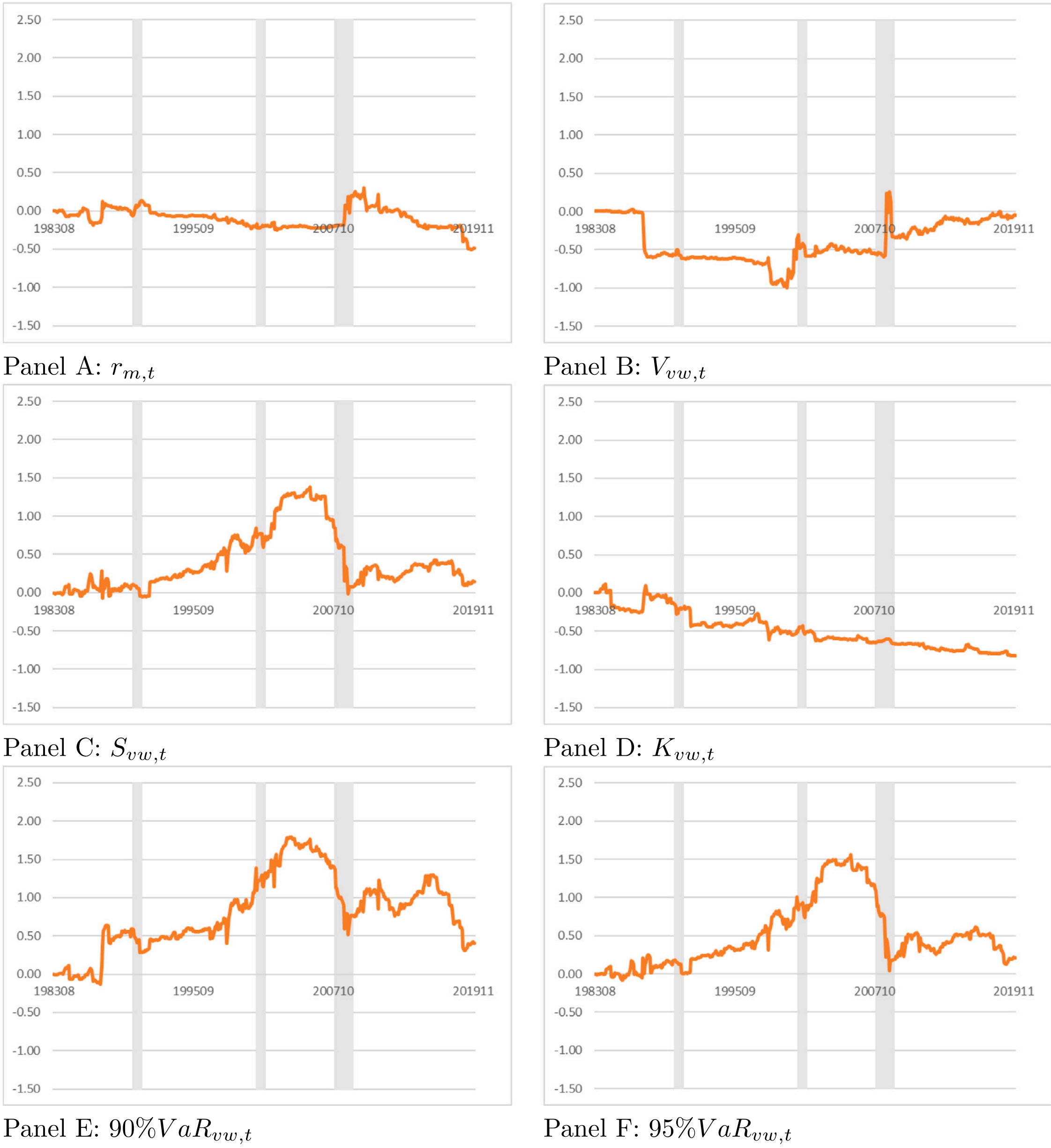

Fig. 1. Performance of the out-of-sample forecasting models. The figure plots the out-of-sample performance of the forecasting model based on each risk measure, calculated as the difference between the cumulative sum of squared errors (SSE) of forecasts based on the 240-month rolling window sample mean minus the cumulative SSE of forecasts based on the risk measure. The out-of-sample evaluation period is from August 1983 to January 2020. National Bureau of Economic Research recessions are represented by shaded bars.

