EFFECTS OF SHORT-TERM THINKING ON INVESTMENT AND IMPLICATIONS FOR MACROECONOMIC POLICIES

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Abstract

In this thesis, after a brief introduction, three papers on the effects of short-term thinking on the economy are presented, where short-term thinking is defined to encompass both short-termistic incentive structures as well as imperfectly forward-looking expectations. The first paper studies the impact of short-termistic incentives for managers of American corporations on their decisions regarding the capital mix of their companies. The second paper presents a New Keynesian model with rigid wages and myopic expectations and assesses the performance of simple monetary policy rules. The final chapter uses a medium-scale dynamic stochastic general equilibrium model to study to what extent multipliers of government investment depend, inter alia, on the forward-lookingness of agents' expectations.

Kurzfassung

In der vorliegenden Arbeit werden nach einer kurzen Einleitung drei Aufsätze zu den Effekten kurzfristigen Denkens auf die Ökonomie betrachtet. Unter „kurzfristigem Denken“ werden sowohl kurzfristige Anreizsysteme als auch unvollkommen vorausschauende Erwartungen subsumiert. Der erste Aufsatz untersucht die Auswirkungen kurzfristiger Anreizsysteme bei Managern US-amerikanischer Unternehmen auf die Wahl ihres Kapitalstocks. Der zweite Aufsatz beleuchtet im Rahmen eines neukeynesianischen Modells mit Lohnrigiditäten die Stabilisierungseigenschaften unterschiedlicher einfacher geldpolitischer Regeln, wenn Wirtschaftssubjekte myopische Erwartungen aufweisen. Im Rahmen des letzten Aufsatzes wird mittels eines dynamischen stochastischen allgemeinen Gleichgewichtsmodells untersucht, inwiefern der Ausgabenmultiplikator auf öffentliche Investitionen davon abhängt, wie vorausschauend die Erwartungen der Wirtschaftssubjekte sind.

Dedicated to Agnes, the love of my life,
and my always supporting family.

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Any remaining errors are my own.

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Chapter 1

Introduction

Motivation

Over the past decades, (macro)economists have generally performed dynamic analyses mostly in models with rational expectations and with geometric discounting of future utility streams. This combination can give rise to agents being extremely forward-looking, which yields a set of peculiar predictions. E.g., in a macroeconomic context, the forward-guidance puzzle (Del Negro *et al.*, 2012) may arise, giving implausibly strong effects for forward guidance at long horizons. This, jointly with other observations, has given rise to additional interest in deviations from rational expectations in macroeconomic models. Here, in particular, approaches that lead to a discounting of, or inattention with respect to the (far-away) future have become more common (Angeletos *et al.*, 2021; Angeletos and Lian, 2022, for reviews, see). In parallel, in recent years, there has been a worry that Western economies become too focussed on short-term profits at the expense of long-run economic value and welfare. In particular, some observers have lamented 'short-termism' in the corporate sector, in particular in the United States (e.g., Koller *et al.*, 2017).

Although these two strands of literature (imperfect – in particular myopic – expectations in macroeconomics and short-termism in firms' behaviour) have evolved in parallel and there is no direct link between the two sets of literature, one can consider both to be a result of 'short-term thinking' in a broader sense, meaning that a relatively large focus is placed on the present and near future, whereas the more remote future (i.e., the medium- and long term) is disregarded. In particular, in both cases, the marginal effects of present-day's decisions on future variables is effectively underestimated, or at least under-represented, relative to textbook models of economic growth and business cycle dynamics.[1] Depending on the type of short-term thinking, this can have structural effects or only affect the dynamic responses of economic variables, e.g., to exogenous shocks.

In particular, a structural incentive-based focus on short-term performance can also result in structural effects that lower investment and growth in the long run (see, Terry (2017); for microeconometric evidence also see Edmans *et al.* (2021, 2017) and Ladika and Sautner (2019)). Hence, short-term thinking can affect the balanced-growth path of

[1]In the case of corporate short-termism through incentives, this is not really an 'underestimation'. Rather, managers and shareholders 'suffering' from short-termistic pressures knowingly sacrifice long-term value. I.e., managers might individually behave optimally, but they offload the costs of short-termism to shareholders and other stakeholders (in particular, labour). Of course, via general-equilibrium effects, managers as consumers might still end up worse off.

the economy, and it can have important welfare implications. This, in turn, would call for corrective structural policies.

On the other hand, even if we assume that incentives per se do not lead to a structural change in investment, short-term thinking in the form of imperfect expectations[2] can have important effects: In that case, agents effectively discount (or, neglect) the medium- and long-run general-equilibrium effects of current events. As argued, inter alia, by Angeletos and Lian (2022), this can dampen or reinforce those general-equilibrium effects *along a transition path*, meaning that deviations from a given steady equilibrium can be quite different than predicted by models without the short-term bias produced by short-term thinking. This is in particular relevant for business cycle-dynamics; and also for the conduct of monetary and fiscal policies, in particular those aimed at regulating business cycle fluctuations.

There is by now a vast literature on both types of short-term thinking, i.e. on short-termism via incentives (for a small review, see the literature review in Chapter 2) or via imperfect expectations (see, in particular, Angeletos *et al.* (2021) and the references therein). However, a number of observations regarding the previous literature should be noted:

1. On corporate short-termism: Studies so far have mostly treated firms' capital as a homogenous good; there has been little focus on the actual composition of the capital mix. However, capital goods are heterogeneous. And one of the more relevant differences across capital goods is their durability (or conversely, depreciation rate): In particular, structures and machinery generally have a longer service life, whereas e.g., capital goods for marketing (advertisement brochures etc.) have only a very limited duration. To the extent that the different capital goods can substitute for each other in generating revenue, one should expect the capital mix to be affected by the degree of short-termism. In particular, since long-lived capital goods derive a lot of their value from future employability in the firm, their value for executives should be strongly dependent on the degree of short-termism. Less durable goods should, however, be more akin to flexible factors, i.e. since the net present value of these investments is more dependent on the near future, short-termism should not affect the value of these investments as much. Hence, ex ante one would expect that more short-termistic firms skew their investment decisions towards shorter-lived capital goods. As a result, not only would short-termism in incentives lead to less investment overall, it would also make that investment overall less durable. In consequence, however, the average depreciation rate of the firm would be higher, leading to higher capital expenditures relative to the capital stock, which can be considered a source of inefficiency.

2. On imperfect expectations: Most studies on the implications of boundedly rational expectations for the conduct of monetary and fiscal policies have been conducted in the context of a simple New Keynesian model augmented by a deviation in terms of expectations (e.g., Farhi and Werning (2019), Nakata *et al.* (2019), Gabaix (2020), Nakata *et al.* (2020), or Benchimol and Bounader (2021)). However, the literature (see, in particular Auclert *et al.*, 2020) has recently pointed out a couple of pitfalls

[2]Note that these imperfect expectations can be a direct result of cognitive discounting as in Gabaix (2020) or other forms of bounded rationality – e.g., k-level thinking as in Farhi and Werning (2019), or it can be due to imperfect information Angeletos and Lian (2018) in general. Also, it could be the result of rational inattention (Sims, 2003).

of this simple model. In particular, this literature has found that other distortions, in particular, wage rigidity, are a key ingredient for an empirically plausible model. Also, so far most of the theoretical studies in this context consider optimal monetary policy very explicitly (i.e., by having the central bank solve a discretionary maximisation problem or the corresponding commitment solution). Little attention has been paid to simple rules (e.g., Taylor rules) that, however are mostly used in empirical work.

3. In addition, the focus of the literature on the effects of imperfect expectations on macroeconomic dynamics has clearly been on monetary policy (especially at the effective lower bound), and – to a smaller extent – on fiscal policy. This is understandable given that in the early 2000s, a general consensus view was that monetary policy should be the prime stabiliser in the short- to medium run. However, the decline of real – and nominal – interest rates across the globe observed from the 1980s to the 2020s,[3] as well as the experiences from the various crisis of the early 21st century (Great Recession of 2007–2009, European debt crisis after 2009, COVID-19 pandemic since 2020, among others) has sparked renewed interest in fiscal policy (Ramey, 2019).[4] So far, analyses of fiscal policy with imperfect expectations have mostly been performed in small-scale models and/or focussed on innovations to pure government consumption. In the context of the COVID-19 pandemic, however, fiscal stimulus has been on public investment, which different than government consumption is often assumed to be productive. Also, public investment is generally found to be affected by significant time delays, with productivity gains often unfolding long after the investment was initiated – but also over a long time horizon (see Ramey, 2021, and references therein). This gives government investment a very distinct profile in terms of time structure and types of macroeconomic effects, which can be quite different than government consumption. Also, the longer time frames involved in public investment should make this type of fiscal policy particularly susceptible to the effects of short-term thinking in the sense of imperfectly formed macroeconomic expectations. After all, if public investment in, e.g., infrastructure, possibly takes years to complete (where this time window can be subject to some uncertainty), and the exact productivity gains achievable are not known, should one expect private agents to incorporate the productivity gains and the full set of future general-equilibrium effects in their considerations, as predicted by rational expectations? Given that there is now some evidence that suggests that expectations in reality are probably not fully rational (Angeletos *et al.*, 2021; Reis, 2021a; Angeletos and Lian, 2022), it seems worthwhile to analyse how (imperfect) expectations affect the macroeconomic implications of public investment, which so far has not been done in the literature.

[3]Some authors, e.g. Schmelzing (2020), argue that the long-run decline in real interest rates has actually taken place across an even longer horizon.

[4]This is partially due to the fact that with low interest rates, in particular, real interest rates r below the growth rate g, government budget constraints are less restrictive, see Blanchard (2019) or Mian *et al.* (2022). In addition, from a theoretical point of view, as the economy is closer to the effective lower bound (ELB) on nominal interest rates, that constraint is expected to be binding more frequently. Theoretical arguments point to the fact that at the ELB, fiscal multipliers are relatively large (see, e.g. Christiano *et al.*, 2011). However, empirical evidence regarding this is mixed, with studies generally finding an multiplier during ELB episodes of up to 1.5, see, e.g. Klein and Winkler (2021), Ramey and Zubairy (2018) or the review by Ramey (2019).

These three points determine worthwhile research questions that will form the basic subjects of interest for this thesis. In particular, we will investigate how short-term thinking affects private firms' investment decisions – and in particular, the composition of their capital mix – and how it affects the effectiveness of macroeconomic policies – in particular, monetary policy and public investment. In a grand scheme of this literature, the research questions we attempt to address are:

1. To what extent does short-term thinking in the sense of short-termism in managers' decisions affect the capital mix of the affected firms? Do we observe a shift towards short-lived capital goods if managers' incentives become more short-term-oriented? What can we infer regarding the efficiency of investment?

2. To what extent does short-term thinking in the sense of imperfectly forward-looking expectations affect the conduct of macroeconomic policies? In particular,

   (a) in the presence of both rigid wages and imperfect expectations, how do simple rules for monetary policy compare against each other in terms of stabilisation characteristics? To what extent is this trade-off affected by the degree of 'forwardlookingness'?

   (b) how are the macroeconomic effects of productive public investment affected by this across different time horizons?

For the part on firms' investment decisions (1), we will adopt a microeconomic approach, with limited macroeconomic interpretation. On the other hand, for the two other sub-questions (2a and 2b), we will switch to a purely macroeconomic perspective; here we conduct theoretical, simulation-based research.

Outline of the thesis and preview of results

The remainder of this thesis is structured in three chapters, each of which consists in an independent research paper. Each chapter is immediately followed by the relevant appendices. Literature references for the specific topics will be given along the way. The Bibliography at the end of the thesis jointly represents the references for the entire thesis. Let us now briefly summarise the contents of the individual chapters, before we begin the actual analysis:

Chapter 2[5] presents an empirical and theoretical analysis of the effects of short-termism on the composition of firms' capital stock, in particular with respect to the durability (i.e., depreciation rate) of different investment goods. For the empirical part, we use an accounting reform (FAS 123 R, which was introduced in 2006) that ultimately altered the incentive packages of a subset of firms' managers: Before the reform, the affected managers received a relatively large share of their remuneration in terms of share options. However, firms did not have to report these payments in a transparent way, which was addressed by the reform; however, despite making incentivisation schemes more transparent, this also led to firms shifting (the relative composition of) their remuneration schemes from (equity-based) options to short-term bonuses. We show that this reduced the average duration of affected managers' remuneration scheme. Also, we find that the firms affected

[5] Chapter 2 is based on joint work with Alexander Schramm of Munich Re and Jan Schymik of University of Mannheim (both: formerly LMU Munich) and has been published as a working paper at numerous stages.

by this change in incentives consequently shifted their investment from durable goods to less durable goods. This, in turn, raised the average depreciation rate of affected firms relative to firms not affected by the firm.

Using a stylised theoretical model, we show how such a change in incentives (towards more short-term remuneration elements) tilts the capital mix chosen by a manager towards shorter-lived capital goods. Using a counterfactual analysis, later augmented by a pseudo-general equilibrium analysis, we show that this is detrimental to firms' productivity – which can also have effects in aggregate.

Chapters 3 and 4 then turn to the other dimension of short-term thinking – imperfect (macroeconomic) expectations – and attempt to answer questions regarding the impacts of macroeconomic policies. In particular, chapter 3[6] tries to answer the question how myopic expectations affect the trade-off between different monetary policy strategies, which we operationalise as simple rules. To address this question, we consider a model that incorporates myopic expectations, inspired by Gabaix's 2020 *Behavioral New Keynesian model* into a set-up with rigid prices and rigid wages. We first show how myopic expectations affect determinacy properties of the model and the power of Forward Guidance. Then, using simulations, we compare the performance of simple rules that represent (purely present-based) inflation-targeting with history-dependent rules for monetary policy, in particular price-level targeting and a number of rules implementing average inflation targeting. We find that for both demand and technology shocks, with myopic expectations and compared to rational expectations, history-dependent strategies lose some of the advantages they have relative to inflation targeting. We, however, find that exponential moving average targeting offers some advantages.

Chapter 4 turns the spotlight from monetary policy to fiscal policy, and particularly to government investment. Again, the effects of imperfect macroeconomic expectations are studied, this time with a focus on the multiplier effects government spending has on output. In order to do this, a medium-scale Dynamic Stochstic General-Equilibrium model based on an extended version Ramey (2021) is presented and augmented with imperfect macroeconomic expectations. Simulation-based evaluations are performed and a particular emphasis is put on the interaction of time delays in public investment projects with imperfect expectations of private-sector agents. We find that myopic expectations raise short-run multipliers, especially for government investment that has long time to build. The effects on long-run multipliers depend on the exact degree of myopia. Also, we study hybrid expectations that also feature a backward-looking term. We study a number of additional model features that can affect the results. In particular, we show that at the effective lower bound, multipliers can be larger than during normal times. With myopic expectations, this effect is somewhat muted, whereas with hybrid expectations multipliers can become implausibly large. Some avenues for future research are discussed.

[6]Chapter 3 is based on joint work with Michael Dobrew, Daniel Kienzler and Rafael Gerke (all: Deutsche Bundesbank; as of March 2022, M. Dobrew was in the process of transitioning to ECB). The project started out as part of the Eurosystem's strategy review while the author was an intern with Bundesbank. A select set of results has previously been published as part of a monthly report of Bundesbank (Deutsche Bundesbank, 2021).

Chapter 2

Capital (Mis)allocation, Incentives and Productivity

Based on joint work with Alexander Schramm[§] and Jan Schymik[*]. A previous version of this chapter was circulated under the name 'Capital (Mis)Allocation and Incentive Misalignment' (Schramm *et al.*, 2021), also published as part of Alexander Schramm's (2021, ch. 1) Ph.D. thesis.

Abstract

This chapter studies the impact of managerial incentives on the allocation of capital in the economy. When incentives distort private returns of capital away from the social optimum, this causes capital misallocation. We document that firms reallocate investments towards short-lived assets when a shift in incentives from equity towards bonuses lowers managerial firm ownership. To evaluate the role of incentives for capital (mis)allocation, we calibrate a dynamic model of firm investments with agency frictions. The pass-through from incentives to investments is substantial: within-firm wedges in the returns across capital goods increase by 1.1 bp for a 1%-shift in incentives.

Keywords: Corporate investment, Capital misallocation, Firm dynamics, Short-term incentives

JEL Codes: E22, G31, D24, D25, L23

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2.1 Introduction

Economists have long claimed that managers are key determinants in explaining the large and persistent differences in productivity levels across businesses (Syverson 2011, Bloom and Van Reenen 2007). However, when decision-makers in firms do not internalise the benefits of long-term investments, this can lead to capital misallocation. In this chapter, we study the impact of managerial incentives on the allocation of capital in the economy and argue that incentive contracts can be a cause of capital (mis)allocation when managerial ownership of the firm is reduced due to a shift in incentives away from equity compensation towards bonuses.

While many durable investment goods have a life span of several years, typical CEO compensation schemes of public US firms feature much shorter vesting periods of the different CEO pay components.[1] The mismatch between the horizon of managers' incentives and the durability of firms' assets suggests that there is a risk that managers opt for investment policies that are biased towards short-term investment goods as these pay off earlier. Consequently, production would be more efficient if capital expenditures were reallocated away from the capital goods with a shorter life span towards more durable capital goods.

We study this mechanism in two ways. First, we document this within-firm misallocation channel empirically. Second, we develop a model of firm investments with agency frictions that rationalises our empirical findings. To quantify the economic importance of this misallocation channel, we calibrate the model to the US economy and find that there is a large pass-through from managerial incentives to the allocation of capital.

In the first part of the chapter, we provide reduced-form empirical evidence that incentive shifts from equity towards bonuses that lower managerial ownership of the firm cause investment shifts towards more short-lived assets. The main identification challenge is that both, incentives and investment policies are endogenous firm choices. We address the endogeneity of incentives by exploiting the introduction of the FAS 123 accounting reform in the US during 2005 as a quasi-natural experiment that made equity compensation relatively costly. The reform of this policy effectively abolished an accounting advantage of equity compensation, as firms were prohibited to expense option compensation to employees at its intrinsic value and had to expense equity compensation at fair value (see Hayes *et al.* 2012). We show that this reform led to a shift in the managerial compensation structure for the treated firms in our sample. While firms could have substituted options for restricted stock, treated firms – that incentivised managers with options prior to the reform – raised bonuses and lowered equity-based compensation relative to their untreated counterparts.[2] This change in the term structure of incentives is also reflected in a shorter duration of executive pay as measured by Gopalan *et al.* (2014). Additionally, firms were allowed to accelerate unvested options to fully vest prior to the compliance date, further increasing short-term incentives. Based on these observations, we argue that the policy reform has contributed to a rise in short-term managerial incentives. Using a within-firm

[1] Gopalan *et al.* (2014) find an average duration of CEO pay of about 1.5 years, computed as the weighted average of the vesting periods of the different components of executive pay including salary, bonus, stocks and options. Following on that, a duration of 1.5 years would correspond to a depreciation rate of 66.7% which by far exceeds the estimates of capital depreciation rates from the literature (see e.g., Nadiri and Prucha 1996).

[2] This is consistent with Hayes *et al.* (2012) who document such a shift of compensation around the introduction of FAS 123R in a setting that is not based on difference-in-differences variation but on overall pay variation over time.

estimator that exploits variation across investment goods that differ in their life span allows us to estimate the impact of this change in incentives on investment distortions within firms. To empirically study the changing investment composition inside firms, we use data on the population of stock-listed firms in the US. Listed firms disclose investment expenditures across different asset categories such that we can exploit variation in durability across asset groups to distinguish between short- and long-term investments, similar to Garicano and Steinwender (2016) or Fromenteau *et al.* (2019). Combining these data on firm investments in land, buildings, machinery, transport equipment, R&D, computer equipment and advertising with information on compensation practices allows us to measure how incentives affect the capital allocation within firms.

We document that the policy-induced shift in incentives created a wedge in investment expenditures. Firms whose managerial incentives where affected by the reform shifted investment expenditures towards assets with a shorter life span compared to other firms. Our within-firm estimator – comparing investment expenditures across categories for treated and untreated firms around the reform of the policy – allows us to estimate a statistically and economically significant effect of incentives on investment policies. Moreover, we document that the observed changes in investment policies tilt capital stocks towards more short-term capital and increase firm-specific depreciation rates. Compared to untreated firms, treated firms invest 6% more into capital goods with a 10 percentage points higher depreciation rate. This shift towards more short-term assets is reflected in a 1.58 percentage-point increase of firm-specific depreciation rates causing substantial refinancing costs related to this decrease in the durability of capital stocks. We also find evidence for a decline in total factor productivity within affected firms.

In the second part of the chapter, we then develop a model of firms facing agency frictions and managers making dynamic investment decisions. Our model rationalises the empirical findings and we calibrate it to quantify the role of incentive contracting for capital (mis)allocation in the US economy. Our model builds on the neoclassical model of dynamic firm investments, similar to the models in Bond and Van Reenen (2007), Cooper and Haltiwanger (2006), Hsieh and Klenow (2009) or Bloom (2009), and we extend it in two dimensions. First, we introduce a decision-maker that faces monetary incentives from a compensation package that is composed of a fixed salary, a bonus on current profits and a share of total equity similar to Nikolov and Whited (2014). The larger is the equity share of firm value that accrues to the decision-maker (i.e. the larger is managerial ownership of the firm), the closer her incentives are aligned with value maximisation.[3] Second, we introduce two types of capital that differ in their durability, measured by different depreciation rates, in the spirit of Aghion *et al.* (2010) or Rampini (2019). Both types of capital are subject to convex capital adjustment costs and firms combine capital and labour to produce output. We show that such a compensation package based on bonuses and equity induces investment short-termism as the decision-makers' optimisation problem mirrors quasi-hyperbolic preferences (i.e. quasi-geometric discounting) which implies time inconsistency. These time inconsistencies are driven in our model by a too strong focus on current profits induced by the combination of bonus payments and equity ownership.[4]

[3]We do not derive the form of optimal contracts but instead approximate contracts that we observe in the data that may or may not be optimal. This approach allows us to identify the effects of changing incentives on firms' investment policies.

[4]Time inconsistencies from hyperbolic discounting have been studied in the context of consumption-saving problems (see e.g. Laibson 1997). Furthermore, the corporate finance literature has also suggested that myopic decision-making can lead to suboptimal equilibria (see e.g. Stein 2003).

We use our model to quantify the economic effects of managerial incentives on capital misallocation within firms and carry out an evaluation of the FAS 123 reform in this regard. We calibrate the model to match specific firm- and sector-level moments for the US economy in a simulated sample of firms prior to the reform and then simulate the effects of an unexpected, persistent shock to decision-makers' incentive structures that resembles the empirical variation around the accounting reform. From a computational point of view, our model shares many similarities with models of quasi-hyperbolic discounting, including the numerical challenges in solving them with Euler-equation-based methods (see Krusell and Smith 2003 and Maliar and Maliar 2005, 2016). As suggested by Maliar and Maliar (2016), we adapt the method of endogenous gridpoints (Carroll 2006) to solve for dynamic firm behaviour. Using this method, we are able to compute the implied effects of the reform on various firm-level variables and compare them to a counterfactual scenario without a change in managerial incentives. Even though the reform had a moderate effect on managerial incentive structures, we find that the pass-through from changes in incentives to changes in investment behaviour is substantial. Our quantification shows that firms respond to the reform with a short-run cut in investments. This is consistent with the empirical findings by Ladika and Sautner (2019) who report a reform-induced investment cut in the years after the implementation of FAS 123R. Importantly, we show that this investment cut is asymmetric across capital goods and the drop in long-term investments is substantially larger which tilts the within-firm allocation of capital toward short-term capital goods. These model-implied investment responses are quantitatively similar to their empirical counterparts and cause a substantial rise in within-firm capital misallocation – the average difference in the rates of return across capital goods increases by 3.7 basis points which corresponds to about 1.1 basis points for an average increase in short-term incentives by 1%. This within-firm shift in the capital mix away from the social optimum boosts short-run productivity due to a cut in investment expenditures but lowers productivity in the long-run. In a general-equilibrium extension, we find that this change in incentives lowered real wages by 0.2%.

Policy-makers, business executives and investors have often warned about the dangers of boosting short-term profits at the cost of long-term value (see e.g. Dimon and Buffet 2018 or Barton 2011). This chapter links the corporate finance literature on managerial incentives and the literature on macroeconomic impacts of capital misallocation and therefore relates to other papers studying short-term behaviour and its consequences for the aggregate economy. We contribute to that literature by identifying a specific microeconomic channel – short-termist incentive distortions – causing misallocation of capital inside firms leading to aggregate output losses. Our work most closely relates to Terry (2017), who shows that short-termist managerial pressures from investors can lower investment and aggregate growth. On the theoretical side, models by Bénabou and Tirole (2016) and Garicano and Rayo (2016) formulate managerial short-termism as an intertemporal version of a multitasking model in which agents must choose between projects that maximise short-term objectives versus projects that maximise long-run objectives. Similar to our model, Aghion *et al.* (2010) study an investment model with two types of capital to analyse the role of credit constraints on the composition of investment. We rely on these ideas in our investment model by letting decision-makers solve an intertemporal optimisation problem with the choice between two types of capital with different durabilities.

Empirically, Edmans *et al.* (2021, 2017) and Ladika and Sautner (2019) find that short-term incentives proxied by vesting equity are associated with a decline in total capital expenditures. Our estimated effects of incentive distortions relate to Ladika and Sautner (2019) or Glover and Levine (2015) who also study the FAS 123R reform. Asker *et al.* (2014) show that private firms, whose management is presumably less prone to short-termism, have substantially higher capital expenditures and are more responsive to investment opportunities. While these studies consider aggregate capital expenditures, our focus is on capital (mis)allocation caused by incentive distortions. Since our estimates are based on within-firm variation across investment categories, we are also able to effectively account for idiosyncratic demand or technology shocks which are absorbed by firm-year fixed effects. These adjustments via within-firm capital (mis)allocation across capital goods also contribute to the literature that discusses and quantifies causes of factor misallocation (see e.g. Hsieh and Klenow 2009, Alder 2016, Kehrig and Vincent 2019, Midrigan and Xu 2014, David and Venkateswaran 2019 or Peters 2020).

The remainder of the chapter is structured as follows. In the following Section, we present empirical evidence on the effect of incentives on capital (mis)allocation. Section 2.3 quantifies these effects based on our model of firm investments. Finally, Section 2.4 concludes. The tables for the main text can be found behind the conclusion as section 2.5, starting on page 33. The appendices offer detailed variable descriptions, additional empirical exercises, a theoretical derivation of the quantitative model used as well as details on the solution algorithm used.

2.2 Empirical Evidence

This section provides empirical evidence how shifts in managerial incentives distort investment decisions and affect the allocation of capital within firms. Since financial incentives are chosen endogenously, our identification strategy exploits the revision of the FAS 123 accounting standard in the US and we study how reform-induced changes in incentives distorted the investment behaviour of publicly traded firms.

2.2.1 Data

Our sample combines annual data on firm investments with executive remuneration data. We focus on the sample of publicly traded US firms from 2002 to 2007 and consider seven broad investment categories which differ along their durability. Following the approach suggested by Garicano and Steinwender (2016) and Fromenteau *et al.* (2019) we consider investments in the following seven categories: land, buildings, machinery, transport equipment, R&D, computer equipment and advertising and assign category-specific depreciation rates listed in Table 2.1.

We directly obtain annual expenses on R&D and advertising from Compustat North America. Data on the remaining categories of Property, Plant & Equipment are provided by Factset. We use a perpetual inventory method to transform stock variables into annual gross investment. Negative investments and missing values are excluded from the analysis.[5] We keep only active firms in the sample and exclude utilities, financial and public sector firms in our baseline estimations as it is standard in the literature (see e.g. Clementi and Palazzo 2019, Ottonello and Winberry 2018).

[5]We show that our results are also valid if we treat negative investment as true negatives or if we set them to zero.

ExecuComp serves as our primary data source for executive compensation. Since CEOs arguably have the largest impact on the investment decisions of firms, we concentrate on the remuneration of the current CEO in the year before the reform (2004) and construct the following three proxies for treatment eligibility: a dummy indicating if the executive was awarded any stock option (*option dummy*), the share of an executive's stock option awards in his total current compensation (*option per TDC*) and his position in the respective distribution (measured in quintiles). We then merge the CEO data with the investments panel. To motivate our empirical strategy, we additionally make use of another data source of executive compensation, which is BoardEx. BoardEx offers a more detailed listing on the individual components and time-structure of manager remuneration than ExecuComp, which comes at the costs of having less matches with our investment sample.[6]

Table 2.2 lists selected summary statistics. Our comprised sample entails about 700 firms. Most of firms' resources are on average spent on machinery, R&D and advertising, whereas a smaller proportion goes into land and IT investment. The relatively high standard deviation and the large heterogeneity in expenditures per category do not only reflect differences in the investment pattern across firms but also imply lumpiness on the firm level as it is well documented in the literature (see e.g. Doms and Dunne 1998). Overall, each investment category seems to play a substantial non-negligible role for the investment policy of a firm. The last two rows of Table 2.2 summarise the firms' compensation policies in 2004. On average, 74% of CEOs were awarded stock options and about a third of total CEO compensation falls to option grants. Thus, awarding stock options is a widely and strongly used method in CEO compensation.

2.2.2 Empirical Strategy

This Section outlines our empirical strategy. We describe how the revision of FAS 123 changed managerial incentives. We then examine how this reform-induced increase in short-term incentives affected the investment behaviour around the reform, in our main analysis.

Institutional Background: Changes in Accounting Rules for Equity Payments

To study the causal effect of short-term incentives on the allocation of capital, we exploit an unexpected and unprecedented change in accounting practices for US firms caused by the revision of FASB Statement No. 123 (FAS 123R). In December 2004, the Financial Accounting Standard Board (FASB) revised this practice that establishes standards to account for transactions in which an entity exchanges its equity instruments for goods or services. The revision then became effective for companies with their first full reporting period beginning after June 15, 2005.

The principal reason for revising this accounting rule was to remove an accounting advantage that affected the issuance of equity-based employee compensation leading to potential misrepresentation of economic transactions. Before the reform, companies were allowed to expense equity compensation to employees at its intrinsic value, i.e. the difference between the stock price on the granting date and the strike price. This had the

[6] See Appendix 2.A.1 for a comprehensive and detailed description of the variables used in the empirical analysis.

consequence that equity-linked compensation could often be granted without causing according accounting expenses. For example, options with a strike price equal to current stock prices had no intrinsic value and therefore did not show up as an expense. After introduction of the reform, firms were obliged to expense option compensation at fair value which effectively abolished this accounting advantage of equity compensation. Other stated reasons for this revision were to simplify US Generally Accepted Accounting Principles (GAAP) and to make them more comparable with international accounting rules by moving towards fair-value accounting.

There are two channels how FAS 123R has shortened the horizon of incentives for option-paying firms. First, as the costs of equity compensation increase, firms might want to substitute towards other forms of incentive compensation such as paying bonuses on profits. As profits are inherently more short-term than equity value, this distorts incentives towards the presence. Second, as part of the reform, the FASB also allowed firms to accelerate unvested options to fully vest prior to the original compliance date in order to swiftly move towards a fair-value accounting for equity compensation. This policy change particularly incentivised firms to accelerate the vesting of slightly in- as well as out-of-money options, which gave rise to an additional source of short-term managerial incentives caused by the reform (see Ladika and Sautner 2019 and Edmans *et al.* 2017). In Subsection 2.2.2, we show that incentives for managers in treated firms indeed became more short-term compared to the incentives that managers of control-group firms faced.

Identification of Within-Firm Distortions in Capital Allocation

To identify the effects of managerial incentives on investment decisions, we compare the investment behaviour of firms that were affected by the reform to the investment behaviour of unaffected firms during the time span around the revision of FAS 123 in 2005. We consider all firms that compensated their CEOs with options in the pre-reform year 2004 as the set of treated firms. We consider these firms as affected for two reasons. First, the costs of equity-linked compensation effectively increased for firms that compensated managers with options before FAS 123R while firms that did not choose to offer options before 2005 did not necessarily face any additional costs. Second, firms that compensated managers with options before FAS 123R were allowed to let these options vest earlier, effectively reducing the duration of executive compensation while non-option-paying firms remained unaffected.

We estimate the following within-firm triple-differences specification where $invest_{ict}$ denotes a measure of investments by firm i in investment category c at time t:

$$invest_{ict} = \beta_1 \times Shock_t \times X_{i,2004} \times \delta_c + \beta_2 \times X_{i,2004} \times \delta_c + \lambda_{it} + \lambda_{c/t} + \varepsilon_{ict}. \qquad (2.1)$$

Our sample includes firms' expenditures on seven investment categories c: advertising, computer equipment, R&D, transportation equipment, machinery equipment, buildings and land. The parameter of interest is β_1 which identifies a distortion in the relative composition of firm investments created by a shift in incentives due to the accounting reform. This parameter is the coefficient of the triple interaction $Shock_t \times X_{i,2004} \times \delta_c$, where $Shock_t$ is a time-specific dummy variable that equals one for years succeeding the reform (i.e. for $t > 2005$) and zero otherwise. Furthermore, $X_{i,2004}$ is our firm-specific treatment indicator, which – depending on specification – measures whether firms granted options to its CEO (baseline specification) or measures the total amount of options granted, both during the pre-reform year 2004. The term δ_c reflects the depreciation for each investment category c. Following the approach used by Garicano and Steinwender (2016) and Fromenteau *et al.* (2019), we either ordinally rank asset categories according to their time to payoff or we directly use the category-specific depreciation rate to distinguish between more long- and more short-term investments.

Importantly, if the revision of FAS 123 induces treated firms to adjust their investment composition towards short-term assets, the coefficient of interest β_1 is expected to be positive. By exploiting the change in incentives triggered by this reform as a quasi-natural experiment, we aim to capture a causal and economically meaningful effect of incentives on within-firm capital (mis)allocation.

The vector λ_{it} contains fixed effects at the firm-year level. These firm-year fixed effects absorb unobserved time-varying firm-specific factors that can affect investment decisions. Notably, these include demand shocks or technology shocks as long as they do not affect short- and long-term investments differently. Hence, our identification is based on within-firm variation across investment categories for a given time period. The vector $\lambda_{c/t}$ contains fixed effects for either investment categories c or for category-year fixed effects ct. In our baseline specifications, we restrict our sample period to the years around the implementation of FAS 123R. Either we consider a smaller time frame from 2002 to 2007 or a more extended time frame from 2000 to 2014.

Since investments are lumpy in their nature, we transform investment expenditures using the inverse hyperbolic sine function $invest_{ict} = \text{arsinh}\left(I_{ict}\right) = \ln\left(I_{ict} + \sqrt{I_{ict}^2 + 1}\right)$ in our baseline estimations. This has the advantage that we include zero investments in our estimations while we get for large investment expenditures $\text{arsinh}\left(I_{ict}\right) \to \ln 2 + \ln I_{ict}$ such that the interpretation is almost identical to a log regression. Alternatively, we also estimate (2.1) with logarithmic transformations or consider the Box-Cox transformation instead of using the inverse hyperbolic sine function.

The Effects of the Reform on Incentives

We begin our empirical analysis by illustrating that the reform indeed induced a shift of the compensation structure towards more short-term compensation for treated firms based on a difference-in-differences estimation. As documented by Hayes *et al.* (2012), the structure of CEO compensation changed substantially around the adoption of FAS 123R. For example, firms reduced the value of equity-linked compensation after the revision and increased bonus compensation at the same time. As described in the previous Subsection, we split our sample into a treatment and a control group where the former includes all firms that have granted stock options in the pre-reform year and the latter comprises all the remaining firms, respectively.[7] After having merged remuneration data provided by BoardEx with our firm-investment panel, we calculate for each firm a manager-specific measure of bonus payments by scaling the amount of bonus paid with total compensation. For the equity share we divide all equity-linked compensation by total compensation. In addition, to better capture the term structure of compensation schemes and therefore to give a more nuanced view of how FAS 123R created short-term incentives for option-paying firms, we also construct a measure of manager compensation duration in the

[7]This difference-in-differences approach is where we deviate from Hayes *et al.* (2012) who study the average effect of FAS 123R on compensation components using panel regressions. Given that our identification strategy outlined in Section 2.2.2 is based on differences in investment practices across firms, which differ by their exposure to the reform, we are interested in the differential adjustment in the firms' compensation structure in response to the reform.

spirit of Gopalan *et al.* (2014), which explicitly accounts for the payout horizon of each compensation component separately.[8]

Our empirical results in Table 2.3 reveal that the reform led to a shift in the CEO compensation structure for our treated sample firms. Compared to non-option-paying firms, we find that treated firms reduced equity-based compensation by about 13 percentage points after the reform was introduced. Furthermore, these firms raised bonus compensation by about 6 percentage points.[9] We argue that this shift of compensation away from equity-linked compensation towards other parts of incentive compensation has contributed to a rise in short-term managerial incentives as bonuses are not tied to underlying long-term equity prices but rather to more current profits. This view is further supported when we focus directly on the duration of compensation packages in Table 2.4. The estimates suggest that the CEOs of treated firms experienced an average reduction in their compensation duration due to the FAS 123 reform by almost 2 months compared to CEOs of untreated firms. Furthermore, CEOs with more durable compensation structures prior to the reform experience larger cuts in compensation duration post reform.

2.2.3 Main Results

Investment Behavior: Tables 2.5 to 2.7 present our main results of estimating Equation (2.1), showing the effects of the reform on firm investments. Table 2.5 outlines the results of the regression analysis when we use the option dummy as treatment variable $X_{i,2004}$. This binary treatment divides our sample into two groups: the treatment group of firms with management affected by the reform and the control group whose management should be less affected by the reform. Besides that, our specifications control for ex-ante differences in investment between firms with different compensation practices by interacting the measure of long-term incentives with the depreciation. Moreover, we include firm-year fixed effects as well as either category or category-year fixed effects. The interaction term of the FAS 123R dummy ($Shock_t$) and the depreciation rate is absorbed by these category-year fixed effects. Standard errors are clustered at the firm-level following Abadie *et al.* (2017).

In the first two columns, we use a simple ordering of categories as a measure of depreciation which follows the ordering of depreciation rates and ranges from 1 (land) to 7 (advertising). We are interested in the coefficient outlined in the first row which is the coefficient of the composite interaction term combining the FAS 123R dummy, the treatment indicator and the depreciation measure. We can infer that our coefficient of interest is positive and significant at the 5%-level in column 1 when we use the ordinal ranking as a measure of asset depreciation. When we include fixed effects at the category-year level in column 2 to control for aggregate trends in certain investment categories, the coefficient of interest hardly changes. In columns 3 to 6 we then assign depreciation rates as a measure of asset depreciation. Again, we estimate a positive coefficient of interest which is significant at the 5%- or 1%-level.[10] This suggests that reform-induced shifts in incentives cause a relative shift in investments towards more short-term assets. Quantitatively, the coefficient suggests that treated firms shift about 6% more investment

[8] That is, duration d of firm i at time t is calculated as $d_{it} = \frac{(bonus_{it}+salary_{it})\cdot 0+\sum_{j=1}^{N}(Restr.stock_{ijt}+options_{ijt})\cdot \tau_j}{(salary_{it}+bonus_{it})+\sum_{j=1}^{N}(Restr.stock_{ijt}+options_{ijt})}$ where τ_j is the vesting period of equity-based component j.

[9] Hayes *et al.* (2012) find an average increase in the bonus share of around 3% around the reform.

[10] Results also remain robust to including fixed-effects at the firm-category level.

to a category with a 10 percentage point higher depreciation rate compared to non-option-paying firms (columns 3 and 4). This result remains robust for an extended time period around the reform between 2000 and 2014 (column 5) or when we include firms from the utility, financial and public administration sectors into the sample (column 6).

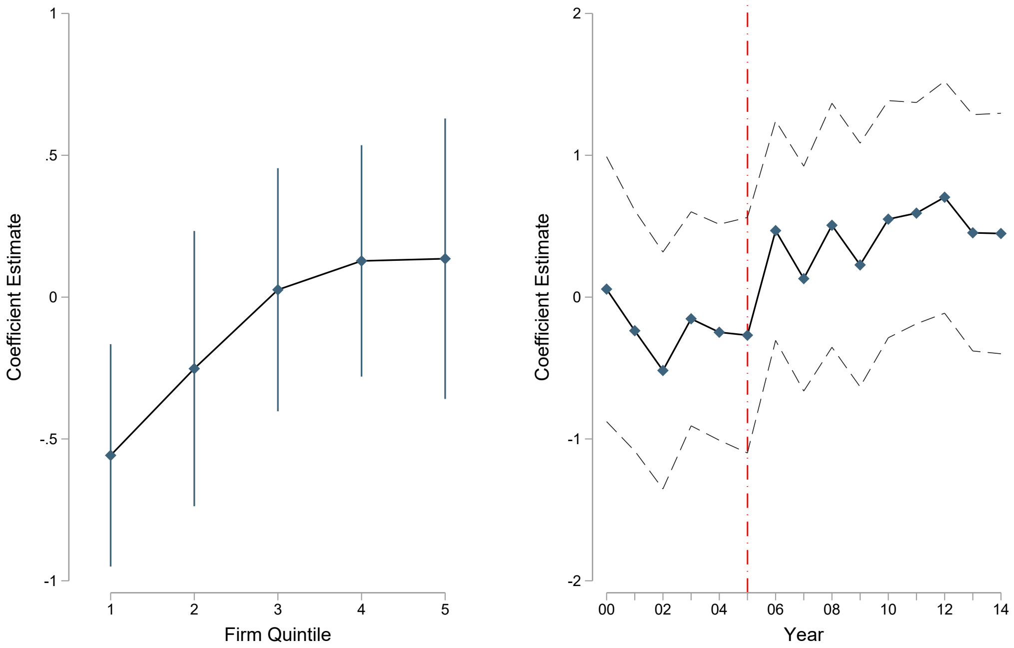

Figure 2.1: Investment Wedges by Treatment Quintiles and Years

Notes: The left graph in the Figure plots jointly estimated quintile-specific coefficients when investments are regressed on the FAS 123R dummy interacted with quintile dummies and depreciation rates. Firm-year and category fixed effects are included, standard errors are clustered at firm-level. Dashed lines illustrate 95% confidence intervals. The null hypothesis of coefficient equality at the bottom and the top quintile can be rejected at the 5%-level ($p = 0.032$). The right graph in the Figure plots time-specific coefficients when investments are regressed on the interaction between an option dummy with year dummies and depreciation rates. Firm-year and category-year fixed effects are included, standard errors are clustered at firm-level. Dashed lines illustrate 95% confidence intervals. The null hypothesis of coefficient equality before versus after the reform can be rejected at the 1%-level ($p = 0.008$).

Next, we use the option share in total compensation as continuous treatment variable $X_{i,2004}$ in Table 2.6. Also with the continuous treatment, results suggest that more affected firms shift more investment towards short-lived categories after the accounting reform. Furthermore, we group firms into quintile spells based on their respective position in the option share distribution and run bin regressions to capture non-linear effects within $X_{i,2004}$. Results are reported in Table 2.7. Again, our coefficient of interest is positive and significant throughout all specifications. The average investment wedge, measured as shift to a ten percentage points higher depreciation rate investment category, equals 1.8% for two adjacent quintiles in our most stringent specification (column 4). This result remains robust for different time horizons and sample sizes (column 5 and 6). To provide evidence that the sign of the average effect is not driven by skewness or outliers of a specific quintile, we also estimate the impact of FAS 123R on the investment mix for each quintile separately by interacting the FAS 123R dummy and the depreciation rate measure with a set of five dummy variables (one for each quintile of $X_{i,2004}$). The left graph in Figure 2.1 plots these five coefficients and illustrates that the distortion towards more short-lived investment categories increases monotonically across quintiles. We can also reject the null hypothesis that the coefficient estimate for the first and the fifth quintile are similar at the 5%-significance-level. Overall, by exploiting the accounting reform, we are able to document that exogenous increases in short-termist incentives induce more short-termist oriented investment decisions.

As a next step, we are going to study if the common trend assumption is likely fulfilled in our empirical setting. If option-paying and non-option-paying firms experience different time trends in their investments even without the accounting reform, we would wrongly attribute the observed investment wedge to the exogenous accounting reform. To rule this out, we regress investment expenditures on the interaction between annual dummies, depreciation rates and the option dummy. The right graph in Figure 2.1 plots the coefficient estimates for each triple interaction and shows that there is a distinct and permanent jump in the investment wedge in the year after the reform. Until 2005 the coefficient of the investment wedge is relatively constant and close to zero which suggests that investment patterns did not systematically differ across treatment and control firms. After 2005 the coefficients then unambiguously shift into positive terrain, remaining at that positive level until the end of our sample. The slight fluctuations between 2007 and 2010 are likely to be driven by turmoils around the Global Financial Crisis. Overall, we can strongly reject the null hypothesis that the average pre-FAS-123R coefficient equals the post-FAS-123R averages at the 1%-level.

Durability of Capital Stocks: Since we considered gross investments as dependent variable so far, the observed relative increase in short-term investments could principally be partly absorbed by the faster depreciation of these investments, such that a reallocation towards a shorter-lived capital stock within the firm does not take place in the end. To explicitly test for the effects on capital reallocation, we construct logarithmised category-specific capital stocks and include them as an alternative dependent variable in our baseline regressions. Physical capital stocks are directly obtained from Factset and intangible capital stocks are determined based on a perpetual inventory method. The results from Table 2.8 demonstrate that the introduction of FAS 123R led indeed to substantial reallocation of capital within firms. On average, option-paying firms increased the stock of a capital category with a ten percentage point higher depreciation rate by 5.2% compared to non-option-paying firms.

Related to that, we further provide evidence that the firm-specific depreciation rate of treated firms went up by the introduction of FAS 123R. To assess this, we construct a depreciation rate for each firm-year based on the relative size of each firm's category-specific capital stocks. Figure 2.2 plots the mean depreciation rate for option-paying firms, non-option-paying firms as well as their difference. While depreciation rates move in parallel until 2004, depreciation rates of option-paying firms fall less than those of non-option-paying firms do, leading to a non-trivial difference between those two groups of firms. Comparing the pre- with the post-FAS-123R depreciation rates suggests that the difference in depreciation rates increased by about 2 percentage points. We then use these firm-year-specific depreciation rates as the dependent variable and run firm-level difference-in-differences regressions. The results in Table 2.9 reveal a substantial cut in the durability of the capital stock for treated firms. Quantitatively, the depreciation rate on the average capital stock of option-paying firms increased by 1.58 percentage points compared to the control group. Ceteris paribus, this decrease in the durability of the capital stock imposes substantial costs on the affected firms. Besides the risk that these firms might suffer from productivity losses due to suboptimal factor composition, firms would have to spend more to retain the same level of capital stock as before the reform.[11] We quantify these extra cost burdens by calculating the additional financing costs required to match the level of the pre-FAS-123R capital stock. Materialised in additional interest payments, we obtain an amount of USD 15.29 per USD 1,000 invested for the affected firms.[12]

In Table 2.A.10 of the Appendix, we additionally report empirical evidence on the misallocation channel of incentives using a model-derived measure of incentives and managerial firm ownership. As an alternative to the reduced-form estimates presented here, we can instrument time variation in managerial incentives with the interaction term $Shock_t \times X_{i,2004}$. These estimates confirm that firms reallocate investments in response to changing incentives.

Total Factor Productivity: In Table 2.10, we study the impact of the reform on total factor productivity. Here, we compute TFPR as regression residuals from regressing log sales on log operating expenses and log assets (as a proxy for the firm-level capital stock) and allowing for industry-specific production function coefficients. The estimates in columns 1 and 2 suggest that treated firms experience a TFPR reduction of approximately 2% which is significant at the 5%-level.

2.2.4 Alternative Channels and Robustness Checks

Firm Size and Other Ex-ante Differences: In Table 2.A.2, we compare firms by treatment status. Treated firms are larger in terms of assets, employment and capital stock, have lower equity volatility and they pay more to their CEOs (in terms of current compensation). We illustrate that the change in investment behaviour was particularly caused by differences in managerial incentives and not by those potentially confounding factors. In principle, larger firms might invest in a different way than their smaller counterparts. In case there is an event in 2006 which affects the investment policy of large firms only, we would run into an omitted variable problem and fail to identify the true relationship between managerial incentives and investment decisions. Equivalently, higher uncertainty – proxied by equity volatility – could incentivise firms to invest more short-term. By explicitly controlling for these confounding factors in Tables 2.A.3 and 2.A.4, we are able to control for these potential confounding channels. We run regressions where we allow for two groups of interaction terms, one including the treatment variable $X_{i,2004}$ and the other including a potential confounding factor. The results in Table 2.A.3 show that the described additional channel via differences in firm size is not present. The triple interaction terms with firm size hardly explain any variation in the data and are insignificant for either proxy of firm size. We can further see that the coefficient magnitude of our original interaction term of interest remains similar. The original point estimate of 0.595 (Table 2.5, column 4) falls slightly to 0.564 when considering employment, to

[11]Given that FAS 123R affects investment decisions via distorted managerial incentives and has no direct impact on the production side of the firm, we argue here that this shift exacerbated effective factor usage.

[12]See Appendix 2.A.3 for details on the calculations.

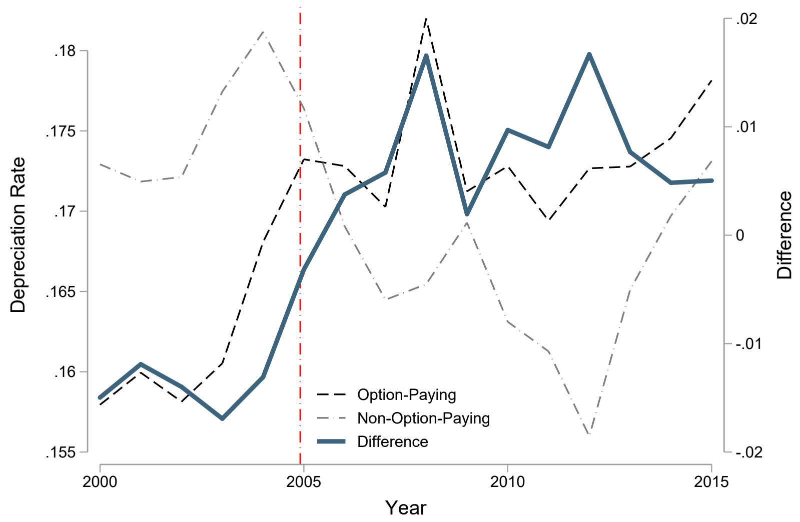

Figure 2.2: Average Firm-Specific Depreciation Rates over Time

Notes: The Figure plots the evolution of raw firm-specific mean depreciation rates for option-paying firms (black), non-option-paying firms (gray) and their difference (bold blue, right axis). Firm-specific depreciation rates are calculated as a weighted mean of category-specific depreciation rates where the weights are firms' capital stocks in the respective categories.

0.586 when considering assets and to 0.584 when considering the capital stock with similar levels of significance. In Table 2.A.4, we show that our effects also remain significantly positive after accounting for ex-ante differences in equity volatility or current CEO pay. Controlling for equity volatility suggests that firms that face more uncertainty also shifted investments towards more short-term assets while the triple interaction with the level of current CEO pay turns out to be insignificant.

Pre-Trends: In order to evaluate if pre-trends are a concern, we conduct a placebo test based on the assumption that the reform was implemented in earlier years. In Table 2.A.5, we estimate the investment distortions if the reform was implemented in 2002, 2003, 2004 or 2006 instead of in 2005. We do this by shifting the treatment variable $X_{i,t}$, the FAS 123R dummy and the sample window. We do not expect these estimates to be significantly different from zero if our baseline estimates identify investment distortions that are specifically caused by FAS 123R. Indeed, the coefficients that identify the investment distortions are insignificant in the placebo treatments.

Career Concerns and CEO Turnover: In general, it might be possible that investment decisions are CEO-specific and that incentives related to career concerns also matter. We would then wrongly attribute changes in the investment mix to changes in the compensation scheme whenever a new CEO enters the firm or whenever a CEO is replaced. We show in Table 2.A.6 that our results are not driven by CEO turnover. Focusing on a subsample that includes only firms with a unique CEO, we are able to rule out that channel. The results in Table 2.A.6 indicate that the effect is even more pronounced when we exclude firms where CEO turnover occurred. The coefficient of interest almost doubles in size and is estimated with higher precision.

Alternative Measurement of Investments: We provide additional empirical evidence that our results do not depend on a specific transformation of the explained investment variable $invest_{ict}$. Instead of applying the inverse hyperbolic sine function to investment expenditures, we run regressions using a log and a Box-Cox transformation in Table 2.A.7 that reveals similar results. We also run robustness checks where we either include negative investments in the analysis or set them to zero. The results remain qualitatively the same, the effect becomes even stronger when we include negative investments (see Table 2.A.8).

R&D Investment and Intangibles: If investments into structures cannot be directly compared with investments into intangibles, this could be an identification threat. In our baseline analysis, we classify intangible investments such as R&D or advertising as rather short-term. While this is internally consistent with neoclassical models of firm investment or calculated average depreciation rates (see Li and Hall 2020), this view is at odds with endogenous growth models where R&D creates ideas which are cumulative. Moreover, intangibles can be subject to different capitalisation rules than structures under US GAAP rules. We address these concerns in Table 2.A.9. In the upper panel of the Table, we omit R&D expenditures. Excluding R&D expenditures slightly increases the magnitude of our estimated coefficient of interest. In the lower panel of the Table, we explicitly control for differences between tangible and intangible investments by adding interactions between a dummy for intangible categories (R&D and advertising), $Shock_t$ and $X_{i,2004}$. Controlling for intangible categories also increases the coefficient of interest.

2.3 Quantitative Analysis

We now present a model of firm investments that rationalises how the shift in managerial incentives away from equity and towards bonus payments affects investments. Our starting point is a standard neoclassical dynamic investment model where firms combine capital and labour to produce output. We extend this model in the following ways. First, we assume that decisions are made by a risk-neutral manager who maximises the present value of her compensation package. This distorts investment decisions away from those predicted by a standard neoclassical model where the manager acts to maximise the value of equity and thus makes decisions that are completely congruent to shareholder interests. Similar to Nikolov and Whited (2014), we consider compensation packages that are composed of a fixed salary, a bonus based on current profits and a share of managerial firm ownership. The larger is the equity share of firm value that accrues to the manager, the more managerial and shareholder incentives are aligned. Second, we introduce two types of capital that differ in their durability in the spirit of Aghion *et al.* (2010) or Rampini (2019), measured by their depreciation rates. Both types of capital are subject to convex capital adjustment costs.

2.3.1 Model

Production: Consider a firm that uses a set of two capital inputs $\mathbf{K}_t = [K_{lt}, K_{st}]$ and labour inputs N_t. Importantly, we assume that the two capital goods differ in their depreciation rates $\delta_l < \delta_s$ such that capital inputs K_{lt} are more durable than capital inputs K_{st}. The firm uses these inputs to produce output Q_t according to a simple Cobb-Douglas production function

$$Q_t = \widetilde{Z}F(\mathbf{K}_t, N_t) = \widetilde{Z}\left(K_{lt}^{\nu}K_{st}^{1-\nu}\right)^{\alpha} N_t^{1-\alpha}, \tag{2.2}$$

where $\widetilde{Z}$ measures the firm's productivity. The firm faces isoelastic demand with elasticity ε:

$$Q_t = BP_t^{-\varepsilon}, \tag{2.3}$$

where B is a demand shifter. Combining the production function with the demand curve yields the following revenue production function:

$$R_t = P_tQ_t = Z^{1-a-b}\left(K_{lt}^{\nu}K_{st}^{1-\nu}\right)^{a} N_t^{b}, \tag{2.4}$$

where we substitute $Z^{1-a-b} \equiv B^{1/\varepsilon}\widetilde{Z}^{1-1/\varepsilon}$ such that Z captures the firm's overall business conditions. We define the terms $a \equiv \alpha(1-1/\varepsilon)$ and $b \equiv (1-\alpha)(1-1/\varepsilon)$ for tractability.

Furthermore, each type of capital is subject to quadratic adjustment costs.[13] That is, using a current capital mix of $\mathbf{K}_t$ and acquiring a future capital mix of $\mathbf{K}_{t+1}$ gives total capital-related costs of

$$C_t^K = \sum_{j\in l,s}\left[\gamma\left(\frac{K_{jt+1}}{K_{jt}}-1\right)^2 K_{jt} + q_j\left(K_{jt+1}-(1-\delta_j)K_{jt}\right)\right], \tag{2.5}$$

with q_j as the unit price of capital good j. Since we will perform partial-equilibrium analyses in what follows, we treat aggregate variables as constant and also set $q_l = q_s = 1$. Furthermore, we abstract from uncertainty regarding $\widetilde{Z}$ and B. The variable factor labour only causes variable costs of wN_t such that overall profits from the operations of the firm in period t are given by

$$\Pi_t = R_t - C_t^K - wN_t. \tag{2.6}$$

Agency Frictions and Incentive Contracts: In this model, we focus on firms with owner-manager separation. As in Nikolov and Whited (2014), we do not derive the form of optimal compensation contracts but instead approximate contracts that we actually observe in the data without making a statement about their optimality.[14] This approach allows us to identify the effects of changing contractual features on firms' investment policies, the allocation of capital and economic activity. Specifically, we assume that the total remuneration for a manager Γ_t is the sum of a fixed salary w_t^f, a bonus b_t that is some proportional share of current profits $b_t = \eta_b\Pi_t$ and equity grants E_t^m proportional to total equity E_t, such that $E_t^m = \eta_e E_t$:

$$\Gamma_t = w_t^f + b_t + E_t^m. \tag{2.7}$$

[13] Empirical adjustment costs are likely neither quadratic nor fully symmetric across different types of capital. In the calibrated version of our model, we have also examined versions with partially irreversible investment and different adjustment cost parameters γ for different capital goods. These variations do not affect our calibration results in a qualitatively meaningful way. Two additional dimensions excluded from the analysis that are potentially important are *i.* to what extent different capital goods can serve as collateral for loans and *ii.* to what extent capital goods can be rented without actually appearing on the firm's balance sheet.

[14] See Murphy (1999) for an empirical survey on CEO compensation packages.

This particular structure of remuneration packages highlights the core mechanism at hand: a part of the remuneration depends on current (short-term) profits, while another part is linked to long-term firm value. To keep the model tractable, we follow Glover and Levine (2015) in assuming that contracts only last for one period and that the manager does not start out with any pre-existing holdings of equity.[15] For future reference, it is opportune to denote managers of the firm by the period t that they are in charge of steering the firm.

Assuming a complete financial market in the background, the market value of equity E_t is given by the discounted stream of expected future cash flows. After taking into account salaries and bonuses for management, the total amount available for dividend payments in each period is given by $(1-\eta_b)\Pi_t - w_t^f$. Furthermore, we let capital markets anticipate that similar remuneration schemes may exist in the future. Hence, if the manager in charge during period $t+1$ is also expected to be awarded a share η_e of equity, shareholders in period t anticipate that the share of future total market capitalisation they hold shrinks by a factor of $1-\eta_e$, leading to share dilution.[16] With complete markets and rational expectations, equity then is valued as

$$E_t = (1-\eta_b)\Pi_t - w_t^f + \frac{1}{1+r}\mathbb{E}_t\left\{(1-\eta_e)E_{t+1}\right\}, \tag{2.8}$$

where r is the relevant market interest rate. After recursive substitution, this becomes

$$E_t = (1-\eta_b)\left[\Pi_t + \sum_{\tau=1}^{\infty}\left(\frac{1-\eta_e}{1+r}\right)^{\tau}\mathbb{E}_t\left\{\Pi_{t+\tau}\right\}\right] - \sum_{\tau=0}^{\infty}\left(\frac{1-\eta_e}{1+r}\right)^{\tau}\mathbb{E}_t\left\{w_{t+\tau}^f\right\}. \tag{2.9}$$

Using (2.9), we can rewrite the value of the manager's remuneration package as

$$\Gamma_t = w_t^f - \eta_e\sum_{\tau=0}^{\infty}\theta^{\tau}\mathbb{E}_t\left\{w_{t+\tau}^f\right\} + \varphi\left[\Pi_t + \beta\sum_{\tau=1}^{\infty}\theta^{\tau}\mathbb{E}_t\left\{\Pi_{t+\tau}\right\}\right], \tag{2.10}$$

where we define

$$\varphi \equiv \eta_b + \eta_e(1-\eta_b), \quad \beta \equiv \frac{\eta_e(1-\eta_b)}{\eta_b + \eta_e(1-\eta_b)}, \quad \theta \equiv \frac{1-\eta_e}{1+r}. \tag{2.11}$$

The term $w_t^f - \eta_e\sum_{\tau=0}^{\infty}\theta^{\tau}\mathbb{E}_t\left\{w_{t+\tau}^f\right\}$ captures the manager's fixed wage and the wage payments of her successors. This term is exogenous to the manager's decision problem such that we may ignore it in the following. This simplifies the model further such that we can consider managers' remuneration packages given by

$$\Gamma_t = \varphi\left[\Pi_t + \beta\sum_{\tau=1}^{\infty}\theta^{\tau}\mathbb{E}_t\left\{\Pi_{t+\tau}\right\}\right]. \tag{2.12}$$

[15]Considering multi-period contracts between managers and owners quickly complicates matters a lot and requires a substantial amount of further structural assumptions. These include *i.* managers' preference relation regarding payoffs at different points in time, *ii.* managers' ex-ante exposure to the firm's performance via preexisting holdings of equity, *iii.* a process linking managers' probability of staying with the firm to firm performance and *iv.* uncertainty about future remuneration packages. All these assumptions on their own would have important consequences regarding the overall term-structure of the managers' decision problem.

[16]The fact that equity-based compensation can lead to share dilution is a well known fact in finance (see, e.g., Asquith and Mullins 1986, Huson *et al.* 2001, Core *et al.* 2002). In the model context, this implies that managers' overall share in market capitalisation would converge to 100% eventually if they were to remain employed infinitely by the firm. This aspect counteracts discounting and could lead to non-trivial time preferences.

Decision-Making: The payout profile represented in (2.12) resembles the preferences that a risk-neutral agent with quasi-hyperbolic time preferences for profits would have. In other words, incentivizing managers with a combination of both, bonuses on current profits and equity payouts induces decision-making that is present biased. Furthermore, managers' optimisation problem in period t_0 inherently depends on the expected behaviour of their successors in future periods and the behaviour of a current manager directly affects the feasible set of outcomes of its immediate successor. Essentially, different generations of managers play a dynamic game with one another: each manager chooses a factor mix $(\mathbf{K}_{t+1}, N_t)$ to maximise her own remuneration taking into account previous managers' decisions and expectations regarding future behaviour. We focus on Markov-perfect equilibria with stationary, smooth strategies, where each manager's decision only depends on her inherited capital stock.

Deriving the demand for the freely adjustable factor labour is straightforward and yields

$$N_t = \left(\frac{bZ^{1-a-b} \left(K_{lt}^{\nu} K_{st}^{1-\nu} \right)^a}{w} \right)^{\frac{1}{1-b}}. \tag{2.13}$$

Equation (2.13) gives a standard labour demand relation equating marginal costs and the marginal revenue product of labour.

In the presence of capital adjustment costs, it is not possible to analytically solve for the policy functions regarding the capital goods. However, we can implicitly characterise a time-invariant policy function, assuming that the policy functions of all managers just depend on the current capital goods and on expectations that future managers will behave in the same way. We denote this function as $\mathcal{K}(\mathbf{K}) = (\mathcal{K}_l(\mathbf{K}), \mathcal{K}_s(\mathbf{K}))$. Here, $\mathcal{K}_j(\mathbf{K})$ is the policy function for capital good $j \in \{l, s\}$. I.e., in period t a manager whose firm starts with capital stocks $\mathbf{K}_t = (K_{lt}, K_{st})$ chooses $K_{j,t+1} = \mathcal{K}_j(\mathbf{K}_t)$. The function $\mathcal{K}(\cdot)$ is then the solution to the manager's first-order conditions. Hence, with a slight abuse of notation, in period t, the policy function will be the solution $\mathbf{K}_{t+1}$ of the following self-referencing characterisation for j:[17]

$$0 = \frac{\partial \Pi_t}{\partial K_{j,t+1}} + \beta\theta \frac{\partial \Pi_{t+1}}{\partial K_{j,t+1}} + \theta(1-\beta) \sum_{k=l,s} \frac{\partial \mathcal{K}_k(\mathbf{K}_{t+1})}{\partial K_j} \frac{\partial V(\mathcal{K}(\mathbf{K}_{t+1}))}{\partial K_k}. \tag{2.14}$$

Here, the term $V(\cdot) := [\Pi_t + \theta V(\mathcal{K}(\mathbf{K}_t))]|_{\mathbf{K}_t}$ represents a recursive continuation value, conditional on the current choice of capital inputs. This capital-specific Euler equation (2.14) takes into account the strategic dependence of future behaviour on current decisions. The first two elements are fairly standard: the first element incorporates the current costs of investment (including the unit prices of capital goods and the marginal adjustment costs), the second term represents the marginal returns in the next period, discounted by $\beta\theta$, adjusted for depreciation. The final term is a peculiarity of our model and other models with quasi-hyperbolic time preferences. This term captures the marginal effect on equity via changes in future investment behaviour. Both, the unknown gradients of the capital policy functions $\frac{\partial \mathcal{K}_k(\mathbf{K}_{t+1})}{\partial K_j}$ for $j, k \in \{l, s\}$ as well as the unknown gradient of the continuation-value function $V(\cdot)$ are relevant to evaluate the effects of future investment on equity value. Whenever managers are compensated with a combination of bonuses and equity (which implies that $\beta \neq 1$), this last term does not cancel out such that this cannot

[17] The derivation of the optimality condition (2.14) is relegated to Appendix 2.B.1.

be solved analytically and requires to be approximated numerically within the calibration exercise.

Discussion: The direct effects of managerial incentives on corporate investments modeled in this chapter are captured by the terms β and θ introduced by the compensation package. The investment policy of a decision-maker that maximises the long-term firm value corresponds to terms $\beta = 1$ and $\theta = \frac{1}{1+r}$. Intuitively, the term $\beta < 1$ induces the manager to behave as if she was solving some quasi-hyperbolic optimisation problem. This behaviour arises from the fact that the compensation structure in (2.7) causes a short-term bias for the manager since current profits are rewarded by both, equity ownership and bonus payments. Increasing the bonus share η_b and lowering the equity share η_e decreases β and increases her bias towards optimizing current profits. Furthermore, the term $\theta < 1$ incorporates a dilution factor arising from the manager taking into account that her equity ownership will be diluted by future managers that will also be incentivised with equity. With equity-based remuneration, share dilution affects long-term investors' holdings of the firm's stock. This implies that for any $\eta_e > 0$, future income streams are more strongly discounted than purely at the market interest rate since $\theta < \frac{1}{1+r}$.

While our model allows for fairly rich dynamics on investment patterns and firms' capital stocks, we abstract from other factors that typically vary over time and affect investment decisions. One of these abstractions is risk-aversion. While it is difficult to measure the extent of an individual manager's risk-aversion, a risk-averse decision-maker could likely have an even stronger preference to tilt the within-firm capital allocation further towards short-term assets as these assets expose the decision-maker to less uncertainty in the future. We also neglect the role of convexity in compensation schemes and the behaviour associated with it. While this simplifies our quantitative analysis, Hayes *et al.* (2012) provide empirical evidence that the change in convexity induced by the reform of FAS 123 had little impact on CEOs' risk-taking behaviour.[18] Another aspect that we abstract from in the baseline quantification is the consideration of general-equilibrium effects. Since factor prices could adjust in general equilibrium, this would explicitly allow for feed-back effects into other decision-makers' investment decisions even though their incentives might have remained unchanged. As a robustness check, we study a general-equilibrium extension of the model that takes these price-effects into account. This general-equilibrium extension comes at the cost that we have to abstract from aggregate dynamics such that we only compare steady-state equilibria.

2.3.2 Model Quantification

With the help of our model, we aim to quantify the effects of a shift in incentives on the capital allocation of firms and its associated economic outcomes. We calibrate the model to match certain features of public US companies and industry characteristics before the introduction of FAS 123R. We then assume an unexpected shock to β that is consistent with what we observe in the data around the reform.[19] Industry-specific information

[18]Related to that, Bebchuk and Fried (2010) discuss how equity-based compensation packages can be designed to achieve strong ties to long-term results.