The default spread (DEF) is calculated as the difference between the Moody's BAA corporate bond yield and the 10-year Treasury bond yield, where the time-series data on monthly BAA-rated corporate bond yield (R_{BAA}) and 10-year Treasury bond yield (R_{10_y}) are obtained from the Federal Reserve statistics release website.

$$DEF_t = R_{BAA,t} - R_{10y,t} \tag{24}$$

The term spread ($TERM$) is the difference between the 10-year Treasury bond yield (R_{10_y}) and the 3-month Treasury bill yield (R_{3m}), obtained from the Federal Reserve statistics release website.

$$TERM_t = R_{10y,t} - R_{3m,t} \tag{25}$$

Table 7

Controlling for business cycle fluctuations. The table reports the estimated parameters from the one-month-ahead two-factor predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$ on the average and market value of each risk measure, the current market return $r_{m,t}$, the dividend yield (DP), the default spread (DEF), the term spread ($TERM$), the stochastically detrended risk-free rate ($RREL$) and the expected component of aggregate illiquidity ($ILLIQ$). The two-sided p-values based on Newey–West adjusted t-statistics with six lags are reported in parentheses. The last two rows report the R^2 and adjusted R^2 values. The sample period is from August 1963 to January 2020.

	(1)	(2)	(3)	(4)	(5)
Constant	0.028	0.034	0.017	−0.272	−0.120
	(0.302)	(0.247)	(0.537)	(0.022)	(0.026)
$r_{m,t}$	0.023	0.080	0.056	0.078	0.082
	(0.601)	(0.100)	(0.232)	(0.105)	(0.093)
$V_{vw,t}$	−0.809				
	(0.085)				
$S_{vw,t}$		−0.118			
		(0.029)			
$K_{vw,t}$			−0.076		
			(0.531)		
$90\%VaR_{vw,t}$				0.240	
				(0.025)	
$95\%VaR_{vw,t}$					0.092
					(0.026)
$V_{m,t}$	0.253				
	(0.762)				
$S_{m,t}$		0.001			
		(0.964)			
$K_{m,t}$			0.024		
			(0.520)		
$90\%VaR_{m,t}$				−0.011	
				(0.698)	
$95\%VaR_{m,t}$					−0.001
					(0.961)
DP	0.003	0.009	0.011	0.012	0.010
	(0.719)	(0.202)	(0.145)	(0.099)	(0.164)
DEF	0.580	0.161	0.169	0.077	0.138
	(0.084)	(0.658)	(0.651)	(0.834)	(0.705)
$TERM$	0.005	0.095	0.104	0.101	0.096
	(0.978)	(0.598)	(0.557)	(0.569)	(0.593)
$RREL$	−0.220	−0.211	−0.219	−0.214	−0.210
	(0.284)	(0.292)	(0.285)	(0.290)	(0.296)
$ILLIQ$	0.001	0.000	−0.002	−0.001	−0.001
	(0.683)	(0.942)	(0.497)	(0.554)	(0.775)
R^2	2.683%	2.739%	1.688%	2.833%	2.784%
$AdjR^2$	1.518%	1.574%	0.511%	1.669%	1.620%

The stochastically detrended risk-free rate ($RREL$) is the difference between the 3-month Treasury bill rate and its 12-month backward-moving average.

$$RREL_t = R_{3m,t} - \frac{1}{12}\sum_{\tau=t-12}^{t-1} R_{3m,\tau} \tag{26}$$

Lastly, the expected illiquidity ($ILLIQ$) is the expected component of aggregate illiquidity, which is obtained as the fitted value of the log illiquidity from the regression:

$$\log(Illiq_{t+1}) = \alpha + \beta \times \log(Illiq_t) + \epsilon_t \tag{27}$$

where the aggregate illiquidity across stocks in month t is computed as $Illiq_t = \sum_{i=1}^{N_t} w_{i,t} Illiq_{i,t}$. The use of the liquidity measure as a control variables is motivated by Bali et al. (2005) who suggest that the predictive ability of average variance is partly explained by a liquidity premium.

Table 7 reports the results of estimating the two-factor regression given by Eq. (11), supplemented by the control variables:

$$r_{m,t+1} = a + b_0 r_{m,t} + b_1 X_{vw,t} + b_2 X_{m,t} + \theta' \mathbf{Z}_t + \epsilon_{t+1} \tag{28}$$

where $\mathbf{Z}_t$ is the vector of control variables at time t. Of the control variables, only the dividend ratio and default spread are statistically significant, and then only in two regressions. The remaining control variables are not significant in any of the regressions, suggesting that the risk measures that we consider are not acting as proxies for macroeconomic determinants of returns. Consequently, the predictive ability of each of the risk measures identified in Table 4 is largely unchanged by the inclusion of the control variables. In particular, average 90% VaR remains the most significant predictor of market returns. In unreported results, we draw similar conclusions when we supplement the single factor regression given by Eq. (10) and the multi-factor regression given by Eq. (12) with the control variables.

Table 8
Sub-sample analysis. The table reports the estimated parameters from the one-month-ahead two-factor predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$ on the average and market value of each risk measure and the current market return $r_{m,t}$. The two-sided p-values based on Newey–West adjusted t-statistics with six lags are reported in parentheses. The last two rows report the R^2 and adjusted R^2 values. Results are reported for the 1963–1991 sub-sample in columns 1–6 in Panel A and 1991–2019 sub-sample in columns 7–12 in Panel B.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Panel A: sub-sample August 1963–October 1991					Panel B: sub-sample November 1991–January 2020				
Constant	0.002	0.011	0.023	−0.278	−0.179	0.014	0.008	0.016	−0.278	−0.119
	(0.784)	(0.008)	(0.467)	(0.116)	(0.028)	(0.000)	(0.013)	(0.389)	(0.057)	(0.083)
r_m	0.051	0.128	0.068	0.089	0.120	0.029	0.054	0.044	0.066	0.059
	(0.320)	(0.041)	(0.194)	(0.092)	(0.052)	(0.658)	(0.414)	(0.515)	(0.348)	(0.363)
$V_{vw,t}$	0.626					−0.969				
	(0.681)					(0.020)				
$S_{vw,t}$		−0.197					−0.065			
		(0.013)					(0.317)			
$K_{vw,t}$			−0.095					−0.109		
			(0.681)					(0.372)		
$90\%VaR_{vw,t}$				0.225					0.242	
				(0.163)					(0.057)	
$95\%VaR_{vw,t}$					0.131					0.072
					(0.034)					(0.155)
$V_{m,t}$	−1.643					1.208				
	(0.489)					(0.212)				
$S_{m,t}$		0.024					−0.015			
		(0.302)					(0.481)			
$K_{m,t}$			−0.027					0.061		
			(0.702)					(0.137)		
$90\%VaR_{m,t}$				−0.004					−0.019	
				(0.927)					(0.586)	
$95\%VaR_{m,t}$					−0.015					0.007
					(0.469)					(0.670)
R^2	0.856%	2.018%	0.849%	1.608%	1.842%	2.091%	1.345%	0.722%	1.481%	1.503%
$AdjR^2$	−0.032%	1.141%	−0.039%	0.726%	0.963%	1.212%	0.459%	−0.170%	0.596%	0.618%

5.2. Sub-sample analysis

Our main analysis is based on the full sample August 1963 to January 2020. Here we examine the temporal stability of our results by considering two equal sub-samples: August 1963 to October 1991 and November 1991 to January 2020. Table 8 reports the results of estimating the two-factor regression given by Eq. (11) for these two samples. In particular, while average skewness is statistically significant at the 5% level in the first sub-sample with a coefficient that is somewhat higher than it is in the full sample, it is insignificant in the second sub-sample with a much reduced coefficient. The picture for average VaR is less clear. In the first sub-sample, 90% VaR is not significant at the 10% level, while 95% VaR is significant at the 5% level. In the second sub-sample, 90% VaR is significant at the 10% level, but 95% VaR becomes insignificant. The sub-sample results thus suggest that the relative importance of skewness and VaR have changed over time. Again, in unreported results, we draw the same conclusions when we estimate the single factor regression given by Eq. (10) and the multi-factor regression given by Eq. (12) for the two sub-samples.

5.3. Alternative risk measures

Our final robustness check is to consider a number of alternative measures of risk, including different definitions of skewness and kurtosis, a non-parametric asymmetry measure, a measure of downside risk, and alternative measures of tail risk.