[19]In this exercise, we do not alter θ to focus ideas purely on the effect of a relative shift in the duration structure of managers' remuneration. That is, in terms of the model we effectively consider a shock to η_b.

is obtained from the US files of the EU KLEMS database for 2003–2005, for firm-level remuneration data we rely on Execucomp and Coles *et al.* (2006).[20]

Calibrating Incentive Contracts: We consider a sample of 1,000 firms that draw a pre- and a post-FAS-123R value for β that match the observed distributions of β in the years 2005 and 2007 from a discretised distribution taking observed transition probabilities into account. The calculation of the structural parameter β follows Equation (2.11) and is determined by the bonus share η_b and the equity share η_e. For the construction of η_b, we scale the sum of bonuses and non-equity incentive compensation by firm sales. The equity share is constructed by scaling managers' equity-linked firm wealth by their employing firms' market capitalisation.[21] We then discretise the distribution of β into ten bins varying from 0.75 to 1.0 in steps of size 0.025. Table 2.12 provides the observed transition probabilities across bins, the changing distribution of β is plotted in Figure 2.3. The histograms illustrate the shift of compensation packages away from equity around the reform: drawing a large value for β became less likely after the reform. Moreover, the transition matrix also suggests that there is substantial path-dependency as the diagonal elements (i.e. the probabilities of remaining within a certain bin) show values between 63.6% and 90.15%. Path dependency seems to matter in particular at the outer bounds of the distribution as the probability of remaining within a bin is highest for the bottom and the top bin. Overall, the sample mean value for β falls by about 2.8 percentage points from 0.918 to 0.890. This decline in β is driven by both, a reduction in the share of equity compensation (η_e) and an increase in the average bonus share (η_b). Moreover, about 70% of the firms remain in the same bin for β, while about 19% move to a bin with a higher value for β and only 11% enter a lower β-bin. Thus, the incentive structure of managers has shifted slightly, but noticeably, in the period around the reform.

In Table 2.A.10 of the Appendix, we link the constructed structural parameter β back to our reduced-form estimates. There, we estimate that reductions in β are indeed associated with a shift of investments towards more short-lived capital goods. Moreover, we use $Shock_t \times X_{i,2004}$ as an instrument for β to confirm that the reform-induced shift in incentives caused a more short-term investment behaviour.

Other Parameters: We assign each firm to a specific industry taking the sise composition of industries in the US according to OECD data on the number of firms by sector into account. We assume that the measure for firms' overall business conditions Z is composed of a industry-wide demand condition $B = B^{ind}$ and a firm-specific TFP component $\tilde{Z} = Z^{firm}$ according to $Z = (B^{ind})^{\frac{1}{\varepsilon}}(Z^{firm})^{\frac{\varepsilon-1}{\varepsilon}}$. For each industry, we use value added as a proxy for revenues,[22] the total stock of both types of capital, average depreciation rates for both types of capital, the average wage paid to employees and the number of employees. For information on the industries used and the corresponding values for the variables, we refer to Table 2.11. Each firm is characterised by a vector of three i.i.d. random draws which determine Z^{firm}, the manager's incentive structure determined by β and the equity ownership share η_e. The wage rate and the depreciation rates for short- and long-term capital goods are directly inferred from the industry draw. We use stan-

[20]See Table 2.11 for an industry overview.

[21]Details on the computation can be found in Appendix 2.C.1.

[22]We could, of course, explicitly consider a production function with intermediate inputs, but this would complicate the analysis without materially affecting the mechanism studied here.

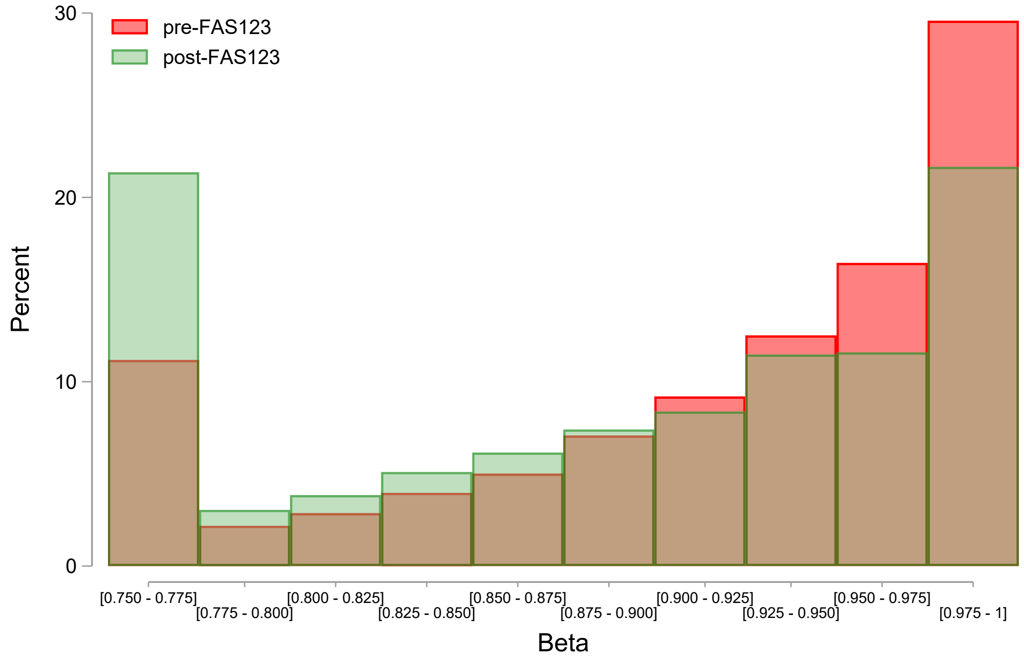

Figure 2.3: Changing Incentives Around FAS 123R

Notes: The Figure depicts the empirical distribution of the β parameter before (red) and after (green) FAS 123R. Distribution overlap is illustrated by the brown area. We group βs into ten bins each ranging 2.25 percentage points. Data is left-censored at 0.75, which applies to 14.39% of the observations.

dard values from the literature for the adjustment-cost parameter γ and the interest rate r.[23]

Next, the scale parameter B^{ind}, the factor shares a and b for capital and labour, and the long-term capital share ν have to be calibrated. Here, we calibrate the parameters $B^{ind}, \alpha, \nu, \varepsilon$ to the benchmark case $\beta = 1$ such that the values for the labour-to-output ratio $\frac{wN}{R}$, the share of long-term capital in total capital $\frac{K_l}{K_s+K_l}$, the capital-to-output ratio $\frac{K_l+K_s}{R}$ and the overall scale of operations R match those of the respective sector in the data.[24]

The idiosyncratic scaling factor Z^{firm} is drawn from a random distribution, where we assume the logarithm of Z^{firm} to be normally distributed around a zero mean and a standard deviation of 0.52, which is what İmrohoroğlu and Şelale Tüzel (2014) find for the productivity dispersion in Compustat data.

We then solve the model for each firm individually. Since the incentive structure in the model features a present-bias ($\beta < 1$) and decision-makers face capital adjustment costs ($\gamma > 0$), our model resembles a quasi-hyperbolic discounting problem such that solving it involves similar challenges as those documented in previous papers on neoclassical

[23] For γ we follow Bloom (2009, Table III) and choose 4.844. The interest rate r is set to 2.98%. A detailed discussion can be found in Appendices 2.C.2 and 2.C.3.

[24] This approach implies that the simulated sample is not exactly representative of the empirical sample because the observed average of the firms' β is below 1. However, this is the only way of calibrating the parameters analytically. Also the relative size of the effects is not altered in a materially important way by this strategy.

growth models with quasi-geometric discounting (e.g. Krusell and Smith 2003, Maliar and Maliar 2016).[25] In particular, as the generalised Euler equation for capital does not have a specific closed-form solution, we resort to numerical methods. Since Euler-equation methods are likely to fail (cf. Maliar and Maliar 2016), we use a version of the endogenous gridpoint method first introduced by Carroll (2006). This method works similar to backward induction: for a fixed number of possible future stocks of both types of capital, one solves the managers' optimality conditions for current stocks. This procedure essentially constructs inverted policy functions from which we can back out the dynamics for each firm.

2.3.3 Results

Relation to the Empirical Estimates: We begin by replicating the reduced-form regressions using our simulated sample. Table 2.13 reports estimates using the simulated sample of firms. In contrast to the empirical sample, these data only contain two types of capital. Furthermore, the treatment indicators used in the estimations here is either a dummy indicating whether the firm experienced a reduction in β or the continuous value of β in the pre-reform period. We find the magnitude of the investment distortion to be very similar compared to the empirical counterparts, even though we did not target the coefficient estimates in the parameterised version of the model. When using the dummy as treatment indicator in columns 1 and 2 of Panel A, we obtain a coefficient of 0.426 which almost equals the counterpart based on the empirical sample (0.595 in columns 3 and 4 of Table 2.5). In the two subsequent columns of Panel A, we consider the respective capital stocks as dependent variable and thereby replicate the reduced-form regressions from Table 2.8 (columns 3 and 4). The coefficients of interest from both regressions are of similar magnitude here as as well. In columns 1 and 2 of Panel B, we then use the continuous treatment variable and again find coefficients of similar size compared to the empirical counterparts given in Table 2.6 (columns 3 and 4). Given the close replication of the empirical estimates, we feel confirmed that our calibration approach is suitable to quantify the effects of managerial incentives on production, investment and capital misallocation.

Capital Adjustments within Firms: In a next step, we use our simulated firm panel to analyse the dynamic within-firm adjustments of capital in response to the shift in incentives. These are depicted in Figure 2.4. The graph at the upper left in the Figure plots investments into short- and long-term capital goods, each normalised by their respective capital stocks. Firms respond to the reform with a short-run dip in investments in both capital goods. This investment dip is consistent with empirical findings by Ladika and

[25] In the case without adjustment costs ($\gamma = 0$), a simple equilibrium is straightforward: since managers' utility is modelled as linear and markets are complete, the choice of $\mathbf{K}_t$ by manager $t-1$ only acts as a level shift to current profits. Hence, manager t's marginal calculations are separate from the current state of the capital stock. As such, the manager could simply choose an arbitrary value of $\mathbf{K}_{t+1}$ irrespective of $\mathbf{K}_t$. If all managers follow such a strategy, the gradients of the policy function are zero everywhere. In anticipation of this, future behaviour cancels out of the model equations and the optimality conditions (2.14) for each capital good $j \in \{l, s\}$ simplifies to

$$1 = \beta\theta \left[\frac{\partial R(K_{lt}, K_{st}, N_t,)}{\partial K_{jt}} + (1 - \delta_j) \right].$$

Sautner (2019), who report a reform-induced investment cut in the years directly after the introduction of FAS 123R. As expected, the results show that this cut in investments is highly asymmetric across both investment goods. Consistent with our empirical findings, the reform causes a distortion in investments across assets with different life span. While short-term investments are only reduced by about 0.5% on average, the drop in long-term investments appears substantially larger around 2.6%. This heterogeneous response in investments results in a shift of the within-firm capital stock towards relatively more short-term capital. This can be observed in the upper right graph of Figure 2.4 which depicts the share of short-term capital in percent of long-term capital goods. On average this fraction is 82.3% in t_0 and increases about 0.7 percentage points in response to the reform.

As gross investment falls in the aggregate, this leads to a reduction in firms' total capital stock by around 1.1% on average, which is illustrated in the lower left graph of the Figure. The lower right graph depicts total gross investment normalised by the total capital stock. Again, one can observe the immediate reduction in the investment ratio (by about 1.1%) directly after the reform that already became apparent in the upper graph showing investment into individual capital goods. Interestingly, the long-run steady-state level of total gross investment relative to the capital stock slightly *increases* compared to pre-reform levels. This effect is driven by the within-firm reallocation of capital. Since the capital composition shifts towards short-term capital goods and these deplete faster, the average depreciation rate of capital increases. Consequently - in relative terms - larger re-investments are necessary.

Misallocation, Output and Productivity: Next, we consider the effects of the change in incentives on misallocation, output and productivity which we illustrate in Figure 2.5. In order to make a statement on the economic relevance of such a relatively mild shift in the within-firm composition of capital stocks, we compute the distortion of marginal revenue products across investment categories within firms, inspired by Hsieh and Klenow (2009). Specifically, we define the marginal product gap within a firm as

$$MPG_t \equiv |MPK_{st} - MPK_{lt}|, \tag{2.15}$$

where MPK_{jt}, $j \in \{l, s\}$ is the sum of the marginal revenue product of a capital good and its resale value $(1 - \delta_j)$ such that the marginal product gap MPG_t captures the wedge in the different rates of return across capital goods within firms. The graph at the upper left of Figure 2.5 plots this measure of within-firm misallocation of capital. It shows that the relatively moderate shift in the composition of capital stocks causes a very substantial rise in within-firm capital misallocation. Since short-term capital goods have higher depreciation rates those capital goods can adjust relatively faster which explains the spike in the marginal product gap followed by a slight reduction afterwards. This can also be seen in the change of the curvature of the relative capital stocks from convex to concave (upper right graph in Figure 2.4). The within-firm wedge in the rates of return across capital goods increases in the long-run by about 3.7 basis points.

We then quantify the effects of the within-firm capital misallocation channel on economic output and total factor productivity. Based on the underlying Cobb-Douglas production function (2.2), output falls by about 0.5% on average. Due to the homogeneity of the production function and labour being a freely adjustable production factor, the observed relative decline in output is similar to the employment change. The graph at the lower left depicts changes in total factor productivity. We define revenue total factor

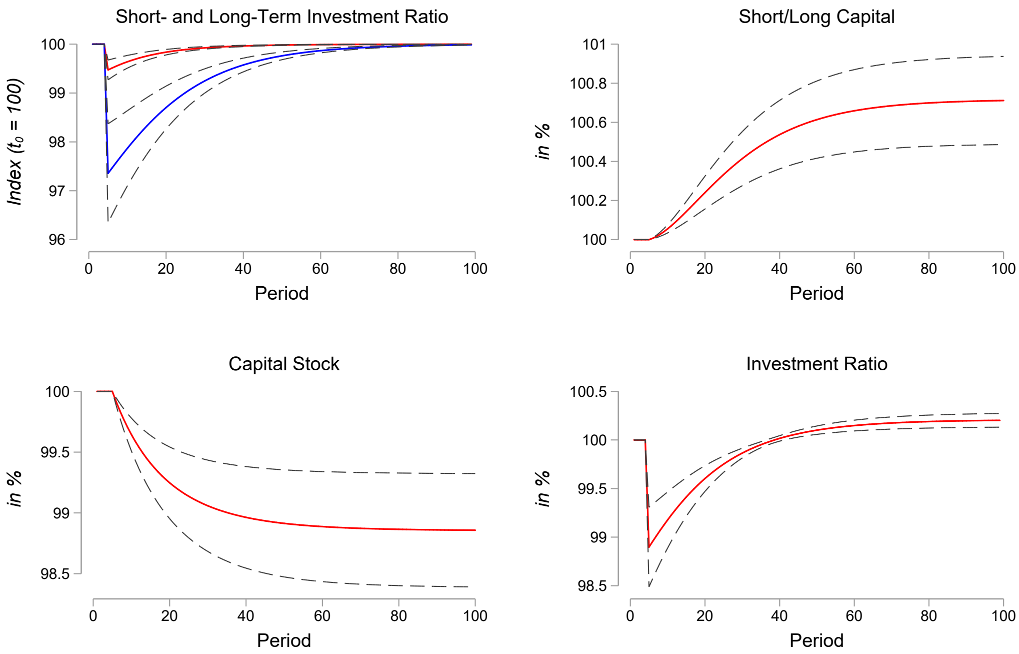

Figure 2.4: Capital Adjustments within Firms

Notes: The Figure depicts the dynamic adjustment process for short-term investment in red and long-term investment in blue (top-left), the capital ratio (top-right), the capital stock (bottom-left) and the total investment ratio (bottom-right). Investment ratios are normalised by their respective capital stocks. We normalise each of the responses with respect to their pre-shock values. The average adjustment is illustrated by solid lines, dashed lines depict 95% confidence intervals.

productivity here as

$$TFPR_t \equiv \frac{R_t}{\left(C_{lt}^{\nu} C_{st}^{1-\nu}\right)^{\alpha} N_t^{1-\alpha}}. \tag{2.16}$$

A short-run spike in productivity by about 0.3% on average becomes evident. This short-run productivity spike is driven by the sudden cut in investments and the according reduction in capital costs. Productivity then declines in the long-run by about 0.2% on average as the within-firm capital mix shifts away from the social optimum. The finding that the motive to raise short-term profits at the expense of long-run macroeconomic growth matters in the aggregate is also in line with Terry (2017) who finds that short-termist incentives cost 6% of output in the long-run. Compared to this finding, the impact of the reform on output is indeed substantial, even though its direct effect on incentives has been moderate.

Finally, we use our model to analyse the effects of the reform on misallocation across firms. Since the FAS 123 reform only affects incentives and investment choices of some managers while other firms remain unaffected, the change in accounting rules is likely to raise misallocation across firms. In the graph at the lower right, we plot the cross-firm dispersion in the capital mix of short- relative to long-term capital by normalizing the standard deviation of the capital ratio across firms with the initial standard deviation before the reform. It is evident that the cross-firm dispersion in the capital ratio increases by about 1.3% after the reform, speaking to the fact that firms become more heterogeneous in terms of their factor endowment. Given that FAS 123R has no direct effect on the marginal productivity of capital goods, such a reallocation of capital across firms should not have been taken place from a social-planner point of view. We therefore interpret this increase in firm heterogeneity with respect to capital endowment as indirect evidence for more cross-firm capital misallocation as firms are more unevenly endowed with short- and long-term capital after the reform.

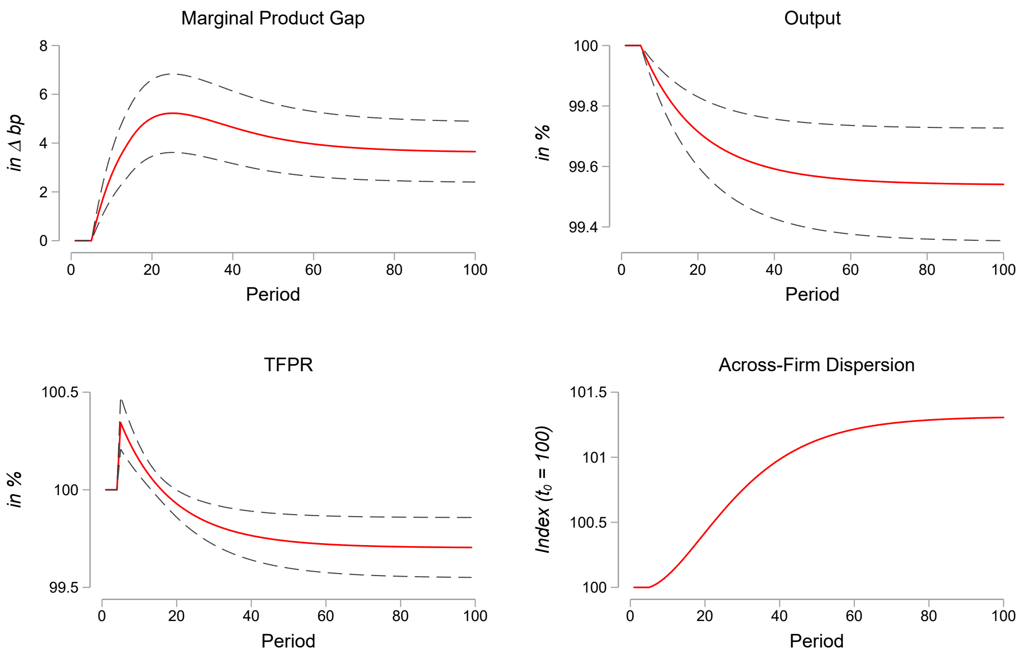

Figure 2.5: Misallocation, Output and Productivity

Notes: The Figure depicts the dynamic adjustment process for the within-firm gap in marginal revenue products of capital goods (top-left), output (top-right), revenue TFP (bottom-left) and the across-firm dispersion (s.d.) of the capital ratio (bottom-right). We normalise each of the responses with respect to their pre-shock values. The average adjustment is illustrated by solid lines, dashed lines depict 95% confidence intervals.

Robustness to General-Equilibrium Effects: We next study to what extent the previous results are robust once we account for general-equilibrium effects. When the reform increases firms' demand for short-term capital goods, some parts of the within-firm misallocation of capital could be mitigated by increases in factor prices. Furthermore, when firms produce at higher marginal costs due to a sub-optimal capital mix, final-good prices might increase leading to lower welfare. At the same time, demand shifts away from short-termist firms because consumers can substitute towards cheaper goods. To study these effects, we use the same sample of firms as before but endogenise factor markets and demand for final goods. In this (pseudo-)general-equilibrium extension, goods produced by the firms within each sector are combined into a CES bundle. The various sectoral bundles are then combined into an aggregate Cobb-Douglas final good. Regarding factor markets, we assume that all costs related to gross investments are created from using labour and we impose factor-market clearing by equating aggregate labour demand with a fixed labour endowment. The demand shifter B^{ind} now becomes an endogenous equilibrium object and we use labour as the numéraire such that the wage rate is normalised to 1 and homogeneous across sectors. Compared to the partial-equilibrium analyses, the disadvantage of this approach is that we can only compare implied aggregate steady states before and after the reform and thus neglect dynamic adjustments around the reform. Details on the treatment of the general-equilibrium effects can be found in Appendix 2.B.2.[26]

As before, firms differ along the following dimensions: each firm is assigned to one out of 13 sectors, which determines most model parameters and the CES basket into which the firm's output is included. Additionally, each firm draws an idiosyncratic TFP, as well as their own β, η_e and η_b, where we use the same transition of firm-specific βs as in the partial-equilibrium setting before.

In Table 2.14, we present the counterfactual effects of our simulated reform on a set of aggregate variables. In each case, the presented numbers are relative changes compared to the steady-state value before the reform. Remember that the shock on managerial incentives induced by FAS 123R has been rather moderate with an average decline in β by roughly 2.8 percentage points (about 1 percentage point if we consider the discretised distribution of β). In the previous partial-equilibrium exercise, this shock was associated with a substantial gap in the marginal products of capital causing a drop in output, capital stocks and a relative shift in investment from long-term to short-term capital goods. These findings carry through to our general-equilibrium analysis here, albeit the effects are quantitatively smaller due to the counteracting general-equilibrium adjustments. Aggregate output drops by about 8 basis points. If we compare the change in aggregate capital stocks, we see that the general-equilibrium change is about one third smaller than the partial-equilibrium change: while the capital stock falls by 0.81% in general equilibrium, it falls by 1.1% in partial equilibrium. Furthermore, the reduction of total investments is somewhat smaller (-0.59%) than the drop in the overall capital stock as firms need to reinvest more frequently due to the shift in the capital mix away from more durable capital goods. This shift can also be observed in the larger decline in long-term investments compared to the decline in short-term investments. Lastly, the general-equilibrium exercise allows us to determine the effects of the reform on the aggregate price level of the final good and hence on the real wage and thus welfare in the economy. Here, we observe an increase in the price level of about 17 basis points, which translates to an equally-sized decline in the real wage caused by the reform.

2.4 Conclusion

In this chapter, we studied how managerial incentives affect the allocation of capital. We provided empirical evidence showing that firms systematically shift investment expenditures towards less durable assets in response to a shift towards more short-term incentives from equity towards bonuses that lower managerial firm ownership. To quantify the impact of such incentive distortions on capital (mis)allocation, we then calibrated a dynamic model of firm investments in which managers determine investment policies and face typical incentive contracts.

[26] In this extension, we abstract from firm entry and exit and still assume managers' remuneration packages as exogenously given. As such, we denote this extension a pseudo-general-equilibrium framework.

Our results indicate that even relatively small deviations in incentives away from equity compensation like those induced by the FAS 123 accounting reform can cause substantial economic distortions. Firms cut their investments into long-term assets and within-firm capital misallocation increased due to a mismatch in decision-makers' private marginal products of capital and social marginal products of capital, causing a fall in output, capital stocks, productivity and real wages. We conclude that corporate decision-makers' incentives are crucial for economic policy-making as managers respond very sensitively to changes in their incentives which affects economic outcomes.

2.5 Tables for Chapter 1

Table 2.1: Assigned Depreciation Rates

Category	*Land*	*Buildings*	*Machines*	*Transport*	*R&D*	*Computer*	*Advertising*
Depreciation	0%	3%	12%	16%	20%	30%	60%

Notes: Assigned category-specific depreciation rates following Garicano and Steinwender (2016) and Fromenteau *et al.* (2019).

Table 2.2: Selected Summary Statistics

Variable	Mean	Std. Dev.	Min	p25	p50	p75	Max	Obs	Sample
Firm-Investment Data									
Land	33.45	192.64	0.00	0.10	1.95	9.99	3929.20	2126	2002 - 2007
Buildings	118.60	526.41	0.00	3.77	15.46	59.81	10 978.46	3027	2002 - 2007
Machines	461.21	2264.74	0.03	20.09	78.71	291.36	78 706.20	2997	2002 - 2007
Transport	143.19	622.46	0.00	0.50	2.16	19.60	7587.88	409	2002 - 2007
Research	282.71	956.11	0.00	2.74	28.33	128.15	12 183.00	2765	2002 - 2007
Computer	101.20	386.99	0.19	9.86	21.49	77.30	7800.70	602	2002 - 2007
Advertising	261.27	663.45	0.00	7.95	40.95	169.00	7937.00	1884	2002 - 2007
Compensation Data									
Option per TDC	0.33	0.27	0.00	0.00	0.32	0.53	0.99	696	2004
Option Dummy	0.74	0.44	0.00	0.00	1.00	1.00	1.00	696	2004

Notes: Investment expenditures are denoted in million USD. Option per TDC is calculated as the value of all granted options divided by total current compensation. Option Dummy takes 1 if any options are awarded, zero otherwise.

Table 2.3: The FAS 123R Accounting Reform and the Structure of Compensation

	Bonus Share			Equity Share		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Interaction with Pre-FAS123 Option Dummy						
FAS123	0.0620***	0.0500***	0.0449***	-0.134***	-0.114***	-0.111***
× Option-Dummy	(0.0171)	(0.0150)	(0.0142)	(0.0224)	(0.0201)	(0.0190)
Panel B: Interaction with Pre-FAS123 Option Share						
FAS123	0.155***	0.139***	0.131***	-0.267***	-0.239***	-0.237***
× Option-Share	(0.0237)	(0.0199)	(0.0204)	(0.0334)	(0.0290)	(0.0293)
Year FE	×	×	×	×	×	×
Firm FE	×	×	×	×	×	×
Observations	3,392	6,638	4,435	3,392	6,638	4,435
No. Firms	578	578	757	578	578	757
Sample Period	2002 - 2007	2000 - 2014	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample			incl. fin. & util.			incl. fin. & util.

Notes: The Table reports the results on the relationship between the FAS 123R reform and the structure of managerial compensation. *Option-Dummy* in Panel A is a dummy that indicates if any options are awarded in 2004. *Option-Share* in Panel B is given by the option share in total compensation in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Bonus Share* is the fraction of bonus payments in total compensation and *Equity Share* is the fraction of equity payments in total compensation (both obtained from BoardEx). Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.4: The FAS 123R Accounting Reform and the Duration of Incentives

	Duration		
	(1)	(2)	(3)
Panel A: Interaction with Pre-FAS123 Option Dummy			
FAS123 × Option-Dummy	-0.156**	-0.174**	-0.104
	(0.0715)	(0.0768)	(0.0701)
Observations	3,392	6,638	4,435
No. Firms	578	578	757
Panel B: Interaction with Pre-FAS123 Duration			
FAS123 × Pre-FAS123-Duration	-0.396***	-0.341***	-0.403***
	(0.0323)	(0.0344)	(0.0378)
Observations	3,373	6,601	4,411
No. Firms	573	573	751
Panel C: Interaction with Pre-FAS123 Duration Quintile			
FAS123 × Pre-FAS123-Duration Quint.	-0.224***	-0.201***	-0.235***
	(0.0203)	(0.0204)	(0.0193)
Observations	3,373	6,601	4,411
No. Firms	573	573	751
Year FE	×	×	×
Firm FE	×	×	×
Sample Period	2002 - 2007	2000 - 2014	2002 - 2007
Sample			incl. fin. & util.

Notes: The Table reports the results on the relationship between the FAS 123R reform and the duration of managerial incentives. *Duration* is measured as in Gopalan *et al.* (2014). *Option-Dummy* in Panel A is a dummy that indicates if any options are awarded in 2004. *Pre-FAS123 Duration* in Panel B is given by the duration of total compensation in 2004. *Pre-FAS123 Duration Quintiles* Panel C are given by the quintile categories of the sample duration distribution in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.5: Incentives and the Durability of Investments - Option Dummy

	Investments					
	(1)	(2)	(3)	(4)	(5)	(6)
Measure of Depreciation:	*Ordering*		*Depreciation Rate*			
FAS123 × Option-Dummy × Depr	0.0478**	0.0480**	0.595**	0.595**	0.693***	0.537**
	(0.0240)	(0.0239)	(0.232)	(0.231)	(0.252)	(0.235)
Option-Dummy × Depr	0.0135	0.0132	-0.292	-0.294	-0.237	-0.454
	(0.0361)	(0.0361)	(0.355)	(0.355)	(0.356)	(0.350)
FAS123 × Depr	-0.0409**		-0.558***			
	(0.0207)		(0.200)			
Investment FE	×		×			
Investment-Year FE		×		×	×	×
Firm-Year FE	×	×	×	×	×	×
Observations	13,422	13,422	13,422	13,422	33,737	14,200
No. Firms	667	667	667	667	684	721
Sample Period	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample						incl. fin. & util.

Notes: The Table reports the results on the relationship between managerial incentives and investment decisions. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation, following an ordinal scale in columns 1 and 2, and expressed in absolute depreciation rates in columns 3 to 6. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.6: Incentives and the Durability of Investments - Option Share in Total Compensation

	Investments					
	(1)	(2)	(3)	(4)	(5)	(6)
Measure of Depreciation:	*Ordering*		*Depreciation Rate*			
FAS123 × Option-Share × Depr	0.0775*	0.0820**	0.711*	0.735*	0.777*	0.678*
	(0.0395)	(0.0395)	(0.391)	(0.391)	(0.417)	(0.385)
Option-Share × Depr	0.0707	0.0682	-0.580	-0.596	-0.508	-0.870
	(0.0613)	(0.0612)	(0.617)	(0.617)	(0.601)	(0.604)
FAS123 × Depr	-0.0315**		-0.353**			
	(0.0160)		(0.158)			
Investment FE	×		×			
Investment-Year FE		×		×	×	×
Firm-Year FE	×	×	×	×	×	×
Observations	13,422	13,422	13,422	13,422	33,737	14,200
No. Firms	667	667	667	667	684	721
Sample Period	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample						incl. fin. & util.

Notes: The Table reports the results on the relationship between managerial incentives and investment decisions. *Option-Share* is given by the option share in total compensation in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation, following an ordinal scale in columns 1 and 2, and expressed in absolute depreciation rates in columns 3 to 6. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.7: Incentives and the Durability of Investments - Option Quintiles

	Investments					
	(1)	(2)	(3)	(4)	(5)	(6)
Measure of Depreciation:	*Ordering*		*Depreciation Rate*			
FAS123 × Option-Quintile × Depr	0.0185**	0.0193***	0.180**	0.185**	0.195**	0.168**
	(0.00718)	(0.00719)	(0.0718)	(0.0718)	(0.0781)	(0.0715)
Option-Quintile × Depr	0.0125	0.0121	-0.0926	-0.0954	-0.0772	-0.150
	(0.0112)	(0.0112)	(0.113)	(0.113)	(0.111)	(0.111)
FAS123 × Depr	-0.0604***		-0.650***			
	(0.0230)		(0.228)			
Investment FE	×		×			
Investment-Year FE		×		×	×	×
Firm-Year FE	×	×	×	×	×	×
Observations	13,422	13,422	13,422	13,422	33,737	14,200
No. Firms	667	667	667	667	684	721
Sample Period	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample						incl. fin. & util.

Notes: The Table reports the results on the relationship between managerial incentives and investment decisions. *Option-Quintile* is the quintile of the option share distribution in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation, following an ordinal scale in columns 1 and 2, and expressed in absolute depreciation rates in columns 3 to 6. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.8: Incentives and Capital Stocks

	Capital Stocks					
	(1)	(2)	(3)	(4)	(5)	(6)
Measure of Depreciation:	*Ordering*		*Depreciation Rate*			
FAS123 × Option-Dummy × Depr	0.0403*	0.0404*	0.513**	0.518**	0.780***	0.438*
	(0.0223)	(0.0224)	(0.224)	(0.225)	(0.288)	(0.226)
Option-Dummy × Depr	-0.0128	-0.0130	-0.472	-0.475	-0.509	-0.551
	(0.0356)	(0.0355)	(0.374)	(0.374)	(0.368)	(0.367)
FAS123 × Depr	-0.0437**		-0.572***			
	(0.0203)		(0.202)			
Investment FE	×		×			
Investment-Year FE		×		×	×	×
Firm-Year FE	×	×	×	×	×	×
Observations	12,690	12,690	12,690	12,690	31,784	13,415
No. Firms	663	663	663	663	681	710
Sample Period	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample						incl. fin. & util.

Notes: The Table reports the results on the relationship between managerial incentives and capital stocks. As dependent variable the natural logarithms of the respective capital stocks are used. Physical capital stocks are directly obtained from Factset. Intangible capital stocks (R&D and Advertising) are determined the following: Initial capital stock of category i equals $k_{i0} = \frac{Invest_{i0}}{\delta_i}$ and the subsequent values are constructed iteratively, where the capital stock of category i at time t equals $k_{it} = k_{it-1}(1-\delta_i)+Invest_{it}$. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation, following an ordinal scale in columns 1 and 2, and expressed in absolute depreciation rates in columns 3 to 6. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.9: Incentives and Capital Stock Depreciation

	Average Depreciation Rate			
	(1)	(2)	(3)	(4)
FAS123 × Option-Dummy	0.0158***	0.0165***	0.0189***	0.0163***
	(0.00549)	(0.00568)	(0.00590)	(0.00584)
Option-Dummy		-0.0118		
		(0.00964)		
Year FE	×	×	×	×
Firm FE	×		×	×
Observations	4,118	4,118	10,261	4,877
No. Firms	700	700	701	831
Sample Time	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample				incl. fin. & util.

Notes: The Table reports the results on the relationship between managerial incentives and investment decisions. We use the firms' average depreciation rates weighted by capital stocks in the individual asset categories as the dependent variable. For each firm i with depreciation-specific capital stocks C in year t the capital-stock-weighted depreciation rate δ_{it} equals $\sum_{c=1}^{C} \delta_c \cdot \frac{cap-stock_{itc}}{\sum_{c=1}^{C} cap-stock_{itc}}$. *Option-Dummy* takes 1 if any options are awarded in 2004, zero otherwise. *FAS123* takes 0 for each year until 2005, 1 afterwards. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.10: Incentives and Total Factor Productivity

	TFPR			
	(1)	(2)	(3)	(4)
FAS123 × Option-Dummy	-0.0278**	-0.0281**	-0.00766	-0.0205*
	(0.0135)	(0.0134)	(0.0194)	(0.0121)
Option-Dummy		-0.000338		
		(0.0241)		
Year FE	×	×	×	×
Firm FE	×		×	×
Observations	4,110	4,111	10,251	4,718
No. Firms	699	700	701	805
Sample Time	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample				incl. fin. & util.

Notes: The Table reports the results on the relationship between managerial incentives and productivity. We use the firms' TFPR as the dependent variable. We compute TFPR as regression residuals from regressing log sales on log operating expenses and log assets, where we allow for industry-specific production coefficients. *Option-Dummy* takes 1 if any options are awarded in 2004, zero otherwise. *FAS123* takes 0 for each year until 2005, 1 afterwards. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.11: Industry-Level Variables

Code	Industry Description	Probability Weight (in %)	Value Added (in Mio USD)	Wage Bill (in Mio USD)	Employment (in Thd)	Wages (in Thd USD)	Depr Rates s	Depr Rates l	Capital Stock (in Mio USD)	$\frac{K_l}{K_s+K_l}$ (in %)
A	Agriculture, forestry and fishing	1.972	138,161	36,249	1,278	28.4	0.13	0.02	462,506	58.02
B	Mining and quarrying	0.332	260,953	62,677	552	113.5	0.14	0.02	1,481,196	92.88
C	Total manufacturing	7.883	1,726,301	930,930	16,226	57.4	0.11	0.03	2,794,956	47.38
D, E	Electricity, gas and water supply	0.563	288,219	90,943	1,566	58.1	0.10	0.02	1,665,872	78.54
F	Construction	12.575	748,735	458,619	8,903	51.5	0.16	0.03	228,765	31.76
G	Wholesale and retail trade	22.269	1,777,411	961,865	19,776	48.6	0.15	0.03	1,367,003	71.11
H	Transportation and storage	3.054	472,378	317,617	5,525	57.5	0.14	0.03	1,051,537	60.11
I	Accommodation and food service activities	7.907	414,118	256,740	8,885	28.9	0.14	0.03	466,458	78.35
J	Information and communication	0.397	698,043	325,449	4,865	66.9	0.13	0.04	1,374,041	70.97
K	Financial and insurance activities	5.023	930,028	505,515	6,637	76.2	0.18	0.04	925,047	61.75
M, N	Other business services	17.840	1,343,732	874,366	10,753	81.3	0.15	0.04	1,056,987	50.14
Q	Healthcare	9.033	929,544	767,055	15,756	48.7	0.15	0.03	940,826	76.25
R, S	Arts, entertainment and recreation	11.153	510,096	332,260	8,142	40.8	0.15	0.05	743,230	84.53

Notes: Industry-specific probability weights are based on the number of enterprises across sectors from the OECD Structural Statistics of Industry and Services database for the year 2005. Other industry-level information is based on U.S. 2003–2005 files from EU KLEMS data. Wage bill is obtained by multiplying the number of people employed times the wage. Depr rate s displays the depreciation rate of the short-term capital stock, which is given by the capital stock-weighted depreciation rates of telecommunication equipment (N11322G), computer hardware (N11321G), transport (N1131G) and other machinery equipment and weapons (N11OG). Accordingly, depr rate l is the industry-specific long-term depreciation rate, that is directly provided by EU KLEMS (depreciation rate for other buildings and structures, N11OG). The last column displays the share of long-term capital in total capital.

Table 2.12: Transition Matrix β Before and After FAS 123R

β pre-FAS-123 in 2005	*β post-FAS-123 in 2007*									
	I 0.75-0.775	II 0.775-0.8	III 0.8-0.825	IV 0.825-0.85	V 0.85-0.875	VI 0.875-0.9	VII 0.9-0.925	VIII 0.925-0.95	IX 0.95-0.975	X 0.975-1
I 0.75-0.775	**90.15**	1.01	1.55	0.97	1.35	1.21	0.53	0.58	0.19	2.46
II 0.775-0.8	13.46	**67.01**	1.92	2.56	1.92	2.88	2.56	1.92	0.96	4.81
III 0.8-0.825	10.59	1.81	**69.00**	3.10	3.36	2.07	2.07	3.62	1.55	2.84
IV 0.825-0.85	7.04	1.85	3.70	**66.67**	3.89	4.44	3.70	3.15	1.30	4.26
V 0.85-0.875	6.98	1.67	2.12	2.73	**67.69**	4.25	5.61	4.10	1.21	3.64
VI 0.875-0.9	5.29	1.53	2.82	2.23	4.35	**65.92**	6.11	5.64	2.35	3.76
VII 0.9-0.925	3.39	1.45	1.36	3.19	3.10	4.94	**63.6**	7.74	6.00	5.23
VIII 0.925-0.95	3.19	0.94	1.38	2.25	2.39	3.41	5.66	**66.06**	7.76	6.96
IX 0.95-0.975	1.80	0.50	0.87	1.49	1.61	3.10	4.34	9.06	**65.32**	11.91
X 0.975-0.1	2.29	0.45	0.58	0.81	1.16	1.81	1.42	3.42	6.93	**81.13**

Notes: The Table reports transition probabilities for FAS-123R-induced changes in β. We group betas into ten bins each ranging 2.25 percentage points. Data is left-censored at 0.75, which applies to 14.39% of the observations. Row i displays for a β grouped in bin i the probabilities of being in bins 1-10 after the reform. Therefore, rows sum up to 100%. Diagonal entries indicate the probabilities for β being unchanged after the reform.

Table 2.13: Simulated Firms - Regression Results

	Investment		*Capital Stock*	
	(1)	(2)	(3)	(4)
Panel A: Interaction with Pre-FAS123 Option Dummy				
FAS123 × Option-Dummy × Depr	0.426***	0.426***	0.400***	0.400***
	(0.0231)	(0.0231)	(0.0199)	(0.0199)
Option-Dummy × Depr	0.651	0.651	0.0341	0.0341
	(0.594)	(0.594)	(0.540)	(0.540)
FAS123 × Depr	-0.0327***		-0.0380***	
	(0.00375)		(0.00458)	
Panel B: Interaction with Pre-FAS123 Option Share				
FAS123 × Option-Share × Depr	0.716***	0.716***	0.744***	0.744***
	(0.0918)	(0.0918)	(0.0993)	(0.0993)
Option-Share × Depr	-7.420**	-7.420**	-8.018***	-8.018***
	(3.093)	(3.094)	(2.878)	(2.879)
FAS123 × Depr	-0.605***		-0.641***	
	(0.0831)		(0.0917)	
Investment FE	×		×	
Investment-Year FE		×		×
Firm-Year FE	×	×	×	×
Observations	4,000	4,000	4,000	4,000
No. Firms	1,000	1,000	1,000	1,000

Notes: This Table reports the results on the relationship between managerial incentives and investment decisions for our simulated panel of 1000 firms. We collapse the data into a pre- and post-reform era, where *FAS123* is a dummy variable indicating the latter period. *Option-dummy* is defined as binary variable which is 1 if a firm experience an actual reduction in its firm-specific β after the reform and 0 otherwise. Accordingly, *Option-share* is proxied by the firm-specific β in the pre-reform period. *Depr* is the measure of depreciation for the two capital goods, which is 3.28 percent for long-term capital and 14.48 percent for short-term capital. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.14: General-Equilibrium Effects: Aggregate Results from Counterfactual Reform

Variable	Change (%)	Variable	Change (%)
Output	-0.08	Price level	0.17
Long-term investment	-0.88	Short-term investment	-0.46
Long-term capital stock	-0.97	Short-term capital stock	-0.51
Overall investment	-0.59	Overall capital stock	-0.81

Notes: The Table shows the effects of the simulated reform on a set of aggregate variables. For each variable, the effect is measured as the percentage change of the steady-state value after the reform relative to the steady-state value before the reform.

Appendices

2.A Empirical Appendix

2.A.1 Variable Descriptions

Table 2.A.1: Variable Descriptions and Data Sources

Variable	Description	Source
Investment Variables		
advertising_{it}	*advertising* represents the cost of advertising media (i.e., radio, television, and periodicals) and promotional expenses in millions USD; Compustat variable name: XAD	Compustat
$\text{R\&D}_{it}$	*research & development expenses* (period t) represent all direct and indirect costs related to the creation and development of new processes, techniques, applications and products with commercial possibilities in millions USD; Compustat variable name: XRD	Compustat
buildings_{it}	*buildings* (period t) - 0.97 × *buildings* (period $t-1$); *buildings* (gross property plant and equipment) represent the architectural structure used in a business such as a factory, office complex or warehouse in millions USD	FactSet
computer_{it}	*computer software & equipment* (period t) - 0.70 × *computer software & equipment* (period $t-1$); *computer software & equipment* (gross property plant and equipment) represents computer equipment and the information a computer uses to perform tasks in millions USD	FactSet
land_{it}	*land* (period t) - × *land* (period $t-1$); *land* (gross property plant and equipment) represents the real estate without buildings held for productive use, is recorded at its purchase price plus any costs related to its purchase such as lawyer's fees, escrow fees, title and recording fees in millions USD	FactSet
machines_{it}	*machinery & equipment* (period t) - 0.88 × *machinery & equipment* (period $t-1$); *machinery & equipment* (gross property plant and equipment) represent the machines and machine parts needed by the company to produce its products in millions USD	FactSet
$\text{transportation equipment}_{it}$	*transportation equipment* (period t) - 0.84 × *transportation equipment* (period $t-1$); *transportation equipment* (gross property plant and equipment) represents the cars, ships, planes or any other type of transportation equipment in millions USD	FactSet
Manager Variables		
$\text{option awards}_{2004}$	the aggregate value of stock options (expressed in thousands USD) granted to the executive during the year as valued using Standard & Poor's Black-Scholes methodology; ExecuComp variable name: *OPTION-AWARDS-BLK-VALUE*	ExecuComp
TDC_{2004}	total compensation (expressed in thousands USD) comprised of the following: Salary, Bonus, Other Annual, Total Value of Restricted Stock Granted, Total Value of Stock Options Granted (using Black-Scholes), Long-Term Incentive Payouts, and All Other Total; ExecuComp variable name: *TDC1*	ExecuComp
bonus share_{t}	this is the ratio between *Bonus* (i.e. an annual payment made in addition to salary) and *Total Compensation*, which is the sum of *Total Direct Compensation* and *Total Equity Linked Compensation*; *Total Direct Compensation* consists of *Salary* and *Bonus*, and *Total Equity Linked Compensation* is the sum of *Value of Shares Awarded*, *Value of LTIP Awarded* and *Estimated Value of Options Awarded*; *Value of LTIP Awarded* is the sum of all cash, equity, equity matched and Option plans received over time where the receipt of these awards is contingent on the company's performance	BoardEx
equity share_{t}	this is the ratio between *Total Equity Linked Compensation* (= *Value of Shares Awarded* + *Value of LTIP Awarded* + *Estimated Value of Options Awarded*) and *Total Compensation*, which is the sum of *Total Direct Compensation* and *Total Equity Linked Compensation*	BoardEx
pay duration d_{it}	duration d of firm i at time t is calculated as $d_{it} = \frac{(bonus_{it}+salary_{it})\cdot 0+\sum_{j=1}^{N}(Restr.stock_{ijt}+options_{ijt})\cdot \tau_j}{(salary_{it}+bonus_{it})+\sum_{j=1}^{N}(Restr.stock_{ijt}+options_{ijt})}$ where τ is the *vesting period* of equity-based component j; *vesting period* is obtained by taking the difference between the *vesting date*, which is the date from which options can be exercised, and the annual report date	BoardEx and Gopalan *et al.* (2014)
$\text{firm-related wealth}_{t}$	firm-specific wealth is the sum of the value of the stock and option portfolio held by the executive; the value of the option portfolio is computed as of the fiscal year end using the Black-Scholes formula; for pre-2006, the values of the three option portfolios are summed up: current year grants, previously-granted unvested options, and vested options; for post-2006, the values of all the tranches of options outstanding are summed up; the value of the share portfolio is computed by multiplying the number of shares (Execucomp: *SHROWN-EXCL-OPTS*) by the fiscal year end price (Execucomp: *PRCCF*); the sum of the two provides the value of the CEO's equity portfolio as of the end of the year	Coles *et al.* (2006) and Core and Guay (2002)
Firm Variables		
total assets_{t}	(log) total value of assets reported for 2004 in millions USD; Compustat variable name: AT	Compustat
employment_{t}	(log) number of company workers in 2004 (in thousands); Compustat variable name: EMP	Compustat
sales_{t}	gross sales in millions USD; Compustat variable name: SALE	Compustat
$\text{market capitalisation}_{t}$	annual arithmetic mean of number of common shares (CSHOC) × daily closing price (PRCCD) in millions USD	Compustat
TFPR_{t}	annual TFPR, based on a 2-digit SIC industry production function	Compustat

Notes: The Table contains descriptions of all empirical variables. Note that the variables firm-related wealth_t, sales_t and market capitalisation_t are used in our quantitative analysis.

2.A.2 Sample Selection

Our empirical sample is pooled from the 1,671 firms covered in ExecuComp. From these, we exclude 90 inactive firms and 655 firms that never reported any investment between the years 1995 and 2015 (neither in Compustat nor in FactSet). We then exclude 78 firms for which Compustat starts reporting financial information only after 2004 and 15 firms for that coverage ends before 2006. This leaves our sample at 833 firms.

2.A.3 Economic Significance: Calculating the Increase in Refinancing Costs

Column 1 in Table 2.9 reveals that for option-paying firms the average depreciation rate increased by 1.58 percentage points compared to non-option-paying firms. Assuming that the durability of the capital stock of non-option-paying firms was not affected by FAS 123R, we map this relative change to an absolute number. We compute the average pre-FAS-123R depreciation rate for option-paying firms, which is 16.81% in 2004. This rate converts into a durability of $2,171$ days ($\frac{1}{0.1681} \times 365$ days) for the capital stock. The FAS-123R-induced depreciation rate for option-paying firms is equal to 18.39% (16.81%+1.58%), which implies a durability for the firms' capital stock of 1,985 days. Therefore, FAS 123R decreased the durability of the capital stock by 186 days. Assuming an annual refinancing interest rate of 3%, this lower durability would be associated with an additional amount of interest payments of USD 15.29 for each USD 1,000 invested ($0.03 \times \frac{186}{365} \times$ USD $1,000$).

2.A.4 Robustness and Additional Results

This Appendix presents several robustness analyses and additional results.

Firm Size and Other Ex-ante Differences: Table 2.A.2 compares treated and untreated firms. Table 2.A.3 includes additional interactions with firm size, using assets, employment or capital stocks as a proxy for the size of firms. Table 2.A.4 includes additional interactions with either equity volatility or current CEO pay.

Pre-Trends: Table 2.A.5 presents placebo treatments for other years and shows that the effect is absent in earlier years before actual treatment occurs.

CEO Turnover: Table 2.A.6 replicates estimates focusing on a subsample that includes only firms with a unique CEO to show that results are not determined by CEO-turnover events. Results indicate that the effect is even more pronounced when we exclude firms where CEO turnover occurred.

Measurement of Investments: Tables 2.A.7 and 2.A.8 show robustness regarding the measurement of investments. Table 2.A.7 replicates our findings based on either Box-Cox transformation or logarithmised investments. Table 2.A.8 replicates results when negative investments are either treated as disinvestments or as 0 expenditures.

R&D Investment and Intangibles: Table 2.A.9 shows robustness regarding the inclusion of R&D and intangibles as investment categories. It replicates results when either R&D investments are excluded or when we include interactions with a dummy that indicates intangible investment categories.

Structural Parameters: Table 2.A.10 exploits time variation in the model-derived parameter β to study its effect on investment and firm-specific depreciation rates.

Table 2.A.2: Summary Statistics on Treated and Untreated Firms

Variable	Option-Paying (Treated, N=515)	Non-Option-Paying (Control, N=181)	*t*-test	*p*-value
Total Assets	8,884	5,585	1.72	**0.09**
Sales	8,062	5,690	1.28	0.20
Capital Stock	4,051	2,100	2.95	<**0.01**
Employment	33.19	17.79	3.20	<**0.01**
Labor Productivity	115.5	102.2	1.17	0.24
Depreciation Rate	0.17	0.18	-1.17	0.23
Intangible Share	0.50	0.52	-0.58	0.57
Investment Rate	0.05	0.04	0.37	0.71
Leverage Ratio	0.20	0.18	1.20	0.23
Liquidity Ratio	0.17	0.16	0.44	0.66
Equity Volatility	0.34	0.38	-2.73	<**0.01**
Current CEO Compensation	1,951	1,558	2.36	**0.02**

Notes: A firm is considered as treated if it has granted stock options to its management in 2004. Summary statistics correspond to 2004 values. *Total Assets*, *Sales* and *Capital Stock* are denoted in millions USD, *Employment* is denoted in thousands. *Labor Productivity* is value added per employee in thousands USD (calculated as (SALE - COGS) / EMP). *Capital Stock* is obtained by summing up category-specific capital stocks for each firm, *Depreciation Rate* is the capital-stock weighted mean of category-specific depreciation rates for each firm. *Intangible Share* is the ratio of intangible investments (sum of advertising and R&D investments) to total investments. *Investment Rate* is capital expenditures (CAPX) relative to total assets (AT). The *Leverage Ratio* is defined as the ratio of total debt (sum of items DLC and DLTT) to total assets. The *Liquidity Ratio* equals the ratio of cash and short-term investments (CHE) to total assets. *Equity Volatility* is the annualized equity-return volatility, calculated as the standard deviation of daily stock returns multiplied by $\sqrt{252}$. Daily returns are calulated as (PRCCD $\times$ TRFD / AJEXDI) relative to the previous day. *Current CEO Compensation* is the current compensation of the CEO in thousands USD (compensation excluding equity).

Table 2.A.3: Robustness: Incentives and the Durability of Investments - Controlling for Firm Size

	Investments					
	(1)	(2)	(3)	(4)	(5)	(6)
Firm Size Measure:	*Employment*		*Assets*		*Capital Stock*	
Measure of Depreciation:	*Ordering*	*Depreciation Rate*	*Ordering*	*Depreciation Rate*	*Ordering*	*Depreciation Rate*
FAS123 × Option-Dummy × Depr	0.0441*	0.564**	0.0498**	0.586**	0.0497**	0.584**
	(0.0242)	(0.235)	(0.0244)	(0.243)	(0.0246)	(0.248)
Option-Dummy × Depr	0.0233	-0.418	0.00847	-0.450	0.00948	-0.406
	(0.0363)	(0.352)	(0.0363)	(0.357)	(0.0368)	(0.365)
FAS123 × Firm Size × Depr	0.00775	0.0554	-0.00214	0.00288	-0.00124	-0.00490
	(0.00670)	(0.0638)	(0.00646)	(0.0623)	(0.00571)	(0.0554)
Firm Size × Depr	-0.0224*	0.264***	0.00745	0.203**	0.00646	0.163**
	(0.0120)	(0.0956)	(0.0117)	(0.0920)	(0.0100)	(0.0800)
Investment-Year FE	×	×	×	×	×	×
Firm-Year FE	×	×	×	×	×	×
Observations	13,348	13,348	13,414	13,414	13,380	13,380
No. Firms	662	662	666	666	664	664
Sample Period	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007

Notes: This Table reports the results on the relationship between managerial incentives and investment decisions. Value of total assets, capital stocks and number of employees are logarithmized and represent 2004 values. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation, following an ordinal scale in columns 1, 3 and 5, and expressed in absolute depreciation rates in columns 2, 4 and 6. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%-, and 10%-level.

Table 2.A.4: Robustness: Incentives and the Durability of Investments - Controlling for Other Firm Differences

	Investments			
	(1)	(2)	(3)	(4)
Firm Control:	*Equity Volatility*		*Current CEO Compensation*	
Measure of Depreciation:	*Ordering*	*Depreciation Rate*	*Ordering*	*Depreciation Rate*
FAS123 × Option-Dummy × Depr	0.0530**	0.701***	0.0461*	0.559**
	(0.0241)	(0.233)	(0.025)	(0.246)
Option-Dummy × Depr	0.00954	-0.484	0.00547	-0.585
	(0.036)	(0.349)	(0.0374)	(0.375)
FAS123 × Firm Control × Depr	0.132**	1.206**	0.00609	0.062
	(0.0614)	(0.552)	(0.0137)	(0.131)
Firm Control × Depr	-0.0855	-3.306***	0.0163	0.600***
	(0.0937)	(0.924)	(0.0223)	(0.196)
Investment-Year FE	×	×	×	×
Firm-Year FE	×	×	×	×
Observations	13,422	13,422	13,352	13,352
No. Firms	667	667	664	664
Sample Time	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007

Notes: This Table reports the results on the relationship between managerial incentives and investment decisions. *Equity Volatility* is the annualized equity-return volatility in 2004, calculated as the standard deviation of daily stock returns multiplied by $\sqrt{252}$. *Current CEO Compensation* is the logarithmized current compensation of the CEO (compensation excluding equity) in 2004. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation, following an ordinal scale in columns 1 and 3 and expressed in absolute depreciation rates in columns 2 and 4. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%-, and 10%-level.

Table 2.A.5: Robustness: Incentives and the Durability of Investments - Placebo Treatments

	Investments				
	(1)	(2)	(3)	(4)	(5)
Treatment Year:	*2002 placebo*	*2003 placebo*	*2004 placebo*	*2005 real*	*2006 placebo*
FAS123 × Option-Dummy × Depr	-0.00223	-0.000425	-0.00348	0.0478**	0.0326
	(0.0282)	(0.0322)	(0.0256)	(0.0240)	(0.0236)
FAS123 × Depr	0.00566	0.00747	0.00446	-0.0409**	-0.0113
	(0.0239)	(0.0289)	(0.0218)	(0.0207)	(0.0200)
Option-Dummy × Depr	-0.0548	0.0358	-0.0237	0.0135	0.0523
	(0.0433)	(0.0405)	(0.0346)	(0.0361)	(0.0337)
Investment FE	×	×	×	×	×
Firm-Year FE	×	×	×	×	×
Observations	12,428	12,689	13,079	13,422	13,538
No. Firms	665	665	666	667	670
Sample Time	1999 - 2004	2000 - 2005	2001 - 2006	2002 - 2007	2003 - 2008

Notes: The Table reports placebo estimates on the relationship between managerial incentives and investment decisions. Compared to the baseline estimation in column 4, we shift the *Option-Dummy*, *FAS123* and the sample time to earlier or later years, accordingly. *Depr* is the measure of depreciation, following an ordinal scale. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.A.6: Robustness: Incentives and the Durability of Investments - CEO Turnover

	Investments					
	(1)	(2)	(3)	(4)	(5)	(6)
Measure of Depreciation:	*Ordering*		*Depreciation Rate*			
FAS123 × Option-Dummy × Depr	0.0970***	0.0964***	1.010***	1.009***	1.244***	0.973***
	(0.0300)	(0.0301)	(0.288)	(0.289)	(0.367)	(0.303)
Option-Dummy × Depr	-0.0384	-0.0388	-0.775	-0.780	-0.772	-0.942*
	(0.0512)	(0.0513)	(0.478)	(0.479)	(0.483)	(0.482)
FAS123 × Depr	-0.0847***		-0.908***			
	(0.0238)		(0.216)			
Investment FE	×		×			
Investment-Year FE		×		×	×	×
Firm-Year FE	×	×	×	×	×	×
Observations	5,939	5,939	5,939	5,939	14,886	6,319
No. Firms	286	286	286	286	292	310
Sample Period	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample	same CEO	same CEO	same CEO	same CEO	same CEO	same CEO incl. fin. & util.

Notes: The Table reports the results on the relationship between managerial incentives and investment decisions. There are only firms included which have been run by the same CEO between 2002 and 2007. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation, following an ordinal scale in columns 1 and 2, and expressed in absolute depreciation rates in columns 3 to 6. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.A.7: Robustness: Incentives and the Durability of Investments - Alternative Transformation of the Investment Variable

	Investments							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Investment Measure:	*Box-Cox Transformation*				*Logarithms*			
FAS123 × Option-Dummy × Depr	0.924**	0.924**	0.934**	0.881**	0.490*	0.490*	0.862***	0.431
	(0.376)	(0.377)	(0.402)	(0.380)	(0.282)	(0.283)	(0.291)	(0.284)
Option-Dummy × Depr	-0.799	-0.800	-0.702	-1.167**	-0.430	-0.431	-0.428	-0.533
	(0.524)	(0.524)	(0.539)	(0.533)	(0.374)	(0.373)	(0.367)	(0.368)
FAS123 × Depr	-0.709**				-0.627**			
	(0.314)				(0.250)			
Investment FE	×				×			
Investment-Year FE		×	×	×		×	×	×
Firm-Year FE	×	×	×	×	×	×	×	×
Observations	13,422	13,422	33,737	14,200	12,400	12,400	31,080	13,106
No. Firms	667	667	684	721	664	664	682	711
Sample Period	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample				incl. fin. & util.				incl. fin. & util.

Notes: This Table reports the results on the relationship between managerial incentives and investment decisions. The following transformation applies to the dependent variable y: $y = ln(x + 0.001)$ for columns $1 - 4$ and $y = ln(x)$ for columns $5 - 8$. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation expressed in absolute depreciation rates. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.A.8: Robustness: Incentives and the Durability of Investments - Allowing for Negative Investments

	Investments							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Investment Sample:	*Include Negative Investments*				*Treat Negative Investments as 0s*			
FAS123 × Option-Dummy × Depr	1.024***	1.039***	1.132***	0.892**	0.707***	0.720***	0.832***	0.603**
	(0.389)	(0.390)	(0.336)	(0.391)	(0.262)	(0.262)	(0.260)	(0.260)
Option-Dummy × Depr	0.0448	0.0393	0.226	-0.156	-0.0289	-0.0346	0.0816	-0.198
	(0.414)	(0.415)	(0.404)	(0.413)	(0.356)	(0.356)	(0.355)	(0.352)
FAS123 * Depr	-1.084***				-0.770***			
	(0.330)				(0.224)			
Investment FE	×				×			
Investment-Year FE		×	×	×		×	×	×
Firm-Year FE	×	×	×	×	×	×	×	×
Observations	15,281	15,281	38,607	16,223	15,281	15,281	38,607	16,223
No. Firms	668	668	686	722	668	668	686	722
Sample Period	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample				incl. fin. & util.				incl. fin. & util.

Notes: This Table reports the results on the relationship between managerial incentives and investment decisions. Columns 1 – 4 treat negative investment as true negatives, whereas in columns 5 – 8 negative values are set to zero. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation expressed in absolute depreciation rates. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.A.9: Robustness: Incentives and the Durability of Investments - Assessing the Role of R&D and Intangibles

	Investments					
	(1)	(2)	(3)	(4)	(5)	(6)
Measure of Depreciation:	*Ordering*		*Depreciation Rate*			
Panel A: Omitting R&D						
FAS123 × Option-Dummy × Depr	0.0570**	0.0588**	0.605**	0.614**	0.885***	0.565**
	(0.0259)	(0.0259)	(0.244)	(0.244)	(0.250)	(0.248)
Option-Dummy × Depr	-0.0271	-0.0283	-0.391	-0.398	-0.351	-0.464
	(0.0350)	(0.0350)	(0.343)	(0.343)	(0.339)	(0.339)
FAS123 × Depr	-0.0663***		-0.694***			
	(0.0229)		(0.217)			
Observations	10,480	10,480	10,480	10,480	26,331	11,037
No. Firms	659	659	659	659	677	704
Panel B: Controlling for Intangibles						
FAS123 × Option-Dummy × Depr	0.0664*	0.0688*	0.760***	0.770***	0.932***	0.707**
	(0.0358)	(0.0360)	(0.277)	(0.276)	(0.312)	(0.284)
Option-Dummy × Depr	-0.0279	-0.0294	-1.007	-1.014	-1.081	-1.235*
	(0.0585)	(0.0588)	(0.675)	(0.678)	(0.671)	(0.652)
FAS123 × Depr	-0.0648**		-0.795***			
	(0.0301)		(0.236)			
Observations	13,422	13,422	13,422	13,422	33,737	14,200
No. Firms	667	667	667	667	684	721
Investment FE	×		×			
Investment-Year FE		×		×	×	×
Firm-Year FE	×	×	×	×	×	×
Sample Period	2002 - 2007	2002 - 2007	2002 - 2007	2002 - 2007	2000 - 2014	2002 - 2007
Sample						incl. fin. & util.

Notes: The Table reports the results on the relationship between managerial incentives and investment decisions. The upper panel omits R&D investments and the lower panel controls for interactions between a dummy that indicates intangible investment categories (R&D and advertising), *FAS123* and *Option-Dummy*. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. *Depr* is the measure of depreciation, following an ordinal scale in columns 1 and 2, and expressed in absolute depreciation rates in columns 3 to 6. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

Table 2.A.10: Beta and the Durability of Investments/Capital Stock Depreciation

	Investments				Depr Rate
	(1)	(2)	(3)	(4)	(5)
Model	OLS		IV		OLS
			1st Stage	*2nd Stage*	
$(1-\beta)\times$ Depr	0.428***	0.416***			
	(0.108)	(0.117)			
FAS123 × Option-Dummy × Depr			0.028***		
			0.004		
$\widehat{(1-\beta)\times}$ Depr				1.849***	
				(0.648)	
$(1-\beta)$					0.027***
					(0.009)
Investment FE	×		×	×	
Investment-Year FE		×			
Firm-Year FE	×	×	×	×	
Firm FE					×
Year FE					×
Observations	29,940	29,940	29,940	29,940	9,015
No. Firms	656	656	656	656	676
Sample Time	2000 - 2014	2000 - 2014	2000 - 2014	2000 - 2014	2000 - 2014
Kleibergen-Paap F-Statistic			60.55		

Notes: The Table reports the results on the relationship between the model-specific incentive measure β and the durability of investments/capital stock depreciation. The calculation of β follows Equation (2.11), details on the computation can be found in Appendix 2.C.1. *Depr* is the measure of depreciation, following an ordinal scale. *Option-Dummy* is a dummy that indicates if any options are awarded in 2004. *FAS123* takes value 0 for each year until 2005 and value 1 afterwards. In columns 1 and 2, we investigate the relationship between the firm-specific β and the durability of investments. In column 4, we address endogeneity concerns related to β by instrumenting $(1-\beta)\times$ *Depr* with *FAS123* × *Option-Dummy* × *Depr*. First-stage results are given in column 3. Column 5 estimates the effect of β on the capital stock depreciation by taking a firm-specific capital-stock-weighted depreciation rate as dependent variable. Standard errors (reported in parentheses) are clustered at the firm-level. ***, **, and * indicate statistical significance at the 1%-, 5%- and 10%-level.

2.B Theoretical Appendix

2.B.1 Derivation of Managers' Optimal Behavior

To derive a manager's decision problem, we express the manager's optimisation problem in recursive form. Formally, manager t chooses an action $a_t = (\mathbf{K}_{t+1}, N_t) \in \mathbb{R}^3_+$ depending on the history of previous managers' decisions $\mathcal{H}_t = (a_s | s < t)$. Denote by s_τ a strategy of manager τ. Manager t's problem in general follows as

$$\begin{aligned} &\max_{a_t} \Gamma_t \\ s.t.\ &(2.4), (2.5), (2.6), (2.12), \\ &\text{given } \mathcal{H}_t \\ &\text{given beliefs regarding } s_\tau, \tau > t. \end{aligned} \tag{B.1}$$

Generally, this type of problem has an extremely large strategy space, and a multitude of equilibria can occur, which can be enforced through trigger strategies etc. This, potentially, makes non-monotonic or discontinuous policy functions sustainable. Although a thorough examination of the strategy space of such a game seems interesting, it is beyond the scope of this chapter. In line with most macroeconomic models, we focus on symmetric, smooth Markov perfect equilibria, where the state of the game is entirely described by a_{t-1}. More specifically, we assume that the variable factor labour is always set optimally within each period such that strategies only effectively map from $\mathbf{K}_t$ into $\mathbf{K}_{t+1}$ and N_t.

Since we are interested in a symmetric equilibrium, we denote the policy function for capital as $\mathcal{K}(\mathbf{K}, \xi)$, i.e. if manager t follows this strategy profile, they will set $\mathbf{K}_{t+1} = \mathcal{K}(\mathbf{K}_t, \xi)$ when faced with a predetermined capital stock K_t. Here ξ is a simple vector collecting the parameters of the model: $\xi = (a, b, Z, \nu, \gamma, \delta_l, \delta_s, \varphi, \beta, \theta, w)$. Likewise, $\mathcal{N}(\mathbf{K}, \xi)$ denotes the policy function for N_t. Note that $\mathcal{K}(\cdot)$ is a vector-valued function with two outputs (one for each capital good), which in turn we denote by $\mathcal{K}_j(\mathbf{K}, \xi), j = l, s$. In particular, we denote

$$\mathbf{K}_{t+1} = \mathcal{K}(\mathbf{K}_t, \xi) := \begin{bmatrix} \mathcal{K}_l(\mathbf{K}_t, \xi) \\ \mathcal{K}_s(\mathbf{K}_t, \xi) \end{bmatrix}.$$

Under this restriction, we can represent manager t's maximisation problem in a recursive way. Here, to save on notation, we drop time indices and follow a common convention in the literature: e.g., we denote by K_j the value of K_{jt} at some arbitrary point in time and by K'_j the value of $K_{j,t+1}$ for $j = l, s$. One can then use a similar approach for all other variables, in particular the current capital mix as $\mathbf{K} = [K_l\ K_s]'$ and the capital mix one period later as $\mathbf{K}'$. First, we can combine equations (2.6),(2.4) and (2.5) to obtain a function for the period-profits, $\Pi = \pi(\mathbf{K}, \mathbf{K}', N, \xi)$:

$$\pi(\mathbf{K}, \mathbf{K}', N, \xi) = Z^{1-a-b} \left(K_l^\nu K_s^{1-\nu} \right)^a N^b - \sum_{j \in \{l,s\}} \left[\frac{\gamma}{2} \left(\frac{K'_j}{K_j} - 1 \right)^2 K_j + K'_j - (1 - \delta_j) K_j \right] - wN \tag{B.2}$$

Next, the value of equity $E(\cdot)$ can be decomposed into current profits and a continuation value, denoted by the function $V(\mathbf{K}', \xi)$:

$$E(\mathbf{K}, \mathbf{K}', N, \xi) = \pi(\mathbf{K}, \mathbf{K}', N, \xi) + \theta V(\mathbf{K}', \xi)$$

where this continuation value is given by

$$\begin{aligned}V(\mathbf{K},\xi) &= E(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi)\\ &= \pi(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi) + \theta V(\mathcal{K}(\mathbf{K},\xi),\xi)\end{aligned}$$

As a result, the value of the manager's remuneration is also a function of their decision according to:

$$\Gamma(\mathbf{K},\mathbf{K}',N,\xi) = \varphi\left(\pi(\mathbf{K},\mathbf{K}',N,\xi) + \beta\theta V(\mathbf{K}',\xi)\right)$$

Using these functional definitions, we can express a particular manager's optimised payoff from (B.1) as

$$\Gamma^*(\mathbf{K},\xi) := \max_{(\mathbf{K}',N)}\{\Gamma(\mathbf{K},\mathbf{K}',N,\xi)\} \tag{B.3}$$

And similarly, the policy functions for the capital mix and labour are given by

$$(\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi)) := \arg\max_{(\mathbf{K}',N)}\{\Gamma(\mathbf{K},\mathbf{K}',N,\xi)\}$$

These policy function thus need to satisfy a set of optimality conditions. In particular, the policy function for labour can be derived analytically as

$$\mathcal{N}(\mathbf{K},\xi) = \left(\frac{bZ^{1-a-b}\left(K_l^{\nu}K_s^{1-\nu}\right)^a}{w}\right)^{\frac{1}{1-b}}. \tag{B.4}$$

This directly follows from the first-order condition

$$\frac{\partial}{\partial N}\Gamma(\cdot) \overset{!}{=} 0 \quad\Leftrightarrow\quad \varphi\frac{\partial}{\partial N}\pi(\cdot) \overset{!}{=} 0 \Leftrightarrow \quad \frac{\partial}{\partial N}\pi(\cdot) \overset{!}{=} 0$$

whereas it is generally impossible to solve for analytical policy functions for the capital goods. At most, the following self-referencing characterisation is possible:

$$\begin{aligned}\mathcal{K}_j(\mathbf{K},\xi) = \Bigg\{K_j'\Bigg|\ & 0 = \frac{\partial}{\partial K_j'}\pi(\mathbf{K},\mathbf{K}',N,\xi) + \beta\theta\frac{\partial}{\partial K_j}\pi(\mathbf{K}',\mathcal{K}(\mathbf{K}',\xi),\mathcal{N}(\mathbf{K}',\xi),\xi) \\ & + \theta(1-\beta)\sum_{k=l,s}\frac{\partial}{\partial K_j}\mathcal{K}_k(\mathbf{K}',\xi)\frac{\partial}{\partial K_k}V(\mathcal{K}(\mathbf{K}'),\xi)\Bigg\}\end{aligned} \tag{B.5}$$

To derive this condition, first note that the first-order condition can be stated as

$$\begin{aligned}&\frac{\partial}{\partial K_j'}\Gamma(\cdot) \overset{!}{=} 0\\ \Leftrightarrow\ & \varphi\left(\frac{\partial}{\partial K_j}\pi(\cdot) + \beta\theta\frac{\partial}{\partial K_j}V(\cdot)\right) \overset{!}{=} 0\end{aligned} \tag{B.6}$$

The envelope condition defining $\frac{\partial}{\partial K_j}V(\cdot)$ is given by

$$\begin{aligned}
\frac{\partial}{\partial K_j}V(\cdot) &= \frac{\partial}{\partial K_j}E(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi) \\
&\quad + \sum_{k=l,s}\frac{\partial}{\partial K_j}\mathcal{K}_k(\mathbf{K},\xi)\frac{\partial}{\partial K_k'}E(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi) \\
&\quad + \frac{\partial}{\partial K_j}\mathcal{N}(\mathbf{K},\xi)\frac{\partial}{\partial N}E(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi) \\
&= \frac{\partial}{\partial K_j}\pi(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi) \\
&\quad + \sum_{k=l,s}\frac{\partial}{\partial K_j}\mathcal{K}_k(\mathbf{K},\xi)\left[\frac{\partial}{\partial K_k'}\pi(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi)+\theta\frac{\partial}{\partial K_k'}V(\mathcal{K}(\mathbf{K},\xi)\right] \\
&\quad + \frac{\partial}{\partial K_j}\mathcal{N}(\mathbf{K},\xi)\frac{\partial}{\partial N}\pi(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi)
\end{aligned}$$

From optimal labour demand, it follows that $\frac{\partial}{\partial N}\pi(\cdot)=0$ such that this simplifies to

$$\begin{aligned}
\frac{\partial}{\partial K_j}V(\cdot) &= \frac{\partial}{\partial K_j}\pi(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi) \\
&\quad + \sum_{k=l,s}\frac{\partial}{\partial K_j}\mathcal{K}_k(\mathbf{K},\xi)\left[\frac{\partial}{\partial K_k'}\pi(\mathbf{K},\mathcal{K}(\mathbf{K},\xi),\mathcal{N}(\mathbf{K},\xi),\xi)+\theta\frac{\partial}{\partial K_k'}V(\mathcal{K}(\mathbf{K},\xi)\right]
\end{aligned} \tag{B.7}$$

Inserting equation (B.6) on the left-hand side and –iterated by one period– on the right-hand side of (B.7) gives equation (B.5).

Finally, by re-inserting time indices and suppressing functional dependencies, we can reformulate equations (B.4) and (B.5) to obtain equations (2.13) and (2.14) in the main text.

2.B.2 Pseudo-General-Equilibrium Effects

To test the mechanism for robustness to general-equilibrium effects, we reuse the firm sample from our quantitative exercise (including the relevant parameters and β-transitions), and assume that the $\mathcal{N}_f = 1,000$ firms inhabit one single economy, divided into the $S = 13$ sectors from Table 2.11. Each sector is denoted by $s = 1,\dots,S$, each firm by $f = 1,\dots,\mathcal{N}_f$. For future reference, we define two mappings that link firms and their industries: firm f's sector is given by $s_f = 1,\dots,S$ and the sector s is composed of a set of firms $F_s = \{f = 1,\dots,\mathcal{N}_f | s_f = s\}$.

Demand

As before, we abstract from aggregate dynamics and we are only interested in the change of steady-state variables.[27] Also, as in the previous section, we use the notation x to

[27] Solving the model with aggregate dynamics would, of course, be feasible, but it would be rather complicated (cf., e.g. Krusell and Smith, 1998) and it is not clear what this would add to the analysis at hand.

represent a variable x's value in the current period and x' (x'') for the value of x one period (two periods) ahead.

A competitive final goods firm produces a final consumption good $\mathcal{Q}$ from the sectoral inputs Q_s according to the Cobb-Douglas production function

$$\mathcal{Q} = \prod_{s=1}^{S} \mathcal{Q}_s^{\psi_s}.$$

Here, the ψ_s are calculated from Table 2.11 as the respective shares of value added that sector s contributes to total value added such that they satisfy $\psi_s \in (0,1)$ and $\sum_{s=1}^{S} \psi_s = 1$.

The corresponding aggregate price-level is thus given by

$$\mathcal{P} = \prod_{s=1}^{S} \left(\frac{\mathcal{P}_s}{\psi_s} \right)^{\psi_s}, \tag{B.8}$$

where $\mathcal{P}_s$ denote sectoral price levels. Following standard logic, each sector thus faces a demand curve

$$\mathcal{Q}_s = \frac{\psi_s \mathcal{P} \mathcal{Q}}{\mathcal{P}_s}. \tag{B.9}$$

The sectoral goods are a CES-aggregate of the individual firms' outputs Q_f according to

$$\mathcal{Q}_s = \left(\sum_{f \in F_s} Q_f^{\frac{\varepsilon_s - 1}{\varepsilon_s}} \right)^{\frac{\varepsilon_s}{\varepsilon_s - 1}}. \tag{B.10}$$

Here, the ε_s directly follow from our calibration exercise above. We assume that firms engage in monopolistic competition. The corresponding sectoral price level based on firms' prices P_f is thus

$$\mathcal{P}_s = \left(\sum_{f \in F_s} P_f^{1-\varepsilon_s} \right)^{\frac{1}{1-\varepsilon_s}}. \tag{B.11}$$

Consequently each firm f in sector s faces the following demand:

$$Q_f = P_f^{-\varepsilon_s} \mathcal{P}_s^{\varepsilon_s} \mathcal{Q}_s. \tag{B.12}$$

Note how this equation compares to (2.3): we can now deduce that in each sector, the demand shifter is given by

$$B_s = \mathcal{P}_s^{\varepsilon_s} \mathcal{Q}_s.$$

This links firms on product markets while we also need to link firms' input usage K_{lf}, K_{sf} and N_f to factor markets.

Firm Behavior

The problem of the firm is still the same as in the partial-equilibrium setup. We only need to add the respective firm and industry subscripts to the various variables in equations (2.2)–(2.14).

For concreteness, we restate these here, dropping time indices and adding subscripts f and s: At a sectoral level, we have the following parameters:

$$a_s = \alpha_s \frac{\varepsilon_s - 1}{\varepsilon_s} \tag{B.13}$$

$$b_s = (1-\alpha_s)\frac{\varepsilon_s - 1}{\varepsilon_s} \tag{B.14}$$

In addition, the following relations characterise each firm's behaviour:

$$Q_f = \widetilde{Z}_f \left(K_{lf}^{\nu_s} K_{sf}^{1-\nu_s}\right)^{\alpha_s} N_f^{1-\alpha_s} \tag{B.15}$$

$$Q_f = B_s P_f^{-\varepsilon_s} \tag{B.16}$$

$$R_f = P_f Q_f \tag{B.17}$$

$$= Z_f^{1-a_s-b_s} \left(K_{lf}^{\nu_s} K_{sf}^{1-\nu_s}\right)^{a_s} N_f^{b_s} \tag{B.18}$$

$$C_f^K = \sum_{j \in l,s} \left[\gamma \left(\frac{K'_{jf}}{K_{jf}} - 1 \right)^2 K_{jf} + \left(K'_{jf} - (1-\delta_{js}) K_{jf} \right) \right] \tag{B.19}$$

$$\Pi_f = R_f - C_f^K - w_s N_f \tag{B.20}$$

$$E_f = (1-\eta_{b,f})\Pi_f + \frac{1}{1+r}\mathbb{E}\left\{(1-\eta_{e,f})E'_f\right\} \tag{B.21}$$

$$\Gamma_f = \eta_{b,f}\Pi_f + \eta_{e,f}E_f \tag{B.22}$$

$$\varphi_f := \eta_{b,f} + \eta_{e,f}(1-\eta_{b,f}), \tag{B.23}$$

$$\beta_f := \frac{\eta_{e,f}(1-\eta_{b,f})}{\eta_{b,f} + \eta_{e,f}(1-\eta_{b,f})}, \tag{B.24}$$

$$\theta_f := \frac{1-\eta_{e,f}}{1+r} \tag{B.25}$$

$$N_f = \left(\frac{b_s Z_f^{1-a_s-b_s} \left(K_{lf}^{\nu_s} K_{sf}^{1-\nu_s}\right)^{a_s}}{w_s} \right)^{\frac{1}{1-b_s}} \tag{B.26}$$

$$0 = \frac{\partial \Pi_f}{\partial K'_{jf}} + \beta_f \theta_f \frac{\partial \Pi'_f}{\partial K'_{jf}} + \theta_f(1-\beta_f) \sum_{k=l,s} \frac{\partial K''_{kf}}{\partial K'_{jf}} \frac{\partial}{\partial K''_{kf}} V_f(\mathbf{K}''_f, \xi'_s) \tag{B.27}$$

Note that now, the continuation value $V_f(\cdot)$ also depends on ξ_s, which is a vector containing the sector-wide and aggregate variables, i.e. $\xi_s = (B_s, w_s, r)$. $V_f(\cdot)$ is now given by

$$V_f(\mathbf{K}_f, \xi_s) := \Pi_f + \theta_f V_f(\mathbf{K}'_f, \xi'_s).$$

Factor Markets

Regarding the labour market, we deviate from the partial-equilibrium calibration before and assume a fixed homogeneous labour supply per household $\bar{N}$ which we treat as numéraire. This means the nominal wage across industries is fixed at $w_s = w = 1$ and the real wage is given by

$$w_{real} = \frac{w}{\mathcal{P}} = \frac{1}{\mathcal{P}}.$$

Since we assume that capital is owned by the firm and there are capital adjustment costs, we need an assumption how this investment is produced. For simplicity, we assume that capital goods are produced using only labour as an input and that the adjustment of capital goods also only requires labour as an input.[28]

I.e., the overall labour demand of firm f is given by

$$\bar{N}_f = N_f + \sum_{j \in \{l,s\}} I_{jf} + \gamma \left(\frac{K'_{jf}}{K_{jf}} - 1 \right)^2 K_{jf}, \tag{B.28}$$

where $I_{jf} = K'_{jf} - (1 - \delta_{js}) K_{jf}$ is the firm's gross investment in capital goods of type j.

Equilibrium

The economy is inhabited by a continuum of ex-ante homogeneous households (of measure 1). In every period, each household is endowed with $\bar{N} = 1$ units of labour that is inelastically offered on a competitive labour market in order to generate income w. Households are assumed to hold equity only indirectly via a competitive mutual fund. In each period, a single household ('manager') is randomly chosen to manage any given firm f, for which they receive the corresponding compensation Γ_f. We assume that managers neglect the effects that their individual decisions have on the mutual fund and – as before – we assume they do not anticipate to manage the firm in the future. We further assume time-separable, homothetic preferences with respect to consumption of a final good, as well as complete markets. This means we do not need to track the distribution of wealth and income to infer aggregate demand dynamics. On a related note, we do not impose any restrictions on how households distribute the Γ_f. In particular, it could be that managers just amass more wealth or that they use an insurance mechanism to distribute managers' income across all households.

For aggregate consumption C in any steady state, we thus end up with a simple relationship: all labour income $w \cdot 1$, managers' remuneration Γ_f and the remaining dividends of firms $\Pi_f - \Gamma_f$ (where Π_f is the operating profit of firm f) are used to fund final consumption. Hence, we have

$$C = \sum_{f=1}^{\mathcal{N}_f} [\Gamma_f + (\Pi_f - \Gamma_f)] + w = \sum_{f=1}^{\mathcal{N}_f} \Pi_f + w.$$

Since we treat labour as numéraire, this becomes

$$C = \sum_{f=1}^{\mathcal{N}_f} \Pi_f + 1. \tag{B.29}$$

[28]One could, of course, also assume that investment goods are produced using the final good, which would allow for input-output relationships to become important. For the sake of simplicity and comparability to the partial-equilibrium setup, we abstract from that. A side benefit is that this way, since both q_l, q_s and w are fixed, the firm is really only linked to the aggregate economy via the demand shifter B_s. This simplifies calculations a lot because the firm's operations scale one-for-one with the the demand shifter. Hence, when solving the model, each firm's problem has to be solved exactly once, and then its chosen quantities only need to be rescaled in order to guarantee market clearing in the aggregate.

To close the model, we impose market clearing on both, goods and labour markets which implies

$$C = \mathcal{Q} \tag{B.30}$$

$$1 = \sum_{f=1}^{\mathcal{N}_f} \bar{N}_f. \tag{B.31}$$

Limitations: Before moving on, it is important to note a few caveats in our general-equilibrium analysis. We abstract here from firm entry or exit, endogenous technological change and input-output relationships which all could certainly alter some aspects of the quantification. We also still treat the remuneration packages as exogenous. However, since we are interested in the effects of changes in remuneration packages per se, we thus consider this to be a reasonable assumption

Experiment

The experiment we conduct in this general-equilibrium setting is very much akin to the one reported for the partial-equilibrium case in the main text. The firms have the same parameterisation as before. The only differences are that $w = 1$ for all firms and that the sectoral demand shifter is endogenous and adapts to ensure that the labour-market-clearing condition holds. Since we abstract from aggregate dynamics here (otherwise the solution algorithm would be a lot more involved), we focus on a steady-state comparison taking the observed changes due to FAS 123R as a permanent 'shock'.

Discussion

The quantified aggregate output drop equals 8 basis points in the general-equilibrium setting, compared to the 50 basis points in the partial-equilibrium setting. Besides differences in sectoral wages, the partial-equilibrium analyses plot means of normalised firm values which cannot be used for the aggregate adjustments in general equilibrium since here, the size differences across firms matter as well. Thus, the behaviour of the normalised aggregate variables presented in Table 2.14 rather resembles the one of a normalised mean *across* firms in the economy. To isolate the general-equilibrium feedback, we therefore also consider a scenario, where we shock the βs but keep B^{ind} constant such that we are still in a partial-equilibrium setting but with homogeneous wages fixed at 1. If we apply this to our sample and consider the same output measure as in the partial-equilibrium setting from before, firms' output shrinks by 0.61% on average which is substantially closer to the 0.50% obtained in the partial-equilibrium analysis with sectoral wage data. In general equilibrium, this overall effect on average firm output is then mitigated in absolute terms due to factor-market competition. Here, firms' output shrinks on average by 0.29% due to the reform. In contrast, if we take size differences across firms into account, the (fictitious) average firm sees its output decrease by 0.42% in the partial-equilibrium setting, whereas the average firm in general equilibrium has an output decrease of 12 basis points. The general-equilibrium effects at the aggregate level are thus broadly in line with the behaviour of the fictitious average firm that we studied in partial equilibrium. However, since consumers substitute demand away from short-termist firms, the effect on aggregate output is about one third smaller (8 versus 12 basis points) compared to the output change for the average firm.

2.C Parameterization and Solution Method

2.C.1 Remuneration Package

As we have derived in Subsection 2.3.1, for the purpose of our analysis we treat β as a structural parameter which is determined solely by the bonus share η_b and the equity share η_e (see Equation (2.11)). Both parameters can be directly inferred from the data relying on different sources which have been widely used in the literature. For η_b, we directly obtain the amount of bonus from Execucomp. Furthermore, due to a change in the reporting requirements for executive compensation after December 2006 we add the amount of non-equity incentive compensation to the bonus, which can be found in the *Plan-Based Awards (PBA)* file. This reclassification of bonuses is stressed by Hayes *et al.* (2012) and we follow their approach. In a next step we scale the amount of bonus with the sales of the firm (obtained from Compustat), i.e. $\eta_b = \frac{\text{Bonus+Non-eq-Targ}}{\text{Sales}}$. For the equity share η_e, we rely on data on the manager's firm-related wealth provided by Coles *et al.* (2006) and Core and Guay (2002), which we divide by the total market capitalisation of the respective firm (obtained from Compustat), i.e. $\eta_e = \frac{\text{Firm-related Wealth}}{\text{Market Capitalisation}}$. We winsorise each parameter $\eta_{b/e}$ at the top and bottom 1%. In a final step, we calculate β by applying Equation (2.11). In Table 2.C.1, we provide summary statistics on the key parameters η_b, η_e and β for our sample.

Table 2.C.1: Summary Statistics on Incentive Contracts

Variable	Mean	Std. Dev.	Min	p25	p50	p75	Max	Obs	Sample
η_b	4.028×10^{-4}	1.502×10^{-3}	0	4.668×10^{-5}	1.468×10^{-4}	3.854×10^{-4}	0.1242	16,320	2005 & 2007
η_e	7.922×10^{-3}	2.142×10^{-2}	1.916×10^{-5}	7.241×10^{-4}	1.946×10^{-3}	5.445×10^{-3}	0.1898	16,320	2005 & 2007
β	9.033×10^{-1}	8.400×10^{-2}	7.500×10^{-1}	8.393×10^{-1}	9.281×10^{-1}	9.758×10^{-1}	1.0000	16,320	2005 & 2007

Notes: The Table reports summary statistics on the bonus share η_b, the equity share η_e and β, which is calculated by applying Equation (2.11).

2.C.2 Other Parameters

Discount Factor: Given the parameters derived above, it would be straightforward to obtain $\theta = \frac{1-\eta_e}{1+r}$. Since we draw individual η_e values for each firm, θ would vary across firms, and thus the entire calibration would differ. To avoid this, for the calibration of parameters, we assume $\theta = \frac{1}{1+r}$, i.e. we here neglect the dilution factor. In the exercise reported in the main text, we, however, include η_e.

For r, we use the real interest rate for the United States from the year 2005, which was 2.981% according to World Bank (2020). While the definition of the proper discount factor is an important ongoing discussion, in our model it seems justifiable to take the (safe, apart from inflation risk) real interest rate as a benchmark since we abstract from both, growth and risk.[29]

[29] The choice of r merits some discussion: in the US, around the time of the reform, the real interest rate fluctuated between a high of 6.845% in 2000 and a low of 1.137% in 2011. This happened against the background of an overall downward trend since the 1980s, which was overlaid between 2005 and 2007 by contractionary monetary policy. Over the years 2000–2009 the (geometric) average real interest rate

Production Function: We take δ_s, δ_l, R, $\frac{K_l}{K_l+K_s}$, $\frac{K_l}{R}$, $\frac{wN}{R}$, and w directly from the sectoral data.

Then, for $\beta = 1$, the steady-state conditions given in the main text can be re-arranged so as to yield direct expressions for the remaining parameters. Combining the two FOCs of individual capital goods, we get

$$\nu = \frac{1 - \theta(1 - \delta_l)}{1 - \theta\left[1 - \delta_s - \frac{K_l}{K_l+K_s}(\delta_l - \delta_s)\right]} \frac{K_l}{K_l + K_s}.$$

Given ν, we can solve the first-order condition of the long-term capital good for a as

$$a = \frac{\frac{1}{\theta} - (1 - \delta_l)}{\nu} \frac{K_l}{R}.$$

Likewise, b directly follows from optimal labour demand as

$$b = \frac{wN}{R}.$$

This allows us to recover ε and α from

$$\varepsilon = \frac{1}{1 - a - b}, \quad \alpha = \frac{a}{a + b}.$$

Finally the scaling parameter B^{ind} can be fixed using the labour demand as well as the production function, which then yield

$$B^{ind} = \left(\frac{w^{\frac{b}{1-b}} R}{b^{\frac{b}{1-b}} \left(K_l^{\nu} K_s^{1-\nu}\right)^{\frac{a}{1-b}}} \right)^{\frac{1-b}{1-a-b}}.$$

Note that our assumptions so far imply that firms within an industry have the same parameters, apart from TFP, θ, and the remuneration package.

2.C.3 Sensitivity Analysis

Adjustment Costs

As we have noted before, the adjustment-cost parameter γ also affects the steady state because it alters the slope of the value function and consequently also the policy functions, whenever $\beta < 1$. To study the sensitivity of our results with respect to different values

in the US was about 3.677%, but for the years 2010–2019 it has fallen to 1.996%; between 2003 and 2008 the figure was 3.309%. It's thus not entirely clear which value one should choose as a steady-state value. However, our results would not change much if we used a different value for r. For private businesses, the discount factor should take into account risk premia (related to, inter alia, idiosyncratic uncertainty and the financing structure of the firm), and thus be smaller. On the other hand, due to technological progress and the growth of the overall economy, a firm should expect the demand shifter as well as its TFP to change over time, changing the size of the firm. I.e., if we reinterpret our model's steady state as a balanced growth path with growth rate g and with the variables of the model properly detrended, the firm's discount factor would effectively be $\theta = \frac{(1-\eta_e)(1+g)}{(1+r)}$, which effectively increases the discount factor. Thus, our measure of the discount factor will most likely be either too high or too low. In fact changing θ (thus, also changing r) has a somewhat similar effect as changing β, per se.

of γ, we either consider a value of γ that equals half its original value ($\underline{\gamma}$) or twice its original value ($\overline{\gamma}$).

Changing γ affects both, the resulting steady-state levels of capital goods and the dynamic response to a change in β. Concerning the steady state, the effect of a change in β is muted with $\overline{\gamma}$. Both capital goods fall by roughly one fifth less in response to a given reduction in β in steady state. Alternatively, capital goods fall by approximately one fifth more in steady state with $\underline{\gamma}$. This also changes the composition of steady-state capital stocks, although only relatively mildly. For example, consider a firm that experiences a reduction in its β from 1 to some lower value. If the firm faces low adjustment costs $\underline{\gamma}$ the fall in the share of long-term capital in total capital is roughly one fourth larger compared to the case with original adjustment costs. In contrast, if the firm faces high adjustment costs $\overline{\gamma}$, the share of long-term capital is less responsive and its fall is diminished by about one fourth. Considering the dynamic impact, we also see very intuitive results. When adjustment costs are higher, firms take longer to reach the new steady state and vice versa. To sum up, higher adjustment costs make capital stocks (and their composition) more rigid, in the sense that they become less responsive to changes in β.

Complementarity of Capital Goods

In the main analysis, we consider a Cobb-Douglas production function which implies that the elasticity of substitution between the capital goods equals one ($\sigma_k = 1$) such that both goods are independent from each other. Here, we consider the sensitivity of our results with respect to perturbations of σ_k. A first intuition is that the closer substitutes the two capital goods are ($\sigma_k \to \infty$), the stronger the differential impact of a change in β should be. On the contrary, the more the two types of capital are complements ($\sigma_k \to 0$), the weaker a differential impact one would expect. While this intuition is correct for most perturbations of σ_k, it comes with one caveat: with perfect substitutes, we are in a knife-edge case. For a range of β values, the firm then fully invests in only one type of capital. Consequently, there will be no within-firm reallocation for certain values of β in the limit $\sigma_k \to \infty$.

In our sensitivity analysis, we consider the range $\sigma_k \in [\underline{\sigma}_k = 0.5, \overline{\sigma}_k = 2]$ and find that our results did not qualitatively change as a drop in β still induces a decline in overall investment and a relative shift between the two capital goods. With $\underline{\sigma}_k$, this effect is weakened by roughly one half which is due to long-term capital falling less and short-term capital falling more than in the Cobb-Douglas case. With $\overline{\sigma}_k$, the effect is increased by about one half.

2.C.4 Numerical Solution Method

To illustrate the solution method, we continue with the notation introduced in the previous section. Since the labour decision in the problem above is simply determined by the first-order condition (B.4), we can write per-period operating profits as a function of $\mathbf{K}, \mathbf{K}'$ only by defining:

$$\pi^*(\mathbf{K}, \mathbf{K}', \xi) = \max_N \{\pi(\mathbf{K}, \mathbf{K}', N, \xi)\}. \tag{B.32}$$

Importantly, this function satisfies

$$\frac{\partial}{\partial K_j'} \pi^*(\mathbf{K}, \mathbf{K}', \xi) = \frac{\partial}{\partial K_j'} \pi(\mathbf{K}, \mathbf{K}', N, \xi), \quad j = l, s.$$

The optimisation problem of the manager can be re-stated in recursive form as

$$\Gamma(\mathbf{K}, \xi) = \max_{\mathbf{K}'} \{\pi^*(\mathbf{K}, \mathbf{K}', \xi) + \beta\theta V(\mathbf{K}', \xi) \tag{B.33}$$

$$\text{s.t. } V(\mathbf{K}', \xi) = \pi^*(\mathbf{K}', \mathcal{K}(\mathbf{K}', \xi), \xi) + \theta V(\mathcal{K}(\mathbf{K}', \xi), \xi)\}. \tag{B.34}$$

Here, the future policy function $\mathcal{K}(\cdot)$ is defined as

$$\mathcal{K}(\mathbf{K}, \xi) = \arg\max_{\mathbf{K}'} \{\pi^*(\mathbf{K}, \mathbf{K}', \xi) + \beta\theta V(\mathbf{K}', \xi)\}. \tag{B.35}$$

Note that we assume that this policy function is time-invariant which results from our focus on symmetric strategies.

Next, to keep the notation concise, define the gradient of a function $f(\mathcal{K}, \xi)$ in terms of elements of $\mathcal{K}$ to be given by

$$\nabla_{\mathbf{K}} f(\mathbf{K}, \xi) = \left[\frac{\partial f(\mathbf{K}, \xi)}{\partial K_l} \ \frac{\partial f(\mathbf{K}, \xi)}{\partial K_s}\right]'.$$

We use similar notation for functions with multiple inputs, and the index of ∇ gives the input the gradient applies to. Then, the first-order conditions (B.6) can be stated as

$$\nabla_{\mathbf{K}'} \pi^*(\mathbf{K}, \mathbf{K}', \xi) = -\beta\theta \nabla_{\mathbf{K}'} V(\mathbf{K}', \xi). \tag{B.36}$$

From (B.32), we can derive

$$\begin{aligned}\nabla_{\mathbf{K}'} \pi^*(\mathbf{K}, \mathbf{K}', \xi) &= -\nabla_{\mathbf{K}'} C^K(\mathbf{K}, \mathbf{K}') \\ &= -\begin{bmatrix} \gamma\left(\frac{K_l'}{K_l} - 1\right) + 1 \\ \gamma\left(\frac{K_s'}{K_s} - 1\right) + 1 \end{bmatrix}.\end{aligned}$$

That is, in terms of any capital good, we obtain a first-order condition

$$\gamma\left(\frac{K_j'}{K_j} - 1\right) + 1 = \beta\theta \frac{\partial V}{\partial K_j'}(\mathbf{K}', \xi).$$

Note that this can be readily solved for K_j:

$$K_j = \frac{K_j'}{1 + \frac{\frac{\partial V}{\partial K_j'}(\mathbf{K}', \xi) - 1}{\gamma}}. \tag{B.37}$$

Equation (B.37) is the central ingredient in the endogenous grid method we apply. This method is best described by Algorithm 1 below.

Essentially, we start with a set of G gridpoints $\tilde{\mathcal{K}}' = (\tilde{\mathbf{K}}_h')_{h=1,\dots,G}$, which represent different outcomes of $\mathbf{K}'$, and an initial (differentiable) guess $\hat{V}_0(\cdot)$ for $V(\cdot)$. By differentiating $V(\cdot)$, we get the gradient at each point in $\tilde{\mathcal{K}}'$. Then applying the backward induction step in (B.37), we can solve for the optimal solution of the previous manager. Next, we update our guess for the continuation value function $V(\cdot)$ according to the profit function and our current guess. One then iterates on this until convergence is achieved.

We implement this algorithm as MATLAB code (tested against MATLAB R2018b and R2020a), which can be found in the replication package.

The figures in this chapter are based on a sample of 1,000 firms with idiosyncratic parameter draws 30-by-30 in the (K_l', K_s')-space. The coordinates of the gridpoints correspond to Chebyshev nodes in a range around the steady state with $\beta = 1$, (which can be computed analytically). To be precise, the grid ranges from 0.3 to 1.2 of the analytical steady state of that parameterisation. As an interpolation scheme $\rho(\cdot)$ we opt for Chebyshev polynomials up to degree 10 in either dimension.[30] Since the endogenous grid method inherently involves interpolation with a changing set of interpolation bases, the domain of the chosen functions was expanded as needed to keep all points within the domain.

Finally, to specify an initial guess for the value function, we follow the following procedure: initially, we consider with a model where β was set to 1, for which a steady state can be derived analytically. As an initial guess of the value function, we simply assumed that the model would converge uniformly to that steady state within a certain period. Using the resulting net present value of profits gives a reasonably accurate initial guess for the case of $\beta = 1$. However, for lower $\beta < 1$, this does not necessarily lead to convergence. For this reason, we first solved the model for the $\beta = 1$ case. Then, we use the final value function computed and use this as an initial guess to solve the model with a slightly lower value of β. Repeating this process while slowly decreasing β yields satisfactory convergence. The entire process is then repeated for all 1,000 (differently parameterised) firms in the sample.

Modification in the Pseudo-General-Equilibrium Exercise: If we want to use the previous algorithm in a general-equilibrium environment, we need to take into account that each firm now also takes aggregate state variables into account. These include in our framework the two aggregate capital stocks, or more precisely their distribution across all active firms. In the related literature with heterogeneous agents or firms (e.g., Krusell and Smith 1998, Khan and Thomas 2013), the distribution of capital across agents or firms becomes an important state variable, which is an infinitely-dimensional object with infinitely many firms or agents and thus needs to be approximated. In our simulated sample, we only use a finite number of firms (1,000) but accounting for this we would still have a 2,000-dimensional state variable for capital goods alone (1,000 firms $\times$ 2 capital goods). Since we are not interested in the dynamics per se, we can simplify matters a lot by only focusing on aggregate steady states.

When the economy at large is in a steady state, we can use our algorithm from before to solve for each single firm. Note that the only aggregate variable relevant for the firm's problem is the industry-level demand shifter B^{ind}. It is straightforward to show that this shifter proportionally scales the scale of the firm. To make this more precise, the policy

[30] We have chosen Chebyshev polynomials because they have preferable interpolation properties compared to other polynomials functions. Also, Splines were considered, but computing the gradient of a spline is a computationally expensive exercise and experiments with cubic splines showed inferior convergence properties. We also experimented with Chebyshev polynomials with a total degree of 30. However, most coefficients with a higher degree are virtually identical to zero. In fact, higher order polynomials present a problem for the algorithm since for these higher order polynomials, the gradient quickly becomes very large in absolute terms, even if the corresponding coefficient is small; this generates additional sources of numeric error, which leads to far worse convergence properties. Given that this method ultimately generates an inverse of the policy function, we eventually have to back the real policy functions out. This final step is done using cubic splines.

Algorithm 1: Version of EGM used in the model solution

Set i_{max} as well as convergence thresholds $\bar{\epsilon}^v, \bar{\epsilon}^{invp} > 0$ for the continuation value and inverse policy, respectively. Pick a parameter vector ξ, a set of gridpoints $\tilde{\mathcal{K}}' = (\tilde{\mathbf{k}}'_g)_{g=1,\ldots,G}$, an initial guess for each of these points, i.e. $\hat{V}_{0,g}$ for $g = 1, \ldots, G$, and an interpolation scheme $\rho(x, X, Y)$ to be used. Find interpolated values $v_0(\mathbf{K}) = \rho(\mathbf{K}, (\mathbf{k}'_g)_{g=1,\ldots,G}, (\hat{V}_{0,g})_{g=1,\ldots,G})$.

Set *continue*=true. set i=1.

while *continue* **do**

- **for** *g=1,…,G* **do**
  - Set $\hat{\mathbf{k}}_{j,i,g} = \frac{\gamma k'_{jg}}{\gamma + \beta\theta \frac{\partial}{\partial \mathbf{K}'_j} v_{i-1}(\mathbf{k}'_g) - 1}$ for $j = l, s$.
  - Set $\tilde{v}_g = \Pi(\mathbf{k}_{i,g}, \mathbf{k}_g, \xi) + \theta \hat{V}_{i-1,g}$.
- Find interpolant $v_i(\mathbf{K}) = \rho(\mathbf{K}, (\mathbf{k}_{i,g})_{g=1,\ldots,G}, (\tilde{v}_g)_{g=1,\ldots,G})$.
- **for** *g=1,…,G* **do**
  - Set $\hat{V}_{i,g} = v_i(\mathbf{K}_g)$.
  - Set $\epsilon^v_{ig} = \left| \frac{\hat{V}_{i,g}}{\hat{V}_{i-1,g}} - 1 \right|$.
  - Set $\epsilon^{invp}_{jig} = \left| \frac{k_{j,i,g}}{k_{j,i-1,g}} - 1 \right|$.
- **if** $\max_{g\in(1,\ldots,G)}\{\epsilon^v_{ig}\} < \bar{\epsilon}^v$ *and* $\max_{j\in(l,s),g\in(1,\ldots,G)}\{\epsilon^{invp}_{ig}\} < \bar{\epsilon}^{invp}$ **then**
  - Set *continue*=false.
- **else**
  - Set i=i+1;

Obtain policy function as $\mathcal{K}(\mathbf{K}, \xi) \approx \tilde{\mathcal{K}}(\mathbf{K}, \xi) := \rho(\mathbf{K}, (k_{i,g})_{g\in\{1,\ldots,G\}}, (\mathbf{k}_g)_{g\in\{1,\ldots,G\}})$

function now depends on the demand shifter as well as on parameters ξ:

$$\mathbf{K}' = \mathcal{K}\left(\mathbf{K}, B^{ind}, \xi\right). \tag{B.38}$$

Notably, it can be shown that the policy functions scale with the demand shifter as follows:

$$\mathcal{K}(\mathbf{K}, B^{ind}, \xi) = B^{ind} \cdot \mathcal{K}\left(\frac{1}{B^{ind}}\mathbf{K}, 1, \xi\right). \tag{B.39}$$

From this, we can directly infer that the steady-state capital stock of the firm directly scales with B^{ind}.

The firm affects the general equilibrium through its factor choices, its output Q_f and its price level P_f. Notably, while a firm's steady-state output Q_f is directly proportional to B^{ind} its price in steady state is fully determined by technology and the relative composition of its factor choices. We have just argued that the entire policy function is scaled up or down by B^{ind} and as a a result, B^{ind} does not affect the relative composition of its factor inputs in steady state. I.e., the steady-state price level of the firm is independent of macroeconomic outcomes. This allows us to solve for the pseudo-general-equilibrium solution in a simple way. For each firm, we can simply solve the firm's problem for an arbitrary B^{ind} and obtain the firm-level steady state. From now on, we only refer to steady-state values of all variables. We can do this exercise for our entire sample of firms, $f = 1, \ldots, 1,000$. As a result, we have a steady-state price level P_f for each firm. The resulting steady-state price level can be used to infer sectoral and aggregate price levels $\mathcal{P}_s, \mathcal{P}$ using (B.11) and (B.8). From (B.9), it is possible to show that the demand shifter in any sector is then proportional to aggregate demand $\mathcal{Q}$ times a function purely dependent on the pricing choices of all firms. As a result, also the quantity produced by any firm, and ultimately factor choices are simply proportional to aggregate demand.

Thus, to derive general equilibrium, we simply obtain all the relevant price levels.

Using (B.9) and (B.12), we can obtain

$$Q_f = \psi_s P_f^{-\varepsilon_s} \mathcal{P}_s^{\varepsilon_s - 1} \mathcal{P} \mathcal{Q}, \tag{B.40}$$

i.e., the output of any firm and hence its factor demand is proportional to aggregate demand.

Here, since prices are fully determined by parameters and firms' incentive structure, we get

$$Q_f = p_f \mathcal{Q}, \tag{B.41}$$

where $p_f = \psi_s P_f^{-\varepsilon_s} \mathcal{P}_s^{\varepsilon_s - 1} \mathcal{P}$ does not depend on $\mathcal{Q}$. From the firm's individual problem, we can derive a steady-state ratio of total labor used to output produced as $n_f = \frac{\bar{N}_f}{Q_f}$, which again is independent of $\mathcal{Q}$. Total labour demand is then given by

$$\bar{N} = \sum_{f=1} \bar{N}_f = \sum_{f=1}^{\mathcal{N}_f} (n_f p_f) \mathcal{Q}.$$

$\mathcal{Q}$ directly follows by imposing market clearing on the labour market. We then scale each firm accordingly, taking into account p_f and n_f.

Chapter 3

Monetary Policy Strategies under Bounded Rationality

Based on joint work with Michael Dobrew,[§] Rafael Gerke[*] and Daniel Kienzler[‡]. Previous results from this project were published as part of a monthly report by Deutsche Bundesbank (2021).

Abstract

This chapter studies the performance of various monetary policy strategies in a New Keynesian model with sticky wages, supply shocks and bounded rationality. We show that sticky wages enhance bounded rationality as a solution to various New Keynesian paradoxes. Enhanced determinacy, a reduced severity of ELB episodes and a limited power of forward guidance imply that policy strategies with a strong history dependence loose their typical advantage. Nevertheless, we uncover that an interest rate rule with a variable degree of history dependence - exponential average inflation targeting - performs remarkably well across different types of shocks and independently of the degree of myopia.

Keywords: Bounded Rationality, Sticky Wages, Monetary Policy Strategies, Zero Lower Bound

JEL Codes: E20, E24, E31, E32, E52

We have benefited from discussions with Gerhard Illing, Assaf Razin and a seminar audience at LMU Munich.

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3.1 Introduction

The long-term decline in the natural real rate[1] and the constraint on conventional interest rate policy imposed by the effective lower bound (ELB) on nominal interest rates has posed a considerable challenge for monetary policy in recent years. Consequently, in the strategy reviews that major central banks conducted recently,[2] an important aspect was how to provide policy stimulus at the ELB. One possibility to achieve this are so-called history dependent strategies like average inflation targeting (AIT) or price level targeting (PLT).[3] These strategies take into account past deviations of prices or inflation from target and can yield powerful additional stimulus during ELB periods through a built-in make-up element. For example, if inflation falls short from its target during an ELB episode, monetary policy will keep interests low even after the ELB ceases to bind in order to make up for these inflation shortfalls. If agents rationally expect higher inflation in the future, this can have stabilising effects on inflation already today via declining real interest rates. This mechanism largely depends on agents being rational and forward-looking. However, recent research on expectation formation has increasingly documented substantial deviations from full-information rational expectations.[4] Thus, a key question for monetary policy makers is whether history-dependent strategies still perform well when expectations deviate from full-information rational expectations. Moreover, ELB episodes are not the only scenarios that history-dependent strategies need to cope with. In particular, supply shocks that may or may not come along with ELB episodes induce a tradeoff between the stabilisation of inflation and real activity that poses a challenge for monetary policy.

In this chapter, we analyse the performance of different optimised history-dependent interest rate rules compared to an optimised non-history-dependent Taylor-type rule in an ELB-constrained economy with boundedly rational agents and a role for supply-side shocks. To that end, we employ a New Keynesian model with sticky prices and sticky wages as in Erceg *et al.* (2000). Moreover, in the spirit of Gabaix (2020), we assume that agents are partially myopic in the sense that they discount expectations of future variables. This weakens the strong positive effects of the expectations channel at the ELB. At the same time, agents are fully aware of the long-term macroeconomic relationships and thus also of the long-term economic equilibrium that arises in the absence of shocks.

Our first main result is that in this setup, for a sufficiently high degree of bounded rationality, a non-history-dependent Taylor-type rule representative of inflation targeting (IT) performs better in terms of welfare than a PLT and an AIT rule, the most prominently discussed variants of history-dependent policy rules. This is true for both supply and demand shocks. In contrast, if the degree of bounded rationality is low, the expectations channel is not crucially weakened and history-dependent rules retain their advantages over non-history dependent rules, as in a rational expectations setting.

In practice, monetary policy makers face considerable uncertainty about the degree of bounded rationality. Therefore, a robust interest rate rule should perform reasonably

[1] See e.g. Brand *et al.* (2018) or Holston *et al.* (2017).

[2] E.g. the Federal Reserve concluded its strategy review in 2019, the Eurosystem in 2021, the Bank of Canada engages in a 5-year returning review process and the Bank of Japan has conducted a smaller policy review in the beginning of 2021.

[3] Other possibilities in the realm of monetary policy include negative interest rates, forward guidance, and asset purchase programmes. In this chapter, we focus exclusively on history-dependent strategies.

[4] For survey evidence, see e.g. Coibion *et al.* (2018). For experimental evidence, see e.g. Afrouzi *et al.* (2021).

well across different degrees of bounded rationality, and across different types of shocks. Essentially, on the one hand, such a rule should exhibit some form of history dependence in order to reap the benefits of this feature in case the degree of bounded rationality is low. On the other hand, it should resemble an IT rule when the degree of bounded rationality is high. In principle, the AIT rule can fulfill these requirements as it performs in between the IT and the PLT rule for both low and high degrees of myopia. However, in particular for trade-off-inducing technology shocks, conventional AIT that features an arithmetic, or simple, moving average of inflation rates exhibits an inherent volatility-inducing character. The reason is that any past deviation of inflation from its target directly affects the average for a given time frame, calling for a compensation by monetary policy in order to achieve its average inflation target. But once the initial deviation drops out of the averaging window, monetary policy is required to compensate the previous compensation to achieve its average inflation target, and so on. Conventional AIT thus has the disadvantage that it requires periodic fluctuations in the inflation rate to achieve its average inflation target.

We propose an exponential AIT (eAIT) rule as an approach which retains the advantages of the conventional AIT rule and avoids its disadvantage of inherently inducing volatility. Under an eAIT rule, monetary policy targets an exponential moving average of the current and past inflation deviations instead of an arithmetic, or simple, moving average as in conventional AIT rules. This implies that the weights of past inflation rates entering the average exponentially decay. That is, past inflation deviations are assigned a higher weight the closer they are to the present period. This is in contrast with the conventional AIT rule where all (past) inflation deviations entering the average receive equal weights. Thus, in comparison to a conventional AIT rule, the higher weights on inflation rates closer to the present "tilt" the character of the eAIT rule towards an IT rule. The effect of this tilting is more pronounced for higher degrees of bounded rationality, as this further reduces the influence that inflation rates further in the past have on expectations about future inflation rates. Consequently, as a second main result, the eAIT rule not only avoids inherently inducing volatility and preserves the history-dependent character of the conventional AIT rule. It also is able to approximate the IT rule when the degree of bounded rationality is high.

To build intuition for the effects of myopia and sticky wages on the performance of history-dependent policy rules, we show that, in our setup, several well-known paradoxes in New Keynesian models are mitigated: excessively large recessions due to the ELB, the existence of a continuum of equilibria if monetary policy is sufficiently passive, and excessively large effects of forward guidance.[5] While Gabaix (2020) shows that bounded rationality in itself contributes to mitigating these paradoxes, we extend this result and document how the interaction between sticky wages and boundedly rational expectations enhances their mitigation. In particular, in our setup, indeterminacy regions, the severity of ELB episodes, and the power of forward guidance decrease compared to a setup with boundedly rational expectations and sticky prices only. The channel that is responsible for the occurrence of the paradoxes is the large impact of expectations of future variables on current economic outcomes. The strong performance of history-dependent policy rules in the traditional New Keynesian model depends on the same channel. Thus, mitigating

[5]The paradox of toil and the paradox of flexibility are in essence not affected by bounded rationality. If anything, the former is aggravated because a given technology shock introduces larger swings in the natural rate. Concerning the latter, higher flexibility still makes the Phillips curve steeper. This means that while the effect of changes of future inflation is lessened, the immediate impact of a given change in the output gap on inflation is still the same.

the paradoxes by weakening this channel also reduces the power of history-dependent policy rules.

The assumption of sticky wages is important in our analyses as they, in conjunction with sticky nominal prices, induce that the real wage can no longer adjust freely in response to shocks. Accordingly, monetary policy needs to decide on the relative roles of price and wage inflation in bringing about real wage adjustments. In such a setup, the divine coincidence disappears for technology shocks as these shocks directly affect the desired real wage. As a result, technology shocks have an effect on economic welfare under alternative policy frameworks. In principle, history-dependent strategies can be welfare-enhancing in the presence of such inefficient shocks as they can contribute to resolving the trade-off between inflation and real activity with the help of expectations.[6] Thus, weakening the expectations channel also reduces the performance of history-dependent rules in the case of inefficient technology shocks.

To compare the performance of the different monetary policy strategies under different degrees of myopia whilst taking into account the ELB, we proceed as follows: For each rule and each degree of myopia M, we span a grid over the rules' parameters. On each grid node, we run a stochastic simulation within which we solve the model with the extended path algorithm as described in Fair and Taylor (1983).[7] These simulations give us probability distributions of the model variables conditional on the type of shock. An advantage of our relatively simple model structure is that we can use the model-consistent welfare loss function to evaluate the performance of each rule. Thus, for each degree of myopia we can compare a set of optimised interest rate rules against each other. We consider values for M in the range of 1 to 0.5, where $M = 1$ coincides with rational expectations.[8]

In the case of technology shocks, for low degrees of myopia, the PLT rule is the welfare-optimal rule whereas the IT rule performs markedly worse and the performance of the AIT rule lies in between. As the degree of myopia increases, the welfare losses of the PLT and AIT rules increase and the IT rule becomes the welfare-dominant policy rule for myopia parameter value $M < 0.8$. In the case of demand shocks, again the history-dependent PLT and AIT rules perform very well in terms of welfare for low degrees of myopia. Again, IT lags behind in terms of welfare for low degrees of myopia. As the degree of myopia increases, the performance of all rules improves, but this improvement is most pronounced for the IT rule, which overtakes the PLT rule for low values of M and closes the gap to the AIT rules.

When it comes to the eAIT rule, we consider a high and a low value for the smoothing parameter in the exponential moving average that enters the average inflation target. A lower value shifts weight to past inflation rates closer to the present. For supply shocks, we find that the eAIT rule performs almost as well as PLT for low degrees of myopia, especially the eAIT rule with a high smoothing parameter. For low values of $M < 0.75$, i.e., in regions where the performance of the PLT rule is markedly worse than that of the IT rule, the version of the eAIT rule with a small smoothing coefficient performs equally well or even slightly better than the IT rule, depending on the exact value of M. For

[6]For example, after a shock that increases inflation and decreases real activity, the expectation that monetary policy will be restrictive in the future to make up (some of) the higher inflation rates today attenuates the inflation increase today, without monetary policy having to amplify the recession in order to restrict the inflation increase.

[7]We implement the computations with the software package Dynare, see Adjemian *et al.* (2021).

[8]$M = 0$ would establish no influence of expectations of future variables whatsoever. We do not consider values of $M < 0.5$ as it is questionable whether such high degrees of myopia would be plausible.

demand shocks, the performance of the eAIT rule is very similar to PLT for low degrees of myopia, and almost indistinguishable from IT for high degrees of myopia.

The chapter draws on several branches of the literature. The way we model agents' myopia is inspired by Gabaix (2020). He takes into account discounting of future deviations from steady state, which he introduces via expectations regarding future variables that are biased towards the steady state. Similar to Erceg *et al.* (2021), we do not micro-found myopia but use a shortcut to capture the idea that expectational effects are attenuated on an aggregate level. This idea has been formalised in different ways in several recent papers. For example, García-Schmidt and Woodford (2019) and Farhi and Werning (2019) introduce different variants of level-k thinking, an approach to bounded rationality that involves a finite number of updating rounds when deducing the behaviour of other agents in the future. Woodford (2019) introduces agents with a finite planning horizon, and Angeletos and Lian (2018) relax the assumption of common knowledge, resulting in a form of myopia on the aggregate level.[9]

Gabaix (2020) shows that in the New Keynesian model with bounded rationality, strict price-level targeting is not the optimal policy anymore in the case of a cost-push shock. This relates to our results concerning a technology shock in the sticky-price-sticky-wage economy. Benchimol and Bounader (2021) study optimal monetary policy under bounded rationality. They show that PLT remains the optimal policy as long as agents continue to form expectations about inflation rationally, even if they are myopic with respect to the rest of the economy. In addition, we consider an economy subject to the ELB which is the prime motivation to consider history-dependent policy strategies in the first place. In that regard, we share a part of the setting of Nakata *et al.* (2019). They analyse optimal monetary policy at the effective lower bound in a New Keynesian model with discounting in the Euler equation as well as the Phillips curve. However, unlike the present paper, they abstract from wage rigidities and only focus on demand shocks. In contrast to Benchimol and Bounader (2021) and Nakata *et al.* (2019), we consider simple rules.

Our paper also relates to other analyses that study history-dependent monetary policies with simple rules, taking into account the ELB. Reifschneider and Williams (2000) track the sum of past deviations of the policy rate from the desired rate due to the ELB and propose to compensate this shortfall in stimulus by keeping the policy rate lower-for-longer. Nakov (2008) studies AIT and PLT rules, among others, in the light of the ELB. In contrast to these papers, we consider boundedly rational agents and study how this feature affects the performance of history-dependent monetary policies. Bernanke *et al.* (2019) analyse history-dependent policy rules in a large model for the US and consider reduced effectiveness of history-dependent policies due to imperfect credibility of the monetary policy framework. Similarly, Coenen *et al.* (2021) use a large model for the euro area to compare history-dependent monetary policy rules.[10] Erceg *et al.* (2021) compare

[9] Other attempts to remedy the implausibly strong forward-lookingness of the standard New Keynesian model include Bilbiie (2021) and Michaillat and Saez (2021). The former considers a simple (analytical) two-agent incomplete-markets formulation of the New Keynesian model, in which agents self-insure against idiosyncratic transitions into worse states. The paper shows that the empirically relevant case of procyclical income risk in fact reinforces forward-lookingness of the model; unrealistic countercyclical income risk mitigates this. Michaillat and Saez (2021) study how wealth preferences (in terms of bond holdings) affect the dynamic properties of the New Keynesian model. They find that wealth preferences can act similar to a discounting term in the Euler equation. Also, they show that this has important implications concerning the dynamics at the effective lower bound.

[10] Their model principally also allows for limited credibility. However, our understanding is that they consider limited credibility only for their analysis of forward guidance, not for their analysis of history-dependent policy rules.

history-dependent policy rule in a medium-scale model for the euro area and additionally consider boundedly rational agents as in Gabaix (2020) as we do. In contrast to these studies, we use a smaller model which allows us to use a model-consistent welfare function in order to optimise the coefficients of our policy rules and produce welfare rankings of the different rules.

Two other papers that we know of also study AIT with an exponential moving average of the inflation rate. Nakata *et al.* (2020) analyse the welfare properties of eAIT under rational and boundedly rational expectations, and determine the optimal smoothing parameter of the exponential moving average in both cases.[11] Their motivation for using an exponential moving average is a technical one.[12] Using an arithmetic moving average involves incorporating a possibly large number of endogenous state variables (the lags of the inflation rate) into the model. This would imply prohibitively large computational costs given their global solution method that fully takes into account uncertainty. In contrast, we use the extended path algorithm to solve the model, a global solution method that makes use of certainty equivalence. This choice allows for considering arithmetic as well as exponential moving averages in the interest rate rule while taking into account the effective lower bound. Honkapohja and McClung (2021) analyse the stability properties of various AIT rules, among them a rule involving the exponential moving average of the inflation rate, under adaptive learning. In contrast, our focus is on the welfare properties of different monetary policy strategies in a setting with myopic agents.

The remainder of the chapter is structured as follows. Section 3.2 presents the log-linearised model, its calibration, and the numerical methods used to solve the model and conduct the grid search for finding the optimal coefficients of the rules. Section 3.3 discusses the implications of the interaction between sticky wages and bounded rationality, in particular with respect to determinacy and the power of Forward Guidance. Section 3.4 performs a welfare comparison of the simple optimised IT, AIT and PLT rules. Subsection 3.4.4 presents eAIT as a robust approach to monetary policy when the degree of bounded rationality is uncertain. Section 3.5 concludes.

This chapter also features two appendices: Appendix 3.A presents the derivation of the model and appendix 3.B contains additional results.

3.2 Model

In this section, we augment a New Keynesian model with sticky prices and sticky wages as in Erceg *et al.* (2000) with bounded rationality in the spirit of Gabaix (2020) and an ELB on the policy rate. We present the model's log-linearised equilibrium conditions and delegate the derivation of the micro-founded non-linear equilibrium conditions to Appendix 3.A.[13]

[11] Their analysis is in the spirit of the policy delegation literature, i.e., the monetary authority derives its decision rule from maximising under discretion an objective function that involves average inflation.

[12] Nakata *et al.* (2020) argue that eAIT can be considered a crude proxy for AIT. Our results below for technology shocks rather suggest that eAIT can behave quite differently than regular AIT.

[13] While we could in principle conduct our simulations presented in later sections with the non-linear equilibrium conditions, we found that linearising them is conducive to the convergence of the extended path algorithm and thus speeds up the simulations. In any case, the key non-linearity in the model, namely the ELB, is fully taken into account in our simulation exercises.

3.2.1 Private Sector

Time is discrete and the private sector of the economy features four agents: households, monopolistically competitive labour unions setting wages and two types of firms. Intermediate goods firms engage in monopolistic competition producing output which in turn is used by a competitive final goods firm to produce the final consumption good. Agents' expectations are crucial in determining equilibrium outcomes and in our model they generally need not be rational. Each sector might form expectations differently and we therefore denote expectations of each sector $s \in \{H, F, U\}$ in period t regarding some variable x_τ in period $\tau > t$ as $\mathcal{E}_t^s[x_\tau]$.

Households' optimisation problem gives rise to a dynamic IS curve relating period-t the (log-linearised) deviation of output y_t from steady state to the same output deviation in the next period, the contemporaneous real interest rate r_t and a demand shock z_t

$$y_t = \mathcal{E}_t^H[y_{t+1}] + \frac{\rho + z_t - r_t}{\sigma}, \tag{3.1}$$

where $1/\sigma$ denotes the elasticity of intertemporal substitution of consumption and ρ is both the rate of time preference and the steady-state real interest rate.

The ex-ante real rate is related to the nominal interest rate i_t and inflation expectations of households π_t^e via the Fisher equation

$$r_t = i_t - \pi_t^e. \tag{3.2}$$

We follow Gabaix (2020) and assume that inflation expectations in the Fisher equation are actually rational, i.e., $\pi_t^e = \mathbb{E}_t[\pi_{t+1}]$, where $\mathbb{E}_t[\cdot]$ is the expectations operator, conditional on information available in period t.[14]

Households supply labour to unions which set wages according to Calvo (1983). Only a fraction $1 - \theta_w$ of wages are optimised in each period. We assume that the remainder of unions have their wages indexed to the steady-state inflation rate π^*. Standard derivations then yield the New Keynesian wage Phillips curve

$$\pi_{w,t} - \pi^* = \kappa_w x_t - \lambda_w \hat{w}_t + \beta \mathcal{E}_t^U[\pi_{w,t+1} - \pi^*], \tag{3.3}$$

relating wage inflation $\pi_{w,t}$ to its expected future value, the current (log-linear) output gap $x_t = y_t - y_{n,t}$, and the real wage gap $\hat{w}_t = w_t - w_{n,t}$. $y_{n,t}$ and $w_{n,t}$ are the natural output and the natural wage, respectively, and will be defined below. The slope parameters κ_w and λ_w are composite parameters depending amongst other things on θ_w and are fully described in appendix 3.A.

We assume sticky prices on the firm side similarly to sticky wages on the union side. Intermediate goods firms are monopolistically competitive and each period only a fraction θ_p can reset their prices. Prices that are not reset are indexed to steady-state inflation. This setup gives rise to the New Keynesian price Phillips curve

$$\pi_t - \pi^* = \kappa_p x_t + \lambda_p \hat{w}_t + \beta \mathcal{E}_t^F[\pi_{t+1} - \pi^*], \tag{3.4}$$

relating the current inflation rate π_t to future inflation, the output gap and the wage gap. κ_p and λ_p again are composite parameters, which depend among other things on θ_p. $\beta = 1/(1 + \rho)$ is the steady-state real discount factor.

[14] It would be straightforward to allow for non-rational expectations in the Fisher equation as well. However, the effect is minuscule compared to behavioural expectations in other equations.

Stickiness in both nominal wages and prices implies that the real wage adjusts only sluggishly and the evolution of the real wage is governed by

$$w_t = w_{t-1} + \pi_{w,t} - \pi_t. \tag{3.5}$$

Introducing sticky wages in conjunction with sticky nominal prices implies that the real wage can no longer adjust freely in response to shocks. Accordingly, monetary policy needs to decide on the relative roles of price and wage inflation in bringing about real wage adjustment. In such a setup, the divine coincidence disappears for technology shocks as these shocks directly affect the desired real wage. As a result, technology shocks have an effect on economic welfare under alternative policy frameworks. Hence, taking into account sticky wages and technology shocks gives a more complete picture of the challenges monetary policy faces in bringing about welfare-optimal outcomes: monetary policy does not only have to cope with demand shocks driving the economy to the ELB but also with the trade-offs induced by supply shocks.

Natural Variables

Natural output and the natural real wage are a function of possibly time-varying total factor productivity a_t and structural parameters. In the linearised models, we can write

$$y_{n,t} = \psi_{y,n} a_t, \quad \text{and} \tag{3.6}$$

$$w_{n,t} = \psi_{w,n} a_t, \tag{3.7}$$

where $\psi_{y,n}, \psi_{w,n}$ are composite parameters explained in appendix 3.A.

Absent price and wage rigidities, a version of the IS curve (3.1) can be used to define the *natural rate of interest* r_t^* as the interest that satisfies

$$y_{n,t} = \mathcal{E}_t^H[y_{n,t+1}] + \frac{\rho + z_t - r_t^*}{\sigma}. \tag{3.8}$$

Note that one can also combine (3.1) and (3.8) to obtain an IS curve in terms of the output gap x_t as

$$x_t = \mathcal{E}_t^H[x_{t+1}] + \frac{r_t^* - r_t}{\sigma}. \tag{3.9}$$

Shock Processes

Our model features two exogenous variables, a demand shock z_t and a technology shock a_t. Each exogenous variable follows its own AR(1) processes with mean $\bar{x}$, persistence ρ_x, and innovation $\varepsilon_{x,t}$ for $x \in \{z, a\}$ such that

$$z_t = (1 - \rho_z)\bar{z} + \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{3.10}$$

$$a_t = (1 - \rho_a)\bar{a} + \rho_a a_{t-1} + \varepsilon_{a,t}, \tag{3.11}$$

where we normalise $\bar{a} = \bar{z} = 0$. Each innovation $\varepsilon_{x,t}$ is drawn from an i.i.d. normal distribution with mean 0 and variance $\sigma_x^2 \geq 0$:

$$\varepsilon_{x,t} \sim \mathcal{N}(0, \sigma_x^2), \quad x \in \{z, a\}, \tag{3.12}$$

where the realisation of the innovation in period t is not known or anticipated until the start of the period, i.e.

$$\mathbb{E}_t[\varepsilon_{x,t}] = \mathcal{E}_t^j[\varepsilon_{x,t}] = 0, \quad x \in \{z, a\}, j =\in \{H, F, U\}$$

3.2.2 Expectations Formation under Bounded Rationality

By now it is well known that New Keynesian models with rational expectations, i.e., $\mathcal{E}_t^j[\cdot] = \mathbb{E}_t[\cdot]$ for $j \in \{H, F, U\}$, give rise to various paradoxes and puzzles. Among these, the forward-guidance puzzle (Del Negro *et al.*, 2012) stands out as the most prominent one. Events like interest rate changes that take place far in the future, but are already known today, have quantitatively large effects on contemporaneous inflation and output. This counter-intuitive result and other puzzles are a direct effect of a very strong expectations channel in the workhorse model. Households under rational expectations are extremely forward-looking and even events occurring in the very distant future substantially matter for their contemporary decision making. This particularly matters for monetary policy strategies that rely on policy reactions that will take place in the future but are already announced and therefore known today.

For this reason, we employ boundedly rational expectations in the spirit of Gabaix (2020).[15] In this approach, agents engage in cognitive discounting. They do not fully understand the world, in particular as regards events that are happening far in the future. Instead, future events get discounted relative to the rational benchmark which leads to myopia regarding future variations around the steady state. In particular, agents form expectations according to

$$\mathcal{E}_t^j[x_{t+1}] = M_j \mathbb{E}_t[x_{t+1}] + (1 - M_j)\bar{x} \tag{3.13}$$

for $j \in \{H, F, U\}$, i.e., they perceive future (state) variables x_{t+1} to be biased towards their corresponding steady state $\bar{x}$.[16]

This modifies the IS curve and the price and wage Phillips curves according to

$$x_t = M_H \mathbb{E}_t[x_{t+1}] + \frac{r_t^* - r_t}{\sigma}, \tag{3.14}$$

$$\pi_t - \pi^* = \kappa_p x_t + \lambda_p \hat{w}_t + \beta M_F \mathbb{E}_t[\pi_{t+1} - \pi^*], \tag{3.15}$$

$$\pi_{w,t} - \pi^* = \kappa_w x_t - \lambda_w \hat{w}_t + \beta M_U \mathbb{E}_t[\pi_{w,t+1} - \pi^*]. \tag{3.16}$$

Boundedly rational expectations imply discounting in the dynamic IS curve as well as in the Phillips curves, which mitigates the expectations channel. In particular, expectations about future variables are less sensitive than in the benchmark rational expectations case whenever $M_j < 1$.[17]

Concerning the natural (flexible-price) version of the economy, first note that natural output and wage are not forward-looking variables and as such, they are not affected by this myopia. Likewise, absent price and wage stickiness, the expectational parameters on the expectations of price- and wage-setters are irrelevant, i.e., M_F, M_U do not affect the natural variables at all. However, the natural rate of interest r_t^* is affected by the discounting in the IS curve. Combining (3.6) and (3.8) yields the evolution of the natural rate

$$r_t^* = \rho + z_t + \sigma(M_H \rho_a - 1)\psi_{y,n} a_t. \tag{3.17}$$

[15] Other recent papers in a similar vein are Benchimol and Bounader (2021), Nakata *et al.* (2019) and Erceg *et al.* (2021). As recently discussed by an Angeletos *et al.* (2021) or Angeletos and Lian (2022), this in general leads to an under-reaction of expectations and real variables.

[16] Note that for the deviations of the various variables from steady-state, we directly have that they are biased toward 0.

[17] One could obtain observationally equivalent results at least on the IS curve by assuming incomplete markets and k-level thinking as in Farhi and Werning (2019), incomplete markets with procyclical inequality as in Bilbiie (2021), or wealth preferences as in Michaillat and Saez (2021).

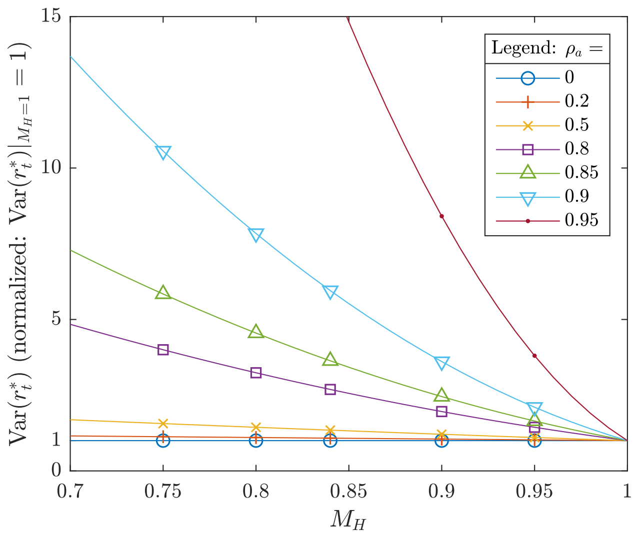

Figure 3.1: Effects of myopia on the variance of the natural interest rate for technology shocks

The variance of the natural rate is then given by

$$\text{Var}(r_t^*) = \text{Var}(z_t) + \sigma^2 \psi_{y,n}^2 (M_H \rho_a - 1)^2 \text{Var}(a_t). \tag{3.18}$$

In particular, for given variances of the shock processes for a_t and z_t, as long as ρ_z and ρ_a are strictly positive (and less than one), the variance of the natural interest rate is strictly decreasing in M_H for $M_H \in [0, 1]$. In other words, with persistent shocks, ceteris paribus, more discounting in the IS equation increases the variance of the natural rate. This increase is stronger the more persistent the shocks are and the lower M_H becomes (the relationship is quadratic in M_H). Figure 3.1 illustrates this by plotting the increase in volatility of the natural interest rate relative to the rational-expectations benchmark across values for M_H and the shock persistence.

Equations (3.14)-(3.16) together with the shock processes fully describe the private sector of the economy. To close the model we have to specify how monetary policy is conducted.

3.2.3 Interest Rate Rules

We compare history-dependent and non-history-dependent monetary policy strategies and operationalise the strategies via simple interest rate rules.[18] Under non-history-dependent interest rate rules, the monetary policy maker only seeks to stabilise current inflation and possibly real economic activity, but fully disregards past actions and economic developments when setting the current policy rate. The most prominent example of such a strategy is IT.

In contrast, under history-dependent rules, inflation developments in the past have to be compensated in order to reach the target variable and thus play a prominent role in

[18] Work related to recent monetary policy strategy reviews by major central banks also chose to operationalise the different kinds of monetary policy strategies with simple interest rate rules, see, e.g., Cecion *et al.* (2021). It thus seems that interest rate rules play an important role in monetary policy practice.

current policy setting. The most prominent examples of such a strategy are PLT, where the target variable is the price level, and AIT, where the target variable is some average of current and past inflation rates.