As an alternative to the skewness measure proposed by Jondeau et al. (2019), we consider the standardised realised skewness coefficient (see, for example, Amaya et al., 2015):

$$\tilde{S}_{i,t} = \frac{\frac{1}{D_t}\sum_{d=1}^{D_t}(r_{i,d}-\bar{r}_{i,t})^3}{(\sqrt{\frac{1}{D_t}\sum_{d=1}^{D_t}(r_{i,d}-\bar{r}_{i,t})^2})^3} \tag{29}$$

We use an analogous definition for standardised realised kurtosis:

$$\tilde{K}_{i,t} = \frac{\frac{1}{D_t}\sum_{d=1}^{D_t}(r_{i,d}-\bar{r}_{i,t})^4}{(\sqrt{\frac{1}{D_t}\sum_{d=1}^{D_t}(r_{i,d}-\bar{r}_{i,t})^2})^4} \tag{30}$$

These measures of standardised skewness and kurtosis, which we use in the calculation of value at risk using the Cornish–Fisher expansion, differ from those based on Jondeau et al. (2019) by a factor of $\frac{1}{\sqrt{D_t}}$ and $\frac{1}{D_t}$, respectively. For both skewness and kurtosis,

Table 9

Alternative risk measures. The table reports the estimated parameters from the one-month-ahead two-factor predictive regressions of the value-weighted CRSP market excess return $r_{m,t+1}$ on the average and market value of alternative risk measures and the current market return $r_{m,t}$. The two-sided p-values based on Newey–West adjusted t-statistics with six lags are reported in parentheses. The last two rows report the R^2 and adjusted R^2 values. The sample period is from August 1963 to January 2020.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Constant	0.009	0.021	0.005	0.008	−0.093	−0.019	−0.078	−0.032	−0.104	−0.083
	(0.001)	(0.224)	(0.022)	(0.009)	(0.017)	(0.415)	(0.041)	(0.231)	(0.015)	(0.034)
r_m	0.085	0.062	0.059	0.030	0.086	0.069	0.074	0.072	0.074	0.085
	(0.071)	(0.174)	(0.250)	(0.513)	(0.068)	(0.145)	(0.103)	(0.127)	(0.098)	(0.069)
$\tilde{S}_{vw,t}$	−0.025									
	(0.029)									
$\tilde{K}_{vw,t}$		−0.006								
		(0.310)								
$ETP_{vw,t}$			0.074							
			(0.752)							
$DV_{vw,t}$				−5.727						
				(0.583)						
$90\% ES_{vw,t}$					0.062					
					(0.029)					
$95\% ES_{vw,t}$						0.009				
						(0.449)				
2nd $min_{vw,t}$							0.063			
							(0.032)			
$min_{vw,t}$								0.017		
								(0.258)		
$p10_{vw,t}$									0.098	
									(0.010)	
$p5_{vw,t}$										0.055
										(0.036)
$\tilde{S}_{m,t}$	0.001									
	(0.800)									
$\tilde{K}_{m,t}$		0.001								
		(0.475)								
$ETP_{m,t}$			−0.015							
			(0.606)							
$DV_{m,t}$				−10.367						
				(0.522)						
$90\% ES_{m,t}$					−0.002					
					(0.783)					
$95\% ES_{m,t}$						0.002				
						(0.579)				
2nd $min_{m,t}$							−0.004			
							(0.523)			
$min_{m,t}$								0.001		
								(0.772)		
$p10_{m,t}$									−0.009	
									(0.246)	
$p5_{m,t}$										−0.003
										(0.710)
R^2	1.386%	0.521%	0.417%	1.218%	1.391%	0.629%	1.106%	0.705%	1.391%	1.112%
$Adj R^2$	0.947%	0.078%	−0.026%	0.778%	0.953%	0.187%	0.666%	0.263%	0.952%	0.672%

we continue to use the mean adjustment $\bar{r}_{i,t} = \frac{1}{D_t - 2}\sum_{d=1}^{D_t-2} r_{i,d}$ to accommodate the turn-of-the-month effect in stock returns identified by Lakonishok and Smidt (1988).

In addition to the parametric measure of skewness described above, we also use an alternative non-parametric measure of asymmetry in stock returns, proposed by Jiang et al. (2020). They define the excess tail probability (ETP) as:

$$ETP_{i,t} = \int_{1}^{+\infty} f(x)dx - \int_{-\infty}^{-1} f(x)dx \tag{31}$$

where the probabilities are evaluated at one standard deviation from the mean.

We also consider a measure of downside risk, which also reflects asymmetry in returns. In particular, we define the downside variance of stock i in month t as

$$DV_{i,t} = \frac{1}{D_t^*}\sum_{d=1}^{D_t}(min(r_{i,d} - \bar{r}_{i,t}, 0))^2 \tag{32}$$

where D_t^* is the number of observations that fall below $\bar{r}_{i,t}$.