We analyse IT in the form of a standard Taylor-type rule. That is, we assume that the monetary policy maker does not know the natural variables and thus bases its interest rate decisions on a simple rule that depends on inflation, the deviation of inflation from target, a constant intercept term, and the deviation of output from its steady state. Thus, the rule reads

$$i_t^* = \rho + \pi_t + \phi_\pi(\pi_t - \pi^*) + \phi_y y_t, \tag{3.19}$$

where i_t^* is the desired nominal rate (which will not be constrained by the ELB), and $\phi_\pi > 0$ and $\phi_y \geq 0$ are coefficients controlling the monetary policy response to deviations of inflation from target and the deviation of output from steady state, respectively.[19]
We contrast the IT rule to several history-dependent strategies. First, we analyse a (flexible) PLT rule of the form

$$i_t^* = \rho + \pi_t + \phi_\pi \hat{p}_t + \phi_y y_t, \tag{3.20}$$

where the log deviation $\hat{p}_t$ of the (log-linearised) price level $p_t = p_{t-1} + \pi_t$ from its target path $p_t^* = p_0^* + t\pi^*$ evolves according to

$$\hat{p}_t := p_t - p_t^* = (p_{t-1} + \pi_t) - (p_{t-1}^* + \pi^*) = \hat{p}_{t-1} + \pi_t - \pi^*. \tag{3.21}$$

Second, we analyse an AIT rule with the general form given by

$$i_t^* = \rho + \pi_t + \phi_\pi \tilde{\pi}_t + \phi_y y_t, \tag{3.22}$$

where $\tilde{\pi}_t$ is the deviation of average inflation from target. Average inflation is measured by a simple moving average (SMA) of the form

$$\bar{\pi}_t|_{SMA,T} = \frac{1}{T}\sum_{\ell=0}^{T-1}(\pi_{t-\ell} - \pi^*), \tag{3.23}$$

which is widely studied in the literature and where T denotes the length of the averaging window. To keep the rules comparable across time windows, we re-normalise the averages such that the response coefficient of inflation on nominal interest rate setting is fixed at ϕ_π. This means that for the simple-moving average, we have

$$\tilde{\pi}_t := T\bar{\pi}_t|_{SMA,T} = \sum_{\ell=0}^{T-1}(\pi_{t-\ell} - \pi^*) \tag{3.24}$$

[19] A simple way of introducing history dependence in an inflation-targeting rule would be to allow for interest rate smoothing,

$$i_t^* = \rho_r i_{t-1}^* + (1-\rho_r)\hat{i}_t,$$

where $\hat{i}_t$ is the target interest rate the central bank would like to set, given current values of inflation and output (replace i_t^* with $\hat{i}_t$ in (3.19)) and $\rho_i \in [0, 1)$ is a smoothing parameter. We did not consider interest rate smoothing in our analysis since we want to clearly distinguish between rules that do and do not exhibit history dependence.

Note that with this specification, in the limit we get inflation targeting with $T \to 1$ and price-level targeting with $T \to \infty$.

For all rules that we study we assume that there is an ELB $i_{ELB} = 0$ on the nominal interest rate, which constrains monetary policy to set the nominal interest rate according to

$$i_t = \max\{i_{ELB}, i_t^*\}. \tag{3.25}$$

3.2.4 Welfare

The welfare criterion we use to compare the different interest rate rules is the expected utility of the representative agent under the rational expectations operator,

$$W = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t)$$

. This follows the behavioural economics literature which views agents as using heuristics in their behaviour but experiencing utility from the actual consumption and leisure stream (Gabaix, 2020).

The welfare loss function can then be written as a linear combination of the expected quardratic deviations of the output gap, price inflation and wage inflation from their respective targets and is given by

$$\mathbb{L} = \frac{1}{2}\left\{ \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) \mathbb{E}\left[x_t^2\right] + \frac{\epsilon_p}{\lambda_p}\mathbb{E}\left[(\pi_t - \pi^*)^2\right] + \frac{\epsilon_w(1-\alpha)}{\lambda_w}\mathbb{E}\left[(\pi_{w,t} - \pi^*)^2\right] \right\} \tag{3.26}$$

where the weights are functions of the underlying structural parameters. Note that these weights are not impacted by myopia.[20]

3.2.5 Calibration

Our baseline calibration mostly follows Galí (2015).[21] Each period in the model corresponds to a quarter. The preference parameter β is calibrated so that the annualised steady-state real rate is 1%. We assume log utility ($\sigma = 1$) and target a Frisch elasticity of labour supply of 0.2 yielding $\varphi = 5$. We set $\alpha = 0.25$ implying decreasing returns to scale. Setting $\epsilon_p = 9$ implies a steady state price markup of 12.5%. The elasticity of substitution between differentiated types of labour ϵ_w is set to a lower value of 4.5 implying an average wage markup of around 30%.[22] The reset probability of prices and wages is set to 0.75. This yields an average price and wage duration of one year consistent with much of the empirical evidence. The inflation target π^* is set so that annual inflation equals 1.9%, consistent with the former target of the ECB.[23]

[20]To formally derive this loss function as a second order approximation to households' discounted utility, we also implicitly assume that the steady state has zero inflation and is efficient, i.e., that there are subsidies in place that undo the distortions arising from market power in the labour and goods market.

[21]We also investigated a calibration as in Gabaix (2020) where the Phillips curves and the IS curve are flatter. The main insights do not change much under this alternative calibration.

[22]In a model featuring unemployment this would imply an average unemployment rate of around 5% which is a standard value in the literature.

[23]Note that since we adjust the shock size below so as to achieve a certain ELB frequency under rational expectations, the choice of π^* does not affect our results in any way: As long as we assume that non-adjusting firms update their prices by steady-state inflation, the choice of π^* in steady state only determines the distance between the steady-state nominal interest rate and its effective lower bound.

Table 3.1: Calibration of parameters

Parameter	Value	Description
β	0.9975	Preference Parameter
σ	1.00	Risk Aversion
φ	5.00	Inverse Frisch elasticity
α	0.25	Production Returns to Scale
θ_p	0.75	Prob. of price resetting
θ_w	0.75	Prob. of wage resetting
ϵ_p	9.00	Elasticity of substitution btw. goods
ϵ_w	4.50	Elasticity of substitution btw. workers
ρ_z	0.85	Persistence demand shock
ρ_a	0.85	Persistence technology shock
σ_a	0.0199	Standard deviation technology shock
σ_z	0.0357	Standard deviation demand shock
π^*	0.005	Annual inflation: 2%

For the persistence of technology and discount factor shocks we use standard values of 0.85. The standard deviations are calibrated to yield an ELB incidence of 20% under rational expectations in the case of inflation targeting.[24] The ELB is set to 0%. We vary the degree of myopia M_j for $j \in \{H, F, U\}$ in a range of 0.5 to 1. This allows us to analyse its impact on New Keynesian puzzles as well as its effects on monetary policy strategies. This range also contains most empirical estimates and, as we show below, constitutes the region with the most interesting interactions.

3.2.6 Numerical Method

We need to solve our model under various monetary policy rules given an occasionally binding constraint due to the zero lower bound and the real wage as a state variable. To do so we rely on the deterministic extended path algorithm first proposed by Fair and Taylor (1983). The extended path algorithm combines the accuracy of deterministic perfect foresight solutions with the ability to provide an accurate account of non-linearities. This crucially relies on the assumption that agents react to random current period shocks but assume that no shocks will occur in future periods, i.e., that the economy asymptotically returns to equilibrium after the current period shocks. Therefore, while in our numerical simulations the economy is hit by shocks every period, agents only act to contemporary ones, i.e., we assume certainty equivalence.

We optimise the response coefficients of the different monetary policy rules for each degree of myopia and each type of shock separately. This ensures a fair comparison in which all the rules perform as best as they can – that is, without implicitly biasing the comparison by assigning sub-optimal parameters to a rule – conditional on the type of shocks and the degree of myopia. We numerically optimise parameters by conducting a grid search for each respective rule and calculate welfare from equation (3.26) using the variances arising from stochastic simulations at each grid point.[25]

[24] In particular, we calibrate shocks such that the ELB frequency under the IT rule is 20% for $\phi_\pi = 0.5$ and $\phi_y = \frac{0.5}{4}$.

[25] The grid in general covers parameter values in the range from 0 to 10 for both parameters (regular

3.3 Implications of Sticky Wages under Bounded Rationality

In this section, we analyse the interactions between sticky wages and boundedly rational expectations. Bounded rationality in itself suffices to solve a number of paradoxes arising in New Keynesian models. We extend this insight and show that wages stickiness in addition to price stickiness considerably enhances bounded rationality as a solution to these paradoxes. Here, we focus on determinacy properties, the severity of zero lower bound episodes and the forward guidance puzzle. These are crucial in the comparison of different monetary policy strategies.[26]

Enhanced Determinacy

The traditional New Keynesian model suffers from the existence of a continuum of equilibria if monetary policy is sufficiently passive. This occurs when the Taylor principle is violated, i.e., when monetary policy acts according to an IT rule but reacts to inflation deviations less than one-for-one ($\phi_\pi < 0$). This exposes the economy to the possibility of self-fulfilling sunspot fluctuations, therefore implying indeterminacy. Panel a) of Figure 3.2 shows such determinacy regions in the ϕ_y-ϕ_π-space when all agents have either rational (solid lines) or boundedly rational (dashed lines) expectations contrasting the case when only prices (red lines) or also wages (blue lines) are sticky.

Under rational expectations, obeying the Taylor principle is crucial to guarantee existence of a unique equilibrium if monetary policy does not react to output deviations ($\phi_y = 0$).[27] This holds irrespective of the degree of stickiness of prices and wages. In the standard model, sticky wages alleviate determinacy issues but only if monetary policy also responds to output deviations ($\phi_y > 0$).

Under bounded rationality and in combination with sticky wages, a number of new results emerge. First, even if monetary policy does not respond to output, the Taylor principle need not be satisfied. Crucially however, the degree to which monetary policy can deviate from the Taylor principle depends on the degree of myopia as well as the degree of wage stickiness. If only prices are sticky and expectations are not sufficiently bounded ($M_j = 0.9$ for $j \in \{H, F, U\}$ as in Figure 3.2), only small deviations from the Taylor principle are possible. In contrast, if wages are also sticky, deviations from the Taylor principle can be quite substantial. Second, this implies that, under sticky wages, a smaller degree of myopia is necessary to guarantee uniqueness of equilibria irrespective of the policy rule in place.[28] Third, with an increasingly stronger reaction to output, the reaction to inflation required to achieve determinacy is increasingly reduced with sticky

grid), where $\phi_\pi \geq 0.5$. For the demand shock, we also admit values up to 1000 (20,50,100,200,500,1000) for both parameters. For the technology shock, we also add gridpoints between 0 and 0.5 and between 10 and 20 for ϕ_π. Note that for the technology shock, the constraint $\phi_y \geq 0$ will be binding, see the discussion below. Hence, in the appendix, we also add grid points with a negative ϕ_y.

[26]In appendix 3.B.1, we additionally analyse the influence of boundedly rational expectations and sticky wages on the Neo-Fisherian paradox, the paradox of toil, and the paradox of flexibility.

[27]This holds for a target inflation rate of zero. If the target inflation rate is above zero, the Taylor principle is – in general – not sufficient to guarantee the existence of a unique equilibrium for $\phi_y = 0$, see Ascari and Sbordone (2014). But with our assumption of perfect indexation for non-optimising firms, this issue becomes irrelevant.

[28]Gabaix(2020) terms this the "strong bounded rationality principle", i.e., the degree of myopia necessary to guarantee determinacy. Under sticky prices only, this principle requires $M = 0.8$, whereas under sticky wages, $M = 0.85$ is sufficient.

wages compared to sticky prices only. This holds for rational expectations as well, but is substantially stronger under myopia. If expectations are boundedly rational, sticky wages cause a particularly pronounced shrinking of the indeterminacy region.

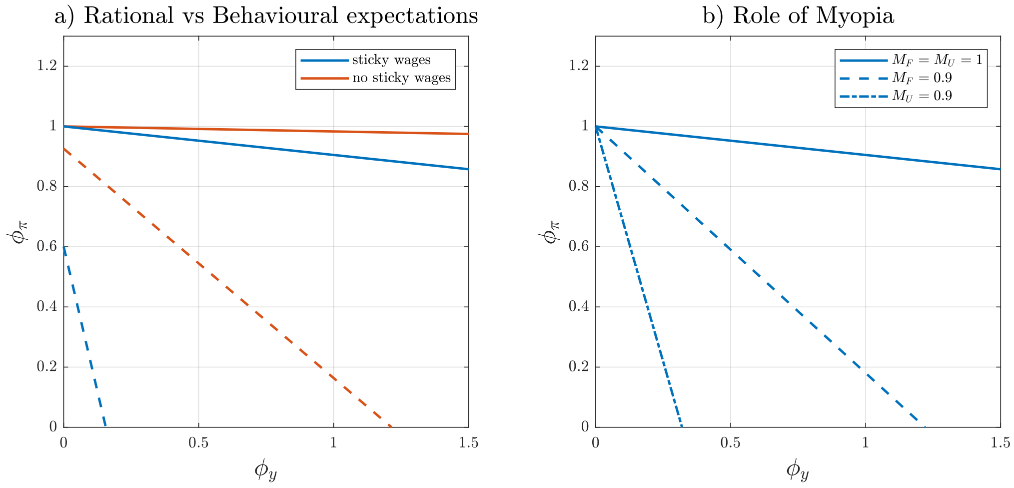

Figure 3.2: Determinacy

Note: The left-hand panel shows determinacy regions as a function of monetary policy parameters comparing the case when only prices or also wages are sticky. Solid lines indicate rational expectations, dashed lines indicate behavioural expectations with all agents exhibiting the same degree of myopia ($M_j = 0.9$ for $j \in \{H, F, U\}$). The right-hand panel shows the influence of partial myopia under sticky wages.

Panel b) of Figure 3.2 breaks down relative contributions of myopia to the improved determinacy properties under sticky prices and wages. We set either $M_F = 0.9$ or $M_U = 0.9$, while keeping the other myopia factors at 1. Myopic firms or unions, i.e., myopia in just the price or the wage Phillips curve, tilts the boundary of indeterminacy without modifying the Taylor principle if the central bank does not react to output gap deviations. However, myopia in the wage Phillips curve requires only little additional responsiveness to output gap deviations ($\phi_y > 0.3$) to guarantee uniqueness compared to myopia in the price Phillips curve ($\phi_y > 0.9$). This implies that myopia on the side of wage unions is more important for eliminating multiple equilibria.[29]

Decreased Severity of the ELB

To study the implications of the ELB under bounded rationality with sticky wages, we conduct the following experiment:[30] Let monetary policy follow the IT rule (19) with $\phi_\pi = 0.5$ and $\phi_y = 0.125$. At t_0, let the natural real interest rate r_t^* drop to -0.20 for exactly $T \geq 0$ periods, which drives the economy to the effective lower bound. After T periods, r_t^* reverts to its steady state $1/\beta - 1$. Figure 3.3 depicts the severity of such an ELB recession (measured as the output gap x_{t_0} in the first period t_0) as a function of the duration of the shock to the natural rate. Panel a) depicts the situation with rational

[29] In line with the previous footnote, myopia in the IS curve only does not alter the determinacy properties.

[30] This experiment is based on the one reported by (Gabaix, 2020, p. 19 et seq.), which in turn is based on ideas put forth by Werning (2011) and Eggertsson and Woodford (2003). The main difference is that we consider a model with sticky wages. Also, compared to the before-mentioned sources we add $\phi_y > 0$, which is, however, inconsequential.

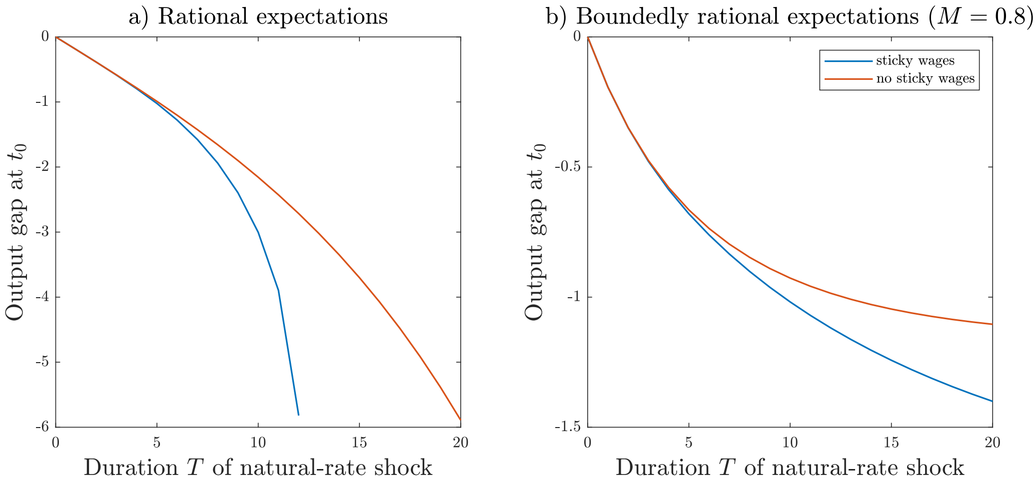

Figure 3.3: Severity of ELB

Note: This figure compares the severity of recessions induced by a constant decline in r_t^* to -0.20 for T quarters under rational (lhs) and boundedly rational (rhs) expectations.

expectations (i.e., $M_j = 1$ for $j \in \{H, F, U\}$), panel b) the corresponding one in a model with boundedly rational expectations and $M_j = 0.8$ for $j \in \{H, F, U\}$. In each panel, we compare the severity of the ELB with and without sticky wages. In all cases depicted, the output gap decreases with the duration of the shock to r_t^*. Under rational expectations, both with and without sticky wages the output gap in the initial period at the ELB decreases without bounds as T increases.[31] Note that, under rational expectations, the unbounded decrease is much stronger with sticky wages as T increases. In contrast, under boundedly rational expectations, the decrease in the output gap is bounded both with and without sticky wages. However, comparing both cases with their corresponding rational expectations counterpart in panel a), it is evident that the decrease in the severity of the ELB is much more pronounced under sticky wages. In fact, the decrease in the output gap is almost as attenuated as in the case without sticky wages (note the difference in the scale of the vertical axis between panels a) and b)) despite the much stronger decrease under rational expectations. It is in this sense that the combination of sticky wages and bounded rationality enhances the resolution of the paradox of excessively large recessions at the ELB compared to a case without sticky wages.

The reason why the same shock to the natural rate is more severe in absolute terms in the model with sticky wages for approximately $T > 5$ is that by adding sticky wages, we add an endogenous state variable, the real wage w_t, which only adjusts sluggishly. At the ELB, as output contracts, the real wage decreases. Given the deflationary pressure on prices, wage inflation has to be even lower than price inflation. Conversely, when exiting the ELB, the real wage has to increase again, which requires $\pi_{w,t} > \pi_t$ for some time. Under our calibration, the price Phillips curve is steeper than the wage Phillips curve, implying that the majority of this adjustment has to occur via lower price inflation. That is, with wage rigidities, there is an extra, endogenous disinflationary force keeping inflation low. If the shock to r_t^* is severe enough and causes a long spell at the ELB, this disinflationary force can endogenously cause this spell to be substantially longer.[32]

[31] In fact, for the experiment considered, with sticky wages and sticky prices, the numeric solver was not able to solve the model anymore for $T > 13$. Note that with a higher ϕ_π, the cut-off level of T increases.

[32] See Figures 3.B.2 and 3.B.3 in Appendix 3.B.2 for more details on this experiment. In particular,

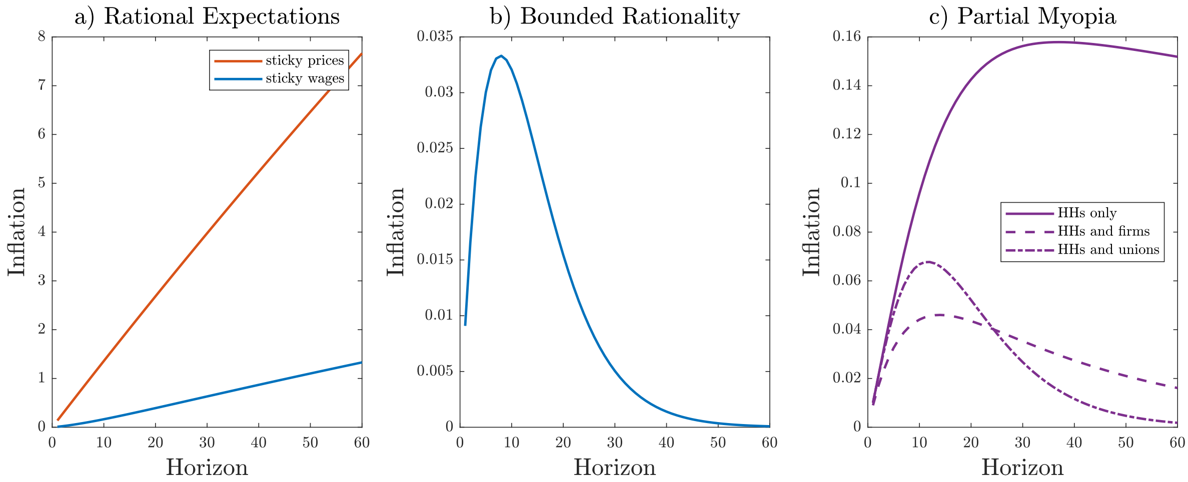

Figure 3.4: Strength of Forward Guidance
The left figure compares the strength of forward guidance in the New Keynesian model with sticky prices only against sticky wages in addition under rational expectations. The middle graph shows the case of sticky wages in addition to sticky prices when all agents are myopic and $M_j = 0.9$ for $j \in \{H, F, U\}$. The right hand graph compares cases of partial myopia.

Reduced Power of Forward Guidance

The canonical New Keynesian model is subject to the well-known forward guidance puzzle (Del Negro *et al.*, 2012). Reductions in the interest rate that take place further in the future have a stronger contemporaneous effect on output and inflation. Panel a) of Figure 3.4 illustrates this. Both the versions with sticky prices only and sticky wages in addition are subject to the puzzle, although the magnitude is lower for the case of sticky wages.

Bounded rationality provides a solution to the forward guidance puzzle. The middle panel of Figure 3.4 shows that this is also the case in the model with sticky wages in addition to sticky prices. Crucial to resolving the puzzle is myopic behaviour of households that leads to discounting in the IS curve (panel b)). Additional myopia on the firm and union side further reduces the magnitude of contemporaneous changes in inflation albeit with nuanced differences. Myopia on the firm side reduces forward-guidance effects in the short and medium term compared to myopia on the union side. The latter however is more important to the quantitative resolution of the puzzle in the long term.

Figure 3.B.2 maps the natural-rate shocks depicted in Figure 3.3 to actual durations at the effective lower bound. Note that for both rational and boundedly-rational expectations, with sticky wages, the ELB duration generally starts to be longer than the pure shock to the natural rate. Note however, with rational expectations, this increase is stronger before the line ends. This, in turn is due to the real wage, which declines more because of the larger output gap. From the right-hand panel, it becomes clear that with boundedly rational agents, the duration at the ELB does not grow as fast – which is mostly caused by a smaller (in absolute terms) output gap. For sake of comparison, the boundedly rational New Keynesian model with only sticky prices does not feature endogenous prolonging of the ELB spells. As such, the duration of the ELB spell coincides with the duration of the shock to the natural rate. Figure 3.B.3 finally compares the actual duration at the ELB (from Figure 3.B.2) and the initial output gap (from Figure 3.3), where the layout of the figure is the same as in 3.3. Obviously, with boundedly rational agents and sticky wages, the output loss in the first period at the ELB is bounded even for very long episodes at the ELB – contrary to rational expectations.

3.4 Monetary Policy Strategies under Bounded Rationality

Having discussed the properties of the New Keynesian model with sticky wages and boundedly rational expectations, we now turn to the implications for monetary policy. We compare the performance of various monetary policy rules across different degrees of myopia using stochastic simulations. To clearly isolate the effects, we consider demand and supply shocks separately.

3.4.1 Technology Shocks

Figure (3.5) shows the welfare comparison of different policy strategies under technology shocks when all agents in the economy are myopic to the same degree. For each degree of myopia, we re-optimise the interest rate rule coefficients so that each strategy can perform in the best way possible for a given economic environment.

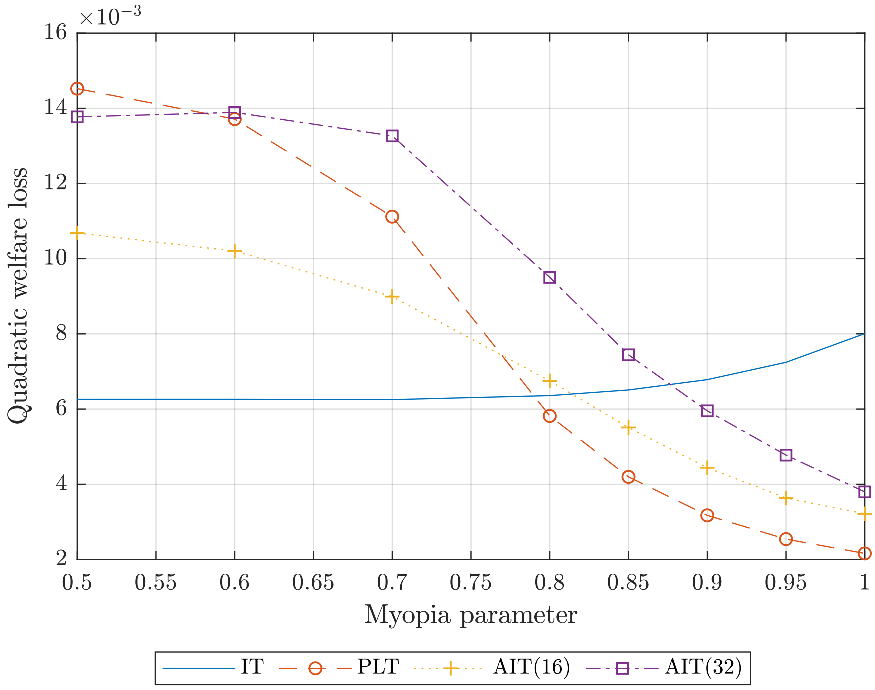

Figure 3.5: Welfare comparison for technology shocks

Under rational expectations and under very mild myopia, the PLT rule yields the lowest quadratic welfare losses, while the IT rule yields the highest welfare losses. Under mild myopia, the properties of history-dependent rules in the rational model that depend on the expectations channel remain in place and optimal policy results carry over to the model with boundedly rational expectations.

Under higher degrees of bounded rationality ($M_H = M_U = M_F < 0.8$), the advantage of the PLT rule and other history-dependent rules turns into a disadvantage. Rankings completely reverse and the IT rule turns out to yield the lowest welfare losses. This is mostly due to the IT rule yielding remarkably stable welfare losses across different degrees of myopia whereas history-dependent rules suffer from the reduced forward-lookingness of agents.

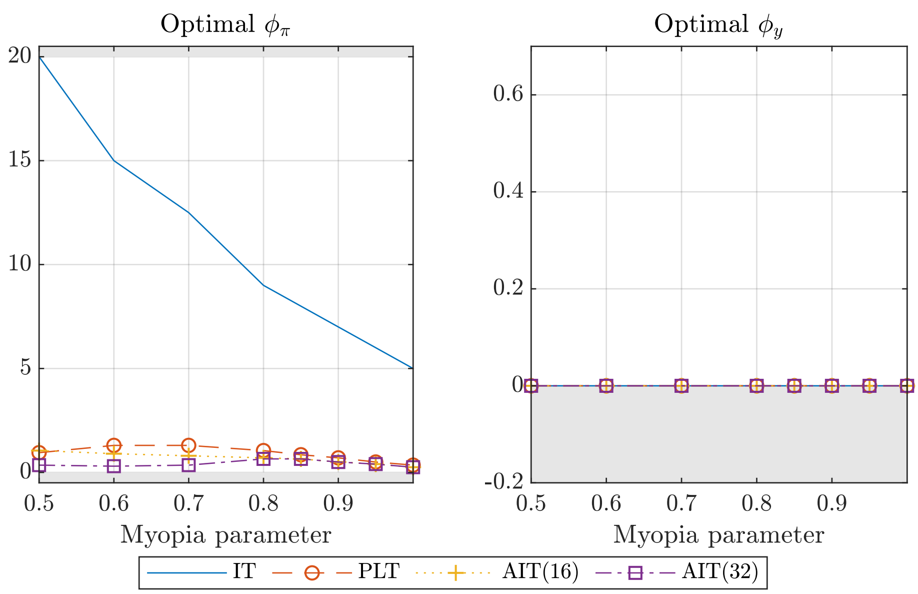

Figure 3.6: Optimal policy rule parameters for technology shocks

Figure 3.6 shows the optimal policy coefficients for each degree of myopia. The grey shaded areas show the bounds of the grid used for the grid search. Under technology shocks, it is never optimal to react to deviations of output from steady state irrespective of the specific interest rate rule.[33] In contrast, the optimal response coefficients for the nominal variable differ markedly between the history-dependent AIT rules on the one side and the non-history-dependent IT rule on the other side. Under the IT rule, it is optimal to react to inflation as strongly as possible, no matter the degree of myopia. In contrast, the optimal response coefficient in the AIT and PLT rules exhibit a lower value throughout the different degrees of myopia. The reason can clearly be seen in Figure 3.7, which shows the variances underlying the welfare rankings. The flat welfare line across degrees of myopia under IT is the result of a monotonic decline of volatility of inflation and wage inflation (whereas the volatility of output monotonically increases). This, in turn, is a result of the declining severity of ELB recessions. For the history-dependent rules, the picture is more nuanced. While dealing very well with the ELB under rational expectations mild degrees of myopia lead to increasing volatility of all variables in the loss function. On the

[33]This is due to the fact that we restrict the response parameter to be positive. In fact, since the Taylor rule is not specified in terms of the output gap, this is little surprising, since with technology shocks, output under-reacts, giving an output gap of the opposite sign relative to the deviation of output from steady state. If we allow the central bank to observe natural variables and set $i_t = r_t^* + \pi_t + \phi_\pi(\pi_t - \pi^*) + \phi_y x_t$ (or a similar variant for PLT and AIT), the performance of the various monetary policy rules becomes nearly identical. In any case, in these scenarios, a TFP shock actually calls for (in absolute terms) large coefficients on the output gap and small ones on the measure of inflation. Similarly, if we stick to the assumption that the central bank does not include natural variables in its Taylor rule, it would be optimal to set a negative coefficient on the output deviation from steady state. This optimal coefficient becomes more negative for smaller values of the myopia parameters (remember, the natural rate reacts more strongly with higher degrees of myopia relative to natural output). I.e., with technology shocks in the EHL framework, output-gap targeting becomes a key ingredient of optimal monetary policy, relating our results to those of Garín *et al.* (2016). Figures 3.B.14 shows the welfare comparison if we allow for negative reaction parameters and also refine the grid for ϕ_π. Figure 3.B.15 depicts the resulting optimal parameters. It should be noted that our main insights are not affected, but it becomes clear that the importance of the output gap increases in the degree of myopia.

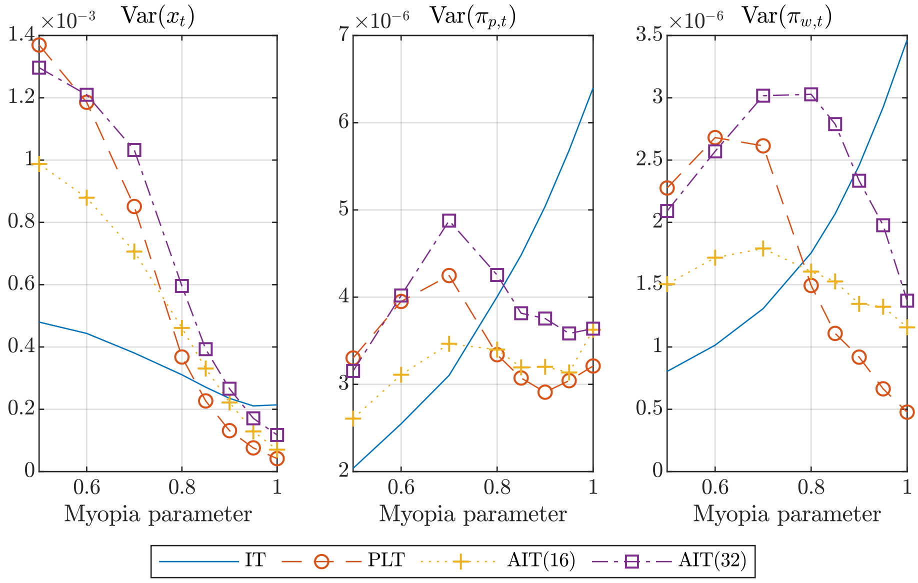

Figure 3.7: Variances for technology shocks

one hand, the effectiveness of the history-dependent component which originally leads to low economic volatility is lowered. On the other hand, myopia also increases the volatility of the natural rate. Pure history-dependent strategies ($\phi_y = 0$) immediately push the economy to the ELB for a larger fraction of time, leading to fewer, but longer spells at the ELB. In addition, this makes the cost-push component of technology shocks relatively more important, raising the costs associated with history-dependent strategies.[34]

Figure 3.8 shows the ELB statistics. Larger degrees of myopia lead to fewer spells at the ELB across all policy rules. This is because myopia increases the volatility of the natural rate subsequently leading to fewer ELB incidences. At the same time the average duration and time spend at the ELB increases for all history-dependent strategies. While ELB periods are less severe under bounded rationality, the effectiveness of the expectations channel is also reduced. History-dependent strategies therefore need to keep nominal rates lower for even longer in order to achieve their target and compensate the reduced effectiveness.[35]

[34]This result is muted if monetary policy reacts to natural variables or if we allow for a negative ϕ_y in the present model. In any case, with technology shocks, as myopia increases, the optimal response shifts from stabilising inflation to stabilising the output gap. See Figure 3.B.15, which depicts the optimal coefficients for the case where we also allow for $\phi_y < 0$.

[35]This is in line with the results obtained by Nakata *et al.* (2019).

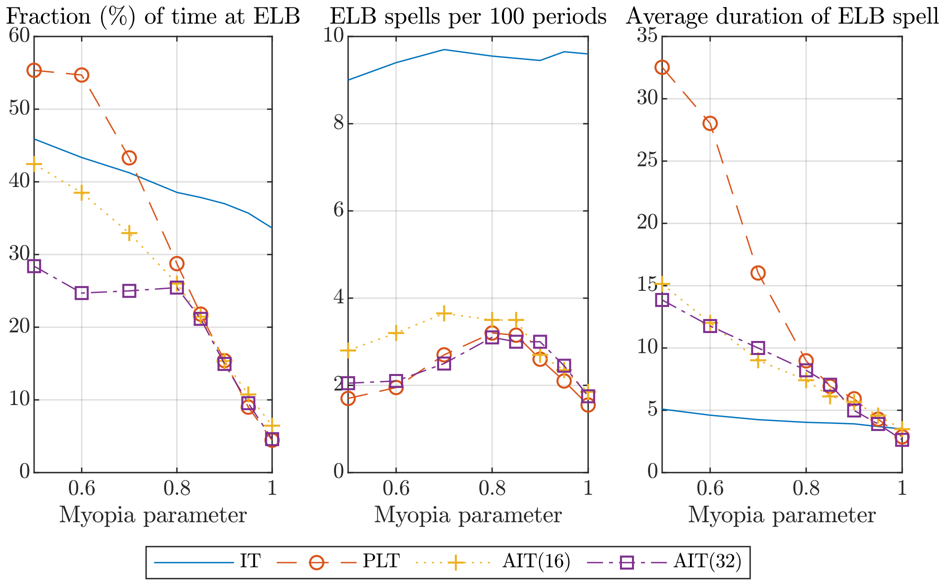

Figure 3.8: ELB statistics for technology shocks

3.4.2 Demand shocks

Figure 3.9 shows the welfare comparison of different policy strategies under discount factor shocks. Under mild forms of myopia, the typical results from the New Keynesian model carry over. History-dependent rules outperform the IT rule due to their inherent lower-for-longer component. In fact, it should be noted that we find the PLT rule, which fully reaps the benefits from the expectations channel, to be the best performing rule by a small margin as long as myopia is not too severe For larger degrees of myopia, the importance of the expectations channel decreases.[36] Consequently, the relative performance of history-dependent strategies worsens. In fact, the IT rule again overtakes the PLT rule and proves to be the better performing strategy by a small margin. However, for low values of M, the different strategies become almost identical in terms of their performance. Note that volatility decreases as agents become more myopic, although the variance of the natural rate *per se* is constant. This is due to the fact that agents neglect the persistence of shocks, making them underreact.[37] As such, myopia per se makes it easier for monetary policy to achieve its goals.

As opposed to technology shocks, the optimised parameters shown in Figure 3.10 exhibit a similar patterns across strategies for demand shocks. For low degrees of myopia, close to rational expectations, it is optimal under all rules to react as strongly as possible to inflation or price gaps and with smaller reaction coefficients to output deviations. For history dependent strategies, this reverses under high degrees of myopia. In order to provide additional stimulus at the ELB and to counter the ensuing recessions, it is optimal to strongly react to the output gap and almost entirely neglect inflation gaps – this also avoids generating additional volatility. Meanwhile, for inflation targeting, myopia mostly

[36] This comes as agents expect inflation to be mean-reverting faster than it actually is.

[37] Such an underreaction or weakening of general-equilibrium effects is a common result of adding bounded rationality; however, certain forms can also lead to overreaction. For a literature review, see Angeletos and Lian (2022) and the references cited therein.

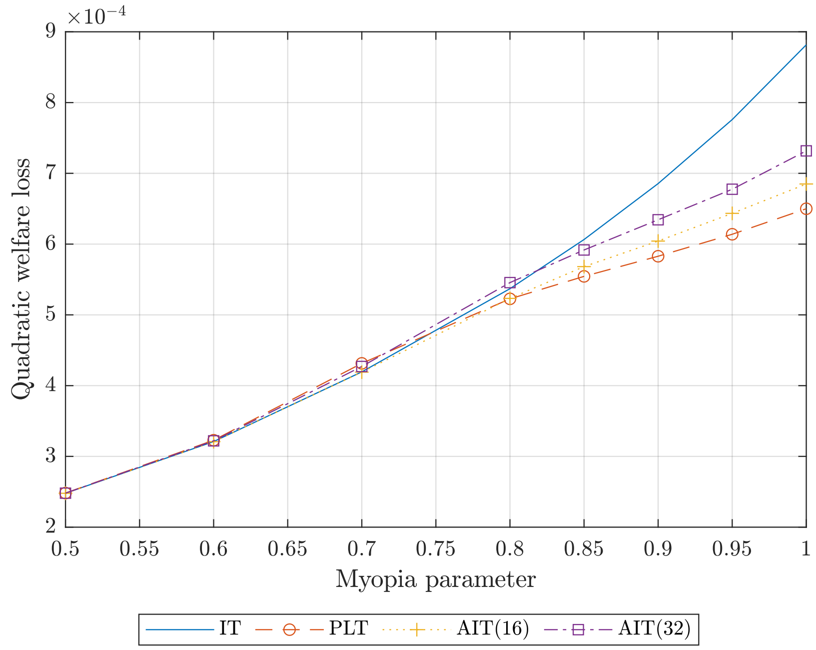

Figure 3.9: Welfare comparison for demand shocks

implies that the central bank should *also* put a greater weight on the output gap, whereas it is still optimal to react as strongly as possible to inflation deviations (coefficient value of $\phi_\pi = 1000$, upper grid boundary).

Likewise, the patterns for the variances underlying the welfare losses shown in Figure 3.11 are broadly similar across the different interest rate rules. Notably, inflation targeting 'catches up' in terms of each variance. Also, as shown in Figure 3.12, the ELB statistics in the simulations are broadly similar (watch for the scaling on the y-axis). Notably, average inflation targeting with a short horizon is a relatively attractive policy here. The most remarkable element of the figure is that the various policies hit the ELB more often (in terms of number of ELB spells per 100 periods), but the average spell becomes shorter.

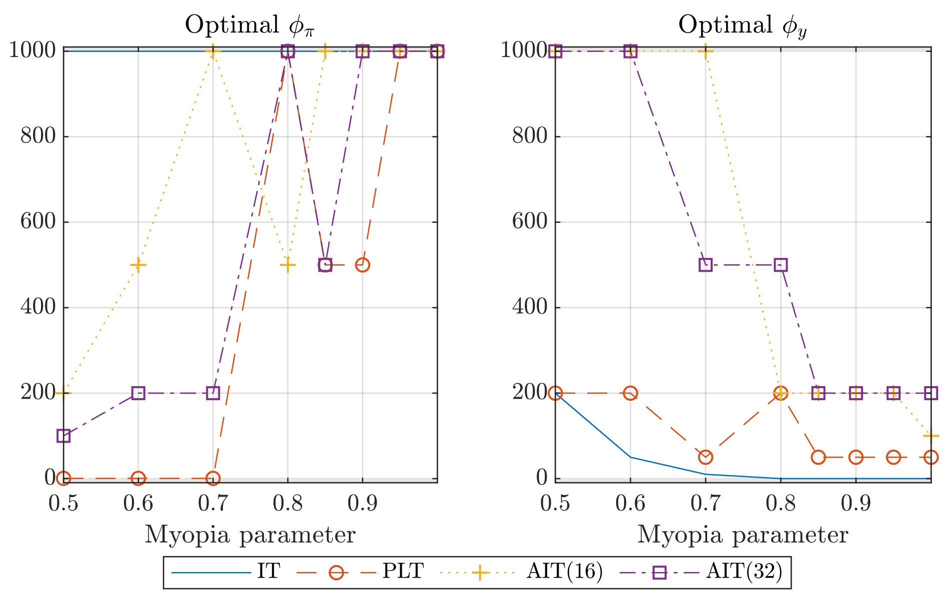

Figure 3.10: Optimal policy rule parameters for demand shocks

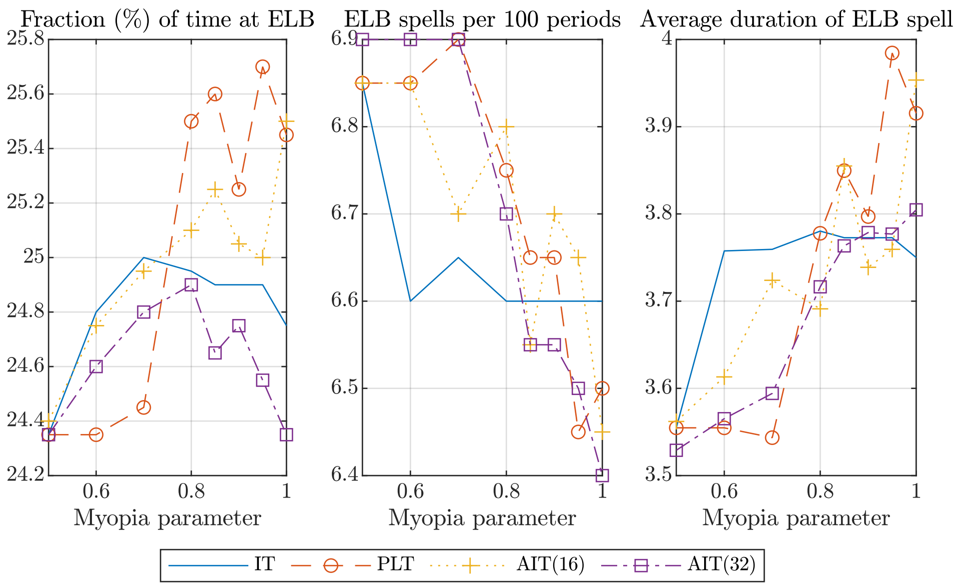

Figure 3.12: ELB statistics for demand shocks

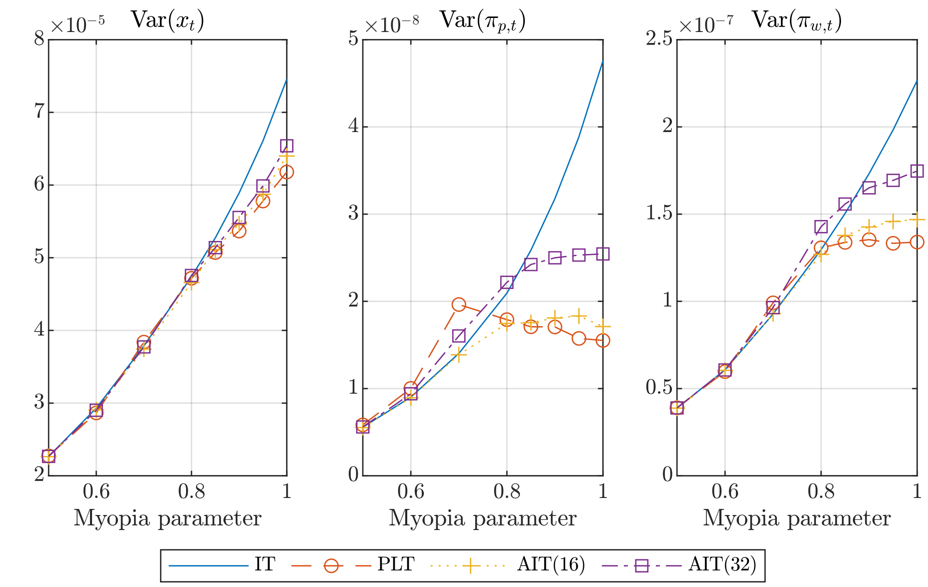

Figure 3.11: Variances for demand shocks

3.4.3 Inspecting the Mechanism

As already known from the discussion in Gabaix (2020), with myopic agents, a stationary price level is no longer the optimal policy in a simple New Keynesian model. Thus, it may come as no surprise that the performance of the PLT rule deteriorates relative to the other rules as myopia increases. The underlying reason is that the power of PLT is intimately related to the effectiveness of the expectations channel, which operates at maximum strength with rational expectations and becomes weaker with increasing myopia. PLT, by making current inflation deviations relevant for all future periods exhibits a very strong expectations channel. This can be advantageous in the case of inefficient technology shocks as it can help resolve the trade-off between stabilising inflation and economic activity.

Note that absent the expectations channel, PLT per se can *increase* the volatility of inflation because it requires that deviations of inflation in one direction be compensated by equally-sized deviations in the other direction in later periods, increasing the overall variance of inflation.[38] With myopia, the expectations channel is significantly weakened, making inflation expectations less relevant for current inflation stability. At the same time, PLT still puts a lot of weight on past errors, i.e., the potentially volatility-increasing effect of PLT is still active.

Hence, AIT – which includes only a limited number of past inflation rates in a simple moving average – in principle should act as a good compromise across different degrees of myopia. First, past deviations from the average inflation target are corrected for in the following periods to some extent, so AIT can harness the expectations channel for low degrees of myopia, although not as strongly as PLT. Therefore, increasing myopia should affect AIT less than PLT. Second, the memory of AIT is limited: After a certain number of periods, past deviations drop out of the averaging window, so the potentially volatility-increasing effects of past deviations for high degrees of myopia are mitigated. Taken together, the AIT rule, while harnessing the expectations channel to some extent for low degrees of myopia, should become more competitive compared to the PLT rule as myopia increases. Indeed, our results in section 3.4 show that this is actually the case.

What contributes to the impression that the AIT rule might be an obvious compromise is that its performance lies in between the performance of the PLT and the IT rule for both low and high degrees of myopia and for both demand and supply shocks. In the case of demand shocks, the AIT rule even comes very close to the best performing rule for low and high degrees of myopia (the PLT rule and the IT rule, respectively; see Figure 3.9). This may come at little surprise since monetary policy does not have to confront a trade-off in the case of demand shocks and we compare optimised rules. However, in the case of trade-off-inducing technology shocks, while the AIT rule again represents middle ground, its welfare losses are markedly higher than those of the best performing rules for low and high degrees of myopia (again the PLT rule and the IT rule, respectively; see Figure 3.5). This casts some doubt on whether the AIT rule would actually be a desirable compromise candidate.

Moreover, a fact that usually receives little attention is that simple moving averages may actually introduce volatility on their own when used as an indicator for monetary policy. The reason is that any past deviation of inflation from its target directly affects the average for a given time frame, but once it drops out of the averaging window this effect reverses. Thus, if monetary policy uses the simple moving average as an indicator, that indicator may well switch signs suddenly.

To illustrate this, consider, a monetary policy maker that pursues an average inflation target with an averaging window of T periods. Assume that in period t_0, starting from a steady state with $\pi_t = \pi^*$ for $t \leq t_0$, inflation π_{t_0} exogenously deviates from its target π^* by an amount $\Delta\pi$. From period $t_0 + 1$ onwards monetary policy has perfect control over the current inflation rate and sets inflation such that the average $\bar{\pi}_{t,T} := \frac{1}{T}\sum_{s=0}^{T-1} \pi_{t-s}$ satisfies $\bar{\pi}_{t,T} = \pi^*$ again. For the sake of illustration, assume that monetary policy sets $\pi_{t_0+1} = \pi^* - \Delta\pi$ in period $t_0 + 1$, fulfilling its target in that period, and over the course of the next periods it keeps $\pi_t = \pi^*$. However, in period $t_0 + T$, period t_0 drops out

[38]This is typically the result in studies that assume backward-looking expectations, as e.g. in Lebow *et al.* (1992).

of the averaging window. Thus, monetary policy would set $\pi_{t_0+T} = \pi^* + \Delta\pi$. In the next period, period $t_0 + 1$ drops out of the averaging window, requiring again a negative deviation from target. This would repeat every T periods, as depicted in Figure 3.13 a). For comparison, we show what would happen under PLT (panel b)) and under IT (panel c)). Under PLT, the increase in period t_0 would be countered with a negative deviation in the following period, but since no period would ever drop out of the averaging window, no further deviation from the target would be required in the absence of an exogenous shock. Under IT, the opposite holds: since there is no averaging window, deviations immediately 'drop out' of memory, i.e., from $t_0 + 1$ onwards, π_t could be set to the target value.

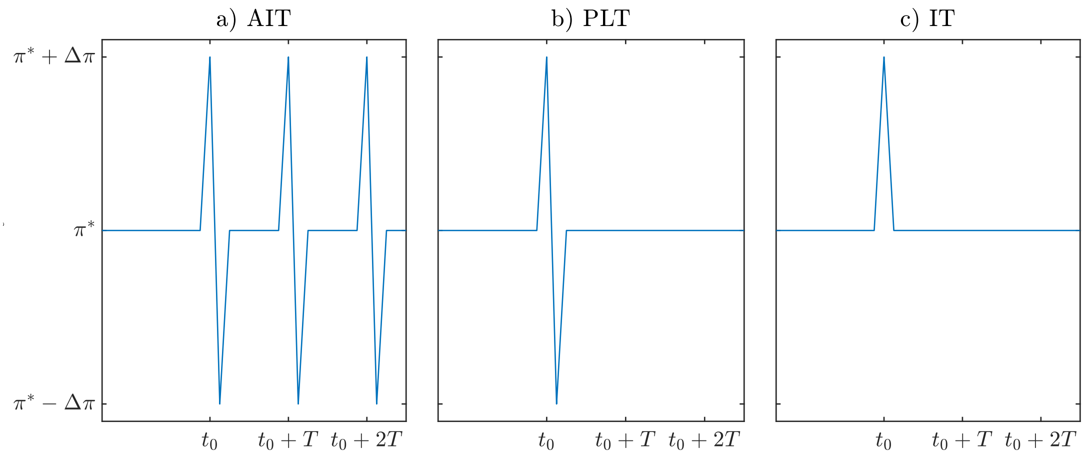

Figure 3.13: Illustration: Cycling behaviour of average-inflation targeting

Hence, in this simple framework, targeting a simple moving average would actually introduce an undamped cycle. Any shock that leads to a deviation of inflation from target would require periodically repeating deviations of inflation from its target. This, by itself, would increase welfare-detrimental variations in inflation. In a fully specified model, and especially if monetary policy only has indirect control of inflation via nominal interest rate setting, these cycles would be mitigated and smoother than depicted in the Figure. However, the underlying mechanism still remains. Note that the described inherent volatility-inducing characteristic of AIT is particularly relevant for shocks that induce a trade-off for monetary policy since in the case of demand shocks, monetary policy can (under ideal circumstances) immediately undo shocks that make inflation deviate from its target and, hence, the periodically repeating pattern of deviations does not occur.

Figure 3.14 shows that this cycling behaviour can also be found in our model. It depicts the impulse responses of nominal interest rates, inflation and the price level to an inflationary shock for a baseline parameterisation and with rational expectations. For clarity, we here set $\phi_y = 0$ and $\phi_\pi = 1$. Similar to what we have seen before, (pure) AIT features oscillating spikes in the nominal interest rate as past periods with high (or low) inflation drop out of the averaging window.[39] This induces volatility in the inflation rate. The drift in the price level for IT or AIT is clearly visible, as is the overshooting of inflation for (longer-run) history-dependent strategies.

To sum up the aforementioned observations, the AIT rule exhibits some desirable features when it comes to the performance across different degrees of myopia. These include

[39] Allowing for $\phi_y \neq 0$ ameliorates the cycling behaviour of AIT strategies to some extent, because it increases the weight on current deviations. However, the general tendency of cyclicality remains.

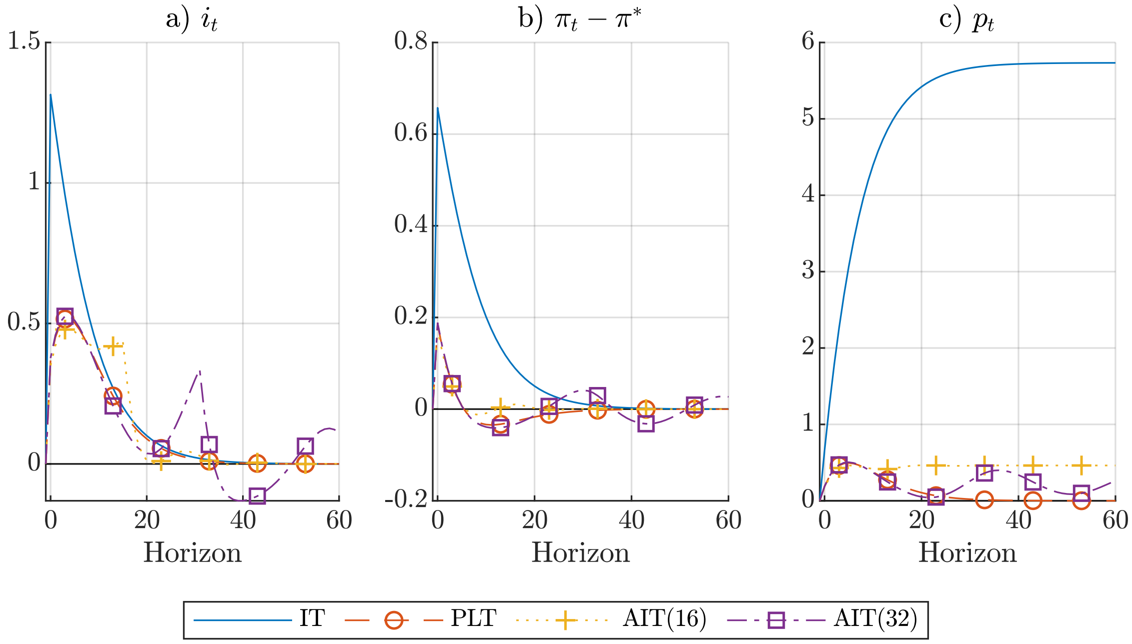

Figure 3.14: Illustration: Cycling behaviour of average-inflation targeting in the model

Note: The figure shows the impulse response functions of the nominal interest rate i_t, the inflation rate π_t and the price level p_t to a inflationary demand shock for $\rho_z = 0.85$, $\phi_\pi = 1$, $\phi_y = 0$, with rational expectations.

history dependence which is beneficial for low degrees of myopia, and a limited memory compared to the PLT rule which is beneficial when high degrees of myopia weaken the expectations channel substantially. In this sense, the AIT rule represents a compromise between the PLT and the IT rule, which are the best performing strategies for low and high degrees of myopia, respectively. However, for trade-off-inducing technology shocks the welfare losses under the AIT rule are markedly higher than those of the best performing rules for the different degrees of myopia, which the inherent volatility-inducing characteristic of AIT is crucially contributing to. As a consequence, a strategy that preserves the aforementioned benefits and avoids the disadvantage of inherently generating volatility has the potential to perform better than conventional AIT across different degrees of myopia and across different types of shocks. In the following section, we present exponential Average-Inflation Targeting (eAIT) as a promising candidate for such a strategy.

3.4.4 Exponential AIT

Characteristics of the Exponential Moving Average

In contrast to the AIT rule, where a simple (or arithmetic) moving average of the inflation rate enters the interest rate rule, the argument that enters the eAIT rule is an exponential moving average of the inflation rate. The exponential moving average is an infinite impulse response filter, i.e., the average is applied to all past observations according to

$$\bar{\pi}_{t|EMA,T_{eAIT}} = \sum_{s=0}^{\infty} \eta_s \pi_{t-s} \quad \text{with } \eta_0 \geq 0 \text{ and } \eta_s = \rho_{eAIT}\eta_{s-1} = \rho_{eAIT}^s \eta_0 \text{ for } s \geq 1. \tag{3.27}$$

Here, $\rho_{eAIT} \in [0, 1)$ is a smoothing parameter. If the weights are normalised such that

$$\sum_{s=0}^{\infty} \eta_s = 1,$$

ρ_{eAIT} and η_0 are linked according to $\eta_0 = 1 - \rho_{eAIT}$. Moreover, equation (3.27) can be restated recursively as

$$\bar{\pi}_{t|EMA,T_{eAIT}} = \eta_0 \pi_t + \rho_{eAIT} \bar{\pi}_{t-1|EMA,T_{eAIT}}, \tag{3.28}$$

which with $\eta_0 = 1 - \rho_{eAIT}$ gives

$$\bar{\pi}_{t|EMA,T_{eAIT}} = (1 - \rho_{eAIT})\pi_t + \rho_{eAIT} \bar{\pi}_{t-1|EMA,T_{eAIT}}. \tag{3.29}$$

Equation (3.29) with appropriately normalised weights nests both a single-period inflation rate for $\rho_{eAIT} = 0$ and the (linearised) law of motion of the price level for $\rho_{eAIT} = 1$. Entering into an interest rate rule, these two special cases would correspond to the IT rule and the PLT rule, respectively.

Note that with the exponential moving average being an infinite impulse response filter, there is actually not a direct equivalent for the averaging window in a simple moving average. Nevertheless, the smoothing parameter in the exponential moving average is sometimes expressed as

$$\rho_{eAIT} = \frac{T_{eAIT} - 1}{T_{eAIT} + 1} \tag{3.30}$$

to define a T_{eAIT}-period exponential moving average.

Figure 3.15 depicts the weights that AIT and eAIT assign to different lags of inflation. For expositional reasons, we assume that $T = T_{eAIT} = 8$. The weights are normalised such that the weight on current inflation is always given by one. For reference, we include IT and PLT in panels a) and b). The weight on all lagged values is zero for inflation targeting and one for price-level targeting. A simple moving average of inflation as in the AIT rule (panel c)) has weights one for lags up to $T - 1$ and zero afterwards. The weights of the exponential moving average in (3.29) with $T_{eAIT} = 8$ in $\rho_{eAIT} = \frac{T_{eAIT}-1}{T_{eAIT}+1}$, depicted in panel d), follow an exponentially declining function across lags. That is, for $0 < \rho_{eAIT} < 1$, past inflation deviations are assigned a higher weight the closer they are to the present period. The relative weight of two adjacent periods in the exponential moving average is always given by

$$\frac{\eta_s}{\eta_{s-1}} = \frac{\rho_{eAIT}^s \eta_0}{\rho_{eAIT}^{s-1} \eta_0} = \rho_{eAIT}.$$

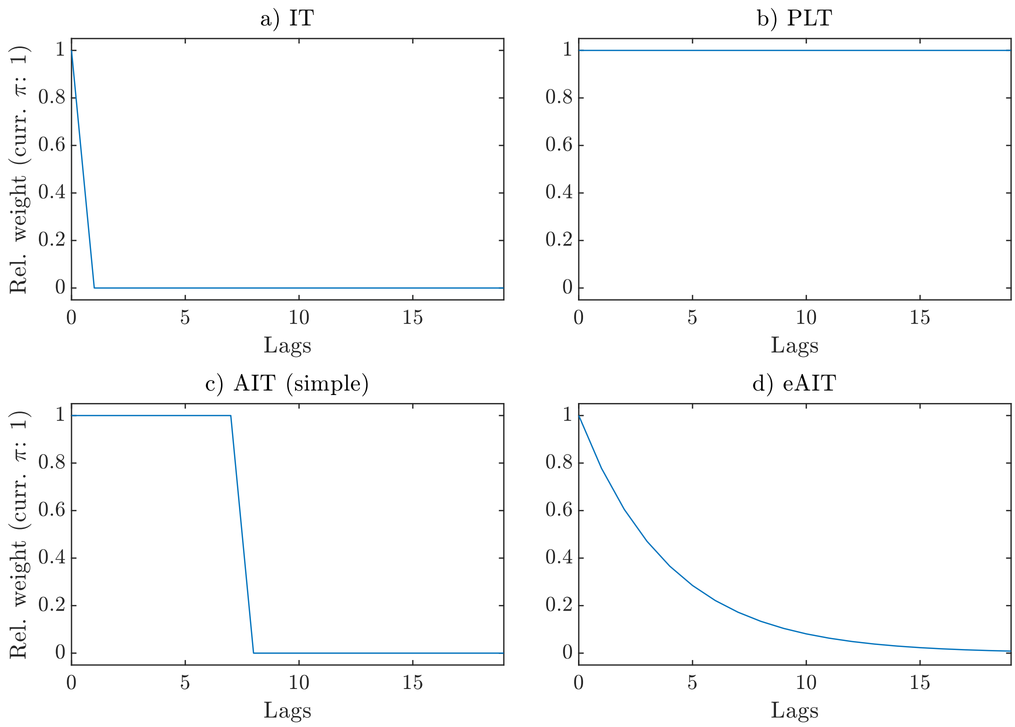

Figure 3.15: Normalised weights on different lags of inflation for different monetary policy rules ($T = T_{eAIT} = 8$ where applicable)

This implies that if a deviation of inflation from target is cancelled out subsequently, no additional adjustments will be necessary to reach the targeted exponential moving average of the inflation rate in the absence of additional shocks.[40]

To illustrate this graphically, consider Figure 3.16. Compared to the simple moving average used in the AIT rule in panel a) (which is the same graph as panel a) in Figure 3.13), the exponential moving average used in the eAIT rule in panel b) does not feature the periodically repeating pattern of inflation deviations. Also, it becomes evident that the response of the central bank targeting an exponential moving average will respond similar to one that stabilises the price level (see panel b) in Figure 3.13). The major difference is that the response in the period after the shock is smaller in magnitude than the shock itself, which is due to the exponential decay in weights. After this initial response, absent further shocks, the central bank could just set the desired inflation rate forever.

The Welfare Performance of the eAIT Rule

In the following, we analyse the welfare performance of an interest rate rule that takes an exponential moving average of inflation as an argument. The rule is given by equation (3.22), where average inflation is measured by the exponential moving average in equation (3.29), and the corresponding smoothing parameter is determined as in equation (3.30).[41]

[40]The underlying reason is that the past error never actually drops out of an averaging window, and thus never shows up with its sign reversed.

[41]To be precise, we operationalise this as

$$\tilde{\pi}_t := \hat{\pi}_t + \frac{T-1}{T+1}\tilde{\pi}_{t-1}$$

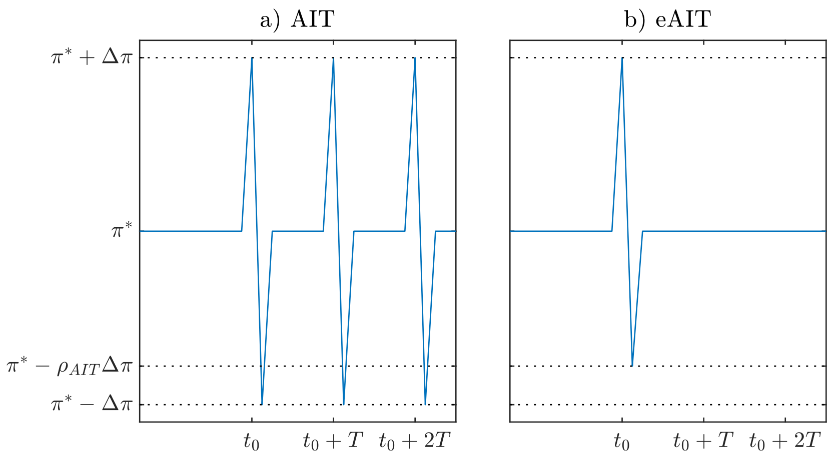

Figure 3.16: Illustration: cycling behaviour of AIT and eAIT

As in previous sections, there is an ELB on the policy rate as in equation (3.25). For a given T_{eAIT}, we optimise the coefficients in the eAIT rule for each degree of myopia and each shock according to the welfare criterion in equation (3.26).

Figure 3.17 shows welfare losses for demand shocks as in Figure 3.9, where now two lines for eAIT are added: one for eAIT with $T_{eAIT} = 16$ (green line with crosses) and one for eAIT with $T_{eAIT} = 32$ (light blue line with diamonds). As described above, the demand shock does not imply a trade-off for monetary policy, and the inherent volatility-inducing characteristic of AIT does not take effect for this type of shock. Hence, the conventional AIT rule already performs very well. In fact, for nearly rational expectations, eAIT (at least the rule with the higher auto-regressive coefficient) even performs worse than conventional AIT.

In contrast, for trade-off-inducing technology shocks, Figure 3.18 shows that the eAIT rules can result in a marked improvement in terms of welfare losses compared to conventional AIT. For low degrees of myopia, the performance of the eAIT rules is very close to the best performing rule, the PLT rule, and markedly better than for the conventional AIT rules. As the degree of myopia increases, the deterioration in terms of welfare losses is much smaller under the eAIT rules than under the PLT rule. Consequently, the eAIT rules exhibit lower welfare losses than the PLT rule for a value of $M_H = M_F = M_U$ below 0.9. In our simulations, eAIT with $T_{eAIT} = 32$ is the best performing rule for values of the myopia parameter between 0.85 and 0.9. However, the differences are relatively small and could be due to numerical imprecision. For even more myopic agents, the welfare losses of this eAIT rule increase and approach those of the IT rule for $M_H = M_F = M_U < 0.7$. However, for $M_H = M_F = M_U < 0.8$, eAIT with $T_{eAIT} = 16$ clearly exhibits the lowest welfare losses. In any case, the welfare losses under the eAIT rule with $T_{eAIT} = 16$ are markedly lower than the ones under the conventional AIT rules for the whole spectrum of degrees of myopia.

Also, our results suggest that eAIT can be regarded as a robust strategy: Welfare losses are minimal or close to minimal among the considered rules across different degrees of myopia and different types of shocks. For demand shocks, eAIT performs almost as well as the best performing rules for low and high degrees of myopia. For technology shocks, eAIT can significantly improve on conventional AIT and its performance is very close or

in equation (3.22). Note that our specification of eAIT is closely related to interest rate smoothing (where past shadow rates are used to smooth current ones).

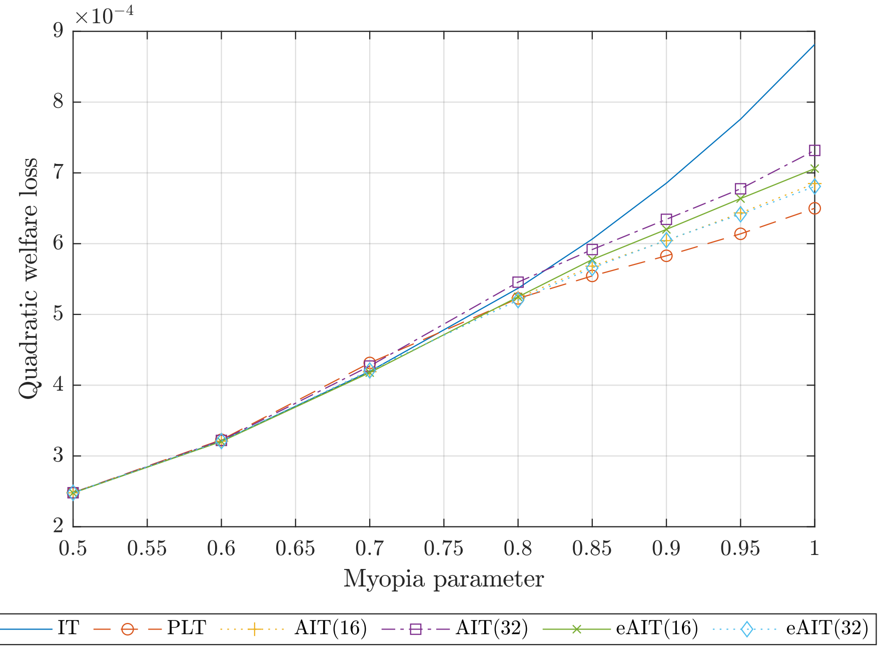

Figure 3.17: Welfare comparison of all monetary policy rules, demand shock

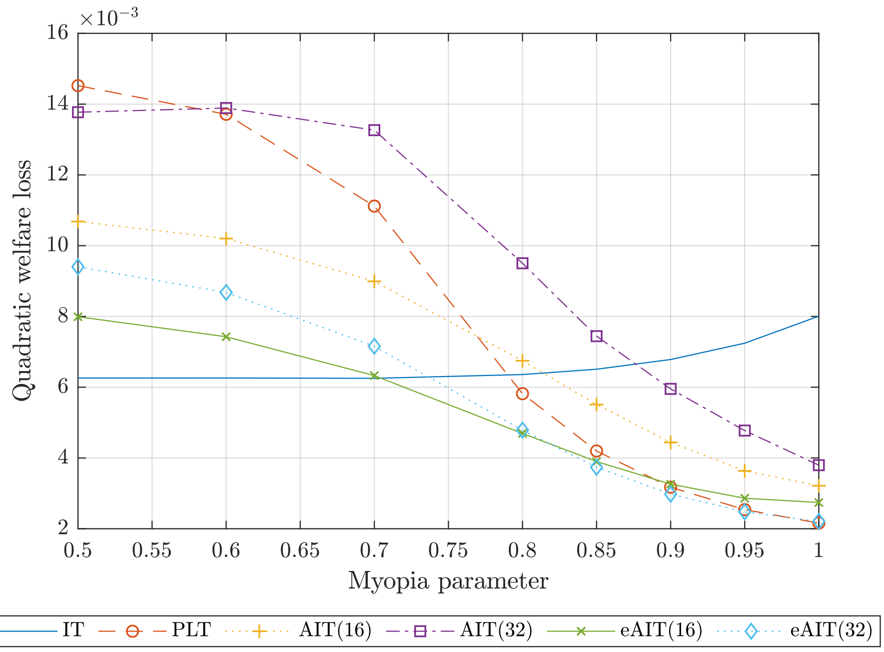

Figure 3.18: Welfare comparison of all monetary policy rules, technology shock

even somewhat better than that of the top performing rules for low and high degrees of myopia. This has three main reasons. First, in contrast to the conventional AIT rule, shock-induced deviations of the inflation rate from target do not trigger periodic policy-induced fluctuations of inflation under the eAIT rule, as described above. This benefits the eAIT rule's performance across the whole spectrum of degrees of myopia – especially for technology shocks, where smoothing out inflation is not optimal. Second, just like the conventional AIT rule and the PLT rule, the eAIT rule exhibits history dependence. This is beneficial when the expectations channel is powerful, i.e., for low degrees of myopia. Note that the history-dependent character of the eAIT rule is more akin to the PLT rule than to the conventional AIT rule since the exponential moving average entering the eAIT rule is an infinite impulse-response filter and, hence, all past inflation deviations are relevant for current and future monetary policy. This is why the eAIT rules come much closer to the PLT rule than the conventional AIT rule for low degrees of myopia in the case of technology shocks (see Figure 3.18). Third, the exponentially decaying weights of the exponential moving average in the eAIT rule imply that inflation deviations are assigned a higher weight, the closer they are to the present period. This 'tilts' the character of the eAIT rule – regarding the performance with myopic agents – towards an IT rule (in which the weight is one on the present inflation rate and zero on past inflation rates). The effect of this tilting is more pronounced for high degrees of myopia as this further reduces the influence that inflation deviations further in the past have on expectations about future inflation rates. In this way, the eAIT rule is able to approximate the IT rule when the degree of myopia is high.

3.5 Conclusion

In a New Keynesian model with sticky prices, sticky wages, an ELB on the policy rate and bounded rationality, we have studied the welfare performance of various simple interest rate rules for monetary policy. History-dependent interest rate rules such as a PLT rule or a conventional AIT rule that features a simple moving average of past inflation rates perform better than an IT rule for both a demand and a supply shock when agents have rational expectations or are mildly myopic. Stronger degrees of myopia reduce the advantages of strongly history-dependent strategies and IT may become the best performing rule. We found that an interest rate rule targeting an exponential moving average of the current and past inflation rates is robust in the sense that it performs very well across different degrees of myopia and types of shocks.

Our analysis comes with some caveats: (i) For purposes of illustration, we restrict our analysis to an economy where only one type of shock is active at any particular point in time. (ii) We assume that different agents in the economy have the same degree of myopia. (iii) Myopia is not influenced by the choice of the chosen interest rate rule. (iv) A couple of additional channels found to be empirically relevant are not operating in the model, especially with respect to capital and investment. Since the purpose of our paper was to illustrate as clearly as possible the interrelations between sticky wages and bounded rationality and their ramifications for different interest rate rules, we did not take into account these aspects in the current paper and leave them for future research. In a next step, we plan to bring the model to the data and estimate the degree of myopia.

Appendices

3.A Derivation of the Full Model

3.A.1 Household Behaviour

There is a continuum of households $h \in [0,1]$ with identical preferences

$$U_{h,t_0} = \mathcal{E}^h_{t_0}\left[\sum_{t \geq t_0} \beta^t d_t \left(u(c_{h,t}) - \frac{\nu}{1+\varphi} n_{h,t}^{1+\varphi}\right)\right], \tag{3.31}$$

where $c_{h,t}$ is h's consumption and $n_{h,t}$ is h's labour supply, φ is the household's inverse Frisch elasticity, d_t is a discount factor shock, and the function governing period utility flow from consumption $u(c)$ is given by

$$u(c) = \begin{cases} \ln(c) & \text{if} \quad \sigma = 1, \\ \frac{c^{1-\sigma}-1}{1-\sigma} & \text{otherwise.} \end{cases}$$

Here σ is both the inverse of the elasticity of intertemporal substitution as well as the coefficient of relative risk aversion.

Denote h's period-t wage by $w_{h,t}$, the wage-tax (or subsidy) by τ_w and the price level by P_t. Each household obtains nominal wage income $(1-\tau_w)W_{h,t}n_{h,t}$, real profits Π_t^F and transfers $P_t\tau_{ht}$ and spends it on final-goods consumption $P_t c_{ht}$ and liquid bonds $P_t b_{ht}$, these liquid bonds then yield a nominal return of $(1+i_t)P_t b_{ht}$ in the next period.

That is, the households's nominal period-budget constraint is given by

$$P_t(c_{ht} + b_t) = (1-\tau_w)W_{h,t}n_{h,t} + P_t(\Pi_t^F + \tau_{ht}) + (1+i_{t-1})P_{t-1}b_{h,t-1} \tag{3.32}$$

and the equivalent in real terms is

$$(c_{ht} + b_t) = (1-\tau_w)\frac{W_{h,t}}{P_t}n_{h,t} + \Pi_t^F + \tau_{ht} + R_t b_{h,t-1} \tag{3.33}$$

where

$$R_t = \frac{1+i_{t-1}}{1+\pi_t}$$

is the real interest factor between periods $t-1$ and t.

We assume that the household acts as a price-taker on all markets except for the labour market. On that market, we assume that households form unions U, which have market power. In particular, each household delegates its wage bargaining to the union, which then also determines the labour supply n_{ht}.

I.e., the household's optimisation problem in period t_0 is given by

$$\max_{(c_{ht},b_{ht})_{t\geq t_0}} U_{h,t_0} \quad \text{s.t.} \quad (3.33) \text{ holds } \forall t$$

Denoting as λ_{ht} h's Lagrange multiplier on (3.33) in t, we obtain the first order conditions:

$$u'(c_{ht}) = \lambda_{ht} \tag{3.34}$$

$$\lambda_{ht} d_t = \beta \mathcal{E}_t^h [d_{t+1} R_{t+1} \lambda_{h,t+1}], \tag{3.35}$$

which with $u'(c) = c^{-\sigma}$ delivers the standard Euler equation

$$d_t c_{ht}^{-\sigma} = \beta \mathcal{E}_t^h \left[d_{t+1} R_{t+1} c_{h,t+1}^{-\sigma} \right]$$

Following Erceg *et al.* (2000), we assume that $(\tau_{ht})_{h\in[0,1],t\geq t_0}$ is such that in each period, all households choose the same level of consumption $c_{ht} = C_t^H \forall t$. Aggregate consumption is then given by

$$C_t = \int_{h\in[0,1]} c_{ht} dh = \int_{h\in[0,1]} C_t^H dh = C_t^H.$$

Also, we assume that all households form the same expectations regarding the future $\mathcal{E}_t^h[\cdot] = \mathcal{E}_t^H[\cdot]$. Then, there is an aggregate Euler equation

$$d_t C_t^{-\sigma} = \beta \mathcal{E}_t^H \left[d_{t+1} R_{t+1} C_{t+1}^{-\sigma} \right] \tag{3.36}$$

and an aggregate stochastic discount factor

$$\mathcal{Q}_{t|t+1} = \beta d_{t+1} C_t^{\sigma} C_{t+1}^{-\sigma} / d_t. \tag{3.37}$$

which satisfies

$$1 = \mathcal{E}_t^H \left[R_{t+1} \mathcal{Q}_{t|t+1} \right] \tag{3.38}$$

3.A.2 Final-Goods Firm

The final good Y_t is produced by a competitive firm, using intermediate inputs $y_{ft}, f \in \mathcal{F}$ according to a production function, where each input in $\mathcal{F}$ with $|\mathcal{F}| = 1$ is produced by one intermediate firm, indexed also with f, according to

$$Y_t = \left(\int_{f\in\mathcal{F}} y_{ft}^{\frac{\epsilon_p - 1}{\epsilon_p}} df \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{3.39}$$

where $\epsilon_p > 1$ is the elasticity of substitution across varieties. With the price of the final good being given by P_t and intermediate input prices given by p_{ft}, profits of the final-goods firm are given by

$$\Pi_t^{\text{final}} := P_t Y_t - \int_{f\in\mathcal{F}} p_{f,t} y_{ft} df \tag{3.40}$$

As is usual with this production function, profit maximisation and a zero-profit condition give demand for each intermediate good as

$$y_{ft} = \left(\frac{p_{ft}}{P_t} \right)^{-\epsilon_p} Y_t \tag{3.41}$$

and an equation for the price level

$$P_t = \left(\int_{f\in\mathcal{F}} p_{ft}^{1-\epsilon_p} df \right)^{\frac{1}{1-\epsilon_p}}. \tag{3.42}$$

3.A.3 Intermediate-Goods Producers

Each intermediate goods firm f uses an aggregate labour input $N_{f,t}$ to produce its intermediate good according to a production function

$$q_{ft} = A_t N_{f,t}^{1-\alpha}, \tag{3.43}$$

where α measures decreasing returns to scale and A_t is time-varying productivity.

The aggregate labour input is priced at the real wage w_t and there is a production subsidy τ_p such that nominal profits of the firm are given by

$$P_t \Pi_{f,t} = p_{ft} q_{ft}(1+\tau_p) - P_t w_t N_{ft} \tag{3.44}$$

Each firm can only re-optimise its price p_{ft} in a given period with a probability $1-\theta_p$. In case the firm cannot reoptimise its price, the price is updated according to $p_{ft} = p_{f,t-1}(1+\pi^*)$. Firms also engage in monopolistic competition, taking into account the demand curves $q_{ft} = y_{ft}$ as in (3.41). I.e., their maximisation problem in period t_0 yields the recursive formulation

$$\Gamma_{f,t_0} := \max_{(p_{ft_0})} \left\{ \begin{aligned} &\mathcal{E}^F_{t_0}\left[\sum_{t \geq t_0} \theta^{t-t_0} \mathcal{Q}_{t_0|t} \left(\frac{p_{ft|t_0}}{P_t} q_{ft|t_0}(1+\tau_p) - w_t N_{ft|t_0} \right) + (1-\theta_p)\mathcal{Q}_{t_0|t_0+1}\Gamma_{f,t_0+1} \right] \\ &\qquad \text{s.t } t \geq t_0 : \\ &\qquad\qquad p_{ft|t_0} = \begin{cases} (1+\pi^*) p_{f,t-1|t_0}, & \text{if} \quad t > t_0, \\ p_{ft_0}, & \text{if} \quad t = t_0, \end{cases} \\ &\qquad\qquad q_{ft|t_0} = A_t N_{f,t|t_0}^{1-\alpha}, \\ &\qquad\qquad q_{ft|t_0} = \left(\frac{p_{ft|t_0}}{P_t} \right)^{-\epsilon_p} Y_t, \\ &\qquad\qquad \mathcal{E}^F_{t_0}[(A_t, Y_t, P_t, w_t)_{t \geq t_0}] \text{ given} \end{aligned} \right\}$$

for the value of an optimising firm Γ_{f,t_0} at the start of a period t_0. Here, the stochastic discount factor $\mathcal{Q}_{t_0|t}$ between non-adjacent periods t_0, t is given by

$$\mathcal{Q}_{t_0|t} = \prod_{s=t_0}^{t-1} \mathcal{Q}_{s|s+1} \quad \text{for } t \geq t_0, t \neq t_0 + 1$$

This gives rise to an optimal pricing decision given by

$$p^*_{ft} = p^*_t = P_t \left(\frac{\epsilon_p}{(\epsilon_p - 1)(1+\tau_p)(1-\alpha)} \frac{\Xi^1_t}{\Xi^2_t} \right)^{\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}}, \quad \text{where} \tag{3.45}$$

$$\Xi^1_t = w_t \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} + \theta \mathcal{E}^F_t \left[\left(\frac{1+\pi_{t+1}}{1+\pi^*} \right)^{\frac{\epsilon_p}{1-\alpha}} \mathcal{Q}_{t|t+1} \Xi^1_{t+1} \right] \quad \text{and} \tag{3.46}$$

$$\Xi^2_t = Y_t + \theta \mathcal{E}^F_t \left[\left(\frac{1+\pi_{t+1}}{1+\pi^*} \right)^{\epsilon_p - 1} \mathcal{Q}_{t|t+1} \Xi^2_{t+1} \right], \tag{3.47}$$

which is the same for all firms optimising in t.

Each intermediate-goods producer also generates real profits

$$\pi_{f_t} = (1+\tau_p)\left(\frac{p_{ft}}{P_t}\right)^{1-\epsilon_p} Y_t - w_t \left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{3.48}$$

and has labour demand

$$N_{ft} = \left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{3.49}$$

The price-level evolves according to

$$P_t^{1-\epsilon_p} = (1-\theta_p)\left(p_t^*\right)^{1-\epsilon_p} + \theta_p(1+\pi^*)^{1-\epsilon_p} P_{t-1}^{1-\epsilon_p}$$

or, in terms of the inflation rate

$$1 = (1-\theta_p)\left(\frac{p_t^*}{P_t}\right)^{1-\epsilon_p} + \theta_p\left(\frac{1+\pi^*}{1+\pi_t}\right)^{1-\epsilon_p} \tag{3.50}$$

3.A.4 Labour Packers

We assume that the labour input used by intermediate goods firms is provided by a competitive labour packer who buys up the labour supplied by the various unions and aggregates it up according to the "production function"

$$N_t = \left(\int_{u\in\mathcal{U}} n_{ut}^{\frac{\epsilon_w - 1}{\epsilon_w}} du\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{3.51}$$

where ϵ_w is the elasticity of substitution across different union's labour input.

With the real price of the aggregate labour output given by the real wage w_t and individual unions' wage rates $w_{u,t}$, we obtain profits

$$\Pi_t^N := w_t N_t - \int_{u\in\mathcal{U}} n_{ut} w_{ut} du. \tag{3.52}$$

Maximising this subject to the production function and zero profits, we obtain a labour demand for individual union's labour supply

$$n_{ut} = \left(\frac{w_{ut}}{w_t}\right)^{-\epsilon_w} N_t \tag{3.53}$$

and an equation for the real wage (index)

$$w_t = \left(\int_{u\in\mathcal{U}} w_{ut}^{1-\epsilon_w} du\right)^{\frac{1}{1-\epsilon_w}}, \tag{3.54}$$

Equations (3.53) and (3.54) can also be expressed in nominal terms as

$$n_{ut} = \left(\frac{W_{ut}}{W_t}\right)^{-\epsilon_w} N_t \quad \text{and} \tag{3.55}$$

$$W_t = \left(\int_{u\in\mathcal{U}} W_{ut}^{1-\epsilon_w} du\right)^{\frac{1}{1-\epsilon_w}}, \tag{3.56}$$

$$\text{where } W_t = P_t w_t \quad \text{and} \tag{3.57}$$

$$W_{ut} = P_t w_{ut}. \tag{3.58}$$

3.A.5 Unions / Wage Setters

Each household is assigned to a union, whose objective it is to maximise its member's utility, taken as given the member's consumption decision. As mentioned before, households' consumption is perfectly insured, however, their labour supply is subject to market clearing at a given wage rate and as such each union takes (3.55) into account. In period t_0, each union discounts a real income stream in period t by $\mathcal{Q}_{t_0|t}$, the disutility of the union stems from the utility function (3.31). Similar to intermediate-goods producers, unions can only reoptimise in any given period with probability $1-\theta_w$, with a probabilty of θ_w their nominal wage is simply adjusted for steady-state inflation π^*.

The dynamic programming problem of an optimising firm in period t_0 can thus be written in recursive form as

$$V_t^u = \max_{(W_{ut_0}^*)} \left\{ \begin{aligned} &\mathcal{E}_{t_0}^u \Bigg[\sum_{t \geq t_0} \theta_w^{t-t_0} \mathcal{Q}_{t_0|t} \left((1-\tau_w) \frac{W_{ut|t_0}}{P_t} n_{ut|t_0} - \frac{\nu}{1+\varphi} C_t^\sigma n_{ut|t_0}^{1+\varphi} \right) \\ &\qquad + (1-\theta_w) \mathcal{Q}_{t_0|t_0+1} V_{t_0+1}^u \Bigg] \\ &\text{s.t } t \geq t_0 : \\ &\qquad W_{ut|t_0} = \begin{cases} (1+\pi^*) W_{u,t-1|t_0}, & \text{if} \quad t > t_0, \\ W_{ut_0}^*, & \text{if} \quad t = t_0, \end{cases} \\ &\qquad n_{ut|t_0} = \left(\frac{W_{ut|t_0}}{W_t} \right)^{-\epsilon_w} N_t, \\ &\qquad\qquad \mathcal{E}_{t_0}^F [(C_t, W_t, N_t, W_t, P_t, \mathcal{Q}_{t_0|t})_{t \geq t_0}] \text{ given} \end{aligned} \right\}.$$

This gives rise to optimal wage setting behaviour

$$(W_{ut}^*)^{1+\epsilon_w \varphi} = P_t W_t^{\epsilon_w \varphi} \frac{\nu \epsilon_w}{(\epsilon_w - 1)(1-\tau_w)} \frac{X_{1t}}{X_{2t}}, \tag{3.59}$$

$$X_{1t} = C_t^\sigma N_t^{1+\varphi} + \theta_w \mathcal{E}_t^u \left[\left(\frac{1+\pi_{w,t+1}}{1+\pi^*} \right)^{\epsilon_w(1+\varphi)} \mathcal{Q}_{t|t+1} X_{1,t+1} \right], \tag{3.60}$$

$$X_{2t} = N_t + \theta_w \mathcal{E}_t^u \left[\frac{(1+\pi_{w,t+1})^{\epsilon_w}}{(1+\pi^*)^{\epsilon_w - 1}(1+\pi_{t+1})} \mathcal{Q}_{t|t+1} X_{2,t+1} \right], \tag{3.61}$$

where

$$(1+\pi_{w,t}) = \frac{W_t}{W_{t-1}} \tag{3.62}$$

The nominal wage index thus evolves according to

$$W_t^{1-\epsilon_w} = (1-\theta_w)(W_t^*)^{1-\epsilon_w} + \theta_w [(1+\pi^*) W_{t-1}]^{1-\epsilon_w}, \tag{3.63}$$

which we can reformulate as

$$1 = (1-\theta_w) \left(W_t^* / W_t \right)^{1-\epsilon_w} + \theta_w \left[\frac{1+\pi^*}{1+\pi_{w,t}} \right]^{1-\epsilon_w}, \tag{3.64}$$

We can express (3.59) in real terms as

$$w_{ut}^* = \frac{W_{ut}^*}{P_t} = \left[w_t^{\epsilon_w \varphi} \frac{\nu \epsilon_w}{(\epsilon_w - 1)(1-\tau_w)} \frac{X_{1t}}{X_{2t}} \right]^{\frac{1}{1+\epsilon_w \varphi}}, \tag{3.65}$$

where the real wage evolves according to

$$w_t = (1 + \pi_{wt})w_{t-1}/(1 + \pi_t) \tag{3.66}$$

and (3.64) can be written as

$$1 = (1 - \theta_w)\left(w_t^*/w_t\right)^{1-\epsilon_w} + \theta_w \left[\frac{1 + \pi^*}{1 + \pi_{w,t}}\right]^{1-\epsilon_w}, \tag{3.67}$$

3.A.6 Fiscal Policy, Market Clearing

We assume that liquid bonds are in zero-net supply, i.e.

$$\int_{h\in[0,1]} b_{h,t} dh = 0 \tag{3.68}$$

Also, the government purchases goods $G_t = g_t Y_t$ and runs a balanced budget. As such, the budget constraint of the government reads as

$$\int_{h\in[0,1]} \tau_{ht} dh + \tau_w \int_{h\in[0,1]} w_{ht} n_{ht} dh = \sum_{f\in\mathcal{F}} \tau_p y_{ft} df = G_t \tag{3.69}$$

Market clearing in the final-goods sector then implies

$$C_t + G_t = Y_t = \int_{f\in\mathcal{F}} y_{ft} df \tag{3.70}$$

or

$$C_t = Y_t(1 - g_t) \tag{3.71}$$

We still need to aggregate up firms' profits from (3.48), taking into account that the final-goods firms and the labour packer make zero profits:

$$\begin{aligned}
\Pi_t^F &= \Pi_t^{final} + \Pi_t^N + \int_{f\in\mathcal{F}} \pi_{f_t} df \\
&= 0 + 0 + \pi_{f_t} df \\
&= \int_{f\in\mathcal{F}} \left((1+\tau_p)\left(\frac{p_{ft}}{P_t}\right)^{1-\epsilon_p} Y_t - \left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \right) df \\
&= (1+\tau_p) \int_{f\in\mathcal{F}} \left(\left(\frac{p_{ft}}{P_t}\right)^{1-\epsilon_p} \right) Y_t - \int_{f\in\mathcal{F}} \left(\left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} \right) \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} df
\end{aligned} \tag{3.72}$$

Similarly, aggregate labour demand is given by

$$N_t = \int_{f\in\mathcal{F}} N_{ft} df = \int_{f\in\mathcal{F}} \left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} df = \int_{f\in\mathcal{F}} \left(\frac{p_{ft}}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} df \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{3.73}$$

Note that for any $x \in \mathbb{R}$, we have

$$s_t^p(x) := \int_{f\in\mathcal{F}} \left(\frac{p_{ft}}{P_t}\right)^x df \tag{3.74}$$

$$=(1-\theta_p)\left(\frac{p_t^*}{P_t}\right)^x + \theta_p \int_{f\in\mathcal{F}} \left((1+\pi*)\frac{p_{f,t-1}}{P_t}\right)^x df \tag{3.75}$$

$$=(1-\theta_p)\left(\frac{p_t^*}{P_t}\right)^x + \theta_p \left(\frac{1+\pi^*}{1+\pi_t}\right)^x \int_{f\in\mathcal{F}} \left(\frac{p_{f,t-1}}{P_{t-1}}\right)^x df \tag{3.76}$$

$$=(1-\theta_p)\left(\frac{p_t^*}{P_t}\right)^x + \theta_p \left(\frac{1+\pi^*}{1+\pi_t}\right)^x s_{t-1}^p(x), \tag{3.77}$$

where, note we have

$$s_t^p(1) = s_t^p(0-\epsilon_p) = 1 \tag{3.78}$$

This allows us to obtain

$$\Pi_t^F = (1+\tau_p)Y_t - s_t\left(\frac{-\epsilon_p}{1-\alpha}\right)\left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{3.79}$$

Similarly, aggregate labour demand is given by

$$N_t = -s_t\left(\frac{-\epsilon_p}{1-\alpha}\right)\left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \tag{3.80}$$

An equilibrium in this economy, given structural parameters $(\sigma, \nu, \rho, \beta, \alpha, \epsilon_p, \epsilon_w, \theta_p, \theta_w)$, policy parameters $(\tau_w, \tau_p, \phi_\pi, \phi_y, \pi^*)$, state variables $(w_{t-1}.W_{t-1}, P_{t-1})$ is a sequence of state-contingent aggregate variabels $(C_t, Y_t, G_t, R_t, \pi_t, \pi_{w,t}, w_t, i_{t-1}, \Xi_{1,t}, \Xi_{2,t}, X_{1,t}, X_{2,t})$

3.B Additional Results

3.B.1 Other New Keynesian Paradoxes

In the main text, we focused on those paradoxes of the standard New Keynesian model under rational expectations that were crucial for the comparison of different monetary policy strategies. For completeness, in this Appendix we also discuss three other puzzles of the New Keynesian model: the paradox of flexibility, the paradox of toil and the Neo-Fisherian puzzle.

Paradox of Flexibility

The paradox of flexibility (Eggertsson and Krugman, 2012) states that with sticky prices (and sticky wages), the ELB becomes more harmful as prices (or wages) become more flexible, i.e. as θ_p, θ_w decrease. The underlying factor is that the slope of the Phillips curves increases with rising flexibility. I.e., a negative output gap requires more deflation, which at the effective lower bound implies a bigger gap between realised real rate and natural rate. This paradox is not resolved by reducing forward-lookingness of agents (see also Gabaix, 2020). However, by breaking the strong link of output gap and inflation rates across periods that one can obtain under rational expectations, this effect is somewhat muted for longer spells at the effective lower bound.

Paradox of Toil

The paradox of toil (Eggertsson, 2010) states that, at the effective lower bound, expansionary supply-side shocks can be contractionary. The reason is that positive TFP shocks are deflationary and are accompanied by a decrease in the natural rate (see equation (3.8)). However, with the economy already at the lower bound, nominal rates cannot drop any further, and the deflationary pressure actually raises the real interest rate. As is evident from (3.8), with bounded rationality, a given TFP shock causes bigger adjustments of the natural rate, this actually makes the paradox of toil more severe. At the same time, since the Phillips curves become less forward-looking, the deflationary effects of persistent TFP shocks are somewhat muted. I.e., overall, the paradox of toil could become more or less severe; however, in any case, it would remain effective.

Neo-Fisherian Paradox

The Fisher equation describes the long-run relationship between nominal and natural interest rates and expected inflation. Permanent increases in the nominal rate imply higher long-run inflation given that the natural rate is exogenously given. The Neo-Fisherian paradox posits that such permanent increases also lead to higher inflation rates in the short-run.

To analyse how Neo-Fisherian the model under bounded rationality and sticky wages is, we follow the experiment by (Gabaix, 2020, p. 38 et seq.). To forecast inflation firms and unions forecast (the same) default inflation $\pi_t^d = (1-\xi)\bar{\pi}_t + \xi\bar{\pi}_t^{CB}$ which is a function of $\bar{\pi}_t$ and $\bar{\pi}_t^{CB}$, i.e. moving averages of past inflation and inflation guidance with $\xi \in [0,1]$. Those averages in turn are given by $\bar{\pi}_t = (1-\eta)\bar{\pi}_{t-1} + \eta\pi_{t-1}$ and $\bar{\pi}_t^{CB} = (1-\eta_{CB})\bar{\pi}_{t-1}^{CB} + \eta_{CB}\pi_{t-1}^{CB}$. At time 0 the central bank announces an immediate, permanent, unexpected

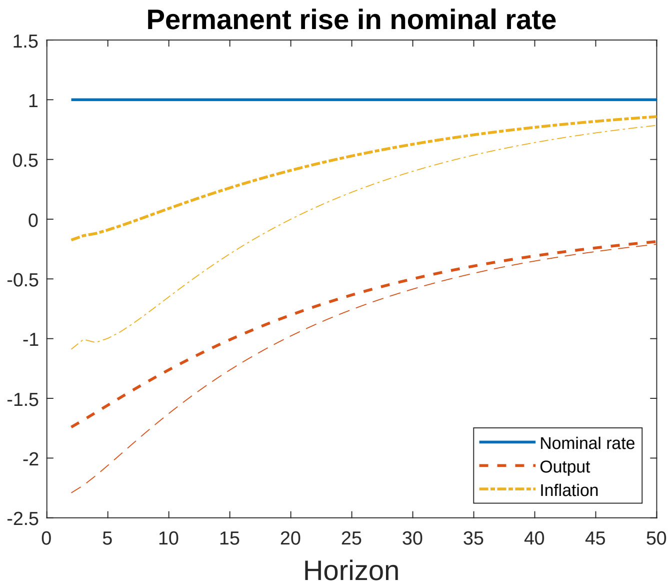

Figure 3.B.1: Neo-Fisherian puzzle

Note: The figure shows the impulse responses to an unexpected permanent 1% increase in the nominal interest rate under sticky wages. Lighter lines indicate the case with sticky prices only.

rise in the nominal rate of 1% and its corresponding target inflation $\pi_t^{CB} = 1\%$.[42] Figure 3.B.1 shows the result of a permanent increase in the nominal rate. The dashed line shows that in the short-run inflation (yellow) reacts negatively to the permanent increase. However, the initial magnitude is quite small especially in comparison to the sticky price model (light dashed). While bounded rationality thus solves the Neo-Fisherian puzzle, it barely does so with sticky wages. In the sticky wage model, convergence to the long-run level is faster.

[42]In addition to the calibration in table 1 we use $\xi = 0.8$, $\eta = 0.5$ and $\eta_{CB} = 0.05$.

3.B.2 Additional Figures for Section 3.3

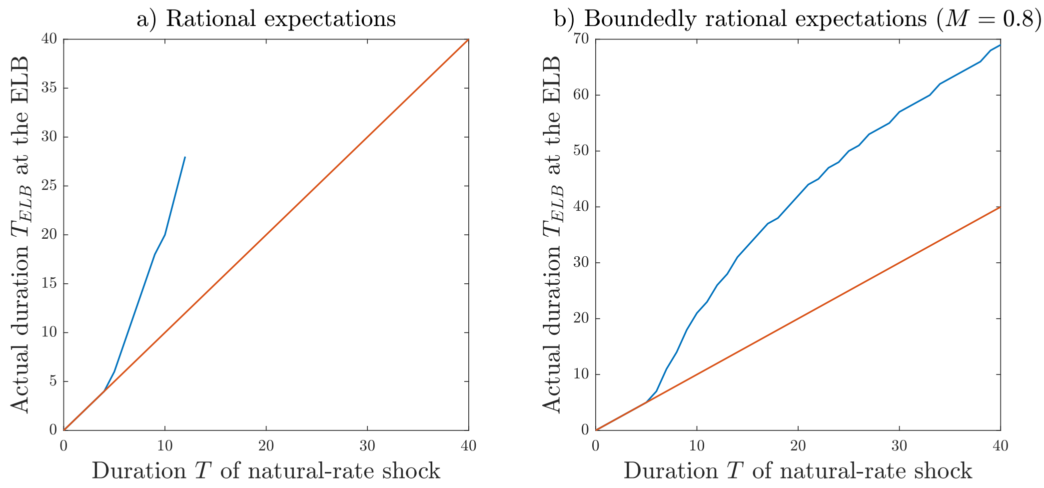

Figure 3.B.2: Severity of ELB: Mapping the natural-rate shock to ELB durations

Note: This figure compares the actual duration of the ELB (T_{ELB}), induced by a constant decline in r_t^* to -0.20 for T quarters under rational (lhs) and boundedly rational (rhs) expectations.

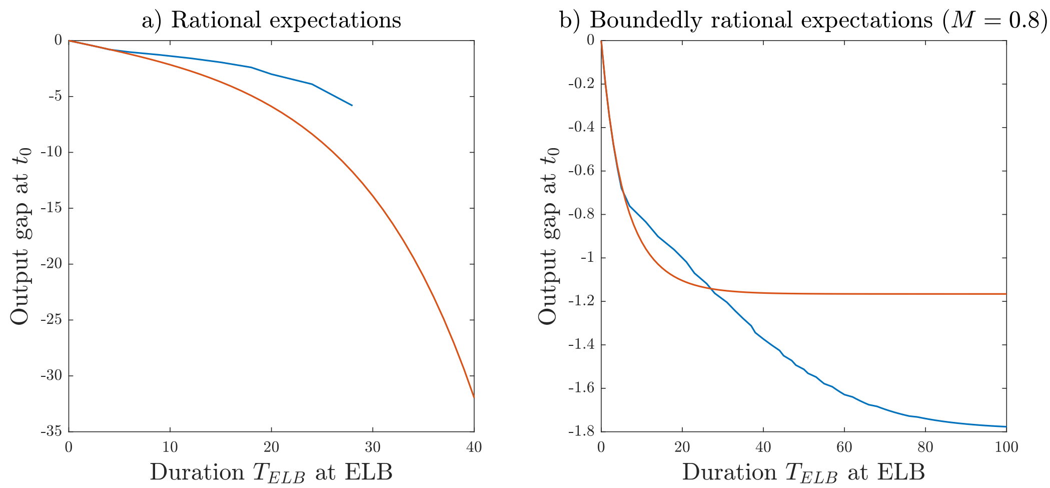

Figure 3.B.3: Severity of ELB: relationship between actual spells at the ELB and initial output gap.

Note: This figure compares the severity of recessions (measured as x_{t_0}) due to a spell at the effective lower bound from t_0 for T_{ELB} periods, induced by a constant decline in r_t^* to -0.20 for T quarters (not reported, see Figure 3.B.2 for the mapping) under rational (lhs) and boundedly rational (rhs) expectations.

3.B.3 Additional Figures for Section 3.4

Simulated Means

Below, a couple of additional results are presented for section 3.4, starting with the mean values of the simulated variables under TFP and demand shocks in figures 3.B.4 and 3.B.5, respectively. In this section, we always show all policy rules from the main text.

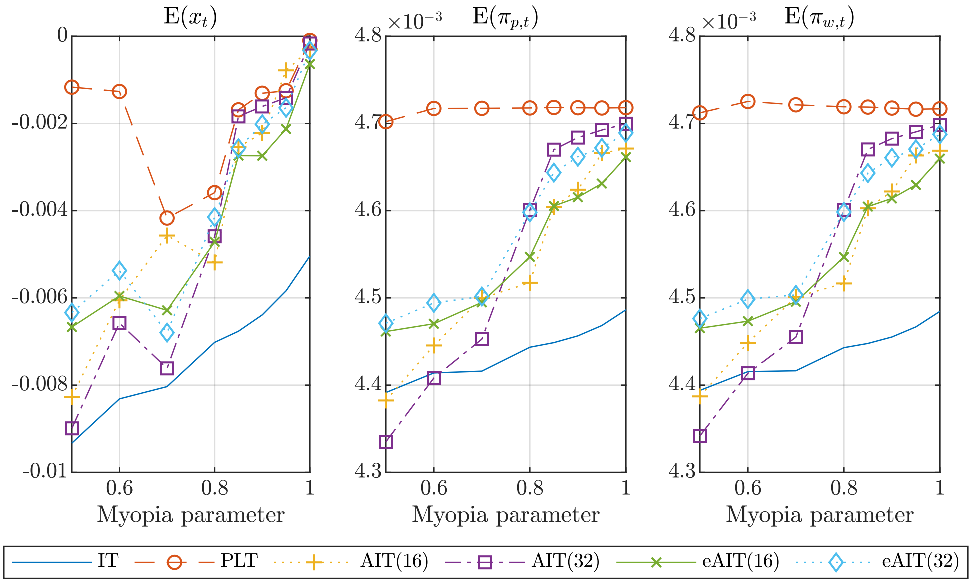

Figure 3.B.4: Means of selected simulated variables, TFP shock

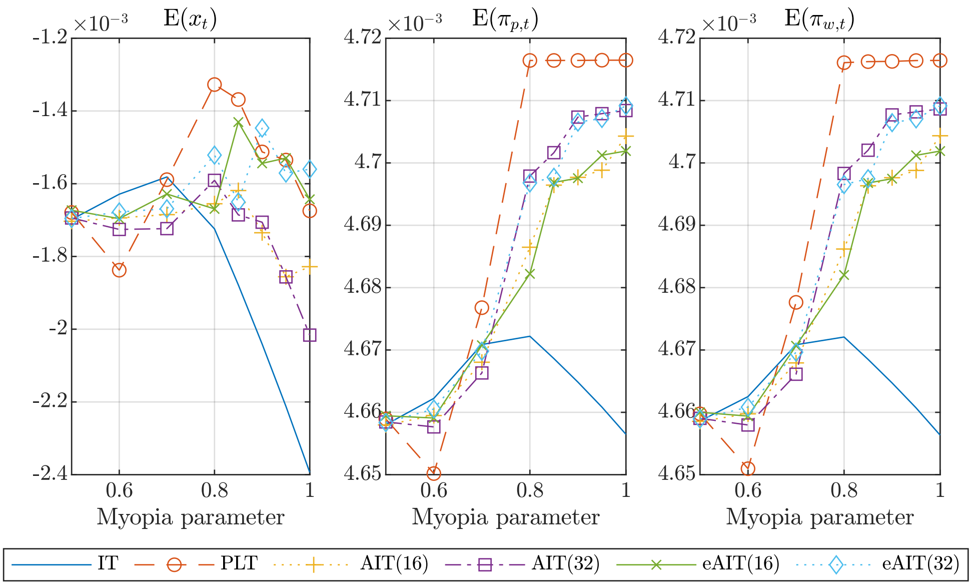

Figure 3.B.5: Means of selected simulated variables, demand shock

It is noteworthy that the presence of the effective lower bound implies that the mean output gap is negative for all simulations. Also, the increased volatility in the natural rate makes the mean output gap larger for TFP shocks for the various AIT or eAIT rules. For IT, this wedge declines with less forward-looking agents, and for PLT there is a non-monotonous behaviour. Notably, it becomes clear that as the TFP shock leads to a lot more time spent at the ELB with history dependent policies, those, in particular PLT, have less effective time (i.e., unconstrained time) to stabilise (average) inflation. Consequently, mean inflation rates start to differ from the target rate as the economy becomes more myopic.

Welfare Purely Based on Variances

Instead of using equation (3.26) to measure welfare losses, often a simple variance-based approximation is used:

$$\mathbb{L}_2 = \frac{1}{2}\left\{ \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) \text{Var}\left(x_t\right) + \frac{\epsilon_p}{\lambda_p}\text{Var}\left(\pi_t^p\right) + \frac{\epsilon_w(1-\alpha)}{\lambda_w}\text{Var}\left(\pi_t^w\right) \right\} \tag{3.81}$$

Figures 3.B.6 and (3.B.7) present the resulting welfare approximations for our experiments conducted in the main text. Note that the overall pattern is unaffected: inflation targeting becomes more, price-level targeting less attractive as myopia increases. However, for technology shocks note that eAIT with $T_{eAIT} = 32$ in this case exactly approaches IT as we make agents more myopic. Moreover, in Figure 3.B.7, it is easier to see that inflation targeting overtakes price-level targeting, also it overtakes it a bit earlier. This is due to the fact that history dependent strategies at the ELB counteract deflationary biases etc., making the mean deviation relevant from a welfare perspective. This, however, is neglected under equation (3.81).

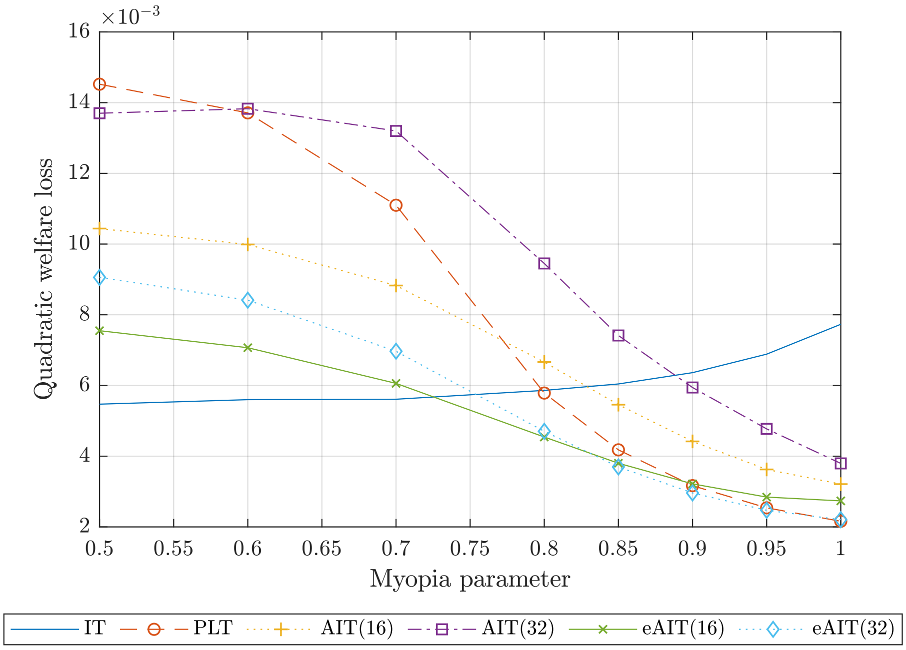

Figure 3.B.6: Welfare comparison for technology shocks, purely variance-based

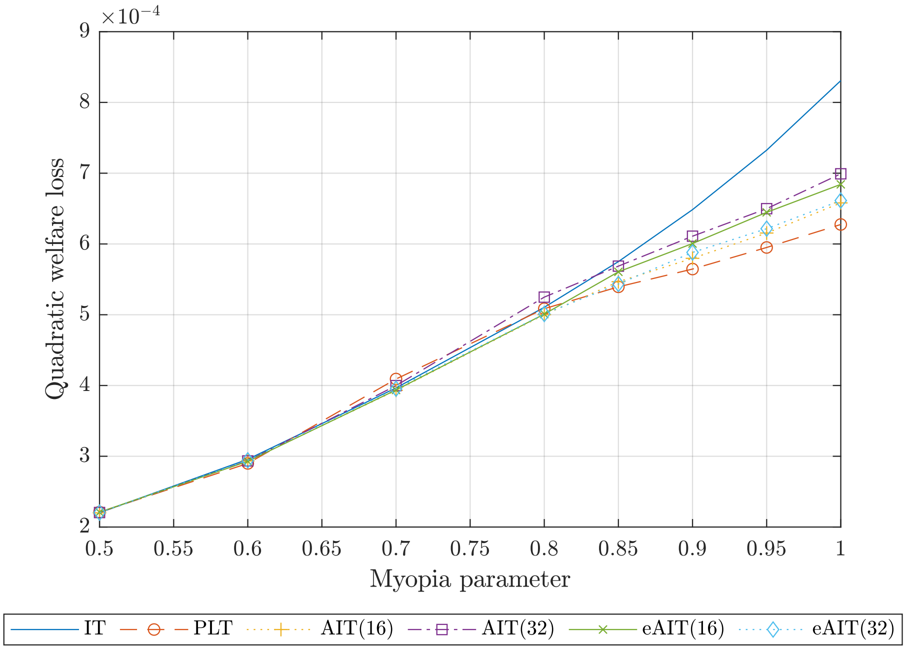

Figure 3.B.7: Welfare comparison for demand shocks, purely variance-based

Optimised Parameters, Variances and ELB Statistics with all Monetary Policy Rules

Now, we present the results for the optimised parameters (Figures 3.B.8 and 3.B.9), the variance plots (Figures 3.B.10 and 3.B.11) and the plots depicting the ELB statistics for the analysis with the eAIT rules included.

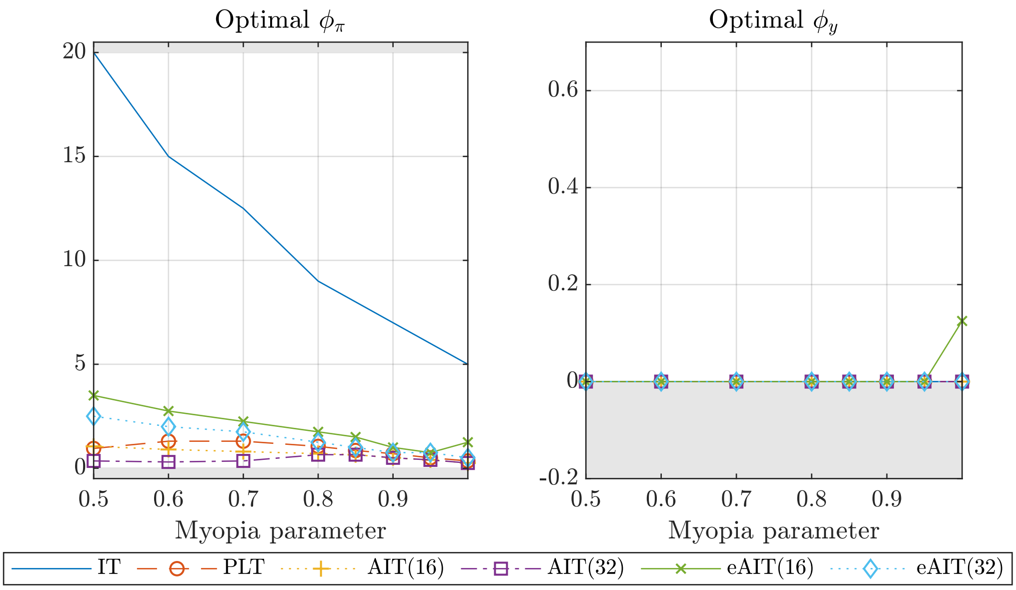

Figure 3.B.8: Optimised parameters, all monetary policy rules, TFP shock

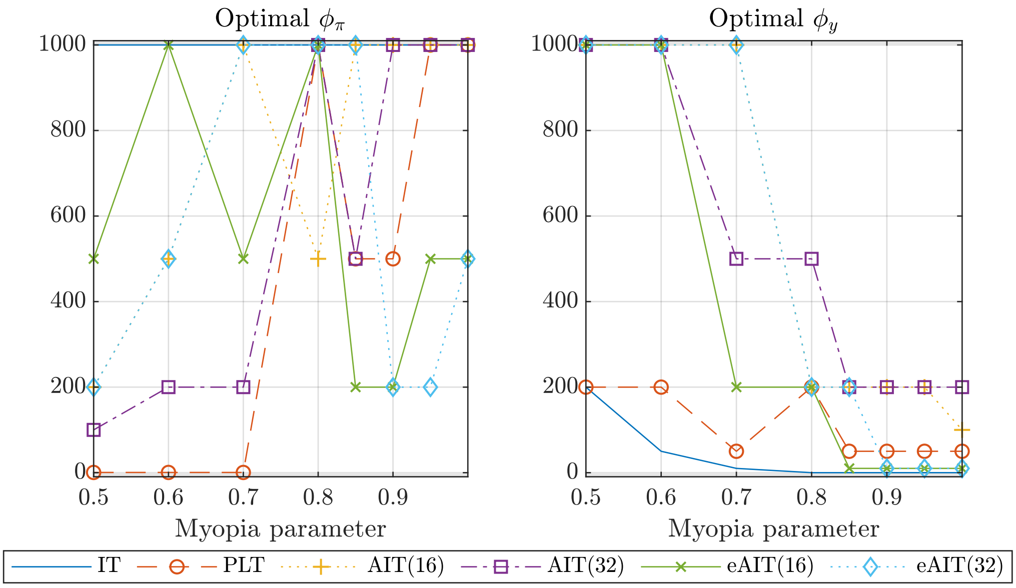

Figure 3.B.9: Optimised parameters, all monetary policy rules, demand shock

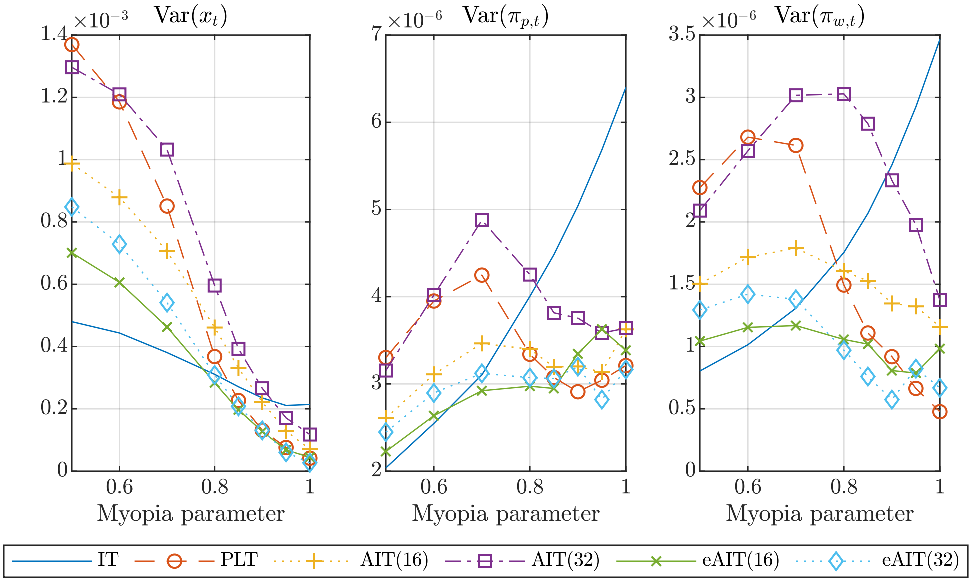

Figure 3.B.10: Variances, all monetary policy rules, TFP shock

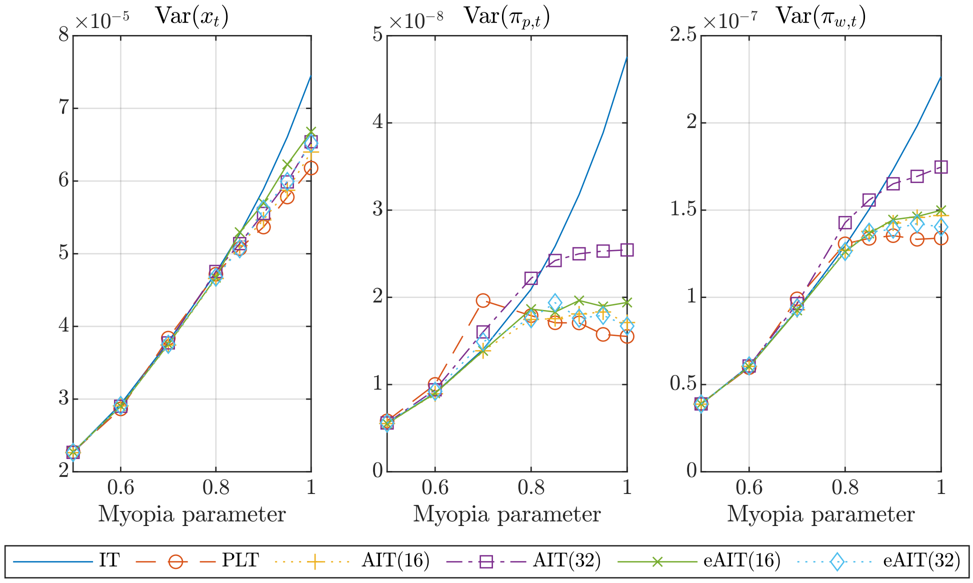

Figure 3.B.11: Variances, all monetary policy rules, demand shock

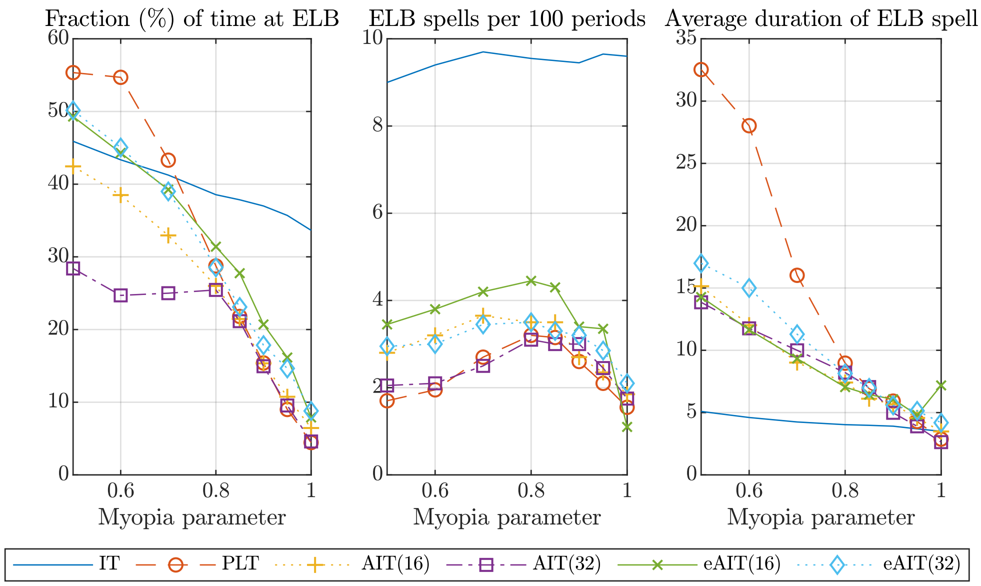

Figure 3.B.12: ELB statistics, all monetary policy rules, TFP shock

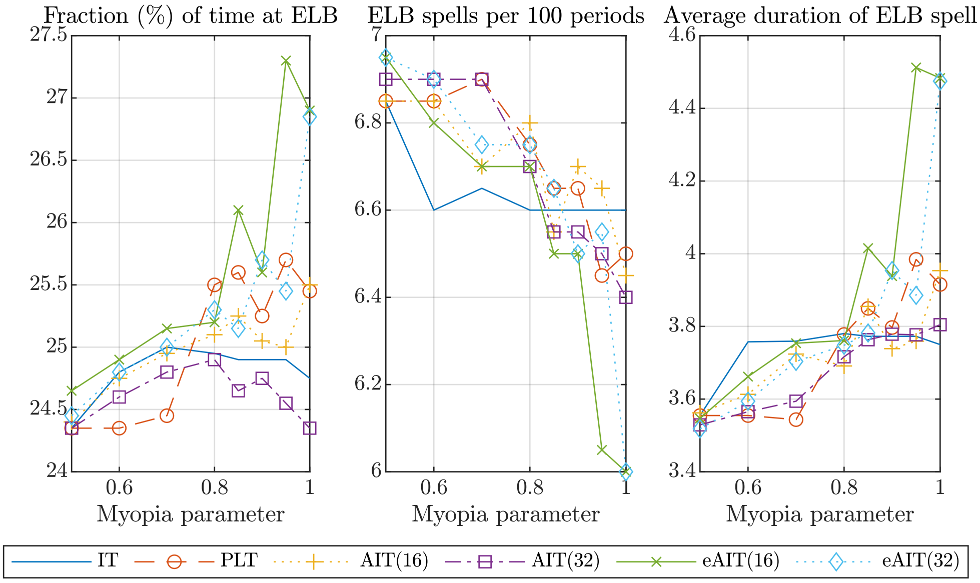

Figure 3.B.13: ELB statistics, all monetary policy rules, demand shock

TFP Shock, Allowing for a Negative Reaction Parameter on Output

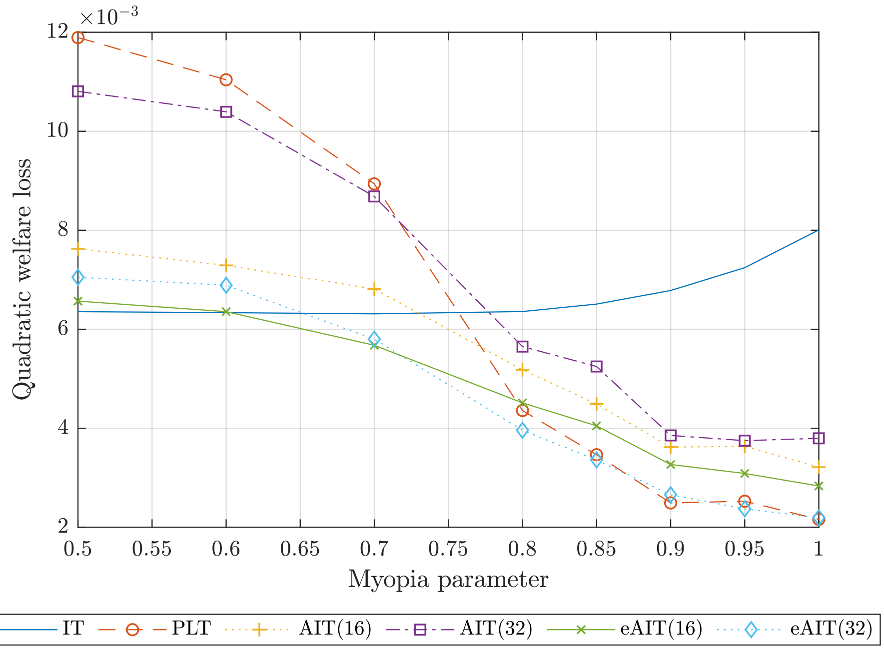

Figure 3.B.14: Welfare comparison for technology shocks, with negative output coefficients allowed

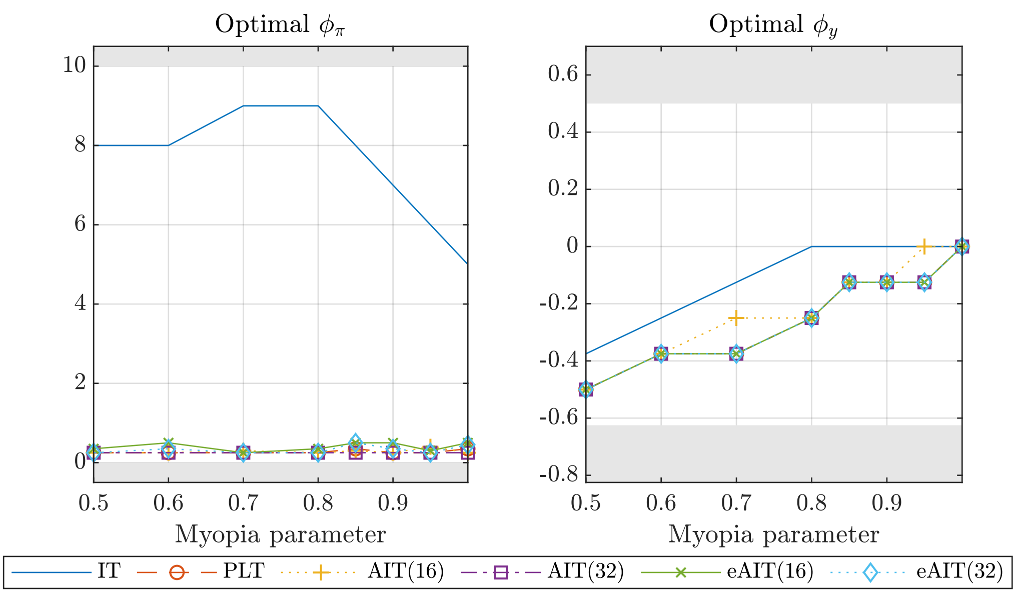

Figure 3.B.15: Optimal reaction parameters, TFP shock, with negative output coefficients allowed

TFP Shock, Keeping Volatility of Natural Rate Constant

As a final result, we also provide (in Figure 3.B.16) the welfare analysis for the TFP shock, when we rescale the variance of the TFP shock so as to keep the variance of the natural rate constant (and allow for negative output coefficients). From the results in the main text, it is evident that this rescaling scales down the variance of natural output by quite a lot. As a result, all variances decrease and the economy hits the ELB less often (for $M < 0.8$, it only hits it with IT or PLT, not with any form of AIT or eAIT). This affects the welfare ranking, making IT and PLT perform relatively worse, even though on average, all rules now become better for lower myopia. However, we still observe that IT 'catches up' with history-dependent strategies. Also, the eAIT rules still perform relatively good.

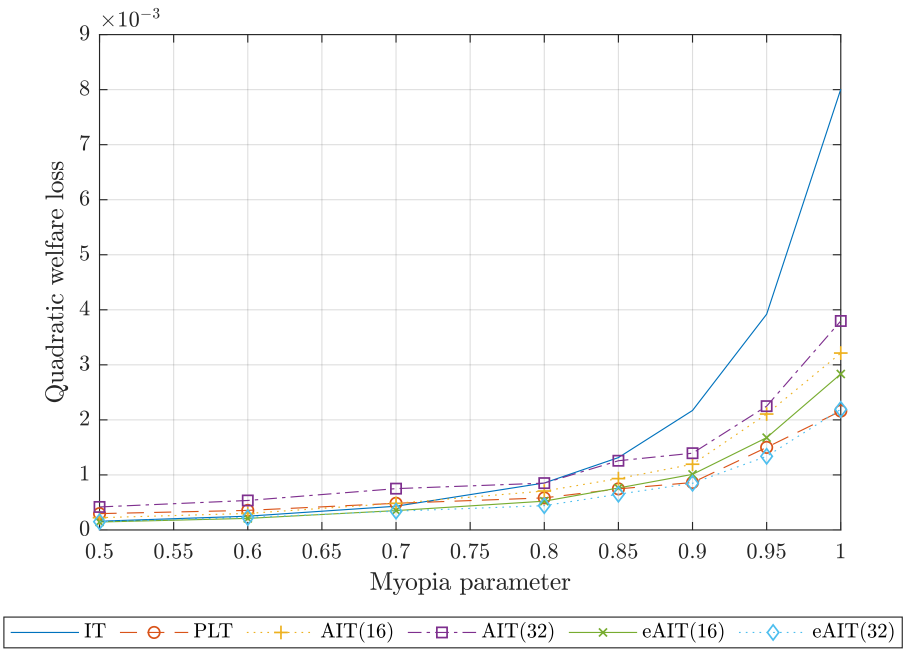

Figure 3.B.16: Welfare comparison for technology shocks, variance of natural rate being kept constant

Chapter 4

The Macroeconomic Impacts of Public Investment with Imperfect Expectations

Abstract

In this chapter, we study the impact that imperfect forward-lookingness of expectations has on the multipliers associated with increased public spending on long-lasting investment projects such as infrastructure. To do this, we perform simulations in a New Keynesian model with limited agent heterogeneity, private and public capital, where the latter features time to build. Here ad-hoc myopic or partially backward-looking (hybrid) expectations are introduced. We obtain that short-run multipliers tend to be larger if agents do not fully incorporate the future into their expectation formation. This effect is stronger for a longer time to build of public capital. We also assess the impact of a number of additional model features on our results.
Keywords: Investment, Fiscal Policy, Expectations, Role and Effects of Psychological, Emotional, Social, and Cognitive Factors on Decision Making, Infrastructures, Public Investment and Capital Stock

JEL Codes: E12, E22, E32, E62, D84, D91, H54

4.1 Introduction

From the 1980s to the early 2020s, real rates in advanced economies declined substantially; also the recovery after the 2008 financial crisis was remarkably slow across Western economies. Also, the effective lower bound (ELB) on nominal interest rates was binding for several years (at the time of this writing, early 2022, it is still binding in the Euro area) forcing major central banks to resort to unconventional monetary policies. As a consequence, Summers (2015) resurrected Hansen's (1939) 'secular stagnation' hypothesis – a concept which ultimately boils down to a persistent excess in the supply of aggregate savings in relation to investment, caused by insufficient demand, which pushes down the real interest rate.[1]

In a secular stagnation, traditional monetary policy becomes less potent, and as a result, many commentators assign a bigger role to fiscal policies, also when it comes to escaping such a situation. Among the suggestions for policy measures in such an environment, apart from distributional issues (see Illing *et al.*, 2018; Mian *et al.*, 2021), a prominent role is frequently assigned to (debt-financed) increases in fiscal spending, in particular public investment to remedy such a situation (see, e.g., Blanchard, 2019; von Weizsäcker and Krämer, 2021). Intuitive reasons are that according to standard economic logic, with low interest rates, the net present value of investments in (public) capital is higher and also that public investment can stimulate growth, at least temporarily.[2]

Coming from a different angle,[3] in the wake of the financial crisis of 2008 and the subsequent sovereign debt crisis of the 2010s, there was a "renaissance in fiscal research" (Ramey, 2019), generating a bulk of both theoretical and empirical work on fiscal multipliers – most of this work, however, focusses on government consumption and taxes. E.g., the important literature survey by Ramey (2019) finds multipliers associated with government consumption to be between 0.6 and 1.0 in normal times and multipliers between -2 and -3 on tax changes.[4] Meanwhile, recent research in dynamic stochastic general equilibrium models (e.g., Bouakez *et al.*, 2017, 2020) points to relatively large multipliers – both relative to 'normal times' and relative to government consumption – on public investment when the economy is at the effective lower bound. In fact, Bouakez *et al.* (2020) argue that – from a normative point of view – in a recession that makes the economy hit the effective lower bound, the government should tilt the composition of its spending towards investment.

At the same time, academic and political commentators lamented a lack of public investment in a couple of Western countries, in particular, in Germany (see, *inter alia*, Bardt *et al.*, 2019; Bach *et al.*, 2020; Behringer *et al.*, 2020; Dullien *et al.*, 2020) and the United States[5].

Figure 4.1 depicts the evolution of the share of gross fixed capital formation undertaken by all levels of government as a share of national GDP for several advanced economies over the last 32 years. In particular, it includes the four major members of the Euro area (Germany, France, Italy, Spain), the United States, Japan and – for reference – the 27 current member states of the EU.[6] Notably, all of the states depicted had ratios between public gross fixed capital formation and GDP between roughly 2 and 6 per cent. However, there is some variation: Germany tended to have a relatively low share, staying

[1]The hypothesis of a secular stagnation caused by insufficient demand is far from universally accepted. In many contemporary analyses, the drop in real interest rates was assumed to be temporary. Also, some authors (*inter alia,* Gordon, 2015; Ramey, 2020) rather name supply-side factors for a slowdown in technological progress as the culprit for the situation.

[2]This likely also holds true if the causes of stagnation are on the supply side (Ramey, 2020).

[3]By 'different angle', we mean that the corresponding literature does not assume a persistent demand deficiency.

[4]For an earlier survey with a focus on fiscal multipliers in a currency union, see Illing and Watzka (2014).

[5]See White House (2021): Fact Sheet: The American Jobs Plan. https://www.whitehouse.gov/briefing-room/statements-releases/2021/03/31/fact-sheet-the-american-jobs-plan/ (Last accessed: 23 February, 2022).

[6]The data have been taken from the automn-2021 vintage of the AMECO database (European Commission, 2021). The series UIGG represents gross fixed capital formation by general government, UVGD is the time series for GDP.

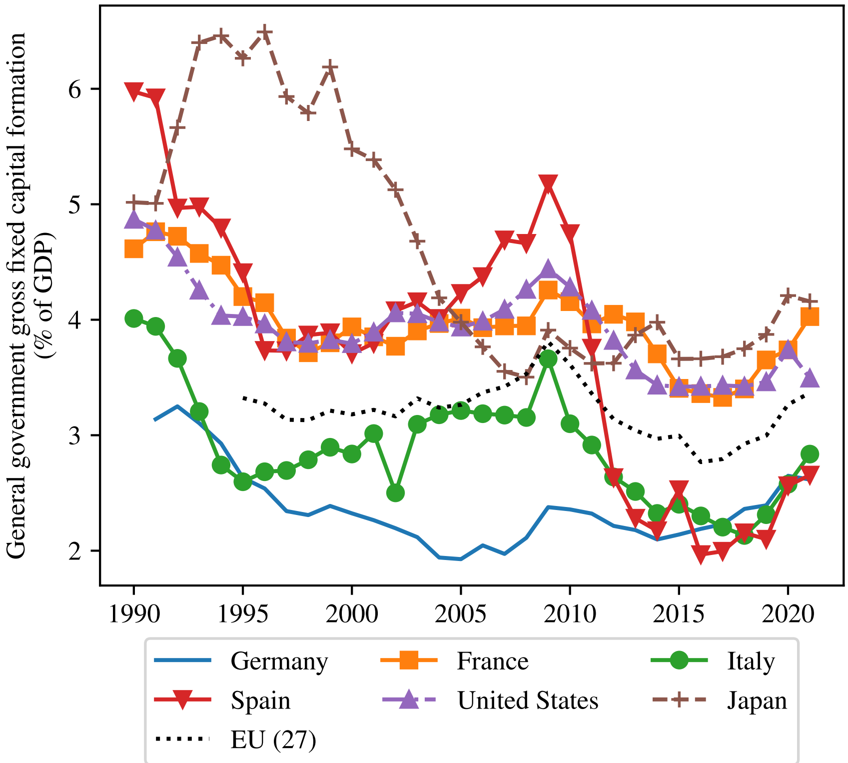

Figure 4.1: General government gross fixed capital formation (% of GDP), selected countries, 1990–2021.
The figure shows the share of gross fixed capital formation by all levels of government in national GDP for a set of advanced economies across the past 30 years.
Source: author's calculations. *Data source*: European Commission (2021), series: UIGG, UVGD.

between 2 and 3 per cent most of the years. This includes a declining trend from 1992 to 2005, followed by a modestly increasing trend since then. Japan, on the other hand, started out as one of the countries with the highest share of public investment according to this metric, reaching more than 6 per cent in the late 1990s. However, since then until 2008, the share had decreased by more than 2 percentage points and stayed broadly constant since. However, given that all other states depicted here have also seen modest declines, Japan turns out to have the highest share of government-sponsored gross capital formation of the states depicted.