To check the robustness of our tail risk measures, we consider two alternative approaches. We first consider the Cornish–Fisher expected shortfall (ES) with confidence levels of 90% and 95%. The standardised Cornish–Fisher ES at the $p\%$ confidence level is given by

$$ES^p_{i,t} = -\{\frac{-\phi(VaR^p_{i,t})}{1-p}[1+\frac{\tilde{S}_{i,t}}{6}(VaR^p_{i,t})^3+\frac{(\tilde{K}_{i,t}-3)}{24}[(VaR^p_{i,t})^4-2(VaR^p_{i,t})^2-1]]\} \tag{33}$$

where $VaR^p_{i,t}$ is defined in Eq. (4). Second, we consider a non-parametric measure of tail risk based on the historical simulation estimate of VaR. Given the limited number of daily observations each month (which is between 19 and 22), we define $min_{i,t}$ and 2nd $min_{i,t}$ as −1 times the minimum and the second minimum daily return, respectively, observed during the month, standardised by the mean $\bar{r}_{i,t} = \frac{1}{D_t-2}\sum_{d=1}^{D_t-2} r_{i,d}$ and the variance $\sigma^2_{i,t} = \frac{1}{D_t}\sum_{d=1}^{D_t}(r_{i,d}-\bar{r}_{i,t})^2$. These correspond approximately to historical simulation value at risk at the 95% and 90% confidence levels, respectively.

As a final check, we also compute exact estimates of the 95% and 90% historical simulation VaR, $HS90\%VaR_{i,t}$ and $HS95\%VaR_{i,t}$, as −1 times the 10th and 5th percentile of the standardised daily returns observed during this month, where the latter are computed using interpolation.

Table 9 reports the results of estimating the two-factor regression given by Eq. (11) for these alternative risk measures over the full sample period from August 1963 to January 2020. The alternative measures of standardised skewness and kurtosis (Columns 1 and 2) yield very similar results to those based on the original measure, suggesting that the results are not sensitive to this adjustment. However, both the average and market non-parametric asymmetry measure (Column 3) are insignificant, suggesting that predictive ability arising from the asymmetry of returns is sensitive to the way it is measured. Similarly, downside variance (Column 4) is also insignificant. 90% expected shortfall (Column 5) is significant at the 5% level. 95% expected shortfall (Column 6), however, is not significant at conventional levels. The second minimum return (Column 7) is significantly positive, while the minimum return (Column 8) is positive but insignificant. Finally, the 90% and 95% interpolated historical simulation VaR (Columns 9 and 10) are significant at the 5% level, further strengthening the evidence of the predictive ability of tail risk for market returns.

6. Conclusion

This paper investigates the ability of the average risk across stocks to predict future market returns. We consider measures of risk that capture the higher moments of the return distribution as well as composite measure of downside risk and tail risk. We show that average tail risk, as measured by value at risk, is a statistically and economically significant predictor of market returns, even after controlling for the tail risk of the market portfolio. Moreover, the relation between tail risk and returns is stronger than the previously documented relation between skewness and returns, which itself is shown to be fragile. In particular, while the predictive ability of average tail risk has strengthened over time, the role of average skewness in predicting returns disappears in the most recent sample. Our results therefore represent further evidence against standard asset pricing models such as the Intertemporal CAPM. We show that our findings are robust to the inclusion of control variables that capture the business cycle effects, suggesting that average tail risk is not simply acting as a proxy for macroeconomic risk factors. They are also robust to different ways of measuring value at risk, and other measures of tail risk such as expected shortfall.

The fact the average tail risk is able to predict future market returns, even after controlling for the tail risk of the market portfolio, suggests that *idiosyncratic* tail risk plays an important role in the risk–return relation. Indeed, given that market tail risk is itself not significant in our regressions, it would appear that it is only the idiosyncratic component of tail risk that is relevant. This result is consistent with the finding reported elsewhere in the literature concerning the role of idiosyncratic volatility in explaining the cross-section of stock returns. A natural area for future research would be to explore the cross-sectional relation between returns and idiosyncratic tail risk.

CRediT authorship contribution statement

Yingtong Dai: Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing. **Richard D.F. Harris:** Conceptualization, Methodology, Formal analysis, Writing – original draft, Writing – review & editing.

Data availability

The authors do not have permission to share data.

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