Note also the sharp decline in this measure of public investment experienced by Spain after the Great Financial Crisis: After a (second) spike at more than 5 per cent of GDP, public investment collapsed almost to 2 per cent of GDP. It should also be noted that the figure shows a local maximum in 2009 for all time series depicted, followed by more or less prolonged U-shaped development until the beginning of the Covid-19 crisis in 2020.

With the advent of the Covid-19 crisis, public investment has come to the forefront of political agenda. In 2021, the United States passed an *Infrastructure Investment and Jobs Act (IIJA)* totalling $1.2 trn, focussing on investment in infrastructure and addressing challenges presented by climate change. Meanwhile, European Union member states also agreed on a recovery package, *'Next Generation EU'*, worth €750 bn, funded by debt issued by the European Commission. This (at least with respect to its scale and funding) in Europe unprecedented package focusses on investments and structural change, with a particular emphasis on a 'green transition' (i.e., taking into account climate change and other environmental aspects) and a 'digital transition'.

There is such a keen interest in the macroeconomic impacts of public investment, both in the short- and also in the long run. Regarding the latter, economic historians have found ample evidence concerning the long-run positive effects of public investment, in particular infrastructure investment. See, inter alia, Gordon's (2017) analysis of the American case or, more recently, the discussion in Frey (2019, in particular, chapter 6). This literature finds strong complementarities between investment in transportation infrastructure (in particular roads) and private investment (capital), leading to a chicken-egg problem regarding causality – but also finding large gains in productivity. Fernald (1999) investigates this in an empirical study across US industries and finds heterogeneous impacts of public investments on roads on productivity, with higher gains in vehicle-intensive activities. However, not surprisingly, he also comes to the conclusion that there are diminishing returns on such infrastructure investment. This means that advanced economies with developed infrastructure should expect lower multipliers than developing countries.

Regarding the short-run impacts of public investment, the recent literature as surveyed by Ramey (2021) points to a key feature of public investment that reduce the multipliers associated with it: Implementation delays, which either simply lead to a delay between decision-making and spending on investments ('time to spend', e.g. administrative procedures) and significant times to complete projects, once started ('time to build'). At the same time, these features can lead to a reversal when the economy is at the effective lower bound on nominal interest rates.(Bouakez *et al.*, 2017, 2020) These analysis were, however, conducted within familiar, medium-scale neoclassical or New Keynesian models that feature a very strong expectations channel – due to rational expectations. Recent research has put the validity of that assumption into question since these assumptions can give rise to a couple of puzzles, including the Forward Guidance puzzle (see previous chapter). Also, some researchers (e.g., Angeletos *et al.*, 2021; Angeletos and Lian, 2022) argue that (full-information) rational expectations are not compatible with existing evidence on the dynamics of expectations. So, instead, recently researchers have been moving into the direction of introducing bounded rationality analyse monetary and fiscal policies (e.g., Farhi and Werning, 2019; Gabaix, 2020).

However, so far, these analyses were limited to simple government-spending shocks – and as such, they may not be very informative in the context of fiscal packages that put a great emphasis on public investments. However, from an intuitive point of view, it seems quite plausible that limited forward-lookingness of expectations, e.g., via bounded rationality interacts with the above mentioned implementation delays, which should affect multipliers, at least in the short run. An aim of this chapter is to evaluate precisely this interaction and fill a gap in the literature by bringing together imperfect forward lookingness and productive government spending. In particular, we consider the following questions: How do (specific) deviations from rational expectations affect the short-, medium- and long-run multipliers of government investment? How does this interact with the time delays associated with public investment? Which other factors (in particular, the effective lower bound) affect this interaction? Due to a lack of data on the relevant expectations, we are restricted to conducting a purely theoretical analysis. Also, we do not strive to give definitive answers at this point, rather we present a number of (rather qualitative) observations.

To answer the above mentioned questions, we modify the medium-scale New Keynesian model from the literature review by Ramey (2021) and we introduce (in an ad-hoc fashion) expectations that potentially feature discounting of future variables and potentially a positive weight on past realisations (see Angeletos and Lian (2022) for a recent discussion which assumption can give rise to such an expectational framework). We conduct a number of simulations, where we vary implementation delays as well as expectational parameters. In addition, we consider a number of additional model features that might be relevant, e.g. whether public spending is tax-financed or debt-financed, agent heterogeneity and the gaps between the real interest rate r and the growth rate g.

Our results indicate that, indeed, expectations can have important effects on multipliers of long-lasting productivity-enhancing government spending such as public investment programmes. In particular, with increasing time to build, the impact multiplier of government investment on output is lower, and – especially with rational expectations – it can even be negative. With less forward-looking expectations, this effect is muted, raising the multiplier even if there are time delays. On the other hand, if expectations are myopic, but there is no backward-looking term, this can also affect long-run multipliers in a non-linear fashion. For small degrees of myopia, a particularity of myopic expectations raises long-run multipliers. For very low values of forward-lookingness, i.e. for sufficiently large degrees of myopia, there is an unambiguous decrease in long-run multipliers. This, however, is a result of myopic expectations not adjusting completely even in the long-run. Both of these results directly imply areas for future research. Expectations that include a lagged term as well as a forward-looking term (which we will call *hybrid expectations*) perform almost identical to rational expectations for low time to build and no time to spend, but for longer time delays, they share some features of myopic expectations.

Also, we find that at if monetary policy does not react to public investment (in particular, if it is constrained by the effective lower bound), the multiplier in the short-run is larger, confirming previous results from the literature. However, here, myopic expectations actually lower the multiplier on impact.

Note that the results from this chapter should in no case be taken as definitive. Rather, we want to argue that the combination of expectations with long-lasting fiscal policies deserves more attention in the future. In that regard, our results should be taken as raising questions that future research should elaborate upon. In particular, we use a rather crude ad-hoc form of deviating from rational expectations, which in subsequent research should be replaced with more realistic models of forming expectations, as introduced, e.g. by Angeletos *et al.* (2021).

This chapter relates to various strands of literature: On the one hand, there is now a vast literature on the effects of government investment in theoretical and empirical macoreconomic models. For a recent survey, see Ramey (2021).[7] That literature generally comes to the conclusion that long-run multipliers on government investment can be large, even though uncertainty bands are quite large. The studies surveyed in Ramey (2021) generally find long-run multipliers between 0.3 and 3.6. On the other hand, short-run multipliers are generally found to be lower than for government consumption. Especially implementation delays can cause significant reductions in short-term multipliers – time

[7] Also see Bom and Ligthart (2014) for a recent meta analysis. They find an output elasticity of public capital of 0.083 in the short run and 0.122 in the long run. Also, in their analysis, their estimates are twice as high for core infrastructure. In a separate meta-analysis, Núñez-Serrano and Velázquez (2017) find a elasticities of 0.13 and 0.16, respectively.

to build or to spend lower the net present value of spending programs and move positive effects on productivity further into the future.[8]

A notable result from that literature is that, in a New Keynesian setting with rational expectations, at the effective lower bound (or with accomodative monetary policy), these results partially turn around: Due to a mechanism inherently related to the 'paradox of toil' (Eggertsson, 2010), Bouakez *et al.* (2017, 2020) find that at the effective lower bound, multipliers on public investment become relatively large and they increase in the time to build. The reason is that technology-enhancing (but also deflationary) effects of enhanced public capital can act contractionary at the effective lower bound. If time to build is sufficiently long and the economy exits from the effective lower bound soon enough, such effects are ruled out, because in normal times the paradox does not prevail. In fact, this result is part of the motivation for approach in this chapter.

Pfeiffer *et al.* (2021) adapt the model of Ramey (2021) to a multi-country, monetary union framework in order to investigate spillover effects of the Next Generation EU instrument.

On the other hand, this paper relates to the vast literature on macroeconomic models without (full-information) rational expectations, which in turn affects general-equilibrium effects (for a survey, see Angeletos and Lian, 2022). In particular, it relates to models that lead to discounting of future variables, such as Gabaix's (2020) behavioural New Keynesian model with cognitive biases or the closely related approaches of combining level-k thinking and incomplete markets (for the later, see, in particular, Farhi and Werning, 2019).[9]

The remainder of this chapter is structured as follows: Section 4.2 briefly presents the main features of the model used for the simulations. The details of the derivation are, however, delegated to the dix. Section 4.3 presents the main experiments conducted as part of the analysis and section 4.4 presents the results from these experiments. After this, in section 4.5 a brief discussion of the results follows and implications for future research are discussed. Finally, section 4.6 concludes.

4.2 Model Exposition

In this chapter, we consider a linearised New Keynesian model with several extensions relative to the simple textbook version (as presented, e.g., by Galí, 2015). The baseline model used here is very similar to the New Keynesian model studied by Ramey (2021), which in itself is an extended version of Galí *et al.* (2007). Below, in subsection 4.2.1, we briefly illustrate the main ingredients in a verbal fashion, before we discuss some of the linearised equations of the model in subsection 4.2.2. The full underlying non-linear model is presented in appendix 4.A, whereas appendix 4.B goes through the linearisation of the equations presented below.

[8] In fact, in the model of Ramey (2021) or the present model, short-run multipliers can even become negative. This is due to the strong expectations channel associated with rational expectations. Our results indicate that myopia reduces the extent of this somewhat troubling implication.

[9] Closely related strings of literature that also deviate from full-information rational expectations include *rational inattention* (first conceived by Sims (2003), for a survey on the current status of research, see Maćkowiak *et al.* (2022)), *sticky information* (Mankiw and Reis, 2002), *reflective equilibrium* (García-Schmidt and Woodford, 2019) and *cognitive hierarchy* (Camerer *et al.*, 2004). Also, introducing incomplete, possibly heterogeneous expectations in the spirit of Morris and Shin (2002, 2006), Woodford (2003), Nimark (2017) or Angeletos and Lian (2018) can reproduce these phenomena. See Angeletos *et al.* (2021) and Angeletos and Lian (2022) for a synthesis and survey.

4.2.1 Quick Overview of Model Features

The model presented below takes into account various developments from the more recent literature on short- to medium-term macroeconomic phenomena. Key ingredients familiar from the mainstream macroeconomic literature are *nominal rigidities* on product and labour market; *capital*, which in the short run cannot be adjusted as easily as labour and gives a meaningful role for investment; agent *heterogeneity*, which allows for significant marginal propensities to consume out of current income. In particular, in the extensions included, we also include a limited form of agent heterogeneity and incomplete markets.

Now, we will explain the model features in a bit more detail.

1. *Overall structure:* The model features a standard private sector for a medium-scale DSGE model: there are households that consume and invest in various assets, monopolistically competitive private intermediate goods-firms setting prices, a final goods firm and monopolistically competitive labour unions setting wages.[10] The policy block of the model consists of a central bank setting the nominal interest rate in order to stabilise inflation and a fiscal authority that levies taxes and potentially issues bonds in order to fund government consumption $\tilde{G}_{C,t}$ – and crucially, public investment $\tilde{G}_{I,t}$, giving total government expenditure

$$\tilde{G}_t = \tilde{G}_{C,t} + \tilde{G}_{I,t} \tag{4.1}$$

Since we consider a simple closed-economy model, the typical following accounting identiy holds: Real GDP Y_t is the sum of total private consumption C_t, private investment $\tilde{I}_t$ and government expenditure $\tilde{G}_t$:

$$Y_t = C_t + \tilde{I}_t + \tilde{G}_t \tag{4.2}$$

2. *Preferences:* We assume that in period t agents (here indexed h) have separable peferences across personal real consumption $\mathcal{C}_{h,t}$, labour supplied $N_{h,t}$ and bond holdings $B_{h,t}$ (normalised by the technology level A_t) of the form

$$U_{ht} = \mathcal{E}_t^H \left[\sum_{s \geq t} \beta^s u_{h,t+s} \right] \tag{4.3}$$

$$u_{ht} = u(\mathcal{C}_{h,t}) - \nu \frac{N_{h,t}^{1+\varphi}}{1+\varphi} + (\bar{\xi} + \xi_{h,t}) B_{h,t}/A_t + \Gamma(\tilde{G}_t) \tag{4.4}$$

$$\text{with } u(C) = \begin{cases} \ln C & \text{if } \sigma = 1, \\ \frac{C^{1-\sigma}-1}{1-\sigma} & \text{otherwise.} \end{cases} \tag{4.5}$$

Here, the discount factor for future utils is $\beta = \frac{1}{1+\rho}$ with ρ being the rate of time preference. $\sigma \geq 0$ measures both risk-aversion and intertemporal elasticity of substitution,[11] $\varphi \geq 0$ is the inverse Frisch elasticity of labour supply. $\nu > 0$ and $\bar{\xi} \geq 0$ are

[10] For reasons of clarity, in the appendix, we also introduce a mutual fund that holds capital and firm shares, and which is held by households. This is a simple modelling device to make some equations shorter; in a more elaborate model, it would also enable us to insure agents from idiosyncratic firms' risk.

[11] Extensions with, e.g., Epstein-Zin preferences (Epstein and Zin, 1989) could potentially be interesting, especially considering the Tractible Heterogeneous-Agent New Keynesian (THANK) structure of Bilbiie (2021) introduced below. However, this is left for future research.

parameters that weigh the contributions of labour and bond holdings, respectively, and ξ_t can be interpreted as a shock to the demand for liquid (and safe) bonds.[12]

Note also that we use a subjective expectation operator (conditional on information available at t) $\mathcal{E}_t^H[\cdot]$ to denote the expectations of households – which does not necessarily reflect the rational (conditional) expectations operator $\mathbb{E}_t[\cdot]$.

3. *Household heterogeneity and incomplete markets:* The model features two types of agents as familiar from the literature on Two-agent New Keynesian (TANK) models in the tradition of Galí *et al.* (2007): i) constrained agents (later indexed c) and ii) unconstrained agents (u). A major difference across these two types of agents is that constrained agents follow a rule of thumb and consume a fixed share of their current income. As is common in the literature, we set that share to 100 %, so that these agents can also be called hand-to-mouth consumers. On the other hand, constrained agents participate in all asset markets (in particular, bonds and equity); their behavior will eventually determine asset prices. Notably, in TANK models, constrained agents typically only receive labour income, whereas capital income and profits are fully reaped by the unconstrained agents. This means that there is income inequality built into this kind of model. With the share of agent type $x \in \{u, c\}$ in the overall population being denoted as s_x and their per-capita consumption in period t as $C_{x,t}$, the total private consumption of the economy is given by

$$C_t = s_u C_{u,t} + s_c C_{c,t} \tag{4.6}$$

In a minor extension to the familiar framework, we adapt the work of Bilbiie (2021), who generalises the TANK set-up to also allow for households' transitions between being constrained and unconstrained. Following this approach, agents' real bond holdings $B_{x,t}$, $x \in \{u, c\}$ also serve as a tool to self-insure against becoming constrained, whereas private capital does not. For this extension, we add the following asssumptions: In each period, a constant share[13] $p_{c|u}$ of unconstrained households becomes constrained *and* loses its holdings of capital and firm shares; the remaining share $p_{u|u} = 1 - p_{c|u}$ remains unconstrained. Likewise, a share $p_{u|c}$ of constrained agents becomes unconstrained and receives the capital and firm shares previously held by the newly constrained households. The remaining $p_{c|c} = 1 - p_{u|c}$ stay constrained. Meanwhile agents are able to keep their bond holdings when switching types. For simplicity, agents of each type pool their overall income and jointly determine consumption choices.[14]

This allows a simple two-agent framework to reflect, in a simple way, a couple of mechanisms found to be relevant in more complicated Heterogeneous-Agent New Keynesian (HANK) models such as Kaplan *et al.* (2018). Also, in line with Bilbiie (2021), we assume that unconstrained agents face an idiosyncratic demand shock (e.g. via an appropriate $\xi_{h,t}$ from above) such that they choose bond holdings equal

[12] *Note:* The shock $\xi_{h,t}$ is, per se, not subject of this paper. However, we include it to perform experiments with public investment at the effective lower bound (ELB) on the nominal interest rate.

[13] An interesting extension for future work could be to allow for fiscal policy to affect the transition probabilities. In the case of long-lasting public investment, this could also affect long-run inequality and interest rates.

[14] See Bilbiie (2021) and the appendix for a more detailed exposition.

to the borrowing limit, which we assume to be zero:

$$B_{c,t} = 0.$$

Taking this into account, the overall per-capita budget constraints of unconstrained and constraint agents are

$$B_{u,t} + C_{u,t} = Y_{L,t} + \frac{Y_{K,t} + Y_{\Pi,t}}{s_u} - \tilde{T}_{u,t} + (1 + r_t)p_{u|u}B_{u,t-1} \tag{4.7}$$

$$\text{and } C_{c,t} = Y_{L,t} - \tilde{T}_{c,t} + (1 + r_t)\frac{p_{c|u}s_u}{s_c}B_{u,t-1}. \tag{4.8}$$

Here, $Y_{L,t}$ is per-capita labour income, $Y_{K,t}$ and $Y_{\Pi,t}$ are total capital income (net of investments) and total profits, and $\tilde{T}_{x,t}$ are agent-specific taxes, which we assume to be non-distortionary. These variables are determined in a standard fashion, and thus will not be discussed here. See the appendix for a derivation. Finally, r_t is the ex-post real interest rate, given by the Fisher equation as

$$1 + r_t = \frac{1 + i_{t-1}}{1 + \pi_t}, \tag{4.9}$$

where i_{t-1} is the nominal interest rate between periods $t - 1$ and t and π_t is the corresponding inflation rate.

Within our model, the alluded precautionary-savings motive leads to a (sort of) liquidity premium on bond holdings in addition to the possible premium generated by wealth preferences $\bar{\xi}$. The Euler equation on bonds is given by

$$1 = \beta\mathcal{E}_t^H \left[\frac{p_{u|u}C_{u,t+1}^{-\sigma} + p_{c|u}C_{c,t+1}^{-\sigma}}{C_{u,t}^{-\sigma}}(1 + r_{t+1})\right] + \frac{(\bar{\xi} + \xi_{u,t})}{A_t C_{u,t}^{-\sigma}}. \tag{4.10}$$

In the model presented, the Euler equation for bond-pricing can be contrasted with the one on other illiquid ("risky") assets, in particular capital, is given by

$$Q_t^A = \beta\mathcal{E}_t^H \left[\left(\frac{C_{u,t+1}}{C_{u,t}}\right)^{-\sigma} (r_{A,t+1} + (1 - \delta_{A,t+1})Q_{t+1}^A)\right], \tag{4.11}$$

where Q_t^A is the price of some asset, $r_{A,t}$ is the rate of return on that asset in the next period and $\delta_{A,t}$ is the depreciation rate of the asset.

Note that, hence, this model features two distinct private stochastic discount factors (SDF): the 'safe' one

$$\mathcal{F}_{t,t+1}^{safe} := \beta\frac{p_{u|u}C_{u,t+1}^{-\sigma} + p_{c|u}C_{c,t+1}^{-\sigma}}{C_{u,t}^{-\sigma}}$$

used to price safe and liquid bonds and the 'risky' one

$$\mathcal{F}_{t,t+1}^{risky} := \beta\left(\frac{C_{u,t+1}}{\mathcal{C}_{u,t}}\right)^{-\sigma}.$$

Note that if $p_{u|u} = 1$, i.e., there are no transitions between idiosyncratic states, the two stochastic discount factors are the same.

4. *Production structure:* We follow most of the New Keynesian literature in assuming a competitive firm produces a final good by combining a fixed, continuous set of intermediate inputs, where each input is produced by a monopolistically competitive firm. Together with the assumption of nominal rigidities á la Calvo (1983, see below), with shocks there will be costly price dispersion, which effectively reduces the aggregate total factor productivity by a factor $1/\Delta_{p,t} \leq 1$. Note, however, that up to first order, close to a balanced growth path, $\Delta_{p,t} \approx 1$; as such, in the linearised model below, this effect drops out. Assuming that each of the intermediate-goods firms has access to the same Cobb-Douglas production function combining labour and capital, given a common technology and a stock of public goods, following Baxter and King (1993) and, more recently, Ramey (2021), the overall production structure can be written as

$$Y_t = \frac{1}{\Delta_{p,t}} A_t^{1-\alpha-\alpha_p} N_t^{1-\alpha} K_t^{\alpha} K_{P,t}^{\alpha_p}, \tag{4.12}$$

Note that the typical logic of the Cobb-Douglas function leads to a real private marginal cost

$$X_t = \frac{1}{A_t^{1-\alpha-\alpha_p} K_{p,t}^{\alpha_p}} \left(\frac{\tilde{W}_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_{K,t}}{\alpha} \right)^{\alpha}, \tag{4.13}$$

where $\tilde{W}_t$ is the real wage and $r_{K,t}$ is the rental rate of capital. Note that by standard results, we obtain factor demands

$$Y_{L,t} := \tilde{W}_t N_t = (1-\alpha)\Delta_{p,t} X_t Y_t \quad \text{and} \tag{4.14}$$
$$r_{K,t} K_t = \alpha \Delta_{p,t} X_t Y_t. \tag{4.15}$$

Note that if we include private investment I_t, we get $Y_{K,t} := r_{K,t}K_t - I_t$. Moreover, overall profits are given by

$$Y_{\Pi,t} = Y_t - \Delta_{p,t} X_t Y_t. \tag{4.16}$$

Note that as in Ramey (2021), we do abstract from corrective taxes; as such, with monopolistic competition we have along a balanced growth path $X_t = X < 1$ and $Y_{\Pi,t} > 0$.

5. *Nominal rigidities:* The model features both sticky prices and sticky wages with a Calvo-1983-like friction as first pioneered in Erceg *et al.* (2000) and introduced into the literature on two-agent New Keynesian models by Colciago (2011).[15] First of all, as known at least since the publication of Keynes's (1936) *General Theory*, with sticky prices, monetary and fiscal policies have stronger real consequences because with only partial adjustments in terms of prices, quantities have to adjust. Also, with sticky prices, the effective lower bound (ELB) on nominal interest rates becomes a meaningful constraint, which enables us to analyze policies there. We have discussed the merits of adding sticky wages in the context of the previous chapter.[16]

[15] This also mirrors the corresponding elements known from the previous chapter.

[16] An additional motive for adding sticky wages in the current model with (at least) two agent types and market imperfections can be found in Colciago (2011) and Ascari *et al.* (2017): It re-establishes a

Here, as there, we follow Calvo (1983) and assume that wage and price setters only reoptimise with certain probabilities $1 - \theta_w$ and $1 - \theta_p$, respectively. In all other periods, they update their wages and prices, respectively with the steady-state inflation rates $\bar{\pi}_w$ and $\bar{\pi}$ respectively. As is standard in the New Keynesian literature, firms and unions are assumed to be monopolistically competitive, facing demand elasticities ϵ_p and ϵ_w, respectively. In steady state, this leads to mark-ups, where firms set mark-up their prices by a factor $\frac{\epsilon_p}{\epsilon_p - 1}$ over marginal costs and unions mark-up real wages by a factor $\frac{\epsilon_w}{\epsilon_w - 1}$ over the average marginal rate of substitution between consumption and labour

$$MRS_t = \frac{\nu N_t^{\varphi}}{s_u C_{u,t}^{-\sigma} + s_c C_{c,t}^{-\sigma}}. \tag{4.17}$$

Away from the steady state, however, there can be wage dispersion and different households have to supply different amounts of hours.

In a non-linear context, this gives rise to blocks of multiple forward-looking equations; when we linearise this below, however, two simple Phillips curves (4.43) and (4.42) emerge.

6. *Capital:* With respect to private capital, we follow Ramey (2021) as well as most of the literature featuring capital in business-cycle models by including variable utilisation and adjustment costs.

   In particular, we follow her in adopting the assumption that investment in private capital is subject to investment-adjustment costs relative to the previous period.[17] I.e., in any given period, let private investment is given by I_t. Then, this raises capital in the next period by

$$I_t \left[1 - S\left(\frac{I_t}{I_{t-1}} - 1 \right) \right]$$

$$\text{with} \quad S(\cdot) \in [0,1], \quad S(g) = S'(g) = 0, \quad \text{and } S''(g) = \kappa_I \geq 0,$$

   where g is the long-run growth rate of the economy.

   Also, we allow for varying degrees of capacity utilisation (in the form of capital utilisation). Letting $U_{K,t}$ be the rate of utilisation of capital(normalised along a balanced growth path), we assume that private fixed capital $\bar{K}_{t-1}$ entering period t can be transformed into $K_t = U_t \bar{K}_{t-1}$ units of effective private capital services. U_t, however, also affects the depreciation rate δ_t of capital via a function:

$$\delta_t = \tilde{\delta}(U_t) \quad \text{with } \tilde{\delta}'(\cdot) > 0, \tilde{\delta}''(\cdot) > 0.$$

couple of standard results from the simple New Keynesian literature such as the Taylor principle. Note that in a slight deviation from Ramey (2021), we follow Erceg *et al.* (2000) in modelling wage stickiness instead of Schmitt-Grohé and Uribe (2006b). In a non-linear context; and also from a normative point of view, there are important differences, but with a linearised model and for purely positive questions, the difference does not matter much. Also, the framework of Erceg *et al.* (2000) is more common in the literature and more in line with the previous chapter. See Schmitt-Grohé and Uribe (2006a) and Born and Pfeifer (2020) for a discussion.

[17] As argued by Angeletos *et al.* (2021), what seems to be "investment-adjustment costs" in macroeconomic data could also be the result of capital adjustment costs with a form of bounded rationality. As a result, it would be interesting to also perform our analysis with non-rational expectations and capital-adjustment costs instead of investment-adjustment costs. However, at this point, such an analysis is beyond the scope of this particular chapter and left for future research.

As a result, the law of motion of private fixed capital is given by

$$\bar{K}_t = (1 - \tilde{\delta}(U_t))\bar{K}_{t-1} + I_t \left[1 - S\left(\frac{I_t}{I_{t-1}} - 1\right)\right] \tag{4.18}$$

From this, standard optimality conditions can be obtained for U_t and I_t:

$$r_{K,t}\bar{K}_{t-1} = q_t \delta'(U_{K,t})\bar{K}_{t-1} \quad \text{and} \tag{4.19}$$

$$\begin{aligned} 1 = q_t &\left(1 - S_I\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} - 1\right) - \frac{\tilde{I}_t}{\tilde{I}_{t-1}} S_I'\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} - 1\right)\right) \\ &+ \mathcal{E}_t^F \left[\mathcal{F}_{t+1}^{risky} q_{t+1} \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\right)^2 S_I'\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t} - 1\right)\right], \end{aligned} \tag{4.20}$$

where q_t is Tobin's q, which can be determined from (4.11) as

$$q_t = \mathcal{E}_t^F \left[\mathcal{F}_{t+1}^{risky} \left(r_{K,t+1} U_{K,t+1} + q_{t+1}(1 - \delta(U_{K,t+1}))\right)\right]. \tag{4.21}$$

For a derivation, see the appendix. Note that we can normalise the equations such that in a stationary equilibrium, we have $U_{K,t} = U_K = 1$ and $q_t = q = 1$.

7. *Monetary policy* For this chapter, we want to focus on fiscal policy and thus keep monetary policy as simple as possible: The central bank sets the nominal interest rate according to simple Taylor rule

$$i_t = \max\{0, \bar{r} + \pi_t + \phi_\pi(\pi_t - \pi^*)\}, \tag{4.22}$$

where $\bar{r}$ is the balanced-growth-path real interest rate, $\phi_\pi > 0$ is a reaction parameter and π^* is the central bank's target inflation rate, which is also the balanced-growth-path inflation rate $\bar{\pi}$ of the model. In the following we will keep some standard values $\phi_\pi = 0.5$ and π^* such that annual inflation is 2 per cent, i.e. $(1 + \pi^*)^4 = 1.02$. For future research, it would be interesting to further investigate interactions between various monetary policy strategies as studied in the previous chapter and longer-term fiscal policies as in the present one.

8. *Fiscal policy* On the other hand, the fiscal block of the model is modestly more complex than in simple New Keynesian models: On the one hand, with government investment, the fiscal authority has a total of four instruments: government consumption, government investment, and taxation of the two agent types.

   Moreover, a crucial feature of public spending, in particular investment, known at least since the important work of Leeper *et al.* (2010) and Ramey (2021), is that it involves significant implementation delays, in particular *time to spend* and *time to build*:[18]

   (a) A first hurdle is often presented by legislative negotiations as evidenced by the American *Infrastructure Investment and Jobs Act (IIJA)*, which was first introduced in early June 2021 (under a different name) and was finally signed into law by U.S. president Joe Biden in mid-November 2021, almost half a

[18] For other relevant factors and for a more detailed discussion, please see section 4.5.

year later. Such delays in appropriating funds and the duration of other administrative procedures (e.g., identification of required investments, planning, expropriations (if required), environmental standards etc.) and protests by local inhabitants, inter alia, can lead to significant times between a project's inception and the beginning of actual physical work on the project. In our model, we capture this by a potential *time to spend*: Funds appropriated in period t only begin to increase public investment after several periods $\mathcal{T}_{1,I} \geq 0$.

(b) A second important fact is that investment (both private and public) can have a significant *time to build*: large infrastructure investments, e.g. bridges, airports or ports often take a couple of years of continuous work before they can finally be used, and thus in terms of the paper become fully productive. In the meantime, often a limited private use can be achieved (think of roads, where a fraction of lanes is open whereas the remainder is part of the construction site). But also, often there is not much use. E.g., in the case of Berlin Brandenburg Airport in Germany, which openend in 2020, most of the newly built airport and rail infrastructure was not at all used until it finally fully opened and traffic from other existing airports in Berlin was moved there. In the context of the current model, we assume that once a project has started being implemented (i.e., after the $\mathcal{T}_{1,I}$ periods have passed and actual construction etc. has started), it takes another $\mathcal{T}_{2,I} \geq 1$ periods to complete the works – after which the investment becomes fully functional.

Following Ramey (2021), we capture these effects in the following manner: A deviation in appropriations to public investment $AP_{I,t}$ leads to additional public investment "*in the pipeline*" after $\mathcal{T}_{1,I}$ periods denoted by

$$\tilde{G}_{I,new,t} = AP_{I,t-\mathcal{T}_{1,I}} \tag{4.23}$$

This translates into spending across the next $\mathcal{T}_{2,I}$ periods according to

$$\tilde{G}_{I,t} = \sum_{s=0}^{\mathcal{T}_{2,I}-1} \chi_{I,s} \tilde{G}_{I,new,t-s}, \tag{4.24}$$

where the $\chi_{I,s} \geq 0$ represent the time structure[19] of the investment.[20]

Finally, the public capital stock evolves according to

$$\bar{K}_{P,t} = (1-\delta_P)\bar{K}_{P,t-1} + \tilde{G}_{I,new,t-\mathcal{T}_{2,I}} \tag{4.25}$$

In the following experiments, we will assume that government consumption simply grows at the growth rate g: $\tilde{G}_{C,t} = \tilde{G}_{C,t-1}(1+g)$ such that $\tilde{G}_{C,t}/A_t = G_C$ remains constant.[21]

[19] An important conceptual issue is that here we treat all investment alike. In reality, macroeconomic public expenditures (at least in larger advanced economies like the U.S. or the euro area) are the sum of (at least) thousands of individual projects with their own timelines. Investigating how this aggregates up is an important question which goes beyond the scope of this chapter. In particular, it does *not* seem reasonable to assume that the time structure of investment across all projects remains constant across time.

[20] In general, in particular without 'steady-state growth' (i.e., with $g = 0$), one might wish to normalise $\sum_s \chi_{I,s} = 1$. However, assuming there are inefficiencies associated with public investment, one could just as well assume $\sum_s \chi_{I,s} > 1$, and realizing that in the model, there might still be some TFP factor associated with public goods provision, also $\sum_s \chi_{I,s} < 1$ could be rationalised.

[21] In the appendix, we also consider multipliers on government consumption, where we also allow for time to spend and various length of execution of appropriations for government consumption.

Considering the funding of the expenditures $\tilde{G}_t = \tilde{G}_{C,t} + \tilde{G}_{I,t}$, the government can set total taxes $\tilde{T}_t = s_u \tilde{T}_{u,t} + s_c \tilde{T}_{c,t}$ or issue real bonds B_t. The law of motion for the latter is given by

$$B_t = \tilde{G}_t - \tilde{T}_t + (1 + r_t) B_{t-1}. \tag{4.26}$$

Note that since constrained agents do not participate in financial markets, we have $s_u B_{u,t} = B_t$.

Here, we want to focus on deviations from a balanced growth path. As such, we assume that, in the absence of shocks, all variables would simply grow at a rate g, – and hence, along a balanced growth path, we have $\tilde{T}_t = T A_t$, $B_t = b A_t$, $\tilde{G}_t = G A_T$, $AP_{I,t} = \bar{ap}_I A_t$ with $T, b, G, \bar{ap}_I \in \mathbb{R}$.

Finally, to close the model, the setting of taxes and their distribution need to be determined: First, below, we consider two regimes for the response of overall taxes to a change in public investment:

(a) a fully tax-financed approach; in particular, we assume that B_t is kept at bA_t at all times,[22] meaning that

$$\tilde{T}_t = \tilde{G}_t + b\left[(1 + r_t) A_{t-1} - A_t\right]. \tag{4.27}$$

(b) a simple tax rule of the form

$$\tilde{T}_t - A_t T = \phi_G(\tilde{G}_t - A_t G) + \phi_B (1 + g)\left(B_{t-1} - bA_{t-1}\right) \tag{4.28}$$

with $\phi_G, \phi_B > 0$ as in Ramey (2021).

Concerning the distribution of taxes across agents, we keep this block as simple as possible and assume that taxes are divided linearly among constrained and unconstrained agents according to the rule

$$\tilde{T}_{u,t} = A_t T_u + \vartheta_u (\tilde{T}_t - A_t T), \tag{4.29}$$

where T_u is again a constant governing the distribution along the balanced growth path. This implies

$$\tilde{T}_{c,t} = \frac{\tilde{T}_t - s_u \tilde{T}_{u,t}}{s_c} = A_t \underbrace{\frac{T - s_u T_u}{s_c}}_{=:T_c} + \underbrace{\frac{1 - s_u \vartheta_u}{s_c}}_{=:\vartheta_c} (\tilde{T}_t - A_t T). \tag{4.30}$$

4.2.2 Linearised Model

Transformed Variables

The model is then reformulated in terms of stationary variables[23] (by dividing most variables by A_t): $c_t = C_t/A_t$, $c_{x,t} = C_{x,t}/A_t$ and $T_{x,t} = \tilde{T}_{x,t}/A_t$ for $x = u, s$, $y_t = Y_t/A_t$, $I_t = \tilde{I}_t/A_t$, $\bar{k}_t = \bar{K}_t/A_t$, $k_t = K_t/A_t$, $\bar{k}_{p,t} = \bar{K}_{P,t}/A_t$, $b_t = B_t/A_t = s_u B_{u,t}/A_t = s_u b_{u,t}$,

[22] An alternative would be to keep the debt-to-GDP ratio B_t/Y_t constant at some value, which we leave for future research.

[23] In the text, we use 'steady state' and 'balanced growth path' interchangeably, although the former is only correct, strictly speaking, with $g = 0$.

$T_t = \tilde{T}_t/A_t$, $G_t = \tilde{G}_t/A_t$, $G_{I,t} = \tilde{G}_{I,t}/A_t$, $G_{new,I,t} = \tilde{G}_{new,I,t}/A_t$ and $w_t = \tilde{W}_t/A_t$. Notably, the variables $r_t, \pi_t, \pi_{w,t}, i_t, r_{K,t}, N_t$ and X_t are already stationary if $g = 0$ or $\sigma = 1$. In the following we assume that either of these conditions is always satisfied.

Then, notably, along a balanced growth path, these variables have stationary values $c_t = c$, $c_{x,t} = c_x$ and $T_x = T_x$ for $x = u, s$, $y_t = y$, $I_t = I$, $\bar{k}_t = \bar{k}$, $k_t = k$, $\bar{k}_{P,t} = \bar{k}_P$, $b_{u,t} = b_t = b$, $T_t = T$, $G_t = G$, $G_{I,t} = G_I$, $G_{new,I,t} = G_{new,I}$, $w_t = w$, $r_t = r$,$\pi_t = \bar{\pi}$,$\pi_{w,t} = \bar{\pi}_w$,$i_t = i$,$r_{K,t} = r_K$, $N_t = N$ and $X_t = X$. Also, crucially, note that the two discount factors $\mathcal{F}_t^{safe}$ and $\mathcal{F}^{risky}$ have the stationary values

$$\mathcal{F}^{safe} = \beta(1+g)^{-\sigma}\left[(1-p_{c|u}) + p_{c|u}\left(\frac{c_c}{c_u}\right)^{-\sigma}\right] \quad \text{and} \tag{4.31}$$

$$\mathcal{F}^{risky} = \beta(1+g)^{-\sigma} \tag{4.32}$$

Also, note that from our assumptions before, we have

$$\Gamma^{safe} := \mathcal{F}^{safe}(1+r) = 1 - \bar{\xi}c_u \leq 1$$

In order to linearise the model, we also introduce the deviations of variables from their stationary values: $\hat{c}_t := c_t - c$, $\hat{c}_{x,t} := c_{x,t} - c_x$ and $\hat{T}_{x,t} := T_{x,t} - T_x$ for $x = u, s$, $\hat{y}_t = y_t - y$, $\hat{I}_t := I_t - I$, $\tilde{k}_t := \bar{k}_t - \bar{k}$, $\hat{k}_t := k_t - k$, $\tilde{k}_{p,t} := \bar{k}_{p,t} - \bar{k}_p$, $s_u\hat{b}_{u,t} \equiv \hat{b}_t := b_t - b$, $\hat{T}_t = T_t - T$, $\hat{G}_t := G_t - G$, $\hat{G}_{I,t} := G_{I,t} - G_I$, $G_{new,I,t} := \tilde{G}_{new,I,t} - G_{new,I}$, $\hat{w}_t := w_t - w$, $\hat{r}_t := r_t - r$,$\hat{\pi}_t := \pi_t - \bar{\pi}$,$\hat{\pi}_{w,t} := \pi_{w,t} - \bar{\pi}_w$,$i_t = i$,$\hat{r}_{K,t} := r_{K,t} - r_K$, $n_t = N_t - N$ and $x_t := X_t - X$. Also, define $\hat{u}_t := U_{K,t-1}$ and $\hat{q}_t := q_t - 1$. Also, as noted before, the variable $\Delta_{p,t}$ is 1 up to first order and can be neglected from here on.

Linearised Equations

We then approximate the model using a first-order Taylor expansion, see Appendix 4.B. The linearised model features a couple of forward-looking equations, e.g. consumption of both types of agents can be found.

Since we want to vary the forward-lookingness of these equations in a simple manner, we group these variables. In particular, we assume that expectations about future consumptions follow the same rules and also expectations about wage and price inflation are formed in the same manner. Denote these groupings as $a \in \{c, \pi, rk, q, I\}$. Then, in the following we replace subjective conditional expectation operators with variable-specific ones, i.e. $\mathcal{E}_{a,t}[x_{t+1}]$ is expectation about variable x in period $t+1$, conditional on information available in period t, where the expectation belongs to group a. We assume that the groups a of similarly formed expectations are

- consumption values $\hat{c}_{u,t}, \hat{c}_{c,t}$, given by households, indexed c,
- inflation rates $\hat{\pi}_t$, by price setters (p),
- inflation rates $\hat{\pi}_{w,t}$, by wage setters (w),
- the return on capital $r_{K,t}$ (rk),
- Tobin's q q_t (q) and
- investment I_t (i), all by investors.

Below we, however, assume throughout that expectations about future Tobin's q and investment are always formed in the same way (they are both part of the investment problem of the same agent). We also assume that expectations with respect to the return to capital rk belongs to this group. We considered varying this, but it did not change our results much. This then gives the following equations, which we use in our analysis: Aggregate demand is linearised as

$$\hat{y}_t = \hat{c}_t + \hat{G}_t + \hat{I}_t, \tag{4.33}$$

where total consumption is given by

$$\hat{c}_t = s_u \hat{c}_{u,t} + (1 - s_u)\hat{c}_{c,t} \tag{4.34}$$

and constrained agents' consumption follows from

$$\hat{c}_{c,t} = w\hat{n}_t + N\hat{w}_t - \hat{T}_{c,t} + \frac{p_{c|u}s_u}{s_c}\frac{1+r}{1+g}\frac{1}{s_u}\left(\hat{b}_t + \frac{\hat{r}_t}{1+r}b\right) \tag{4.35}$$

with a linearised Fisher equation

$$\frac{\hat{r}_t}{1+r} = \frac{\hat{i}_{t-1}}{(1+r)(1+\bar{\pi})} - \frac{\hat{\pi}_t}{1+\bar{\pi}}. \tag{4.36}$$

The Euler equation on bonds (4.10) can be linearised as

$$\frac{\hat{c}_{u,t}}{c_u} = \Gamma^{safe}\left[(1-\bar{\eta}_c)\mathcal{E}_{c,t}\left[\frac{\hat{c}_{u,t+1}}{c_u}\right] + \bar{\eta}_c\mathcal{E}_{c,t}\left[\frac{\hat{c}_{c,t+1}}{c_c}\right] - \frac{1}{\sigma}\left(\frac{\hat{r}_{pre,t}}{1+r} - \varepsilon_t^D\right)\right], \tag{4.37}$$

where $\bar{\eta}_c \in [0,1)$ is a function of c_c, c_u and $p_{c|u}$ as determined in the appendix, ε_t^D is a liquidity preference shock (related to the $\xi_{h,t}$ from before) and

$$\hat{r}_{pre,t} := \mathbb{E}_t\left[\hat{r}_{t+1}\right].$$

Here, $\hat{r}_{pre,t}$ is the deviation of the ex-ante real interest rate from its steady value. Note that in line with Gabaix (2020) and the related literature, here we assume that the ex-ante real rate includes the actual rational expectations in any case, differentiating it from all the other forward-looking variables. Moreover, the Euler equation on capital (4.21) can be written as

$$\hat{q}_t - \sigma\frac{\hat{c}_{u,t}}{c_u} = -\sigma\mathcal{E}_{c,t}\left[\frac{\hat{c}_{u,t+1}}{c_u}\right] + \mathcal{F}^{risky}\left(\mathcal{E}_{rk,t}\left[\hat{r}_{K,t+1}\right] + (1-\bar{\delta})\mathcal{E}_{q,t}\left[\hat{q}_{t+1}\right]\right). \tag{4.38}$$

Meanwhile, private marginal costs follow

$$\frac{x_t}{X} = (1-\alpha)\frac{\hat{w}_t}{w} + \alpha\frac{\hat{r}_{K,t}}{r_K} - \alpha_p\frac{\hat{k}_{p,t}}{k_p} \tag{4.39}$$

giving linearised factor demands

$$\hat{u}_t + \frac{\tilde{k}_{t-1}}{\bar{k}} + \frac{\hat{r}_{K,t}}{r_K} = \frac{x_t}{X} + \frac{\hat{y}_t}{y} \quad \text{and} \tag{4.40}$$

$$\frac{n_t}{N} + \frac{\hat{w}_t}{w} = \frac{x_t}{X} + \frac{\hat{y}_t}{y}. \tag{4.41}$$

Moreover, as is typical of this kind of model, a linear New Keynesian Phillips curve can be obtained

$$\frac{\hat{\pi}_t}{1+\pi} = \kappa_p \frac{x_t}{X} + \beta \mathcal{E}_{p,t}\left[\frac{\hat{\pi}_{t+1}}{1+\pi}\right], \tag{4.42}$$

where the slope of the Phillips curve κ_p is a standard composite parameter.[24] Likewise, due to wage stickiness, we can also derive a New Keynesian Wage Phillips curve

$$\frac{\hat{\pi}_{w,t}}{1+\pi} = \kappa_w mrs_t + \beta \mathcal{E}_{w,t}\left[\frac{\hat{\pi}_{w,t+1}}{1+\pi}\right], \tag{4.43}$$

where

$$mrs_t = \varphi \frac{n_t}{N} + \sigma\left(s_u c_u^{-\sigma}\frac{c_{u,t}}{c_u} + s_c c_c^{-\sigma}\frac{c_{c,t}}{c_c}\right) - \frac{\hat{w}_t}{w} \tag{4.44}$$

measures the gap between (1) the *average* marginal rate of substitution between consumption and leisure of all households and (2) the real wage, and the slope parameter κ_w is another standard composite parameter.[25]

The private economy features two main state variables: the real wage and private capital; the real wage evolves according to

$$\hat{w}_t = \hat{w}_{t-1} + \frac{\hat{\pi}_{w,t}}{1+\bar{\pi}_w} - \frac{\hat{\pi}_t}{1+\bar{\pi}}, \tag{4.45}$$

whereas the law of motion for private fixed capital is given by

$$\tilde{k}_t = \frac{1-\bar{\delta}}{1+g}\tilde{k}_{t-1} - \frac{\bar{k}}{1+g}\delta_1 \hat{u}_t + \hat{I}_t \tag{4.46}$$

Here, capacity utilisation $\hat{u}_t$ is governed by the linear analogue to (4.19):

$$\hat{r}_{K,t} \approx \hat{q}_t \delta'(1) + \delta''(1)\hat{u}_t, \tag{4.47}$$

while the equation governing investment (4.20) can be rewritten as

$$\hat{q}_t = \kappa_I (1+g)^2\left[\frac{\hat{I}_t - \hat{I}_{t-1}}{I}\right] - \mathcal{F}^r (1+g)^3 \kappa_I \frac{\mathcal{E}_{I,t}\left[\hat{I}_{t+1}\right] - I_t}{I} \tag{4.48}$$

Finally, the Taylor rule is the only equation not fully linearised, instead it is given by

$$\hat{i}_t = \max\{\phi_\pi \hat{\pi}_t, -i\} \tag{4.49}$$

[24] Here, as shown in the appendix, the exact formula is given by

$$\kappa_p := \frac{(1-\theta)\left(1-\theta\beta(1+g)^{1-\sigma}\right)}{\theta}.$$

[25] Here, as shown in the appendix,

$$\kappa_w := \frac{(1-\theta_w)(1-\beta\theta)}{\theta(1+\epsilon_w \varphi)}.$$

Turning to fiscal variables, public expenditure is given by

$$\hat{G}_t = \hat{G}_{I,t} + \hat{G}_{C,t}, \tag{4.50}$$

where, as already alluded to, we set $\hat{G}_{C,t} = 0$ for our experiments.[26] Meanwhile, total public investment is given by

$$\hat{G}_{I,t} = \sum_{s=0}^{\mathcal{T}_{2,I}-1} \frac{\chi_{I,s}}{(1+g)^s} \hat{G}_{I,new,t-s}, \tag{4.51}$$

where time to spend is represented by

$$\frac{\hat{G}_{I,new,t}}{G_{I,new}} = \frac{ap_{I,t-\mathcal{T}_{1,I}}}{\bar{ap}_I}. \tag{4.52}$$

and the law of motion for the stock of public capital is given by

$$\hat{k}_{p,t} = \frac{1-\delta_p}{1+g}\hat{k}_{p,t-1} + \frac{\hat{G}_{new,I,t-\mathcal{T}_{\in,\mathcal{I}}}}{(1+g)^{\mathcal{T}_{\in,\mathcal{I}}}}. \tag{4.53}$$

Similarly, the law of motion for the level of government debt is linearised as

$$\hat{b}_t = \hat{G}_t - \hat{T}_t + \frac{1+r}{1+g}\left[\hat{b}_{t-1} + \frac{\hat{r}_t}{1+r}b\right], \tag{4.54}$$

where $\hat{T}_t$ is determined by the fiscal regime as discussed before, i.e., either

1. in a fully tax-financed approach by requiring

$$\hat{b}_t = 0 \tag{4.55}$$

at all times, or

2. with simple tax rule of the form

$$\hat{T}_t = \phi_G \hat{G}_t + \phi_B \hat{b}_{t-1}. \tag{4.56}$$

Taxes are distributed across agents as

$$\hat{T}_{x,t} = \vartheta_x \hat{T}_t, \quad x \in \{u, c\} \tag{4.57}$$
$$\text{s.t.} \quad s_u\vartheta_u + s_c\vartheta_c = 1.$$

Jointly with assumptions about the formation of expectations as well as shock processes for $\varepsilon_{ap,I,t}$ and $\varepsilon_{D,t}$, this completes the description of the model.

[26]Appendix 4.C.2 also shows some results for government consumption with the same time structure as our public investment from the main text.

4.2.3 Structure of Expectations

In the following, we consider three types of expectations: As a benchmark, we use the familiar rational-expectations case – in which case expectations of group a are given by

$$\mathcal{E}_{a,t}\left[\hat{z}_{t+1}\right] = \mathbb{E}_t\left[\hat{z}_{t+1}\right], \tag{4.58}$$

where $\hat{z}_t$ is the deviation of some variable z_t from its steady state and $\mathbb{E}_t[\cdot]$ is the objectively correct expectations operator, conditional on information available in period t.

Similar to Gabaix (2020), we then consider a variation where expectations are biased towards the steady state, meaning that in our model

$$\mathcal{E}_{a,t}\left[\hat{z}_{t+1}\right] = M_a^f \mathbb{E}_t\left[\hat{z}_{t+1}\right] \quad \text{with} \quad M_a^f < 1. \tag{4.59}$$

We simply call these expectations *myopic* below.

Note that, especially for the inflation expectations in the two Phillips curves, the factors M_p^f, M_w^f would – in general – be a rather complicated function of underlying parameters, see the discussion in Gabaix (2020). However, this would overly complicate the analysis since with price and wage stickiness, a number of additional assumptions would have to be placed on how exactly the cognitive bias affects expectations.[27]

A crucial issue with this kind of expectation formation that will become quite clear below is that with persistent shocks and endogenous state variables, they tend to be too rigid for the purposes of this chapter. Also, as forcefully argued by Angeletos *et al.* (2021), purely myopic expectations are not fully compatible with existing evidence on a couple of expectational variables. Instead, they propose a model where agents face noisy information and overextrapolate individual signals. In their models, this leads to both discounting of forward-looking terms in the expectation formation and a positive weight on past realisations. Based on their results, we also consider 'expectations'[28] of the form

$$\mathcal{E}_{a,t}\left[\hat{z}_{t+1}\right] = M_a^f \mathbb{E}_t\left[\hat{z}_{t+1}\right] + M_a^b \hat{z}_{t-1} \quad \text{with} \quad M_a^f < 1, M_a^b > 0. \tag{4.60}$$

We call this type of expectation formation *hybrid* – not least because the backward-looking terms generate phenomena that previously were themselves introduced as ad-hoc

[27] Gabaix (2020) assumes that price setters perceive the optimal price level to be geometrically biased towards the steady-state value. I.e., price setters in his model perceive the deviation of the price level $\hat{p}_t$ j periods in the future as $\mathcal{E}_{p,t}[\hat{p}_{t+j}] = \bar{m}^j \mathbb{E}_t[\hat{p}_{t+j}]$. This implies that the impact of next period's inflation for periods even further in the future is also underestimated. As such, the impact of future inflation deviations is underestimated by a factor smaller than $\bar{m}$. Translating this to sticky wages is not straight forward: Open questions that arise are: If wage setters' expectations are biased towards some value, how are they biased in that direction? Is the real wage biased to its steady state value? Or the evolution of nominal wages? And in either case, to what extent does this affect expectations regarding wage and price inflations? Applying the expectation formation of Gabaix (2020) as is to sticky wages thus requires important additional assumptions. A consequence could be that a term reflecting price inflation enters the wage Phillips curve. Studying this should be a priority for future work.

[28] Note that, as argued by Angeletos and Huo (2021), Angeletos *et al.* (2021) or Angeletos and Lian (2022), the true model of the economy has to be thought of as richer; but they find that their model admits a solution akin to the one presented in equation (4.60). They call this 'as-if' solution. Note that Angeletos and Huo (2021, appendix F) show how observed investment-adjustment costs in a model can result from capital adjustment costs with their biases. For consistency with the work of Ramey (2021), we instead assume structural investment-adjustment costs and adopt imperfect expectations regarding future investment. Future research should redo our analysis with capital-adjustment costs instead of investment-adjustment costs to check the results.

remedies, e.g. habit formation in consumption or inflation indexation (see the literature on the hybrid Phillips curve).

Notably, as Angeletos and Huo (2021) and Angeletos *et al.* (2021) argue, the parameters M_a^f and M_a^b in a model would be endogenous to policy and several other aspects of the model. They propose ways of estimating them for simpler models and specific shocks. However, from an empirical point of view, it is not straightforward to estimate them for this somewhat larger model. In addition, we are only interested in the effects limited forward-lookingness has on the macroeconomic impacts of public investment, so we abstract from this issue and simply consider a number of possible values in an ad-hoc fashion.

Also, since multiple agents form expectations about different variables, deviating from rational expectations is – in and of itself a multidimensional problem. Lacking empirical data, we here start using the assumption that all expectations are formed *in the same way*, i.e. for all groups a of expectations, we have $M_a^f = M^f$ and $M_a^b = M^b$.[29]

4.3 Main Experiments Conducted

In the following, we consider simple shocks to the appropriations to public investment that exogenously raise appropriations in a given period and thus lead to increased spending on public investment a given number of periods later and finally leads to increased public capital. Here, we vary both the timing of investment (i.e. the *time to spend* $\mathcal{T}_{\infty,\mathcal{I}}$ and *the time to build* $\mathcal{T}_{\in,\mathcal{I}}$) as well as the parameters on agents' expectations M^f, M^b seen before.

In addition, we also consider various model features that can influence the results, in particular:

1. Whether or not the economy is at the effective lower bound when public investment is increased. The latter is achieved by setting $\varepsilon_{t_0}^D \neq 0$.[30]

2. The funding of the increase in public investment – purely tax-funded or also funded via increased bond issuance. We start the analysis within a model where the government always runs a balanced budget and does not emit any bonds – not even in steady state. We then also allow for bonds to be issued to (partially) fund additional investment. Then, we allow for positive steady-state public debt.

3. Also, we contrast the simple two-agent New Keynesian framework with the more general THANK approach of Bilbiie (2021).

4. Also, we start by considering an economy without wealth in the utility, i.e. $\bar{\xi} = 0$, and with zero long-run growth $g = 0$, which resembles the baseline New Keynesian literature, we then discuss the implications of introducing $\bar{\xi} > 0$ in such a way to reduce the 'steady-state' real interest rate r, and in a final step we introduce $g > 0$ together with $\bar{\xi} > 0$ such that in steady state $r < g < r_K - \delta$ as in Reis (2021b).[31]

Appendix 4.C presents a couple of additional experiments and their results.

[29]See Appendix 4.C.4 for a limited analysis of imperfect expectations in only a subset of varibles.

[30]Simulations are run with Dynare version 4 (Adjemian *et al.*, 2021). In case of the simulations away from the ELB, we use the simple stochastic solver; for simulations at the ELB, we use the extended-path algorithm of Fair and Taylor (1983).

[31]Note that Reis (2021b) does not include wealth preferences in his model. In fact, his model framework rather resembles Bilbiie's (2021).

4.3.1 Parametrisation

The model is calibrated at a quarterly frequency. Also, we normalise the model in such a way that annual output is equal to one. I.e., in steady-state, $y = 0.25$.

In terms of parametrisation, we use several parameters directly from Ramey (2021), some of which are quite familiar from the literature. For the utility parameters σ and β, we use standard values 1 and 0.99, respectively. For the Calvo parameters for wages and prices (θ_w and $theta_p$), we use the standard value 0.75, which implies an average duration of wages or prices of one year. However, for the shares of constrained and unconstrained agents, for consistency with the THANK framework, we use the number of Bilbiie (2021), 0.275, which is lower than the (high) 0.5 in Ramey (2021). Also, we set $\alpha_p = 0.065$, this is the lower plausible bound mentioned by Ramey (2021), even though she uses a lower value for her main set of simulations.[32] Finally, we use a share of productive investment in GDP of 2.5%, which is lower than what Ramey (2021) uses (3.5%). This is due to the fact that her number includes defense spending, even though she argues that this spending might be less productive. According to her, 2.5% is roughly the share of non-defense infrastructure spending, which we will use in the following.[33] Following Ramey (2021), we set the share of overall government spending in total GDP equal to 17.5%, in line with recent data for the US. The elasticities of prices and wages ϵ_p, ϵ_w are set to 6 each, implying a 20% markup in each case. An overall capital share in income of 30% is then achieved by setting $\alpha = 30\% \cdot (120\%) = 0.36$. Following Ramey (2021, p. 227), we set the inverse Frisch elasticity of labour supply to $\varphi = 0.25$, a relatively low value that, however, allows the model to mimic an economy with some slack.[34] The adjustment The investment adjustment cost parameter κ_I is set to 5.2, the depreciation rates of private and public capital are given by $\bar{\delta} = 0.015$ and $\delta_P = 0.01$, respectively. Also, given $q = 1, U = 1$, the steady-state return to capital can be immediately shown to be $r_K = \frac{1}{\beta} + \bar{\delta} - 1 \approx 0.025$, which is also the steady-state derivative $\delta_1 := \delta'(1)$ of the depreciation function with respect to u_t. Following Ramey (2021), we assume a curvature of that function such that $\delta_2 := \delta''(1) = 2\delta_1$.[35] Also, we chose steady-state price inflation consistent with 2% p.a., and we set a steady-state ratio of constrained to unconstrained consumption of $c_c/c_u = 0.8$.[36] Concerning government debt, we consider first a scenario without any government debt ($b = 0$), later we will also consider a steady-state debt-to-(annual)-GDP level of $b/4y = 100\%$. Regarding policy apart from public investment,

[32]Note that she extensively discusses the impact α_p has on the multiplier.

[33]Note that, in line with Ramey (2021), on can derive an 'optimal ratio' of public capital to output from a simple neoclassical growth model (with only 1 quarter to build and no time to spend) as

$$\frac{k_P^*}{y} = \frac{\alpha_p \beta (1+g)^{-\sigma}}{1 - \beta(1+g)^{-\sigma}} \tag{4.61}$$

and an accordingly an optimal government investment share

$$G_I^*/y = \frac{k_P^*}{y} \frac{\delta_P + g}{1+g}. \tag{4.62}$$

Our assumptions on the parameters imply that $G_I < G_I^*$, which in line with the results in Ramey (2021) makes the multiplier larger in absolute terms.

[34]Ramey (2021) also shows that the multiplier is decreasing in φ. In this model, we are not really interested in the absolute size of the multiplier per se, and thus stick with this value.

[35]Note that the published version of Ramey (2021) uses inconsistent notation with respect to δ_2.

[36]The distributional assumption does not affect our results much. However, in the THANK model, this allows the transition probabilities to be set to higher values.

we set $\phi_\pi = 0.5$ for monetary policy, and $\phi_G = 0.1$ and ϕ_B for fiscal policy, all following Ramey (2021). Table 4.1 presents the main parameters and steady-state relationships used to calibrate the model.

Steady-state values Note that steady-state wage inflation follows from $\hat{\pi}_w = (1 + g)(1 + \bar{\pi}) - 1$. From $X = \frac{\epsilon_p - 1}{\epsilon_p}$ and (4.15), we get k and $\bar{k}$. Then from the law of motion of capital I follows.

Similarly, from government investment and consumption, we can directly obtain k_p. Using the production function and real marginal costs, w and N. c follows from $c = Y - I - G$, and due to the distributional assumption, individual c_u, c_c are directly obtained.

Together with the parameters given, the steady state restrictions can be used to pin down r, ν, T_u, T and T_c.

Concerning the distribution of non-steady-state taxes ϑ_c,ϑ_u across agents, from the condition (4.17) (and also for the THANK specification of (4.10)), we immediately see that even lump-sum taxes are only non-distorting (in the sense that they do per se affect incentives) if relative changes in taxes are proportional to steady-state consumptions of agents. Put another way, taxation is non-distortionary if $\vartheta_c c_c^{-\sigma} = \vartheta_u c_u^{-\sigma}$ as in Galí *et al.* (2007), which essentially mimics a linear tax scheme (in terms of marginal tax rates), just without the effects on incentives.

Table 4.1: Common parameter values for the calibration

Parameter	Value	Description
Varied parameters		
M^f, M^b	$\in \{0,1\}$	Expectational parameters, $M^f + M^b \leq 1$
$\mathcal{T}_{1,I}$	≥ 0	Time to spend
$\mathcal{T}_{2,I}$	≥ 1	Time to build
$\chi_{I,s}$	$\frac{1}{\mathcal{T}_{2,I}}$	Time-structure of investment $s \in \{0, \ldots, \mathcal{T}_{2,I}\}$
g	≥ 0	Long-run growth rate
$\bar{\xi}$	≥ 0	Preferences on bond holdings
$p_{c\|u}$	≥ 0	Probability of becoming constrained
Structural parameters		
σ	1.00	Intertemporal elasticity of substitution, standard value
β	0.99	Discount factor in the utility function, standard value
θ_p	0.75	Price stickiness, standard value
θ_w	0.75	Wage stickiness, standard value
s_c	0.275	Share of constrained agents (from Bilbiie, 2021)
In particular, from Ramey (2021):		
ϵ_p	6.00	Elasticity of substitition, product market (markup: 20%)
ϵ_w	6.00	Elasticity of substitition, labour market (markup: 20%)
φ	0.25	Inverse Frisch elasticity of labour suppy
α	0.36	Capital share in production function
α_p	0.065	Share of public capital in production function
κ_I	5.20	Investment adjustment cost parameter
$\bar{\delta}$	0.015	Steady-state depreciation rate, private capital
δ_P	0.010	Depreciation rate, public capital
δ_2	0.050	Second derivative of depreciation function with respect to u_t
Steady-state relationships		
y	0.25	steady-output level (normalised)
G/y	0.175	Total government spending as a fraction of GDP
G_I/y	0.025	Fraction of public investment in GDP
b/y	$\in \{0,4\}$	Public-debt-to GDP ratio (annually 0 or 100%)
c_c/c_u	0.80	Consumption inequality, arbitrary value
$(1+\bar{\pi})^4$	1.02	Annual inflation rate 2%
Policy parameters not related to government investment		
ϕ_π	0.50	Reaction coefficient, Taylor rule, standard value
ϕ_G	0.10	Reaction coefficient of taxes to government spending in (4.56) (Ramey, 2021)
ϕ_B	0.33	Reaction coefficient of taxes to sovereign debt in (4.56) (Ramey, 2021)

Note: The table presents the parameters and steady-state values used to calibrate the baseline model. Deviations from this calibration are discussed in the text, when appropriate.

4.3.2 Shock Considered

The shock to public investment consists in a persistent increase in appropriations to public investment of the following form:

$$\hat{ap}_{I,t} = \rho_{AP,I}\hat{ap}_{I,t-1} + \varepsilon_{ap,I,t}, \tag{4.63}$$

where following Ramey (2019) and Leeper *et al.* (2010), we set $\rho_{AP,I} = 0.95$. In all of the simulations considered below,

$$\varepsilon_{ap,I,t} = \begin{cases} \Delta_{ap} > 0 & \text{in period } t = t_0, \\ 0 & \text{otherwise.} \end{cases}$$

In particular Δ_{ap} is extremely small relative to output such that the linearisation is not a very stringent assumption in this regard,[37] but it effectively limits the analysis to rather small shocks without putting the assumptions underlying linearisation into question. In order to evaluate larger spending programmes like the American *Infrastructure Investment and Jobs Act* or the European *Recovery and Resilience Facility*, ultimately, a non-linear model is required. This is left for future research.

In addition, for the experiments at the effective lower bound, we assume that in period t_0 a persistent shock to ε_t^D (with autoregressive parameter 0.85) occurs that pushes the economy to the effective lower bound for exactly 8 periods. This means that the shock size is recalibrated for different parametrisations (also of expectations) of the model. We calibrate this for the economy without the shock to investment spending and keep these simulations as a counterfactual to obtain multipliers, see below. In a second simulation, we turn the shock to public investment on – where notably both shocks coincide in timing. As a result, appropriations rise in the first period of the spell at the effective lower bound.

In this and the following section, we present several impulse response functions and similar graphs. For ease of readability, in these, we scale the results from our simulations such that the increase in appropriations $\hat{ap}_{I,t}$ is equal to one in the first period. In order to simplify the interpretation, the reader might understand this to mean an increase in appropriations of one percent of annual GDP. The following figures are also labelled accordingly.

We are now in a position to present the backbone of the analysis: An exogenous increase in appropriations to public investment. Figure 4.2 presents it for the case where $g = 0$ for three different values of time to spend $\mathcal{T}_{\infty,\mathcal{I}}$ and time to build $\mathcal{T}_{\in,\mathcal{I}}$: once with no time to spend and a time to build of one quarter (which is the same as assumed for the private economy).

The first panel shows the evolution of appropriations to public investment, which as just stated equals 1 in the period of the shock and then decays with persistence 0.95 for all three simulations.[38]

As shown in the second panel, this translates into an increase in public investment. Without time to spend, investment immediatly responds, whereas with time to spend the response is lagged by a given amount of time (cf. the red line with circles and the yellow line with squares). Also, with higher time-to build, expenditures rise more slowly

[37]To be precise, in the numerical simulations, we assume that appropriations to public investment are increased by 0.5% relative to their steady state value in the first quarter. Given that $G_I/y = 0.025$, the size of the simulated shock in the first quarter thus amounts to 0.0125 percent of quarterly GDP.

[38]Appendix 4.C.3 also shows similar results for the case with an on-off shock to appropriations to public investment.

as spending is distributed across more periods. As such the peak in public investment lower, which in this experiment is due to the fact that new appropriations after the initial shock are lower than those of the initial shock.

The third panel finally shows the response of public capital. Without time to spend and a one-quarter time to build, public capital increases with a one quarter delay. Due to the persistent shock to investment, it keeps increasing, albeit at a lower rate, peaking after roughly ten years, after which it decays slowly. In our model, with $g = 0$, the same behaviour can be obtained regardless of time to build or spend. However, it is shifted by the sum of both (see the other two lines in the panel).

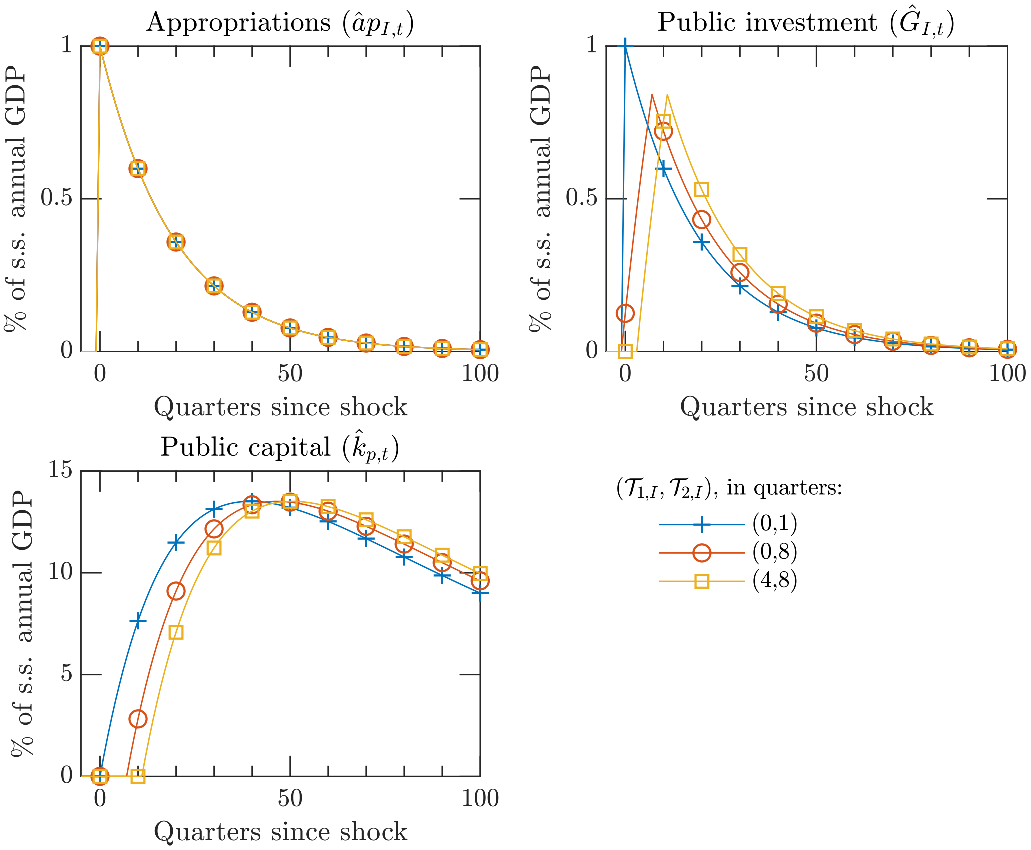

Figure 4.2: Example of exogenous shock considered in the experiments
Note: The figure shows the impulse responses of appropriations, realised public investment and public capital stock to a shock to appropriations to public investment of (normalised) size 1 based on the linear model with zero growth ($g = 0$).

4.3.3 Measuring the Multiplier

A major issue due to the long-service life of public capital is that even not fully persistent shocks to public investment can have long-lasting effects on output and other variables.

For the case $\mathcal{T}_{1,I} = 0$, it is straightforward to derive an 'impact multiplier' on y_t as

$$m^y_{Impact} := \frac{\Delta_x(\hat{y}_{t_0})}{\Delta_x(\hat{G}_{I,t_0})},$$

where $\Delta_x(z_t)$ is the appropriate deviation of some variable z_t in period t. In particular, for the experiments without the effective lower bound, it is simply $\Delta_x(\hat{y}_t) = \hat{y}_t$. On the other hand, for the experiment at the ELB we compare a scenario with both liquidity-preference shock and public-investment shock to one with only the liquidity-preference shock. Then,

$$\Delta_x(\hat{y}_t) = \hat{y}_t|_{\text{both shocks in } t_0} - \hat{y}_t|_{\text{only } \varepsilon_t^D \neq 0 \text{ in } t_0}.$$

Similar to Ramey (2021) we call this 'short-run multiplier'.[39]

However, this neglects the long-run output-enhancing effects of public investment, which unfold after some time. In order to capture these, one could calculate the simple integral across a given number of periods of changes. Taking into account that the economy possibly grows between periods, this would amount to a τ-period 'integral'-based multiplier

$$m_{Integral}^{y,\tau} := \frac{\sum_{s=0}^{\tau-1}(1+g)^s \Delta_x(\hat{y}_{t_0+s})}{\sum_{s=0}^{\tau-1}(1+g)^s \Delta_x(\hat{G}_{I,t_0+s})}.$$

The drawback of this approach would be that future deviations have the same weight as current ones, i.e. there is no discounting of future variables.

In any case, such a calculation requires us to find the undiscounted integral of government investment. Based on the examples previously illustrated in Figure 4.2 with $g = 0$, Figure 4.3 depicts the values of these integrals across time horizons. Time to spend simply shifts the curves around (compare the red line with circles and the yellow line with squares). However, time to build does not purely shift the integral (compare blue line with plus signs to the red line with circles): For the first few periods, with time to build, the integrals grows more slowly, before also increasing faster. In any case, the integral approaches $1/(1-\rho_{AP,I}) = 20$ times the initial shock eventually.

But it would be desirable to take into account the timing of changes in output when calculating multipliers. Thus, following the seminal contribution of Mountford and Uhlig (2009), as also used by Ramey (2021), we include a kind of dynamic present-value multiplier at horizon $\tau > 0$:

$$m_{PV}^{y,\tau}((d_s)_{s=0,\dots,\tau-1}) := \frac{\sum_{s=0}^{\tau-1}(1+g)^s d_s \Delta_x(\hat{y}_{t_0+s})}{\sum_{s=0}^{\tau-1}(1+g)^s d_s \Delta_x(\hat{G}_{I,t_0+s})}, \tag{4.64}$$

where $D = (d_s)_{s=0,\dots,\tau-1}$ is some sequence of discount factors.

In the present context, there are a couple of ostensibly 'natural' choices for the discount factor:

- The 'safe' stochastic discount factor

$$d_s = \begin{cases} 1 & \text{if } s = 0, \\ \prod_{\varsigma=0}^{s}\left[\beta(1+g)^{-\sigma}\frac{p_{u|u}c_{u,t_0+\varsigma}^{-\sigma}+p_{c|u}c_{c,t_0+\varsigma}^{-\sigma}}{c_{u,t_0+\varsigma-1}^{-\sigma}}\right] & \text{if } s > 0, \end{cases}$$

- the 'risky' stochastic discount factor

$$d_s = \begin{cases} 1 & \text{if } s = 0, \\ \prod_{\varsigma=0}^{s}\left[\beta(1+g)^{-\sigma}\left(\frac{c_{u,t_0+\varsigma}}{c_{u,t_0+\varsigma-1}}\right)^{-\sigma}\right] & \text{if } s > 0, \text{ or} \end{cases}$$

[39] In fact, Ramey instead uses an appropriations-based multiplier that replaces $\hat{G}_{I,t}$ with $\hat{ap}_{I,t}$, also for the dynamic multipliers below. This gives different figures, especially in the short-run, because appropriations rise immediately, whereas investment only reacts over time. However, since the flow of investment is the actual economic variable, we consider it as the baseline. In the appendix, we will also report appropriations-based multipliers.

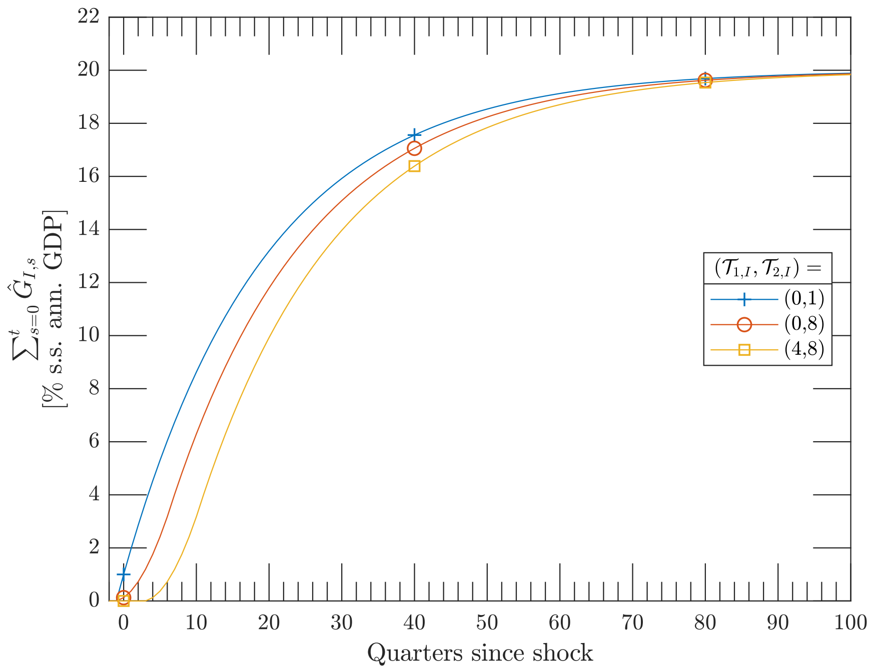

Figure 4.3: Cumulative sum of public investment across time horizons

Note: The figure shows the cumulated sum of realised public investment from Figure 4.2 in response to a shock to appropriations to government investment of size 1 in period t_0. It is based on simulations of the linear model with zero growth ($g = 0$).

- the 'social' stochastic discount factor

$$d_s = \begin{cases} 1 & \text{if } s = 0, \\ \prod_{\varsigma=0}^{s} \left[\beta(1+g)^{-\sigma} \frac{s_u c_{u,t_0+\varsigma}^{-\sigma} + s_c c_{c,t_0+\varsigma}^{-\sigma}}{s_u c_{u,t_0+\varsigma-1}^{-\sigma} + s_c c_{c,t_0+\varsigma-1}^{-\sigma}} \right] & \text{if } s > 0. \end{cases}$$

Of course, in a more realistic set-up, one could use the ex-ante expected values of these variables. From a budgetary point of view, the safe discount factor might be the usual choice, because absent uncertainty it equals the inverse of the real interest factor

$$\prod_{\varsigma=1}^{s} \frac{1}{1 + r_{t_0+\varsigma}}.$$

However, as argued by Reis (2021b), especially with low interest rates (and in particular with $r < g$), it seems more appropriate to use the market interest rate, at least when pricing government debt. On the other hand, from a normative point of view, it might seem preferable to also consider that there is heterogeneity, which is incorporated by using the social discount factor.[40]

In any case, with a small enough shock to public investment (also taking into account that we are considering a linearised model), we can also resort to using the steady-state versions of these discount factors. Both the 'risky' and the 'social' discount factor share the same steady state value

$$d_s = \bar{d}_s := \left(\beta(1+g)^{-\sigma}\right)^s, \quad s \geq 0$$

[40] Note that a benevolent planner would not just use this discount factor – instead also the elasticities of individual consumptions to aggregate variables would be considered.

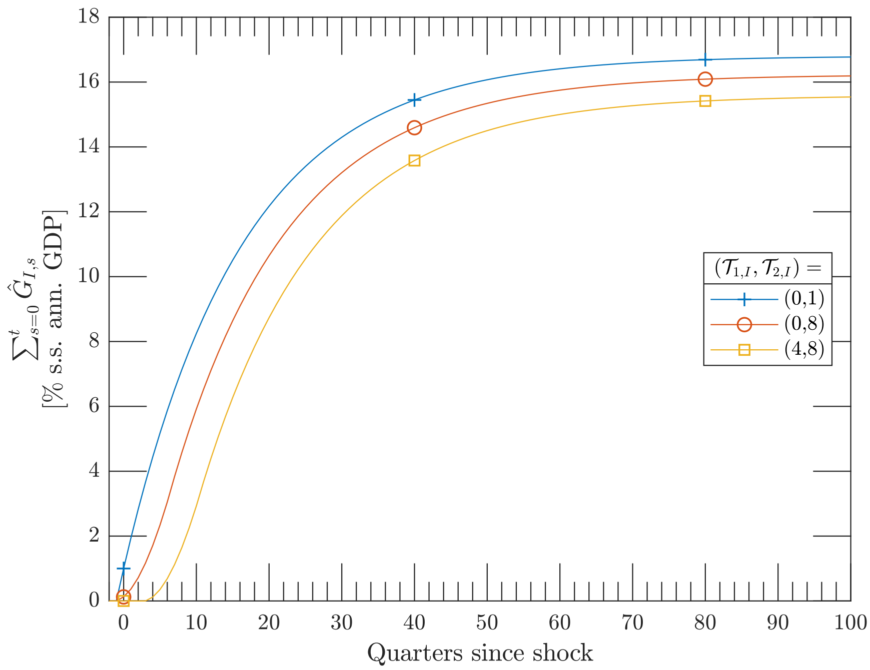

Figure 4.4: Discounted cumulative sum of public investment across time horizons using $\beta(1+g)^{-\sigma}$ as the discount factor
Note: The figure shows the discounted cumulated sum of realised public investment from Figure 4.2 in response to a shock to appropriations to government investment of size 1 in period t_0. It is based on simulations of the linear model with zero growth ($g = 0$). The discount factor β has been used in generating the figure.

This also makes the resulting multipliers of the experiments scale-invariant, i.e., as long as linearisation is a valid approximation, the resulting multipliers do not depend on the size of the shock.

Hence, in the following section we will present the multipliers discounted using this discount factor. Figure 4.4 shows the present discount value of the integrals across durations as previously shown in Figure 4.3, now using the above-mentioned steady-state discount factor.

The initial response looks similar to the figure from before – however, remarkably, for longer time horizons the different timing structures become obvious: With time to build and time to spend, respectively, the overall present value becomes smaller. Also, due to discounting, the present value is always smaller than $1/(1-\rho_{AP,I})$.

4.4 Main results

We are now in a position to investigate the effects of the different expectations considered and the time structure of investment in more detail.

Ultimately, we want to present a comparison of a larger set of various possible constellations of expectations and across other model features. But, for pedagogical reasons, we first start the analysis by considering three different types of expectations: (i) rational expectations, (ii) myopia with a forward-looking dampening parameter of $M^f = 0.9$ and (iii) hybrid expectations with a forward-looking parameter $M^f = 0.9$ and a backward-looking parameter $M^b = 0.1$.

For these cases, in subsection 4.4.1, we next move step by step from impulse response functions to dynamic multipliers. For this, we also continue to use the examples started in section 4.3. In order to do this, keep in mind that, in subsection 4.4.1 and 4.4.2, we throughout focus on the case without growth ($g = 0$), no wealth preferences ($\bar{\xi} = 0$), a standard HANK framework ($p_{u|c} = 0$) and we assume that there is no government debt ($B_t = 0$ at all times, tax policy is given by (4.55)). Also, here we abstract from the Effective lower bound. We turn to these model features in subsection 4.4.3.

4.4.1 From Impulse Responses to Multipliers, No Debt

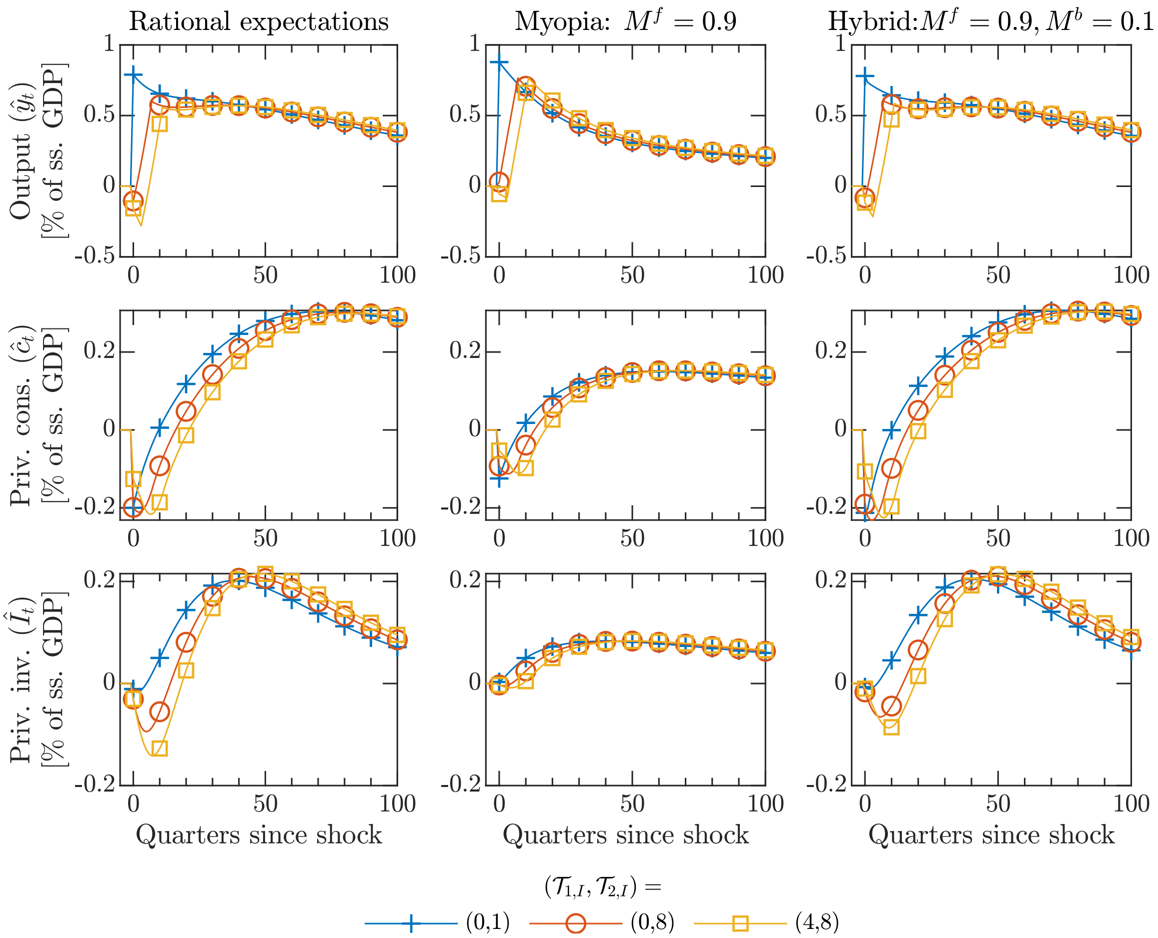

Figure 4.5: Impulse response functions of output as well as private consumption and investment to a government investment shock.
Note: The figure shows the impulse responses of output, private consumption and private investment in response to a shock to appropriations to government investment of size 1 in period t_0 (Figure 4.2) for different forms of forming expectations. The response variables are ordered along the rows of the figure, different expectational formulations along the columns. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences.

Figure 4.5 presents a matrix of impulse responses for aggregate output, private consumption and private investment (rows of subplots, in that order) across the three types of expectations (columns, in the order mentioned before) considered.[41] In each subplot, three impulse responses are shown for the three different types of timing of public investment familiar from the previous section.

A striking result is that, across expectations, the impulse response function for output on impact in this comparison is clearly positive without time to spend and a time to build of one quarter, but for the other constellations depicted the output response is negative or close to zero. This in and of itself is not a generic result of time to build or time to spend *per se*, rather of the time horizons shown. The underlying mechanism is known at least since the work of Leeper *et al.* (2010), Boehm (2020) or Ramey (2021): In standard, New Keynesian models, government investment crowds out private investment, as shown in the last row. The underlying reason is that the government uses resources that would have otherwise been used by the private economy. To fund this, it taxes private agents, which directly causes a negative wealth effect – and in consequence households cut consumption. So far, the logic is identical to the one for government consumption; but here, future increases in public capital act as a positive news shock, which increases future expected output. The combined effect is an increase in the real rate (through increased growth) and expected future *deflationary pressures* due to improved technology.[42] The higher interest rate leads to a crowding out of investment, which according to Boehm (2020) is particularly sensitive to interest-rate changes because of the long service life of capital. With a longer time to spend and/or build, this effect becomes stronger as more periods are affected before a positive wealth effect from higher public capital materialises, leading to a smaller initial output response.[43]

For longer time horizons, the output response on impact even becomes negative. In the next subsection, we will briefly discuss the constellations of time to build and time to spend that lead to this behaviour, given a set of expectations. On the other hand, with rational expectations, as the increase in public capital materialises, investment is elevated beyond the initial value for quite a long time. At the peak of public capital, investment also peaks and then gradually reverts to its steady-state value, which is a result of consumption smoothing.

In any case, note that with rational expectations, the crowding-out effect on private investment clearly becomes larger with longer delays; and also the maximum interim negative impact on investment is shifted into the future. General equilibrium effects then reduce consumption even further.[44]

Consumption, on the other hand, recovers somewhat faster and stays elevated for longer. As a result of the temporary increase in private investment after several years, private capital formation rises, and in consequence, aggregate output also remains elevated for a long time.

Turning to the case with myopia, i.e., a sort of discounting of future realisations of

[41] The impulse responses for a number of other variables can be found in Figures 4.C.15–4.C.17 in appendix 4.C.

[42] Compare Figure 4.C.16 in the appendix, which depicts the impulse responses for interst rates and inflation rates corresponding to the experiments currently discussed.

[43] This mechanism also relates intimately to the Forward Guidance puzzle.

[44] As shown in Figure 4.C.15 in the appendix, the brunt of the costs is borne by the constrained agents. Unconstrained agents, by reducing investment can shelter their consumption temporarily. However, constrained agents are borrowing constraint and as such reduce consumption one-to-one with reduced labour income. This weakens aggregate demand further. On the other hand, as output peaks in later periods, constrained agents only reap parts of the benefits, because capital income increases, too. In effect, income inequality rises.

variables, in the second column of Figure 4.5, we see that the result regarding crowding-out is significantly muted. Even with relatively long delays, investment barely drops below the steady-state value. This is reinforced by a smaller decline in private consumption which is due to a similar effect. Since both consumption and investment decline less on impact, the impact response of output is larger than for rational expectations for any timing structure considered. However, it remains strictly below one.

After a few periods, the impulse response functions for the various timing structures become almost indistinguishable. However, the muting of the initial fall in investment and consumption is mirrored by a corresponding muted expansion later on: investment becomes positive earlier than with rational expectations, but at peak it only reaches a fraction of the response we obtained with rational expectations. This results in significantly lower capital accumulation in the long-run. Correspondingly, with myopic agents, output drops faster than with rational expectations. I.e., here myopia dampens effects in either direction.[45]

This result, however, highlights an issue with the expectations á la Gabaix (2020) in the present context: They are rigid even in the presence of very persistent shocks, rendering them susceptible to the Lucas-(1976) critique. At some point, expectations would have to adjust to the changed public capital stock. In that context, several variations might be considered in future work, in particular adaptive learning. In this chapter, we take a simpler route and instead also add a backward-looking term to the expectations operator, as depicted in the third column of Figure 4.5.

At a first glance, this renders the responses very similar to the ones we obtained with rational expectations – and only minor differences appear to emerge. The most salient expection it the investment response: With significant time to build and/or spend, it still is negative for a few periods, but it does not fall as deep – rather it takes somewhat longer to recover. In any case, another way of looking at the results may give insights as to in what manner this kind of expectations actually affects macroeconomic outcomes. Therefore, we next derive the multipliers. As an intermediate step, Figures 4.C.23 and 4.C.24 in appendix 4.C show the undiscounted and discounted integrals corresponding to the methods established in the previous section and applied to the variables depicted in Figure 4.5. This, together with the corresponding integrals shown in Figures 4.3 and 4.4, allows us to derive the two dynamic multipliers

$$m^{\chi,\tau}_{Integral} \quad \text{and} \quad m^{\chi,\tau}_{PV}\left(\left(\beta^s(1+g)^{-\sigma s}\right)_{s=0,\dots,\tau-1}\right),$$

for each of the variables $\chi = y, c, I$.

Note, however, that for the case with time to spend, an impact multiplier cannot be calculated for the period when the first appropriations are generated, because then, no investment has yet taken place. As an alternative, one could of course use another denominator for the multiplier. We leave such an analysis for future research and instead perform the calculations starting only in period $t_0 + \mathcal{T}_{1,I}$. The response of output during the intervening periods is, however, reflected in the expressions according to the formulae presented before.

Figures 4.6 and 4.7 present the resulting multipliers for the currently analysed cases for the first 100 years (400 quarters).

[45] As discussed by Angeletos and Lian (2022), bounded rationality and similar modelling devices can dampen or strenghthen general-equilibrium effects. In line with their argumentation and that of Gabaix (2020), additional discounting generally acts in a dampening manner.

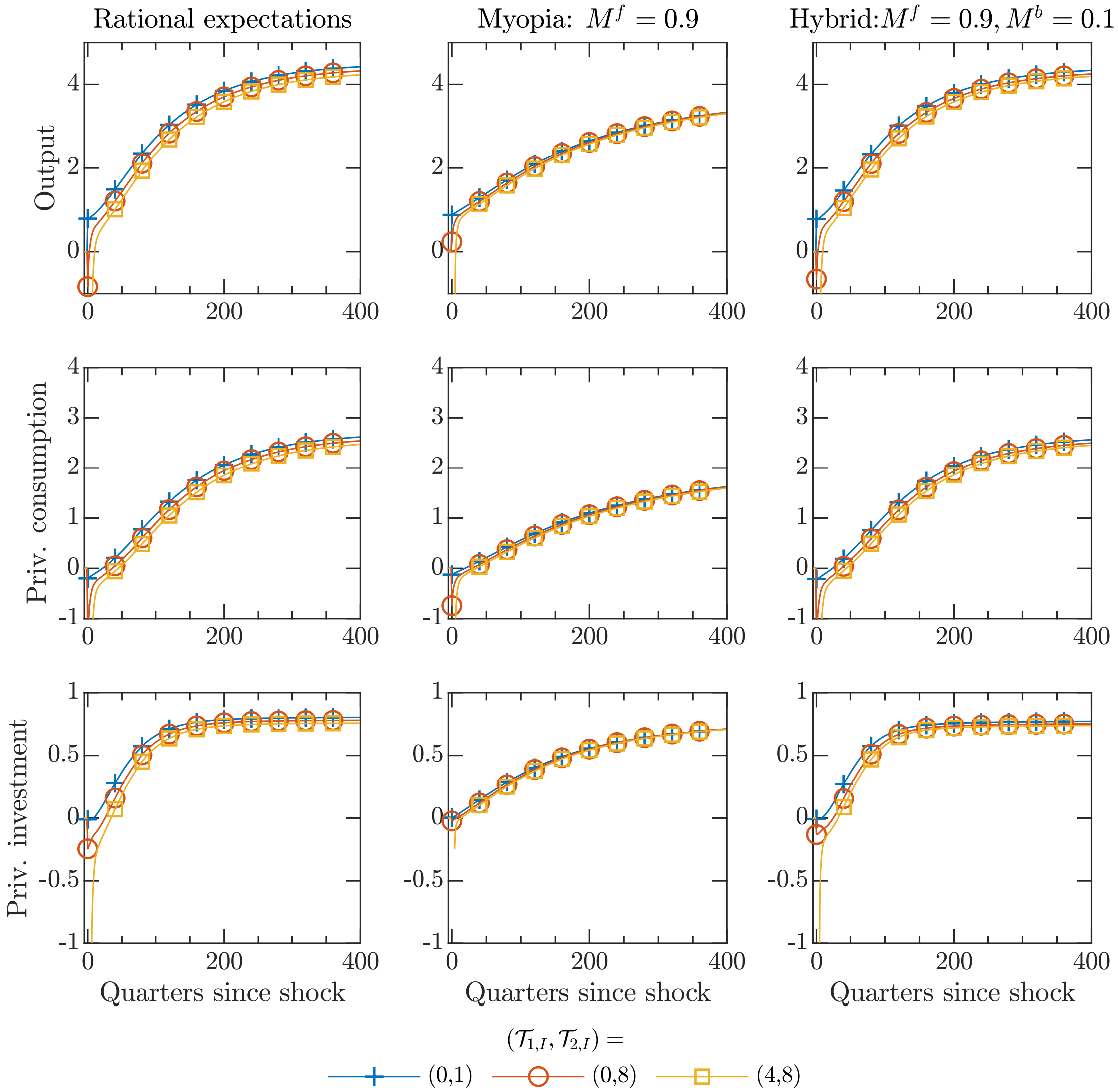

Figure 4.6: Government investment multiplier across periods, undiscounted sums, baseline calibration

Note: The figure shows the cumulated impulse responses of output, private consumption and private investment in response to a shock to appropriations to government investment of size 1 in period t_0 for different forms of forming expectations. The responses of output, consumption and investment is ordered along the rows of the figure, different expectational formulations along the columns. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences.

As is evident from Figure 4.6, which shows the multipliers based on the undicounted integral, these multipliers are increasing over time from the low or even negative values on impact. After 400 quarters, the multiplier on consumption and output is still increasing across all constellations of timing and expectations. And also, the long-run output multiplier measured by this measure can be quite high, exceeding 4 in the case of rational or hybrid expectations and 3.5 in the case of myopic expectations. I.e., each dollar spend on, e.g., infrastructure would cause another 2.50 to 3 dollars in private expenditure in the long-run, most of which in form of consumption. Note that unlike the other two types of expectation formation, the multiplier on private investment with myopic expectations is only starting to stabilise at the end of this window. Remarkably, that particular long-run multiplier is very similar to the one obtained with rational or hybrid expectations.

Figure 4.7 next presents the multipliers from the same simulations, but now using the discount factor $\beta(1+g)^{-\sigma}$ between periods. As was to be expected, the overall shape does not change too much. However, long-run multipliers are significantly smaller: for rational expectations and the hybrid expectations, they stay below 2.5 on output, for the myopic ones below 2. This is a testament to the fact that government spending happens earlier than the gross of the positive private economy effects materialise. Also note that for that reason, long-run multipliers are shorter with time to build and/or spend.

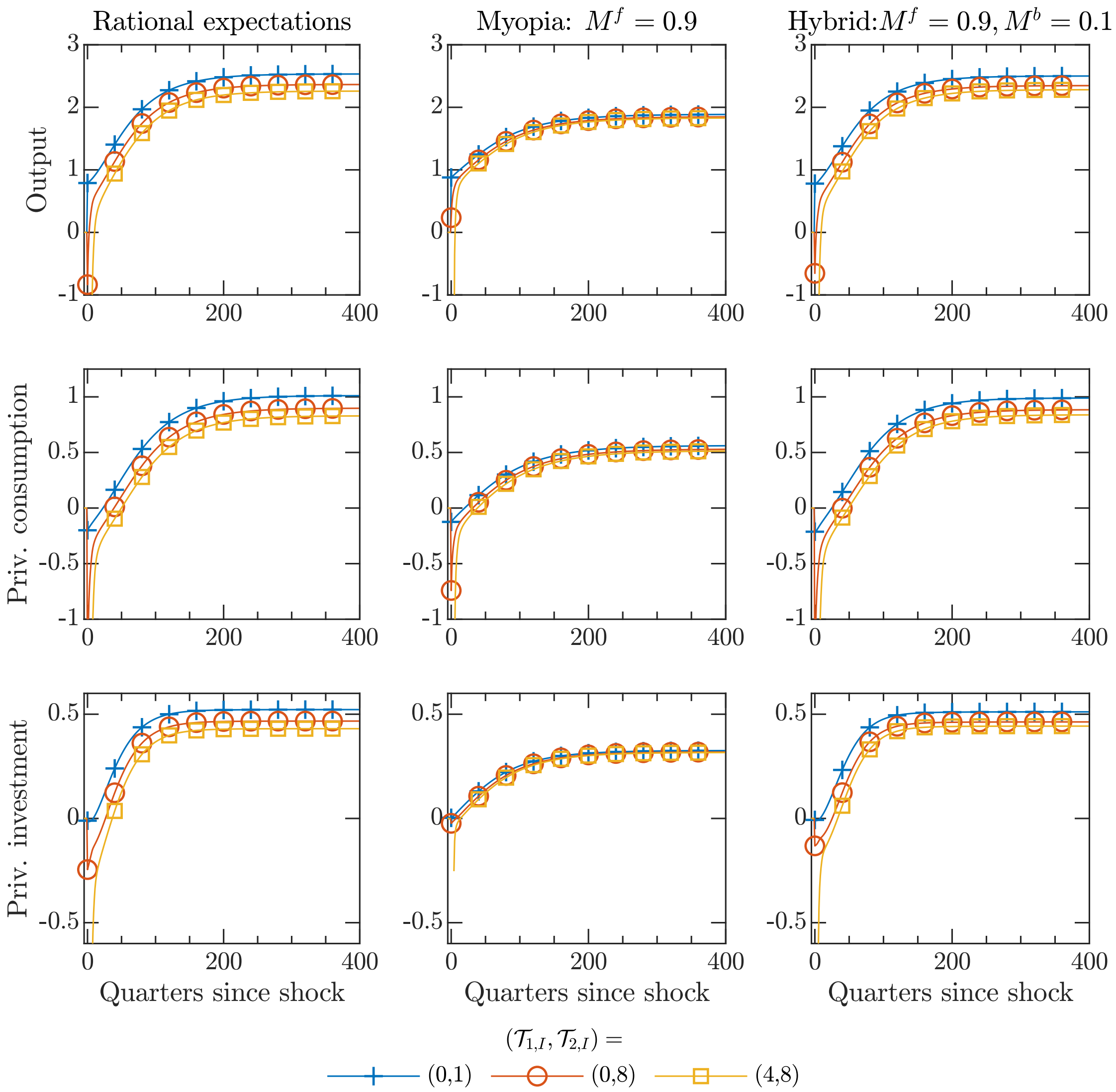

Figure 4.7: Government investment multiplier across periods, discounted sums, discount factor: $\beta(1+g)^{-\sigma}$, baseline calibration

Note: The figure shows the discounted cumulated impulse responses of output, private consumption and private investment in response to a shock to appropriations to government investment of size 1 in period t_0 for different forms of forming expectations, where the discount factor $\beta(1+g)^{-\sigma} = \beta$ was used. The responses of output, consumption and investment is ordered along the rows of the figure, different expectational formulations along the columns. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt $b_t = 0$ and no wealth preferences.

4.4.2 Investigating the Impact of Expectations on Multipliers

However, an issue that arises with the illustrations of the results presented so far is that it is rather difficult to assess differences across expectations and/or the timing structure of public investment. However, precisely the interaction of these two features is of some interest. In order to facilitate the analysis of this interaction and to allow for a larger variety of specifications for expectations and the timing structure, we next compare the multipliers at specific points in time across specifications. We first start out with only time to build and no time to spend, i.e. we keep $\mathcal{T}_{1,I} = 0$ at first.

Only Time to Build, no Time to Spend

In particular, Figure 4.8 presents the discounted output multiplier of an increase in public investments at different horizons across many specifications of expectations and the timing of public investment.[46] Each panel represents a certain interval of periods after the initial shock, where we plot the response in the first quarter (on impact) as well as the cumulated net present value of the output response across the first 2, 5, 25 and 100 years. Along the horizontal axis, we vary time to build while abstracting from a time to spend, i.e. we keep $\mathcal{T}_{1,I} = 0$, plotting the resulting cumulative multiplier along the vertical axis. Each line represents a given type of expectations, with parameters M^f varied between 0.7 and 1 and M^b between 0 and 0.2.

The figure depicts a remarkable variation in multipliers across specifications at different time horizons, which we will discuss now:

First, focus on panel a), depicting the impact multiplier in the very first period in which any spending occurs. A remarkable first observation is that without time to spend and only a one-quarter time to build, the type of expectations do not matter much for the impact multiplier: It is always slightly below. However, for larger delays, significant differences can be detected.

On impact, as we have already discussed, with rational expectations, the multiplier is decaying in the time-to-build parameter for $\mathcal{T}_{2,I} \leq 13$, and for $\mathcal{T}_{2,I} \leq 7$, this decline is almost linearand rather steep. For a time-to-build of roughly a year, the impact multipier is roughly zero, but for a time to build of 2.5 years or more, the impact multiplier is almost -1.5.[47] This means that for each dollar spend by the government in the initial period, private agents cut spending by 2.50 dollars in that period. Notably, the multiplier then starts to become slightly larger again for even longer time to build.

Note that for less forward-looking agents, the impact multiplier does not become as negative: With myopia as in Gabaix (2020), for $M^f = 0.9$, the impact multiplier still de-

[46]For the sake of better comparability with Ramey (2021), we also include Figure 4.C.25 (in the appendix), a figure depicting the appropriations-based multiplier from equation (4.69). In the short-run, the appropriations-based multiplier is smaller than the investment-based one for longer time to build – which is by construction, since initially only a fraction of appropriations translate into actual investment. However, the underlying observations are similar. Also, arguably, the investment-based multiplier figures are a bit easier to read.

[47]Note that a negative multiplier is mostly a result of the time structure of spending: due to time to build and the persistent shock, public spending, and taxation is increasing over time, giving a negative personal income profile for private agents. Confer Appendices 4.C.2 and 4.C.3 for a shock to government consumption with a similar time structure and an on-off shock to public investment. Even with government being shocked, the impact multiplier can be negative with a persistent, stretched-out shock. On the other hand, with an on-off shock to appropriations, the multiplier is positive throughout. Note however, that in each case, myopic expectations dampen the negative 'effect' of $\mathcal{T}_{2,I}$ on impact multipliers.

cays with time to build, but crucially, it only turns mildly negative for a time to build of 10 quarters or more. For lower values of M^f, however, the multiplier always remains positive, and for $M^f = 0.7$, the impact multiplier actually remains almost constant at about 0.9 across time to build. As mentioned before and extensively discussed by Angeletos and Lian (2022), this illustrates that bounded rationality can dampen the responsiveness of the model, and in this case, at least in the short run, reduce general equilibrium effects.

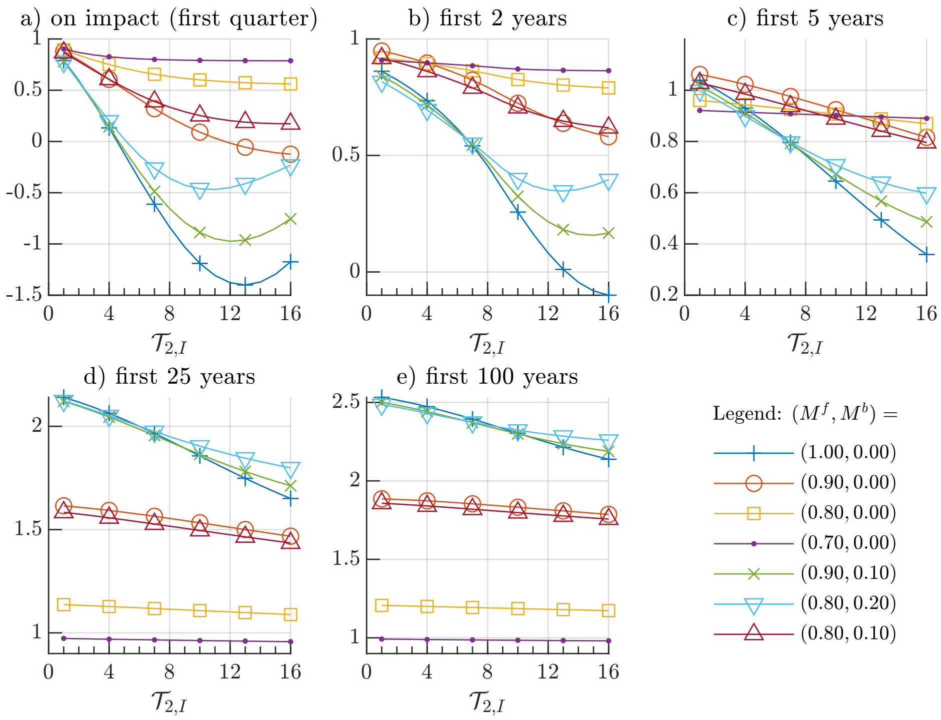

Figure 4.8: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multipliers of government investment on output after a shock to appropriations in period t_0 for different forms of forming expectations. The multiplier is calculated according to equation (4.64). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences.

Three types of hybrid expectations are also presented: Two where the weights M^f and M^b sum to 1, but with varying forward- and backward-looking weights, and one where the sum is less than that.

Note that with small values for time to build, expectations with $M^f + M^b = 1$ behave reasonably similar to rational expectations. However, if the time to build is longer than a year, the combination of a backward-looking element with myopia translates into a muted impact response. The impact multiplier is still decreasing in the time to build, but it levels out for smaller values of $\mathcal{T}_{2,I}$ and then starts to increase again. Also, as was the case with pure myopy, this muting is stronger with less forward-looking expectations, In fact, for very long time to build (4 years, which is the maximum time considered here), the impact multiplier is almost the same as with purely myopic expectations and $M^f = 0.9$.

Note also that with a backward-looking element, and $M^f + M^b < 1$ (e.g., the line with upwards-pointing triangles), a broadly similar result emerges: With $M^f + M^b = 0.9$, and short time to build, the impact multiplier is almost the same as with purely forward-looking myopic expectations and $M^f = 0.9$.

Next, consider the other panels of the Figure: Panel b) shows the discounted multiplier across the first two years (i.e., including the first quarter and periods up to 7 quarters later). The first thing to note is that at this point, all multipliers have risen by a certain amount, but the ordering is still similar: With rational expectations, the cumulative multiplier is still negative for very high values of $\mathcal{T}_{2,I}$. But for most other simulations, the cumulative multiplier is positive at this point, even with rational expectations. This overall development is due to three factors: (1) With longer time to build, the government investment takes time to reach its maximum value (remember Figure 4.2) – this reduces the denominator in the expression for the multiplier, especially if output does not fall proportionately. (2) For $\mathcal{T}_{2,I} < 7$, at this point the (first) investment projects have finished, and consequently, a positive wealth effect has materialised, pushing output and the multiplier up by quite a substantial amount. (3) For $\mathcal{T}_{2,I} > 8$, but close to 8, this positive wealth effect is also in the near future, and output is starting to increase as a result.[48] Note, however, that all multipliers are still below 1; as a result, up to this point, the present-value of the private sector-response (as measures as private consumption plus investment) is clearly negative up to this point.[49] Notably, purely myopic expectations still yield the highest multipliers. And for longer times to build, multipliers are increasing with increasing myopia – but note that for low values of $\mathcal{T}_{2,I}$, the other myopic expectations and the hybrid one with $M^f + M^b = 0.9$ start to overtake the one with $M^f = 0.7$. This, again, is a testament of the fact that myopia tends to dampen responses of variables in both directions – and here it becomes clear that myopia can also dampen the positive wealth and income effects. Meanwhile the response of hybrid expectations with $M^f + M^b$ is still between those of rational expectation.

Moving to panel c), which shows the situation across the first 5 years, we see a pattern emerge: The cumulated output multiplier for very myopic expectations $M^f = 0.7$ increases very slowly relative to the previous panel. With the other types of expectations, the cumulated multiplier start to become larger, and in particular larger than one. Also note the general reversal for low values for the time to build $\mathcal{T}_{2,I}$: Even under rational expectations, the cumulative multiplier starts to approach 1 at this point in time. Also, note that, at this point in time, myopic preferences with the highest value for M^f and low time to build have the highest multiplier. However, as shown in Figure 4.C.26[50] in appendix 4.C.5, as time progresses, the cumulative multiplier rises mostly for rational expectations and the closely related hybrid expectations with $M^f + M^b = 1$, which is due to long-lasting effects on capital accumulation. This is also illustrated in panels d) and e): After 25 years, respectively 100 years, we see the pattern emerge that we presented in the previous section: long-run government investment projects have very high long-run multipliers, whereas it is significantly smaller with myopia. In fact, for $M^f = 0.7$, the long-run response is almost negligible: The cumulative multiplier approaches 1, but never exceeds it. Less myopic expectations do allow for higher long-run multipliers, but there

[48]See the discussion in appendix 4.C.1 for a deeper explanation.

[49]See Figures 4.C.5 and 4.C.5 and the discussion in appendix 4.C.1 for a breakdown of the effects on the multiplier.

[50]Figure 4.C.26 essentially presents the same data as Figure 4.8, but for effects in the medium term (i.e. at horizons of 6–10 years). The results presented there resemble those of Figure 4.8, but it shows more clearly that the multiplier obtained with rational expectations or with $M^f + M^b = 1$ rise beyond those of purely myopic expectations.

is still a significant difference. Also note that the effect of time to build on long-run multipliers becomes almost linear (but remains negative): This is mostly due to the different time structure of the effects on output etc. which affects the net present value differently across simulations.

Yet another way to look at these multipliers is presented in Figure 4.9, which focusses on purely forward looking, but myopic expectations with $M^f \leq 0$, $M^b = 0$ and essentially depicts another dimension of the interaction between expectations and time to build for the case without time to spend, already shown in Figure 4.8. Again, in each subplot, the cumulated discounted sum of the output response divided by the cumulated discounted sum of public investment is shown at different points in time. Here, however, unlike in the previous figure, we plot the M^f parameter on the horizontal axis, whereas each line represents one value for the time-to-build parameter $\mathcal{T}_{2,I}$. The subplots can each be interpreted as follows: Starting from the right-hand edge ($M^f = 1$, rational expectations), what happens to the discounted multiplier (at a fixed time horizon) if the economy becomes less forward looking in a sense similar to Gabaix (2020)? Starting again, in the first panel (on impact), we see the previously presented result: As the economy becomes less forward looking (lower M^f), agents neglect the (general-equilibrium) effects in the next period, and as a result, the multiplier increases. For longer time to build, the multiplier is initially smaller (or even negative), which is due to the crowding out of investment. However, if agents are less forward-looking, they neglect the subsequent increases in output, investment and the deflationary pressures eventually generated by productive public capital. This counteracts the crowding-out effect. Nonetheless, note that for $\mathcal{T}_{2,I} = 16$, the impact multiplier is even a bit smaller (more negative) for M^f close to 1 than with rational expectations. The reason behind this will become clearer after we discuss the other panels. In particular, in panel b) (discounted sum across the first two years), we see that the discount factor is generally increasing with lower M^f, but you see that there is an M^f close to, but below 1, for which the multiplier at this point in time is maximised with a short time to build $\mathcal{T}_{2,I} = 1$. After five years, the multiplier shows a maximum for various time-to-build parameters at $M^f \approx 0.95$, before falling below the value obtained under rational expectations. After 25 years – and even more so after 100 years – that spike in the cumulative discounted multiplier becomes more and more pronounced.[51]

This is consistent with the fact that bounded rationality can in fact dampen or amplify general-equilibrium effects and the overall response of the economic system to shocks (Angeletos and Lian, 2022).

[51]Figures 4.C.28–4.C.31 in the appendix show the relationship between M^f, respectively M^b and multipliers for hybrid expectations. In particular, in Figures 4.C.28 and 4.C.29, we vary the forward-looking component M^f and adjust M^b according to $M^b = 1 - M^f$ (Figure 4.C.28) or $M^b = 0.9 - M^f$ (Figure 4.C.29). In Figures 4.C.30 and 4.C.31, we instead keep M^f constant (at 0.9 in Figure 4.C.30 and at 0.8 in Figure 4.C.31) and vary M_b from 0 to $1 - M^f$. The results point to a spike in medium- to long-run multipliers for $M^f + M^b$ between 0.9 and 1.0. Apart from this, it is notable that with lower M^f and $M^b = 1 - M^f$, multipliers also become larger with lower M^f. Note, however, that with $M^b = 1 - M^f$ and M^f close to 0.5, the economy starts cycling after a shock. Hence, one should not put too much faith in simulations with $M^f < 0.6$ and $M^b = 1 - M^f$.

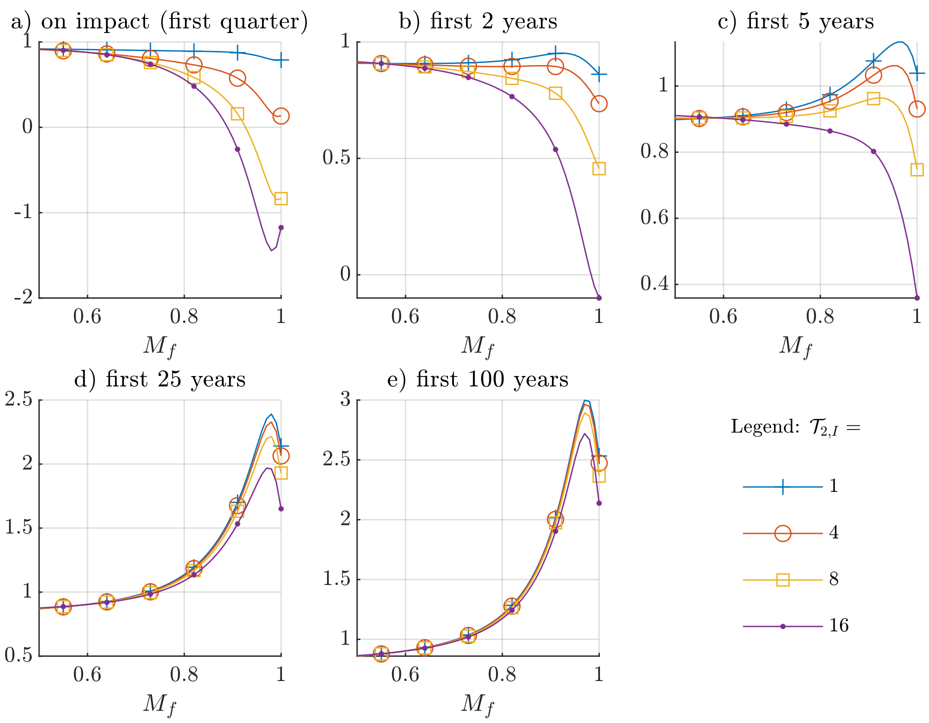

Figure 4.9: Government investment multipliers for different values of $\mathcal{T}_{2,I}$ across different values of M^f for purely myopic expectations, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multipliers of government investment on output after a shock to appropriations in period t_0 for different forms of forming expectations. The multiplier is calculated according to equation (4.64). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. $M^b, \mathcal{T}_{1,I} = 0$ for all simulations.

But where does this long-run amplification for high values of M^f come from? The underlying reason is that $\hat{q}_t$ becomes more sensitive to changes in aggregate output, or more precisely in the consumption of unconstrained households. To see this, let us reformulate equation (4.38) for the case $M^b = 0$:[52]

$$\begin{aligned}\hat{q}_t &= \frac{\sigma}{c_u}\left(\hat{c}_{u,t} - \mathcal{E}_{c,t}\left[\hat{c}_{u,t+1}\right]\right) + \mathcal{F}^{risky}\left(\mathcal{E}_{rk,t}\left[\hat{r}_{K,t+1}\right] + (1-\bar{\delta})\mathcal{E}_{q,t}\left[\hat{q}_{t+1}\right]\right) \\ &= \frac{\sigma}{c_u}\left(\hat{c}_{u,t} - M_c^f \mathbb{E}_t\left[\hat{c}_{u,t+1}\right]\right) + \mathcal{F}^{risky}\left(M_{rk}^f \mathbb{E}_t\left[\hat{r}_{K,t+1}\right] + (1-\bar{\delta})M_q^f \mathbb{E}_t\left[\hat{q}_{t+1}\right]\right) \qquad (4.65) \\ &= \frac{\sigma}{c_u}\mathbb{E}_t\left[\sum_{j=0}^{\infty}((1-\bar{\delta})M_q^f)^j\left(\hat{c}_{u,t+j} - M_c^f \hat{c}_{u,t+j+1}\right)\right] \\ &\quad + \frac{M_{rk}^f}{M_q^f(1-\bar{\delta})}\mathbb{E}_t\left[\sum_{j=1}^{\infty}((1-\bar{\delta})M_q^f)^j \hat{r}_{K,t+j}\right]. \qquad (4.66)\end{aligned}$$

[52]Complementary to this discussion, one can come to this conclusion by looking at the IRF plots for $\hat{q}_t$, $\hat{y}_t$, $\hat{I}_t$ and $\hat{c}_t$. These are depicted for various values of (purely myopic) $M^f < 1$ in the appendix in Figures 4.C.19–4.C.22.

Now, focus on the term

$$\sum_{j=0}^{\infty}((1-\bar{\delta})M_q^f)^j\left(\hat{c}_{u,t+j}-M_c^f\hat{c}_{u,t+j+1}\right).$$

Consider for the moment a completely persistent shock that raises $c_{u,t}$ (and thus $\hat{c}_{u,t}$) permanently by Δc_u. Then the term above becomes

$$\sum_{j=0}^{\infty}((1-\bar{\delta})M_q^f)^j\Delta c_u\left(1-M_c^f\right)=\frac{\Delta c_u\left(1-M_c^f\right)}{1-(1-\bar{\delta})M_q^f}. \tag{4.67}$$

For $M_c^f=1$, i.e. with rational expectations with respect to future consumption, a permanent increase in consumption does not show up in the determination of $\hat{q}_t$. However, with $M_c^f<1$ the term in (4.67) does not vanish, i.e. a persistent increase in consumption leads to higher value of q_t, which via its effects on investment and utilisation can lead to additional capital formation and thus raise output in the long run. This is a somewhat troubling implication of the model proposed, e.g., by Gabaix (2020) and (as can be easily shown) generally extends to models that feature $M_c^f+M_c^b<1$ (i.e., they can be a general cause of concern for models with some form of bounded rationality as Angeletos and Huo (2021) or Angeletos *et al.* (2021). These authors do actually solve forward-looking investment and asset-pricing problems. However, they assume the discount factor to be a constant β, which would lead to some issues when applied here).[53] In fact, a case could be made for some adaption á la adaptive learning (Evans *et al.*, 2009; Evans and McGough, 2020) or another slow-moving backward-looking element. However, we leave this for future research.[54]

Adding Time to Spend

With time to spend $\mathcal{T}_{1,I}>0$, as appropriations are increased in period t_0, there are no actual outlays until period $t_0+\mathcal{T}_{1,I}$, i.e., we have $\hat{G}_{I,t}=0$ for $t=t_0,\dots,t_0+\mathcal{T}_{1,I}-1$. As such, in the very short-run (until the first money flows), we cannot calculate the multiplier in the fashion we have used so far (see equation (4.64)). So, instead, we resort to using an appropriations-based multiplier

$$m_{PV,ap}^{y,\tau}((d_s)_{s=0,\dots,\tau-1}):=\frac{\sum_{s=0}^{\tau-1}(1+g)^s d_s\Delta_x(\hat{y}_{t_0+s})}{\sum_{s=0}^{\tau-1}(1+g)^s d_s\Delta_x(ap_{I,t_0+s})} \tag{4.69}$$

[53]Indeed, when deriving the model, a related observation was one reason for including a backward-looking element.

[54]In appendix 4.C.5, we report the results from a crude robustness check: For this check, we replace the Euler equation on capital, (4.38) by

$$\hat{q}_t=\mathcal{F}^{risky}\left(\mathcal{E}_{rk,t}\left[\hat{r}_{K,t+1}\right]+(1-\bar{\delta})\mathcal{E}_{q,t}\left[\hat{q}_{t+1}\right]\right), \tag{4.68}$$

i.e. we assume that the discount factor of investors is time-invariant (which in and of itself could be considered a type of bounded rationality), we shut down the strong increase in $\hat{q}_t$ due to an increase in consumption. We then rerun the experiments reported so far in this slightly reduced model. Figures 4.C.33 and 4.C.34 report the results in a manner similar to Figures 4.8 and 4.9, respectively. Although the quantities depicted in 4.C.33 are different (in particular, multipliers can be negative and quite large in absolute terms on impact for high values of $\mathcal{T}_{2,I}$ and rational expectations), the overall pattern remains: myopic expectations raise multipliers on impact. In addition, seen together with Figure 4.C.34, it becomes clear that in the model with (4.68), myopia strictly lowers medium to long-term multipliers.

for periods before any actual payments have been carried out.[55]

Figure 4.10 shows the resulting cumulated multipliers for different values of $\mathcal{T}_{1,I}, \mathcal{T}_{2,I}$ under the various expectations considered before. In particular, it shows the cumulated discounted multiplier at horizon $\mathcal{T}_{1,I}-1$ for different values of $\mathcal{T}_{1,I}$; each subplot represents a different time to build – which represents the upcoming time frame of actual investment before public capital actually increases. In a sense, the cumulated multiplier plotted along the vertical axis measures the accumulated negative output deviation that happens before any actual investment takes place. It is notable that the figures look comparable to the ones where time to build was on the horizontal axis, i.e. with rational expectations output drops the furthest, whereas with hybrid expectations, the cumulative multiplier is less negative; but is least negative with (strong) myopia. In fact, with myopic agents the absolute size of the short-run cumulated multiplier is decreasing in time to spend $\mathcal{T}1, I$. With rational expectations (at least, for low values of time to build $\mathcal{T}_{2,I}$), the multiplier is increasing in $\mathcal{T}_{1,I}$.[56] The underlying reason is that unconstrained agents want to save for the future increase in taxes that will happen in order to fund additional public investment. Agents who discount the future break Ricardian equivalence and do not save (enough). As observed before, hybrid expectations tend to lie in between.

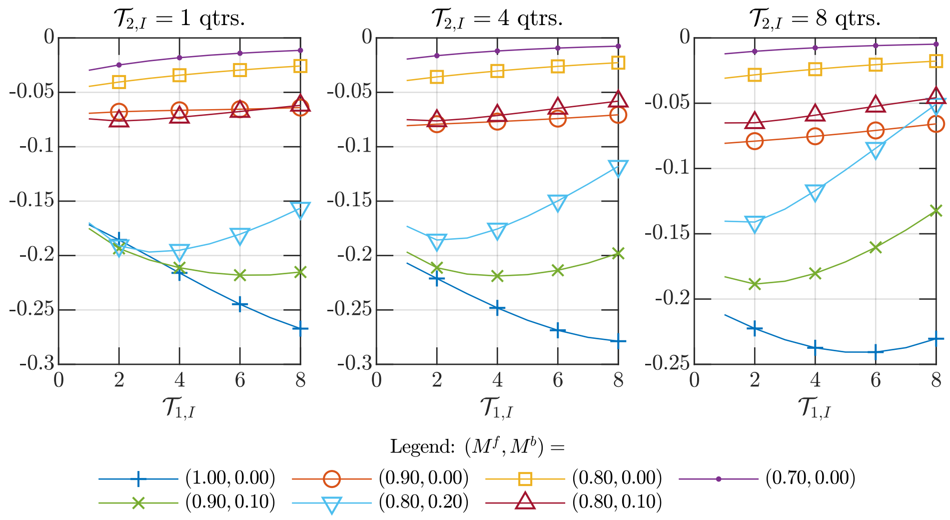

Figure 4.10: Appropriations-based multiplier with time to spend, last period before first payment

Note: The figure depicts the appropriations-based multiplier from equation (4.69) in period $t_0 + \mathcal{T}_{1,I} - 1$ for a shock to appropriations in period t_0 with time to spend $\mathcal{T}_{1,I} > 0$. Each panel represents a different time to build $\mathcal{T}_{2,I}$. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences.

Note that the time to build $\mathcal{T}_{2,I}$ also plays some role, even before any investment has taken place. In the end, however, time to spend mainly acts to lower short-run multipliers

[55] It would also be possible to derive the appropriations-based multiplier in the medium- or long-run. However, in our model all appropriations eventually become government investment, only the timing differs slightly.

[56] For longer time to build, this relationship is no longer monotonic. I.e., with rational expectations, there is a minimum in the multiplier for $\mathcal{T}_{1,I} = 6$ qtrs., after which the depicted multiplier starts rising again. See also Figure 4.C.18 in the appendix, which shows the impulse response function of output in the short run for various values of $\mathcal{T}_{1,I}$, given expectations and values for $\mathcal{T}_{2,I}$.

by adding negative output responses before any public spending has taken place. In the long-run, the effects of time to spend are negligible.[57]

Hence, limited forward-lookingness of expectations also dampens the negative effects of time to spend on short-run multipliers of government investment. However, one should note that overall, the pattern obtained for time to build is very similar to the one obtained for time to build: Short-run multipliers become less sensitive to implementation delays. This 'biases' short-run multipliers before any spending has occured towards zero. Conditional on that, once spending on a project has started, the limited forward-lookingness makes government-investment multipliers more akin to regular government consumption multipliers. Agents (partially) neglect the productivity- and output-enhancing effects of additional public capital in the future and instead mainly react to current income effects.

Summing Up the Results So Far

A key takeaway from this discussion is that there is an important interaction between the formation of expectations and time to build or spend in determining government investment multipliers:

1. In the short run, the more forward-looking agents are (i.e. the higher M^f), the smaller is the short-run multiplier of public investment. This effect is stronger with a longer time to build. Also, with hybrid expectations (i.e. a backward-looking term) and $M^f + M^b = 1$, this effect is only present for longer running investment projects.

2. However, in the medium and long term, multipliers on government investment can become quite large. With rational expectations, the discounted cumulative multiplier can exceed 2. Myopia here has a non-trivial role: For low degrees of myopia, pure myopia ($M^f > 0.95, M^b = 0$) can actually raise long-run multipliers relative to rational expectations, whereas for higher degrees of myopia, long-run multipliers decrease in the extent of myopia. Also note that in the long run, the multiplier is mostly determined by the sum $M^f + M^b$. I.e., a backward-looking term in expectations may retard the initial response, but in the long run this does not matter as much.

3. Following from the previous point, we can deduce that the sum $M^f + M^b$ ultimately reflects the degree of dampening of general-equilibrium effects in our model as discussed by Angeletos and Lian (2022). The result that low degrees of myopia can raise long-run multipliers is however due to a particularity of combining myopia in the form of expectations biased toward the steady state with an Euler equation for capital. This interaction should be explored more thoroughly in future research.

Note that these results are not due to choice of the net present-discounted value multiplier: Figure 4.C.27 in appendix 4.C.5 presents the cumulative, undiscounted integral multiplier for the same periods as Figure 4.8.[58] In appendix 4.C.1, we further expand on the results.

[57]To see this, look at Figure 4.C.32 in the appendix: The figure shows the government-investment multiplier as depicted in Figure 4.8 before. The two differences are that, here, we have positive time to spend of a year ($\mathcal{T}_{1,I} = 4$). Consequently, the 'impact' multiplier is not well defined. Hence, instead, we use the discounted cumulative multiplier for the period $t_0 + \mathcal{T}_{1,I}$. Note that time to spend mainly acts to decrease short-run multipliers (panels a) and b)), while leaving medium-and long-run multipliers generally unaffected.

[58]An analysis of Figure 4.C.27 implies: The patterns remain the same, although magnitudes change in accordance with the results presented in the previous subsection.

4.4.3 Adding Other Dimensions to the Analysis

In the main text, we continue by introducing additional dimensions to the analysis, as set out before. This should also be considered a robustness analysis. In the following, we abstract from time to spend ($\mathcal{T}_{1,I} = 0$) and only focus on time to build $\mathcal{T}_{2,I} > 1$. One might be inclined to also investigate the extension with time to spend. However, the main results can also be illustrated without it. Hence, we leave it to future research at this point.

Government Debt

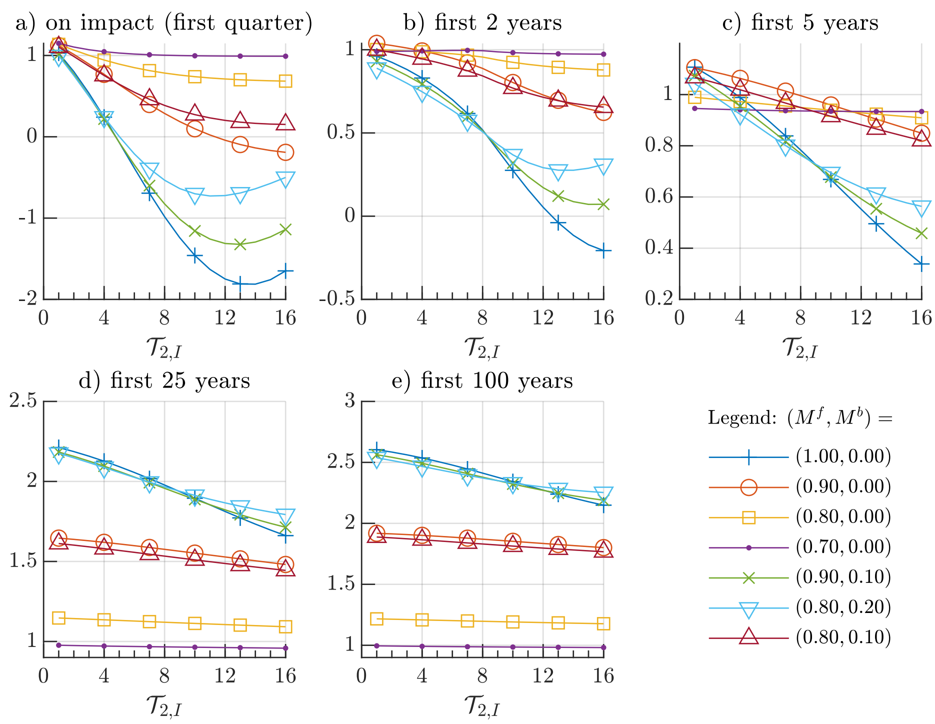

Figure 4.11: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, with bonds issued

Note: The figure shows the discounted cumulated multipliers of government investment on output after a shock to appropriations in period t_0 for different forms of forming expectations. The multiplier is calculated according to equation (4.64). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no steady-state government debt and no wealth preferences. Government taxation is given by (4.56).

As a first step, we allow for the government to issue bonds while keeping steady-state debt at $b = 0$. I.e., fiscal policy is given by (4.56), but in steady-state nothing else has changed. Figure 4.11 presents the results in the same fashion as Figure 4.8. The overall patterns remain broadly the same. In fact, for short times to build allowing for government debt to fluctuate has a slightly positive effect on multipliers throughout, a somewhat negative one with longer times to build and rational expectations. However, compared to the differences across expectations, these differences are tiny and do not affect the general observations from before. Similarly, if we keep the flexible bond issuance from equation (4.56), but increase the debt level to $b = 4y$ such that the (annual) debt-to-GDP ratio is given by 100%, broadly in line with recent trends in the Western world, the fundamental pattern is also unchanged – see Figure 4.12. The data it presents correspond to those in Figure 4.11 except for the higher steady-state debt (and consequentially, a different composition of households' income).[59] Note, however, that adding steady-state debt to the model raises multipliers across the board, in particular for myopic expectations.

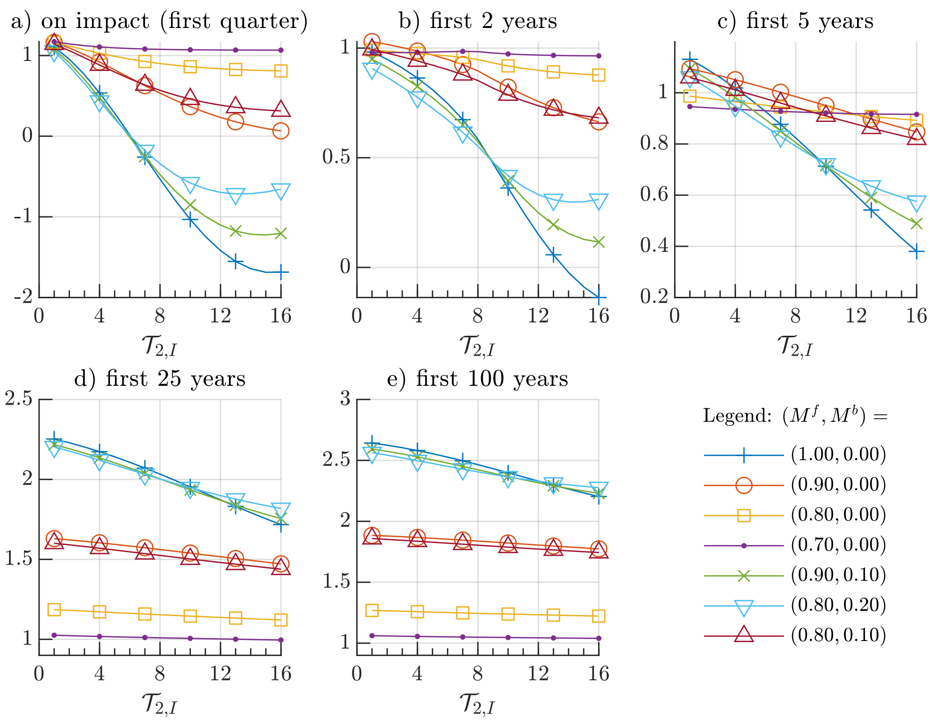

Figure 4.12: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, with bonds issued. Steady-state debt: $b = 4y$

Note: The figure shows the discounted cumulated multipliers of government investment on output after a shock to appropriations in period t_0 for different forms of forming expectations. The multiplier is calculated according to equation (4.64). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), steady-state government debt of 100% of annual GDP and no wealth preferences. Government taxation is given by (4.56).

Effective Lower Bound

Given the experiences of major advanced economies in the aftermath of the financial crisis of 2008, a particularly interesting dimension to add to the model is the effective lower bound (ELB) on nominal interest rates, i.e., a situation where monetary policy cannot perform its role via traditional interest rate setting because it cannot lower its policy rate any

[59] Our recalibration strategy that keeps C_c/C_u constant means that – in the TANK model – the steady-state distribution of taxes is adjusted. For unconstrained agents, this implies $\Delta T_u = -\Delta\left(\frac{r-g}{1+g}b\right)$, where the Δs describe the steady-state adjustments. The major impact is that in the linearised model, with $b \neq 0$, the interest rate has a direct income effect on unconstrained agents.

further – although it still would like to. Effectively, at the ELB, nominal rates are pegged to a constant value, which can alter the dynamics of the model drastically. From earlier Keynesian models, it is well known that if the interest rate does not respond to changes in output, this increases the multipliers associated with government spending. In traditional, backward-looking models, this is mostly due to very interest-rate-sensitive private investment – a constant interest rate then reduces crowding out. In a dynamic forward-looking context, i.e., in particular in Dynamic Stochastic General Equilibrium models, various New Keynesian 'paradoxes' emerge. In recent years, the 'forward-guidance' puzzle has received a lot of attention. For the current analysis, in contrast the paradox of toil (Eggertsson, 2010) is the most relevant one: According to that paradox, at the ELB, positive productivity shocks can be contractionary. Since public investment is generally assumed to be productive, e.g., infrastructure investment projects that finish during an ELB spell will lead to negative effects on output in these models. Conversely, according to those models, destroying infrastructure while at the effective lower bound would raise output. This in and of itself is a troubling implication of New Keynesian models.

In that regard, the results of Bouakez *et al.* (2017, 2020) are not surprising. They combine a New Keynesian model with public investment projects that have longer-running time to build and evaluate the projects' multipliers at the ELB. They find that, for longer time to build $\mathcal{T}_{2,I}$, the immediate productivity-enhancing and deflationary impact of additional infrastructure becomes less relevant. This weakens the impact of the productivity-enhancing effects, and thus leads to a higher overall multiplier. This is also reinforced by the fact that the expected positive income shock in the future (when the project has finished) raises the 'natural real rate'. This reduces the pressure faced by the central bank to lower its policy rate and also acts expansionary. Ramey (2021) criticises the implications derived at the ELB because the mechanism behind the paradox of toil also implies that at the ELB, distortionary taxation can be expansionary. However, Bouakez *et al.* (2017, 2020) also note that for larger shocks that lift the economy off the ELB earlier, this increase in the multiplier is weakened – which is mostly due to the fact that then the effects illustrated so far become relevant again.

How do imperfectly forward-looking expectations affect this mechanism? We gauge this using particular set of simulations. Remember from section 4.3 that for each constellation of expectational parameters M^f and M^b, we find a demand shock ε_{D,t_0} that pushes the economy from the steady-state to the ELB from the initial period t_0 for exactly 2 years (8 quarters).[60] We then assume that in this environment, also in period t_0, appropriations for public spending are raised by 0.5% (which is 0.0125% of quarterly GDP in the baseline calibration), a shock small enough not to move the economy from the effective lower bound. This shock is, once again, highly persistent (with autoregressive coefficient 0.95).

Figure 4.13 depicts the multipliers obtained from these simulations – the figure can be interpreted in the same way as Figure 4.8.[61] In fact, it shows the same calibration with $b = 0$ and no bond issuance in order to fund the investment projects. Looking at the results, a very striking first impression can be obtained: Hybrid expectations actually increase the multiplier enormously when at the effective lower bound, at least in the

[60] Note that with different expectational parametrisations, the economy's dynamic response to all shocks is affected. As a result, the counterfactuals are slightly different in terms of state variables etc. There does not seem to be a clear way to better make such simulations comparable.

[61] Figure 4.C.41 in the appendix depicts the corresponding appropriations-based multiplier. Note that this multiplier is actually still declining in $\mathcal{T}_{2,I}$.

inital period and for times to build that are between one and three years. In fact, with $M^f = 0.8, M^b = 0.2$, the multiplier is larger than 30 for a time to build of roughly 2 years. A couple of observations are noteworthy with respect to: The peak of this increase in multipliers in the initial period occurs at a time to build that is consistent with the projects being finished just after the ELB spell has ended, reflecting the previous discussion. Second, for even longer time to build, the multiplier rapidly decays, in accordance with what we have just discussed; for $\mathcal{T}_{2,I} > 15$, it even becomes negative again. Note that the extremely large multiplier associated with this type of expectations for longer times to build is partially due to a lower initial government investment. In fact, if we instead plot the pure impulse response function of output across $\mathcal{T}_{2,I}$ for the inital period, we see that the output response for $\mathcal{T}_{2,I} = 8$ is only slightly larger than for $\mathcal{T}_{2,I} = 1$, see Figure 4.C.36 in Appendix 4.C.5.

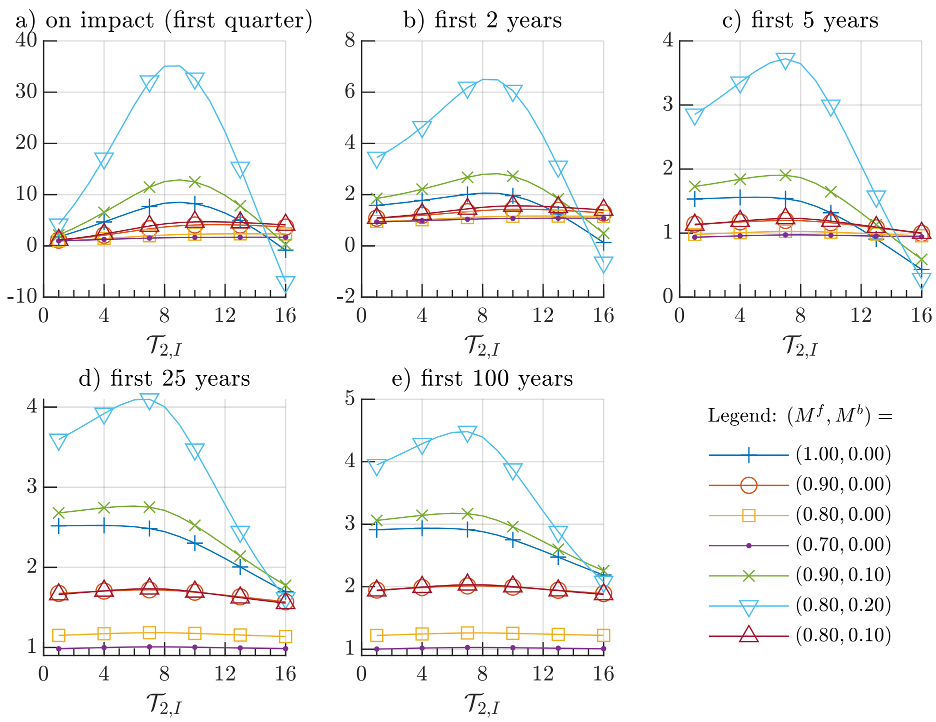

Figure 4.13: Government investment multipliers at the ELB for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, no bonds issued. Steady-state debt: $b = 0$

Note: The figure shows the discounted cumulated multipliers of government investment on output after a shock to appropriations in period t_0 for different forms of forming expectations. The multiplier is calculated according to equation (4.64). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment.

Note that a similar pattern also obtains with the other hybrid expectations, where $M^f + M^b = 1$: The government multiplier in the initial period is, however, highest for a time to build of 9 quarters, which is one period longer than the ELB spell, this peak in the multiplier is at above 10. Meanwhile, with rational expectations and myopic expectations (or with hybrid expectations and $M^f + M^b = 0.9$), the impact multiplier is markedly smaller.[62]

For rational expectations, purely myopic expectations and for $M^f + M^b$, the impact multiplier is still quite larger than without the ELB (except for long time to build and rational expectations). In fact, for most simulations, the multiplier is positive, but within a more 'conventional' range: For rational expectations, the impact multiplier is roughly 2 for a time to build of 1 period – however it rises to roughly 8 with a time to build of 2 years. This again confirms the results of Bouakez *et al.* (2017, 2020). Compared to this, myopic expectations see a somewhat muted multiplier for short time to build. But, strikingly, with myopic expectations, the impact multiplier still rises in the time-to-build parameter. Here, for $M^f = 0.9$, it peaks out at a value for $\mathcal{T}_{\in,\mathcal{I}}) = 11$ with a multiplier of roughly 4. Even more myopic expectations imply a lower impact multiplier that is nonetheless increasing in the time to build. Hybrid expectations with $(M^f, M^b) = (0.8, 0.1)$ have a slightly higher multiplier than myopic ones with $M^f = 0.9$. As we observed before without the effective lower bound, the myopic expectations considered tend to mute the response of the economy to shocks. However, since myopic expectations per se do not resolve the paradox of toil (see also the previous chapter or Gabaix (2020)), it is not an utter surprise that we still observe the somewhat counterintuitive result that the impact multiplier is larger with higher time to build. But, on the other hand, we can also see that this effect is muted. In addition, note that with myopic expectations, at the ELB, the impact multiplier is always positive.

Next, turning to medium- and longer-run multipliers: The fact that the impact multiplier is so large translates into larger medium- to long-term discounted cumulated multiplier values that also display a similar pattern. However, in the periods following the shock, the cumulative multiplier actually declines.[63] Here, this decline is due to the fact that the output response is front loaded: At the ELB, the government investment shock acts as a combined positive demand and news shock that in the initial period lifts output more than in latter periods, at least when expectations are strongly forward-looking. In the following periods, as more and more government investment is being conducted, the denominator in the term for the cumulative multiplier expands faster than output, pushing the multiplier down. Once the economy leaves the ELB, the logic from the previous subsection can be applied again: With rational expectations and hybrid ones that have $M^f + M^b = 1$, the increase in public and private capital becomes self reinforcing. As was the case before without the ELB, purely myopic expectations do not accomodate this persistent effect. As a result, the long-run multiplier stays below the one obtained under rational expectations.

Notably, the large short-run multiplier at the ELB also elevates the calculated (discounted) long-run multipler,[64] and mostly so for those investments that first finished around the time the economy left the ELB. For rational expectations and $M^f = 0.9, M^b =$

[62] Indeed, the scaling in Figure 4.13 might make it rather hard for the reader to distinguish the various lines and read off the multiplier values. Therefore, Figure 4.C.35 in Appendix 4.C.5 shows the same data as Figure 4.13, but omits the two hybrid expectations with $M^f + M^b = 1$. Also, Figure 4.C.36 shows the deviations from the counterfactual (as before, scaled by the deviation in initial appropriations).

[63] For a depiction of the decline in the multipliers while at the effective lower bound, see Figure 4.C.37 in Appendix 4.C.5 that depicts the discounted cumulated multipliers at intervals of six months for the first two years, i.e. during the ELB period. It should be noted that the multipliers decline almost proportionately. However, note that the shapes of the multipliers across $\mathcal{T}_{2,I}$ remain broadly identical (although scaled down) for all forms of expectations considered.

[64] Note that this is also true for the undiscounted integral, see Figure 4.C.39 in Appendix 4.C.5.

0.1 it settles at roughly 3 for $\mathcal{T}_{2,I} \leq 8$, for the expectations with $M^f + M^b = 0.9$ at roughly 2. For even more myopic expectations, the long run-multiplier is again severely muted.

Agent Heterogeneity

Allowing for household heterogeneity along the lines of Bilbiie (2021) does not affect the fundamental results much, either. To see this, we set the transition probability $p_{c|u} > 0$. More precisely, we set $p_{c|u} = 0.025$, i.e., each quarter, 2.5% of unconstrained agents becomes constrained. Figure 4.14 shows the discounted values of multipliers in the by now familiar fashion for the case that there is no debt (and no debt issued to fund the additional spending). The results are still broadly in line with the results obtained before in a classical TANK setup (cf. Figure 4.8). Note however, that the impact response for rational expectations or the closely related hybrid expectations with $M^f + M^b = 1$ is less negative in the THANK setup than in the TANK setup. In fact, for very long time to build, with hybrid expectations, the impact multiplier is larger than with some purely myopic expectations.

The underlying reason here is the precautionary savings motive. In both the TANK and the THANK model, due to the increased taxation, all agents consume less – but due to the impact of an upward sloping consumption profile (after all, government investment here also acts as a positive news shock regarding future productivity), the unconstrained want to reduce their savings and 'borrow' against the expected future income. The subsequent drop in investment and overall output leads constrained agents to reduce their consumption even further. In addition, both the THANK and the TANK framework have one hard-wired inherent mechanism that comes into play: The shares of constrained and unconstrained agents is fixed, and so is their share in aggregate output before taxes. Constrained agents only reap the benefits of labor income, whereas unconstrained agents also receive profits and capital income. As a result, income inequality increases or decreases – also in the long run – unless we have a knife-edge parameter constellation. For our given calibration, in a THANK environment, however, the constrained's consumption rises less in the long run than the unconstrained's, raising income inequality.[65] Future research should try to also shed light on the detailled transmission at the microeconomic level. In particular, in order to assess questions of inequality, analyses with HANK models á la Kaplan *et al.* (2018) will be instrumental.

Overall, in the present calibration, the precautionary savings motive in THANK mitigates the immediate fall in output, as unconstrained agents do not cut consumption and investment as much initially – which then keeps unconstrained agents from cutting their consumption etc. Notably, this effect is weaker for myopic agents – and as a result, impact and short-run multipliers for these types of expectations do not change by much. However, in the long-run, the multipliers still approach nearly the same overall (discounted) multiplier as within the TANK framework for each type of expectations considered.

Adding debt to the THANK model does have non-trivial implications: Without any government debt, there is a precautionary-savings motive, but no means to actually perform savings for the idiosyncratic state transition to being constrained (remember that we abstract from private liquidity and assume that agents cannot take capital with them when they become constrained). Hence, by allowing for the government to issue bonds according to equation (4.56), we allow the government to satisfy the households' precautionary concerns. This, however, also reduces the effect on the multiplier. Figure 4.15

[65] The underlying reason is that we assumed progressive taxation in steady state.

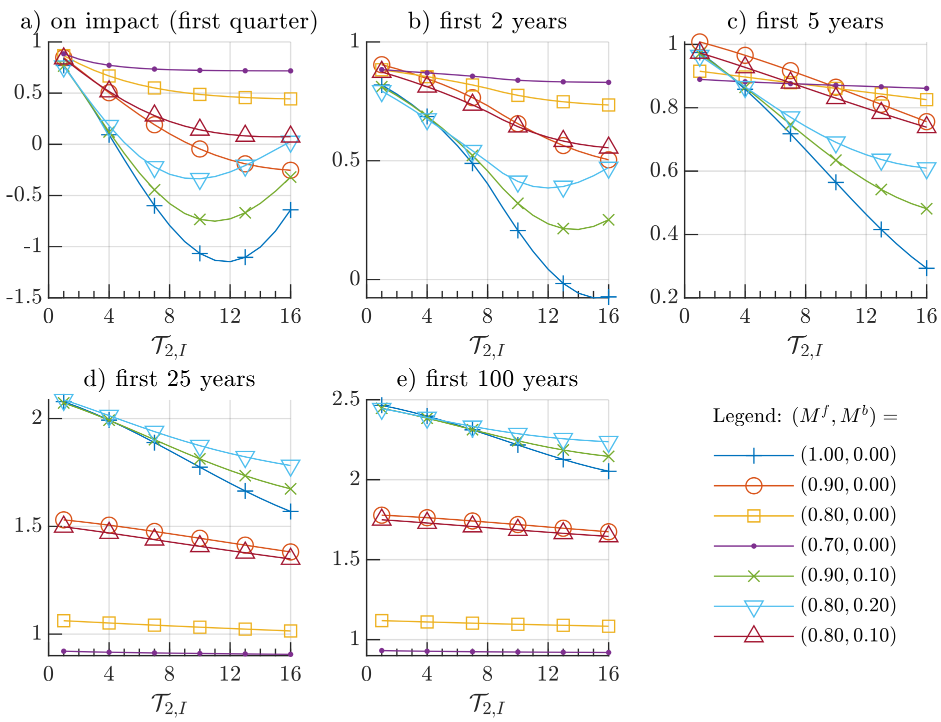

Figure 4.14: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, THANK model, no debt.
Note: The figure shows the discounted cumulated multipliers of government investment on output after a shock to appropriations in period t_0 for different forms of forming expectations. The multiplier is calculated according to equation (4.64). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear THANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences.

shows the corresponding short-, medium- and long-run multipliers.[66] Again the main effects other than the ones previously established can be seen in lower short-run multipliers. The differences to the corresponding Figure 4.11 from before (and underlying data) are miniscule. However, they point exactly in one direction: With the THANK structure, multipliers with the government issuing debt are slightly lower than with a pure TANK setup, but only for rather forward-looking agents (rational expectations, hybrid expectation). With myopia, on the other hand, this negative effect is somewhat remedied, and instead the positive income effect of debt on the unconstrained leads to slightly higher multipliers. Here, however, the exact calibration matters, which we delegate to future research.

[66]Figure 4.C.42 in the appendix shows that, as before, adding steady state government-debt raises short-run multipliers also in the THANK framework, whereas it affects long-run multipliers little.

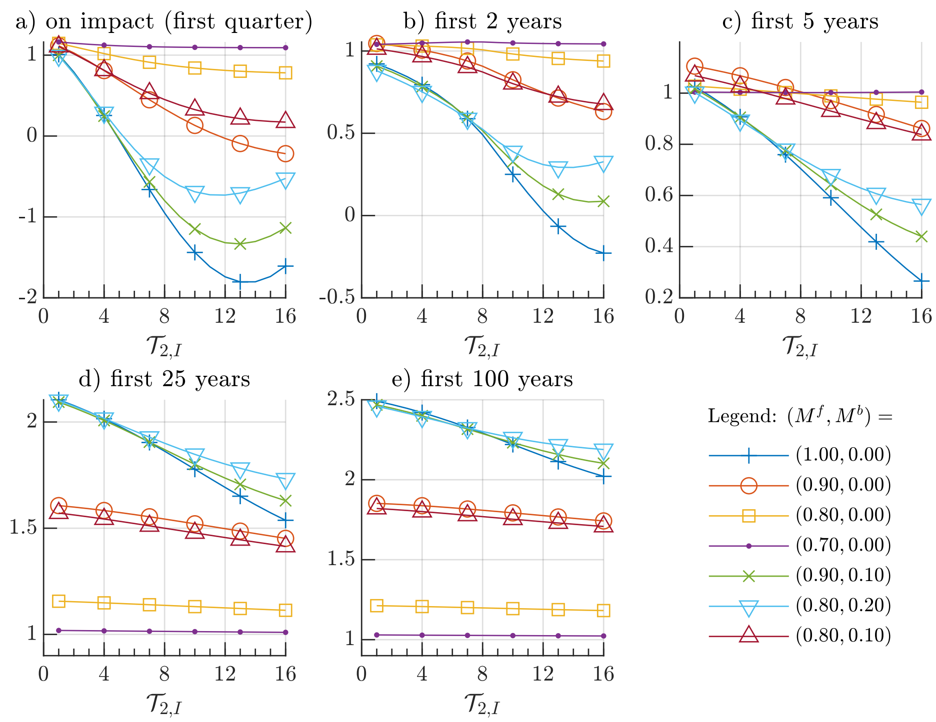

Figure 4.15: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, THANK model, bonds issued, no stady-state debt ($b = 0$).
Note: The figure shows the discounted cumulated multipliers of government investment on output after a shock to appropriations in period t_0 for different forms of forming expectations. The multiplier is calculated according to equation (4.64). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences.

Accommodating $r < g$ in the Model

As already discussed before, the THANK model with inequality necessarily implies that steady-state real interest rates r are smaller than the net return on capital $r_K - \delta$. However, while it is possible to push c_c/c_u so low or $p_{c|u}$ so high that $r < g$ holds in the steady-state, this also makes the model too forward-looking in the sense that there are then violations of the Blanchard-Kahn conditions. On the other hand, a convenience yield via wealth preferences (as used e.g., by Illing *et al.* (2018) or Mian *et al.* (2021, 2022)) (via $\bar{\xi} > 0$) can push steady-state interest rates below the growth rate easily. Also, as shown by Michaillat and Saez (2021), this can resolve some anomalies at the effective lower bound. In the context of our linearised model, this will not affect the insights in an important way. The reason is that $r - g$ immediately affects only the debt dynamics. Since we have already seen before that debt (at least introduced along the lines explored before) per se does not affect our results in a meaningful way, pushing steady-state r down (even below zero) does not affect the results much.[67]

[67]See figures 4.C.43-4.C.45 in the appendix: The figures compare the cumulated discounted multipliers across calibrated steady-state values for r (and implied $\bar{\xi}$) for the baseline TANK model without growth. The different rows show the value of the multipliers at different points in time, the columns different values of the time to build. Figure 4.C.43 shows the situation without debt, Figure 4.C.44 the one with

As a final experiment, we also considered varying steady-state growth g. Here, we kept the other parameters and steady-state relationships G/y, G_I/y, c_u/c_c constant – and in particular, we keep r constant (at 0.5% annualised, achieved via appropriate $\bar{\xi}$). In our model, we assume that all government investment is subject to the same time-to-build constraint, and as a result, for longer time to build, the ratio of public capital to government investment[68] (and hence, also to output) is lower.[69] This in turn increases the multipliers we obtain because the public capital stock becomes lower. In fact, if we were to estimate this model, one would have to account for this effect to obtain reliable estimates of α_p. However, again, if we look at the multipliers, we see that adding growth (and $r < g$) does not affect our insights (see Figure 4.C.46 in the appendix)[70]. We do not consider this experiment to be an ideal one (it would be better to also keep some other variable relationships constant). But there is no clear indication what exactly to keep fixed.

Future research should also consider that g and r might be endogenous objects. In a full model, public investment (e.g., in research or educational infrastructure) might actually have larger effects by altering trend growth.

4.5 Discussion

4.5.1 Summary of Results

Now it is time to briefly summarise our results, before discussing where they lead us. The main message of the results obtained in the previous section is that there is an important interaction between the time to build on government capital formation and limited forward-lookingness when it comes to the multiplier effects additional public investment has on output. Public investment acts as a combined We have seen that in the short run, the multiplier associated with public investment can be quite small and even negative for long time to build when agents are rational or have expectations with sufficiently high weights on forward- and backward looking elements (meaning $M^f + M^b \approx 1$). We have also seen that also in the long run, multipliers with myopic preferences might be significantly different than those obtained with rational expectations. We have observed that at the effective lower bound, (especially short-run) multipliers associated with government investment can be quite large, especially if they also contain backward-looking elements. Regarding the other additional dimensions considered in the previous section – government debt, agent heterogeneity and the $r - g$ gap, our results indicate some interaction, too, although it is quite limited.

In any case, researchers that wish to estimate government spending (in particular investment) multipliers should (as far as possible) take into account the implementation

debt at 100% of annual GDP and with bonds issued to finance public investment. Overall, although there is some effect of r on the multiplier, it is smaller than the effect of the different expectations considered. This changes, obviously, if we also use the safe discount factor ($\frac{1}{1+r}$), as illustrated in 4.C.45. Then, the multiplier tends to decrease in r. Note, however, that this effect only becomes noticeable after several years, because before that both government investment and the output response are positive.

[68]Given our assumptions from above, that ratio is given by $\frac{k_P}{G_I} = \frac{a p_I/(1+g)^{-(\mathcal{T}_{1,I}+\mathcal{T}_{2,I})}}{(1+g)^{-\mathcal{T}_{1,I}} \sum_{s=0}^{\mathcal{T}_{2,I}-1} \chi_s (1+g)^{-s}} \frac{1+g}{\delta_P}$.

[69]Similarly, with $g > 0$, private investment is higher and the private capital stock lower, relative to output.

[70]Note that in the figure, due to the effects g has on the ratio of k_P to G_I, comparisons across columns (different values of $\mathcal{T}_{2,I}$ should be made with caution.)

delays, and more crucially account for the effects on expectations. Isolating the effects of public investment on expectations will, however, be a somewhat complicated affair.

Next, let us turn to discussing some features of our model, before we give an outlook on which other aspects of public investment should be interesting from a macroeconomic point of view.

4.5.2 Simplifications of the Model

The stylised model presented above contains various simplifications, many of which could potentially affect the results. Obvious ones concern the assumed functional forms and parametrisation, which clearly can be debated and call for an estimated version of the model. Also, some methodological choices prevent certain relevant analyses. In particular, by only considering a linearised model, we neglect that the investment multiplier may depend on the size and also sign of the shock; also uncertainty does not play any role. In addition, the ad-hoc assumptions, in particular on expectations, make the model and its implications vulnerable to the Lucas (1976) critique. However, we do not view the quantitative implications as definitive answers on the magnitude of public-investment multipliers, but rather as a starting point for a discussion on the interaction of expectations and long-run government stimuli.

In that regard, one should note that uncertainty about public investment in and of itself is an interesting point, which could affect multipliers – especially when considering that expectations may not be fully rational: In particular, there can be uncertainty about cost overruns (including follow-up costs), time delays and the productivity of public capital. Ultimately, ex post, an investment project may be abandoned – think of the various 'bridges to nowhere' found in Western economies. However, even if pursued, investment projects may generate large additional costs without a corresponding productivity gain, both during construction or thereafter – consider the Berlin Brandenburg airport:[71] That airport faced large cost overruns and unexpected time delays, mostly due to construction errors. This caused additional expenditure related to the construction itself and to legal issues. When the airport finally opened in 2020 amid the Covid-19 pandemic, it could barely be used by passengers. Consequently, it generated unexpected losses and expenditures for the public sector.[72] In fact, such factors may themselves be reasons why agents expectations' about public investment projects are biased, especially considering that personal experience appears to affect (macroeconomic) expectation formation.[73]

In addition, a few more basic conceptual points remain: In the model presented above, we assumed that public capital is a homogeneous good and that there is a single-layered government deciding on it. In the following, we will present a couple of remarks on these assumptions.

4.5.3 Heterogeneity of Public Capital

We modeled public capital as a homogeneous good, which it clearly is not in reality: Roads and railway connections, transportation infrastructure and energy distribution systems are distinct from each other. Each type of infrastructure has unique features, and they

[71]See various media, reports, e.g. by Deutsche Welle (Kinkartz, 2020, in German).

[72]See, e.g., European Commission (2022).

[73]See the results of Malmendier and Nagel (2015) on the effect of inflation experiences on inflation expectations.

have non-trivial relationships with each other and with private sector activity. E.g., road and rails transportation can be considered as substitutes for one another; at the same time, roads form a particular type of public good in a complementary relationship with private road vehicles. There can be significant path dependencies, too, depending on the composition of both public and private capital. Also, investment projects can have distinct scales (also timelines in the sense of *time to build* or *time to spend*), ownership and cost structures, uncertainties etc.

In any case, once one starts thinking about heterogeneity in the context of the model, various layers appear interesting: heterogeneity of public investments, heterogeneity of firms and sectors, and heterogeneity of households. From the literature on Heterogeneous-Agent New Keynesian (HANK) models, (Kaplan *et al.*, 2018), it is now well understood that agent heterogeneity (at the household and/or the firm level) can have non-trivial effects on transmission channels of monetary and fiscal policies. To the extent that the above-mentioned heterogeneities interact,[74] they may affect macroeconomic outcomes, too.

In that regard, an important result from the literature is that the productivity-enhancing effects of public (infrastructure) investment can be heterogeneous across sectors according to their exposure, see, e.g. Fernald (1999). Likewise, different types of public investment can have different macroeconomic effects, see Bom and Ligthart (2014) and references therein. In addition, Cox *et al.* (2020) find that (generic) government spending in the U.S. affects sectors and firms in a heterogenous fashion. Likely this is also true for government investment. Also, different agents may benefit from different investments in quite distinct ways because not all agents may actually use said infrastructure in the same way (e.g., an airport may only be used by wealthier households; whereas poorer households only benefit indirectly). Here, apart from immediatly recognizable 'economic' factors like effects on income and labour supply, indirect effects via time-savings (Gallen and Winston, 2021) can play an interesting role. Given these differences in exposure, agents may also differ in important ways in terms of their formation of expectations regarding public investment. Clearly, much work is still to be done to understand these various heterogeneities and how they affect the transmission of fiscal (and monetary) policies.

A more subtle form of heterogeneity of public capital concerns even more indirect factors such as environmental sustainability (both of the public capital and the private-economy outcomes related to it). E.g., transportation infrastructure may negatively affect ecosystems and contribute to climate change in some form. These indirect costs need to be internalised. Similarly, even though the impact of infrastructure itself may be limited, the effects of private use (e.g., usage of an airport by planes, of a road by cars) should be considered, too. In terms of our model, this might affect (especially long-run) multipliers of individual projects, but also of macroeconomically relevant investment packages.

In this context, one should note that strong networks effects within a given type of infrastructure may generate lock-in effects.[75] E.g., a strong reliance on road transportation in the past can cause the private sector to align its capital (and technology) accordingly. This, in turn, raises the perceived need (and thus, likely, associated short- to medium-term multiplier) for road infrastructure. This is related to what Gordon (2017) calls a 'chicken-egg problem' between (certain types of) private and public capital. When a long-term

[74]By 'interact', we particularly mean that a significant correlation exists as in Auclert's (2019) seminal analysis of the joint distribution of marginal propensities to consume and income elasticities.

[75]See, e.g., Klitkou *et al.* (2015) for an analysis of lock-in mechanisms in the context of road transport.

transition seems desirable, this can generate significant trade-offs for public investment because the immediate short- to medium-run benefits are not evident. Generally, expectations management seems to be an important factor in steering private investment – in particular if agents are not fully rational or there is asymmetric information (which likely is the case in this context).

One could consider the prominent focus on the 'green transition' block within the Next Generation EU framework[76] as an attempt at taking all of this into account. At the time of writing this thesis, however, the overall effectiveness cannot be fully assessed. It will certainly be the subject of various studies in upcoming years.

4.5.4 Heterogeneity of Decision-makers

In addition, in our model, we have attributed the entire decision process to a consolidated government and a single positive shock to public investment. Also, we have abstracted from other interactions between public investment and other fiscal variables (except for overall tax level and/or debt). In reality, however, there are various layers of public administrations, each of which might react to others in a particular way. In particular, in many states, e.g., the U.S. or the larger members of the EU, a large share of public investment is performed by subnational or regional entities. There may as such be important spillover effects across localities. E.g., Cohen and Paul (2004), find important spillover effects across U.S. states concerning infrastructure investment. Gordon (2017, p. 15) points out that, historically, these spill-over effects (in that case of paving roads) can actually entail a 'free-rider' problem, which leads to under-provision of public capital. In the Euro area, spillover effects across member states have typically been found to be positive, but small, in normal times; and larger with a binding ELB (Alloza *et al.*, 2019). More recently, using the framework of Ramey's (2020) model, Pfeiffer *et al.* (2021) estimate significant positive spillovers, which are also in line with arguments put forth by Picek (2020). These analyses for the Euro area generally suppose that government investment has an indirect spill-over via increased income in one nation and consequently higher import demand from the other members. However, here, relating to the point made in subsection 4.5.3, not all capital is created equal. In particular, some infrastructure investments may actually directly facilitate trade with other nations. In that case, there can be larger direct spill-over effects.

Another aspect that should be kept in mind is that decisions at one level of government might impact decisions by others. E.g., with respect to the Next Generation EU funds created by the EU Commission, Gros *et al.* (2021) stress the importance of 'additionality', meaning to what extent public investment by the higher layers actually increases overall public investment. In particular, they worry that funds from the higher layer of government (here, the EU) are seen by lower layers (the member states) as an incentive to cut back their own investment. In fact, which a homogenous capital goods, investments by different public actors could be seen as substitutes. If of course, there are various complementary types of public capital, and the higher layers of government invest (or foster investment) into one that has been undersupplied, there might be direct incentives to raise investment in others, too. At a regional level, e.g., Dörr and Gäbler (2020) find that a set of German municipalities used the completion of a federal infrastructure project (in their case the building of a motorway) to raise local property taxes. In a macroeconomic

[76] See the national *Recovery and Resilience Plans* or the Bruegel dataset (Darvas *et al.*, 2022) for details.

model accounting for richer heterogeneity, this might also have macroeconomic relevance by affecting savings decisions.

Lastly, in our model, the private sector took on a quite passive role: There is a single exogenous shock, and public capital is fully exogenous to the private sector, which only reacts. In fact, there are multiple channels through which private agents can affect the choices of investment projects pursued at all states of implementation (e.g, elections, lobbying, protests, to name a couple). In some cases, especially with public-private partnerships or some extent of privatisation, this role can be quite an active one. Note that for the interactions of different levels of government and for the involvement of the private sector, the structure of expectations will again be a relevant feature.

4.5.5 Expectations

Regarding the expectations used in our model, we are well aware that the ad-hoc formulation is a relatively crude proxy and as such should not be taken at face value. In particular, in the main text, we have assumed that all expectations behave in the same way. Also, we have assumed that the expectations feature constant parameters. This stands in some contrast with the results of Angeletos *et al.* (2021), who come to the conclusion that the parameters M^f and M^b are actually endogenous objects that depend on, inter alia, multipliers. In the end, a successful path towards obtaining a better model will require estimation. The methodology presented by Angeletos *et al.* (2021) offers one possible avenue towards that goal, which also seems appealing because their model of expectations is more parsimonious than, e.g., Sims's (2003) rational inattention. Here, also the comments by Reis (2021a) should be taken into account, who stresses that disagreement among agents might be a central ingredient in a model of macroeconomic expectations. At the same time, it also seems worthwhile applying a rational-inattention model or one with adaptive learning to a macroeconomic model with public investment.

4.6 Conclusion

In this chapter, we have used a Two-Agent New Keynesian model (or its close cousin, the Tractable Heterogeneous Agent New Keynesian model of Bilbiie (2021))to illustrate that imperfect macroeconomic expectations can have significant impacts on the multipliers of persistent productive government investment increases with implementation delays. In particular, we have shown that short-run multipliers tend to be higher if agents' expectations feature discounting of the future, compared to rational expectations. This effect is stronger if public investment features a long time to build (or other implementation delays). Long-run multipliers, however, depend on the overall type of expectations. With purely myopic preferences, long-run multipliers can be larger than with rational expectations if myopia is limited, whereas for stronger myopia, the long-run multiplier is clearly lower. We also discussed the results with hybrid expectations, i.e., those that also feature a backward-looking term. In the short-run, these can behave similar to myopic ones (for long time to build), but in the long run, the sum of the weights on forward- and backward-looking elements of expectations matters most. In the context of our model, we have found that, at the effective lower bound, government investment multipliers are larger than in normal times, mirroring known results from the literature. However, we have found that with myopic expectations, these multipliers are somewhat smaller than with rational expectations, unless time to build is long. On the other hand, if expectations also feature a backward-looking term, the short-run multiplier can become quite large. Also, we have found that the somewhat counterintuitive result of Bouakez *et al.* (2017) – that, at the effective lower bound, multipliers increase with time to build – also extends to models with imperfectly forward-looking expectations. We have also investigated the interactions of our results with a couple of other dimensions, including government debt, agent heterogeneity and the $r - g$ gap. In our simulations, the effects of these other model features on our results where limited, but as mentioned in the discussion, richer set-ups might give them a meaningful dimension. In fact, this is one promising path for future research.

Appendices

4.A Derivation of Nonlinear Model Equations

The model economy is populated by eight types kinds of agents: Constrained households, unconstrained households, monopolistically competitive labour unions and intermediate goods firms, capital-goods producers, a final-goods firm, a central government and a central bank conducting monetary policy. Time runs forever in discrete periods starting at $t = t_0$. The model presented here is more general than the one used in the main text: In particular, we allow for time-varying TFP growth g_t, for debt held by the constrained agents as well as for a negative (disruptive) effect of public investment on existing public capital.

4.A.1 A Brief Comment on the Mathematical Notation Used in this Model

As is usual in the literature, we assume that all endogenous variables – collected in some vector $\mathbf{x}$ – are actually not functions of time t per se, but rather of some state vector $\boldsymbol{\xi}(t)$ (which in turn can be dependent on previous values of itself and $\mathbf{x}$) at that point in time, where there exists a function f_X mapping $\boldsymbol{\xi}$ into the space of $\mathbf{x}$. In order to save on notation, we suppress this dependence and rather only write $\mathbf{x}_t$. It should be understood, however, that actually $\mathbf{x}_t = f_X(\boldsymbol{\xi}(t))$.

In addition, to save on space, in the following, we often slightly abuse notation in that we use continuous indices for variables. E.g., we often write $x_{i,t}$, where the index i may be an element of a continuous set I and hence, strictly speaking $x_t(i)$, or as just discussed, $x(i, \boldsymbol{\xi}(t))$ could be considered more appropriate.

In the following, we often make use of the Lebesgue integral. In particular, consider some set E with the appropriate σ-algebra $\mathcal{A}$ and measure $\mu(x)$ defined for elements x of E. $\{E, \mathcal{A}, \mu\}$ then form a measure space, for which we write the Lebesgue integral of some function $f : E \to \mathbb{R}_0^+$ as

$$\int_{x \in E} f(x) d\mu.$$

Next consider some subset $F \subset E$, then we can define the measure

$$\omega_F(x) := \mathbb{1}_{x \in F}, \tag{4.70}$$

where $\mathbb{1}_x$ is the indicator function

$$\mathbb{1}_x = \begin{cases} 1 & \text{if } x \text{ is true.} \\ 0 & \text{otherwise.} \end{cases}$$

In the following, we will often use a Lebesgue integral to only refer to the elements of such a subset and write

$$\int_{x\in E} f(x)d\omega_F, \text{ or, as just discussed, } \int_{x\in E} f_x d\omega_F. \tag{4.71}$$

This implies, in particular, for some constant $c \geq 0$

$$\int_{x\in E} c\, d\omega_F = c\int_{x\in E} d\omega_F = x|F|. \tag{4.72}$$

4.A.2 Households

The modelling of the household sector closely follows Bilbiie (2021), who generalises the simple Two-Agent New-Keynesian (TANK) framework familiar from the literature to a so-called Tractable Heterogeneous-Agent New-Keynesian model (THANK). In either type of model, there is a continuum of infinitely-lived households, which we – without loss of generality – here normalise to the unit interval. In the following, we thus index individual households by $\hbar \in \mathcal{H} := [0, 1]$. Also, in both types of models, households are either constrained (subscript c for group-specific variables) or unconstrained (u). Let the set of all unconstrained households be given by $\mathcal{H}_{u,t} \subseteq \mathcal{H}$, the set of constrained households is then given by $\mathcal{H}_{c,t} = \mathcal{H}\backslash\mathcal{H}_{u,t}$.

However, unlike TANK, in THANK, households switch types randomly, i.e., in general, $\mathcal{H}_{u,t} \neq \mathcal{H}_{u,t+1}$. In particular, in period t, the unconstrained anticipate becoming constrained in the next period $t+1$ with some probability $p_{c|u}$ and remaining unconstrained with probability $p_{u|u} = 1 - p_{c|u}$. Likewise, constrained agents become unconstrained and remain constrained with probabilities $p_{u|c}$ and $p_{c|c} = 1 - p_{u|c}$, respectively.[77] For $(p_{c|u}, p_{u|c}) \in (0, 1]^2\backslash\{(1, 1)\}$, this implies a stationary distribution with a share s_u of agents being unconstrained and $s_c = 1 - s_u$ being constrained, where

$$s_u = \frac{p_{u|c}}{p_{c|u} + p_{u|c}}.$$

Correspondingly, we can define sets of households according to their status:

$$\mathcal{H}_{y|x,t} = \{\hbar \in \mathcal{H} | \hbar \in \mathcal{H}_{y,t} \cap \mathcal{H}_{x,t-1}\} \quad \text{for } (x, y) \in \mathcal{H}^2,$$

which we can call changers if $y \neq x$ and remainers if $y = x$.

For $p_{c|u} = p_{u|c} = 0$, agents never switch types and we are thus in a traditional TANK environment.[78]

As is standard in virtually all business-cycle models, we assume that households derive utility from their lifetime streams of consumption $\mathcal{C}_{\hbar,t}$ of a homogeneous final good and leisure, i.e. they derive disutility from working $\tilde{N}_{\hbar,t}$ in period t. In keeping with the literature, we assume that the utility function of each agent is separable both across time periods and between consumption and labour supply.

In a minor extension to most of the existing literature and as a departure from Bilbiie's (2021) formulation, we also allow for (separable) wealth preferences. These have found increasing usage in macroeconomics as a tool to model a convenience yield or liquidity

[77]Formally, let $p_{x|y}$ denote $\text{Prob}(h \in \mathcal{H}_{x,t+1} | h \in \mathcal{H}_{y,t+1})$ for $(x, y) \in \{u, c\}^2$.

[78]In this chapter, we actually keep s_u constant for all model specications and set $p_{c|u} = p_{u|c} = 0$ for TANK, whereas for THANK we select a value for $p_{c|u}$ and directly obtain $p_{u|c} = \frac{s_u p_{c|u}}{1-s_u}$.

premium on bonds. Papers that use this in some form or another, are Illing *et al.* (2018) or Mian *et al.* (2021, 2022). However, we restrict these preferences to only cover real liquid bond holdings $\mathcal{B}_t^{\hbar}$, not fixed capital or firm shares.[79] As another extension, we allow for (exogenously given) positive TFP growth.

Preferences

The utility function of each agent in period t is assumed to be given by the recursive formulation

$$\mathcal{U}_{\hbar,t} = u_{\hbar,t} + \beta \mathcal{E}_t^{\hbar}[\mathcal{U}_{\hbar,t+1}] \tag{4.73}$$

Here β measures time preferences, where the rate of time-preference $\rho = \frac{1-\beta}{\beta}$ is assumed to be identical across agents, $\mathcal{E}_t^{\hbar}[x_{t+s}]$ is $\hbar$'s subjective expectation of some variable x_{t+s}, conditional on information available at t, and $u_t^{\hbar}$ is a period utility flow given by

$$u_{\hbar,t} = u(\mathcal{C}_{\hbar,t}) - v(\tilde{N}_{\hbar,t}) + w(\mathcal{B}_{\hbar,t}, A_t, \xi_t) \tag{4.74}$$

$$\text{with} \quad u(c) = \begin{cases} \ln(c) & \text{if } \sigma = 1, \\ \frac{c^{1-\sigma}-1}{1-\sigma} & \text{otherwise;} \end{cases} \tag{4.75}$$

$$v(n) = \nu \frac{n^{1+\varphi}}{1+\varphi}; \tag{4.76}$$

$$w(B, A, \xi) = (\bar{\xi} + \xi) A^{-\sigma} B \tag{4.77}$$

Here, as is usual, $\sigma \geq 0$ measures the intertemporal rate of substitution (as well as the relative risk aversion) of consumption, $\varphi \geq 0$ is the inverse Frisch elasticity and ν is a simple weighting parameter. The quasi-linear specification of liquidity-wealth preferences $w(\cdot)$ in equation (4.77) needs a bit more explanation: $\bar{\xi}$ plays a similar role as ν for this sub-utility, ξ_t is a temporary liquidity-premium shock. In this chapter, this shock is only relevant to the extent that it will be used to push the economy to the effective lower bound (ELB) on the nominal interest rate. As is now well known from previous work on this matter (see, e.g., Ono, 2001; Illing *et al.*, 2018), quasi-linear wealth preferences in combination with real growth lead to permanent stagnation. The reason is that consumption demand becomes saturated, whereas liqudity preferences are insatiable. This is a special case of a wider phenomenon: Loosely speaking, whenever the marginal utility from consumption shrinks to zero faster than the marginal utility of liquid bond holdings as the economy grows, a wedge occurs in the usual Euler equation between the rate of time preferences ρ and the real rate of interest r, with $r < g$. Especially when an effective lower bound on the nominal interest rate i is relevant, this becomes a recipe for underemployment. While certainly an interesting question to consider, in this chapter, we focus on a stationary model and we leave analysing public investment during a permanent and permanently worsening stagnation episode for future research. To avoid such a situation and guarantee scale invariance, Mian *et al.* (2021) introduce relative (status) preferences, where individual agents have preferences w.r.t. their consumption and wealth *relative to* the corresponding aggregate variables. However, this complicates the resulting Euler equations because these then include both individual as well as aggregate variables. As a

[79] The reason for this modelling choice is, as in Bilbiie (2021), that fixed capital is regarded as less liquid. See in particular Kaplan *et al.* (2018) for a full blown Heterogeneous-Agent New-Keynesian (HANK) model.

short-cut, we here use the ad-hoc assumption that the utility from a given stock of real bond holdings declines with higher levels of TFP A_t according to $A_t^{-\sigma}$. As we shall see below, this makes the first-order conditions of the model stationary.

Budget constraints

By working, each household $\hbar \in \mathcal{H}$ – irrespective of its type – obtains nominal work income $P_t Y_{\hbar,L,t}$, where P_t is the price-level of the final consumption good and $Y_{\hbar,L,t}$ is $\hbar$'s real labour income. Following Erceg *et al.* (2000), we assume that households delegate both the amount of labour supplied $N_{\hbar,L,t}$ and the determination of labour income to differentiated labour unions (discussed in more detail in subsection 4.A.4), but that all households enter into an insurance contract that ensures identical net labour income $Y_{\hbar,L,t} = \tilde{Y}_{L,t}$ for all agents $\hbar \in \mathcal{H}$.[80] In addition, we require that all members of a particular union work the same amount of hours, regardless of being constrained or not. This allows us to parsimoniously introduce wage stickiness into this model.

In order to fund consumption, households also have access to a liquid-bonds market and, if they are unconstrained, to a mutual fund that invests into physical capital and firm shares.

Each agent enters the period with a stock of nominal bond holdings $P_{t-1}\mathcal{B}_t^{\hbar,In}$ which earn a nominal interest rate i_{t-1} which has been set in the previous period. At the end of the period, households can choose to keep holding bonds $P_t\mathcal{B}_{\hbar,t}$, where the real level of bond holdings $\mathcal{B}_{\hbar,t}$ is bounded by $\mathcal{B}_{\hbar,t} \geq \bar{b}_t A_t$.[81]

To capture differences in liquidity across assets, we follow Bilbiie (2021) in assuming that only unconstrained households are able to actively participate in capital markets, i.e., only they can invest amounts $P_t Q_t^S S_{\hbar,t}$ into shares of the mutual fund, where $P_t Q_t^S$ is the price of the shares and $S_{\hbar,t}$ is the respective amount. Without loss of generality, the total supply S of shares is normalised to 1 at all times:

$$\int_{h\in\mathcal{H}} S_{\hbar,t} d\omega_{\mathcal{H}} = S = 1.$$

This generates a flow income of $P_{t+1}\tilde{\Pi}_{\mathcal{F},t+1}$ per share in the next period if the agent is still unconstrained. If an unconstrained agent becomes constrained, they lose all shares, which are redistributed to newly unconstrained agents by an insurance scheme. Finally, there are real lump-sum transfers $\tilde{T}_{u,t}, \tilde{T}_{c,t}$ for constrained and unconstrained agents, respectively.

Thus, the budget constraints of constrained households is

$$\begin{aligned} P_t\left[\mathcal{C}_{\hbar,t} + \mathcal{B}_{\hbar,t} + Q_t^S S_{\hbar,t}\right] = P_t &\left[\tilde{Y}_{L,t} - \tilde{T}_{u,t} + (\tilde{\Pi}_{\mathcal{F},t} + Q_t^S) S_{\hbar,t-1}\right] \\ &+ (1+i_{t-1})P_{t-1}\mathcal{B}_{\hbar,t}^{In}, \qquad \hbar \in \mathcal{H}_u, \qquad (4.78) \end{aligned}$$

[80] Conceptually, this mirrors the treatment in Bilbiie (2021) in that agents are heterogeneous from the point of view of preferences and capital-market participation, unlike the traditional HANK literature in the spirit of Kaplan *et al.* (2018), where agents are hit by idiosyncratic productivity shocks. In terms of welfare analysis, this change might not be trivial as in the model presented here, individual agents may not be on their individually optimal labour supply schedule. This is, however, less of a problem since we abstract from normative questions here.

[81] Again, this constraint is dependent on the level of technology A_t in order to make the problem stationary.

where for simplicity, for previously constrained unconstrained households we take into account redistribution of mutual-funds shares by setting

$$S_{\hbar,t-1} = \frac{\int_{h\in\mathcal{H}} S_{h,t-1} d\omega_{\mathcal{H}_{c|u,t}}}{p_{u|c}s_c} \quad \text{for } \hbar \in \mathcal{H}_{u|c,t}. \tag{4.79}$$

Likewise, for constrained agents the budget constraint is given by

$$P_t\left[\mathcal{C}_{\hbar,t} + \mathcal{B}_{\hbar,t}\right] = P_t\left[\tilde{Y}_{L,t} - \tilde{T}_{u,t}\right] + (1+i_{t-1})P_{t-1}\mathcal{B}^{In}_{\hbar,t}, \quad \hbar \in \mathcal{H}_c. \tag{4.80}$$

Next, we assume that agents are able to insure *within each type* (not across!) by transferring bond holdings between themselves. Assume that this yields identical bond holdings across agents within a given type, then we have:

$$\mathcal{B}^{In}_{\hbar,t} = \frac{\int_{h\in\mathcal{H}} \mathcal{B}_{h,t-1} d\omega_{\mathcal{H}_{x,t}}}{s_x} =: B^{In}_{x,t}, \quad \forall \hbar \in \mathcal{H}_{x,t} \text{ for } x \in \{c,u\} \tag{4.81}$$

Together with the above-mentioned budget constraints, this yields a symmetric problem within each type, leading to identical choices for each agent within a given type.

Let this identical choice be denoted as

$$\left.\begin{aligned} \mathcal{C}_{\hbar,t} &= C_{x,t}, \\ \mathcal{B}_{\hbar,t} &= B_{x,t} \end{aligned}\right\} \quad \text{for } \hbar \in \mathcal{H}_x \text{for } x \in \{u,c\} \tag{4.82}$$

Also, all unconstrained agents choose the same amount of shares in the mutual funds:

$$S^{\hbar}_t = S_t \quad \text{for } \hbar \in \mathcal{H}_u. \tag{4.83}$$

Therefore, equation (4.81) simplifies to

$$\begin{aligned} B^{In}_{x,t} &= \frac{p_{x|x}s_x B_{x,t-1} + p_{x|y}s_y B_{y,t-1}}{s_x} \\ &= p_{x|x}B_{x,t-1} + \frac{p_{x|y}s_y}{s_x}B_{y,t-1}, \quad (x,y) \in \{(x,y) \in \{c,u\}^2 | x \neq y\}. \end{aligned} \tag{4.84}$$

Also, equations (4.78) and (4.80) can be rewritten as

$$C_{u,t} + B_{u,t} + Q^S_t S_t = \tilde{Y}_{L,t} - \tilde{T}_{u,t} + (\tilde{\Pi}_{\mathcal{F},t} + Q^S_t)S_{t-1} + (1+r_t)B^{In}_{u,t}, \tag{4.85}$$

$$C_{c,t} + B_{c,t} = \tilde{Y}_{L,t} - \tilde{T}_{c,t} + (1+r_t)B^{In}_{c,t}, \tag{4.86}$$

where, as usual, the real interest rate from $t-1$ to t is given by the Fisher equation

$$1 + r_t = \frac{1+i_{t-1}}{1+\pi_t} = \frac{(1+i_{t-1})P_{t-1}}{P_t}. \tag{4.87}$$

Here, we also introduce the rate of inflation

$$\pi_t \equiv \frac{P_{t-1} - P_t}{P_t}.$$

Optimal household behaviour

We are now in a position to characterise optimal household behaviour. For simplicity, we follow Bilbiie (2021) and assume that a pseudo-benevolent planner tries to maximise a utilitarian welfare function, subject to the market frictions discussed before and taking into account the behaviour of other agents in the model. The planner's problem can be stated as:

$$\max_{(C_{u,t},C_{c,t},B_{u,t},B_{c,t},\mathcal{W}_t,\mathcal{U}_t,S_t)_{t\geq t_0}} \Bigg\{ \mathcal{W}_{t_0}$$
$$\begin{aligned}
\text{s.t. } \forall t \geq t_0: \quad \mathcal{W}_t &= \mathcal{U}_t + \beta \mathcal{E}_t^{HH}\left[\mathcal{W}_t\right] \\
\mathcal{U}_t &= \sum_{x\in\{u,c\}} s_x \left[u(C_{x,t}) - v(N_t) + \boldsymbol{w}(B_{x,t}, A_t, \xi_{x,t})\right] \\
C_{u,t} + B_{u,t} + Q_t^S S_t &= \tilde{Y}_{L,t} - \tilde{T}_{u,t} + (\tilde{\Pi}_{\mathcal{F},t} + Q_t^S)S_{t-1} + (1+r_t)B_{u,t}^{In}, \\
C_{c,t} + B_{c,t} &= \tilde{Y}_{L,t} - \tilde{T}_{c,t} + (1+r_t)B_{c,t}^{In}, \\
B_{u,t}^{In} &= p_{u|u}B_{u,t-1} + \frac{p_{u|c}s_c}{s_u}B_{c,t-1}, \\
B_{c,t}^{In} &= p_{c|c}B_{c,t-1} + \frac{p_{c|u}s_u}{s_c}B_{u,t-1}, \\
B_{c,t}, B_{u,t} &\geq \bar{b}_t A_t, \\
S_t &\in [0, 1/s_u],
\end{aligned}$$
$$\text{given } \mathcal{E}_t^{HH}[(A_t, \tilde{Y}_{L,t}, \tilde{N}_{\hbar,t}, \tilde{\Pi}_{\mathcal{F},t}, r_t, \tilde{T}_{u,t}, \tilde{T}_{c,t}, Q_t^S)_{t\geq t_0}], B_{u,t_0}^{In}, B_{c,t_0}^{In} \Bigg\} \tag{4.88}$$

The first-order conditions of choice variables for $t \geq t_0$ give $\forall x \in \{u, c\}$

$$C_{x,t} : u'(C_{x,t}) = \Lambda_{x,t} \tag{4.89}$$

$$B_{x,t}: \quad \Lambda_{x,t} = \beta \sum_{y\in\{u,c\}} p_{y|x}\mathcal{E}_t^{HH}\left[(1+r_{t+1})\Lambda_{y,t+1}\right] + \frac{\partial \boldsymbol{w}(B_{x,t}, A_t, \xi_{x,t})}{\partial B} + \mathcal{M}_{x,t} \tag{4.90}$$

$$S_t: \Lambda_{u,t}Q_t^S = \beta\mathcal{E}_t^{HH}\left[(Q_{t+1}^S + \tilde{\Pi}_{\mathcal{F},t+1})\Lambda_{y,t+1}\right] - \mathcal{M}_{S1,t} + \mathcal{M}_{S0,t}. \tag{4.91}$$

Here, the $\Lambda_{x,t}$ are Lagrange multipliers on type-x budget constraints in period t, $\mathcal{M}_{x,t}$ on the respective borrowing constraint, whereas $\mathcal{M}_{S1,t}$ and $\mathcal{M}_{S0,t}$ are the two multipliers on the bounds for S_t. The first order conditions w.r.t to the $\Lambda_{x,t}$ just lead to the budget constraints given above. Finally, the complementary slackness conditions are given by:

$$\mathcal{M}_{x,t}(B_{x,t} - \bar{b}A_t) = 0, \quad \mathcal{M}_{x,t} \geq 0, \quad B_{x,t} \geq \bar{b}A_t, \tag{4.92}$$
$$\mathcal{M}_{S1,t}(1/s_u - S_{1,t}) = 0, \quad \mathcal{M}_{S1,t} \geq 0, \quad 1/s_u \geq S_t, \tag{4.93}$$
$$\mathcal{M}_{S0,t}S_{0,t} = 0, \quad \mathcal{M}_{S0,t} \geq 0, \quad S_t \geq 0, \tag{4.94}$$

Market clearing in the market for shares will imply that $S_t = 1/s_u$ (Q_t^S will adjust), but with a slack condition, which implies:

$$Q_t^S = \beta\mathcal{E}_t^{HH}\left[\frac{\Lambda_{u,t+1}}{\Lambda_{u,t}}(Q_{t+1}^S + \tilde{\Pi}_{\mathcal{F},t+1})\right] \tag{4.95}$$

We are now in a position to express the stochastic discount factor for (risky) private investments between periods t and $t+1$ as

$$\mathcal{F}^r_{t+1} := \beta \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} = \beta \frac{u'(C_{u,t+1})}{u'(C_{u,t})} = \beta \left(\frac{C_{u,t+1}}{C_{u,t}} \right)^{-\sigma}, \tag{4.96}$$

allowing us to write the pricing equation (4.95) as

$$1 = \mathcal{E}^{HH}_t \left[\mathcal{F}^r_{t+1} \frac{Q^S_{t+1} + \tilde{\Pi}_{\mathcal{F},t+1}}{Q^S_t} \right]. \tag{4.97}$$

In fact, a similar Euler equation to (4.97) could be given for any type of illiquid capital.

On the other hand, assume that constrained agents are hit with such a negative shock to their liquidity preferences (i.e., a positive demand shock) that they try to reduce their liquid bond holdings to the minimum amount possible. Then,

$$B_{c,t} = \bar{b}_t A_t, \tag{4.98}$$
$$C_{c,t} = \tilde{Y}_{L,t} - \tilde{T}_{c,t} + (1 + r_t) B^{In}_{c,t} - \bar{b} A_t \tag{4.99}$$

at all times, implying that constrained agents will never price bonds. Ultimately, we assume that unconstrained agents will always be on an interior solution, yielding an Euler equation

$$u'(C_{u,t}) = \beta \sum_{x \in \{u,c\}} p_{x|u} \mathcal{E}^{HH}_t \left[(1 + r_{t+1}) u'(C_{x,t}) \right] + \frac{\partial}{\partial B} \mathcal{w}(B_{u,t}, A_t, \xi_{u,t}) \tag{4.100}$$

Akin to equation (4.96), we can use this to define a stochastic discount factor for safe liquid bonds as

$$\begin{aligned} \mathcal{F}^s_{t+1} &:= \beta \sum_{x \in \{u,c\}} p_{x|u} \frac{\Lambda_{x,t+1}}{\Lambda_{u,t}} \\ &= \beta \sum_{x \in \{u,c\}} p_{x|u} \frac{u'(C_{x,t+1})}{u'(C_{u,t})} \\ &= \beta \sum_{x \in \{u,c\}} p_{x|u} \left(\frac{C_{x,t+1}}{C_{u,t}} \right)^{-\sigma}. \end{aligned} \tag{4.101}$$

This can be used to simplify the Euler equation (4.100) to

$$1 = \mathcal{E}^{HH}_t \left[\mathcal{F}^s_{t+1} (1 + r_{t+1}) \right] + \frac{\frac{\partial}{\partial B} \mathcal{w}(B_{u,t}, A_t, \xi_{u,t})}{u'(C_{u,t})} \tag{4.102}$$

Stationary equilibrium

In order to allow for nonzero trend growth in the economy, i.e. $A_t = (1+g_t)A_{t-1}$ with mean $g_t = g \neq 0$, we need to express the equations (4.84), (4.96) – (4.99), (4.101) and (4.102) in terms of stationary variables. For this, define

$$b_{x,t} := \frac{B_{x,t}}{A_t},\ b^{In}_{x,t} := \frac{B^{In}_{x,t}}{A_t},\ T_{x,t} := \frac{\tilde{T}_{x,t}}{A_t} \text{ and } c_{x,t} := \frac{C_{x,t}}{A_t} \quad \forall x \in \{c, u\}$$

as well as

$$\tilde{y}_{L,t} := \frac{\tilde{Y}_{L,t}}{A_t},\ q_t^S := \frac{\tilde{Q}_t^S}{A_t},\ \tilde{\pi}_{F,t} := \frac{\tilde{\Pi}_{F,t}}{A_t}.$$

We can then express the mentioned equations as

$$\begin{aligned}
b_{x,t}^{In} &= p_{x|x}\frac{B_{x,t-1}}{A_t} + \frac{p_{x|y}s_y}{s_x}\frac{B_{y,t-1}}{A_t} \\
&= \frac{A_{t-1}}{A_t}\left(p_{x|x}\frac{B_{x,t-1}}{A_{t-1}} + \frac{p_{x|y}s_y}{s_x}\frac{B_{y,t-1}}{A_{t-1}}\right) \\
&= \frac{1}{1+g_t}\left(p_{x|x}b_{x,t-1} + \frac{p_{x|y}s_y}{s_x}b_{y,t-1}\right), \quad x,y \in \{u,c\}, x \neq y && (4.103) \\
\mathcal{F}_{t+1}^r &= \beta\left(\frac{C_{u,t+1}}{C_{u,t}}\right)^{-\sigma} = \beta\left(\frac{c_{u,t+1}A_{t+1}}{c_{u,t}A_t}\right)^{-\sigma} = \beta\left((1+g_{t+1})\frac{c_{u,t+1}}{c_{u,t}}\right)^{-\sigma}, && (4.104) \\
1 &= \mathcal{E}_t^{HH}\left[\mathcal{F}_{t+1}^r(1+g_{t+1})\frac{q_{t+1}^S + \tilde{\pi}_{\mathcal{F},t+1}}{q_t^S}\right] && (4.105) \\
b_{c,t} &= \bar{b}_t && (4.106) \\
c_{c,t} &= \tilde{y}_{L,t} - T_{c,t} + (1+r_t)b_{c,t}^{In} - \bar{b}_t && (4.107) \\
&= \tilde{y}_{L,t} - T_{c,t} + \frac{1+r_t}{1+g_t}\left(p_{c|c}\bar{b}_{t-1} + \frac{p_{c|u}s_u}{s_c}b_{u,t-1}\right) - \bar{b}_t && (4.108) \\
\mathcal{F}_{t+1}^s &= \beta\sum_{x\in\{u,c\}} p_{x|u}\left(\frac{C_{x,t+1}}{C_{u,t}}\right)^{-\sigma} \\
&= \beta\sum_{x\in\{u,c\}} p_{x|u}\left((1+g_{t+1})\frac{c_{x,t+1}}{c_{u,t}}\right)^{-\sigma} && (4.109) \\
1 &= \mathcal{E}_t^{HH}\left[\mathcal{F}_{t+1}^s(1+r_{t+1})\right] + c_{u,t}^{\sigma}(\bar{\xi}+\xi_t) && (4.110)
\end{aligned}$$

Note that for future reference, we follow Gabaix (2020) and Angeletos *et al.* (2021), inter alia, in replacing equation (4.110) with

$$1 = \mathcal{E}_t^{HH}\left[\mathcal{F}_{t+1}^s\right](1+r_{pre,t}) + c_{u,t}^{\sigma}(\bar{\xi}+\xi_t), \tag{4.111}$$

where

$$r_{pre,t} := \frac{1+i_t}{1+\mathbb{E}_t[\pi_{t+1}]} - 1 \tag{4.112}$$

is the (correct) ex-ante real interest rate, abstracting from inflation uncertainty.

Excursion: Generating persistently low interest rates

From (4.102), it is clear that if $\frac{\partial}{\partial B}\boldsymbol{w}(\cdot) = 0$, in a stationary equilibrium (with perfect foresight), we would have $\mathcal{F}_{t+1}^s = \frac{1}{1+r_{t+1}}$ as in any standard New Keynesian model. Comparing equations (4.97) and (4.102), it becomes clear that in a stationary environment the real interest rate r_t is lower than the return rate on illiquid capital $\tilde{\pi}_{F,t}/q_{t-1}$ in the long run if

$$\mathcal{F}_t^s \geq \mathcal{F}_t^r \quad \Leftrightarrow \quad u'(C_{c,t}) > u'(C_{u,t}) \quad \Leftrightarrow \quad C_{c,t} < C_{u,t}. \tag{4.113}$$

Moreover, if we set $A_t/A_{t-1} = 1+g$ and consider a balanced growth path with $C_{x,t} = c_x A_t$ for $x = u, c$, we have stationary values

$$\mathcal{F}^s = \beta \frac{p_{u|u}(c_u(1+g))^{-\sigma} + p_{c|u}(c_c(1+g))^{-\sigma}}{(c_u)^{-\sigma}}$$

$$= \beta(1+g)^{-\sigma}\left[p_{u|u} + p_{c|u}\left(\frac{c_c}{c_u}\right)^{-\sigma}\right] \tag{4.114}$$

$$= \beta(1+g)^{-\sigma}\left[p_{u|u} + (1-p_{u|u})\left(\frac{c_c}{c_u}\right)^{-\sigma}\right] \tag{4.115}$$

Note that this implies that even without wealth preferences as in Illing *et al.* (2018) or an overlapping-generations structure, one can obtain a steady situation with $r < g$ if

$$1 + r < 1 + g \tag{4.116}$$

$$\Leftrightarrow \quad \frac{1}{1+g} < \frac{1}{1+r} = \mathcal{F}^s = \beta(1+g)^{-\sigma}\left[p_{u|u} + (1-p_{u|u})\left(\frac{c_c}{c_u}\right)^{-\sigma}\right] \tag{4.117}$$

This is satisfied for

$$(1+g)^{\sigma-1} < \beta\left[p_{u|u} + (1-p_{u|u})\left(\frac{c_c}{c_u}\right)^{-\sigma}\right] \tag{4.118}$$

Note that the right-hand side increases in $\frac{c_c}{c_u}$ and – as long as the former is smaller than 1 – also in $1 - p_{u|u}$. This means that, in theory, even without wealth preferences, sufficient idiosyncratic risk can reduce the safe interest rate below the growth rate of the economy in a THANK framework, as already discussed by Bilbiie (2021) for the special case $g = 0$. The mechanism (illiquidity, idiosycnratic risks, and incomplete markets) is the same as the one identified by Reis (2021b). However, as is evident from the discussion in Bilbiie (2021) and as can easily be verified numerically, if condition (4.118) holds, then, one generally faces *compounding in the aggregate Euler equation* and the model (or a version stripped of all other ingredients) presented here is not determinate anymore – not even locally due to violations of the Blanchard-Kahn conditions.

With $\frac{\partial}{\partial B}\boldsymbol{w}(\cdot) > 0$ close to a stationary equilibrium, a stationary version of (4.102) can be written as

$$1 - \frac{\frac{\partial}{\partial B}\boldsymbol{w}(b_u A_t, A_t, \xi_u)}{u'(c_u A_t)} = [\mathcal{F}^s(1+r)]$$

with $B_{u,t} = b_u A_t$, yielding

$$1 + r_t = \frac{1 - \frac{\frac{\partial}{\partial B}\boldsymbol{w}(b_u A_t, A_t, \xi_u)}{u'(c_u A_t)}}{\mathcal{F}^s} < \frac{1}{\mathcal{F}^s}$$

This way, it is generally possible to reduce r below g without numerical issues (see, e.g. Illing *et al.* (2018) or Mian *et al.* (2022)). Also, as pointed out by Michaillat and Saez (2021) in the context of the basic New Keynesian model, wealth preferences introduce discounting in the dynamic IS curve; this mimics to some extent the imperfect forward-lookingness generated by boundedly rational expectations as introduced by Gabaix (2020) and used in this chapter as well as the previous one.

4.A.3 Production Structure and Price Stickiness

The firm sector of the model economy follows (inter alia) Ramey (2021) and the majority of medium-scale DSGE models in the literature: a final-goods firm produces the final good for consumption and investment from differentiated intermediate inputs. These differentiated intermediated inputs are each produced by monopolistically competitive firms, which are subject to staggered price-setting as in Calvo (1983), and which use labour and capital as private inputs. Capital-goods producing firms buy a share of the final goods, produce physical capital with it and sell it to the mutual fund, whereas intermediate goods producers' labour demand faces the supply.

Final goods firm

The final good Y_t, which can be used for consumption and investment is produced by a competitive firm. To do this, it uses intermediate inputs $Y_{f,t}$, $f \in \mathcal{F}$ with the set of available intermediate inputs $\mathcal{F}$ being constant and for simplicity normalised to $\mathcal{F} = [0,1]$. Here $f \in \mathcal{F}$ denotes a given intermediate input. For simplicity, we assume that the final-goods firm has access to a constant-elasticity-substitution (CES) production function

$$Y_t = \left[\int_{f \in \mathcal{F}} Y_{f,t}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{4.119}$$

with elasticity of substitution across inputs $\epsilon > 1$.
Letting P_t denote the nominal price of the final good (which we also take as the overall price level of the economy) and $P_{f,t}$ the price of individual intermediate inputs $f \in \mathcal{F}$, we can derive the standard results for such a CES firm: The relative demands for two intermediate inputs $f_1, f_2 \in \mathcal{F}$ (with $f_1 \neq f_2$) is

$$\frac{Y_{f_1,t}}{Y_{f_2,t}} = \left(\frac{P_{f_1,t}}{P_{f_2,t}} \right)^{-\epsilon},$$

the demand for a given intermediate input $f \in \mathcal{F}$ depends on the final-goods output as

$$\frac{Y_{f,t}}{Y_t} = \left(\frac{P_{f,t}}{P_t} \right)^{-\epsilon}; \tag{4.120}$$

likewise, the price P_t depends on the prices $P_{f,t}$ of individual intermediate inputs according to

$$P_t = \left[\int_{f \in \mathcal{F}} P_{f,t}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \tag{4.121}$$

and the final-goods firm generates zero profits in every period.

Intermediate-goods firms

Intermediate-goods firms take the final-goods firm's behaviour in (4.119), (4.121), and in particular (4.120) into account. Let $f \in \mathcal{F}$ also denote the single producer of intermediate input $f \in \mathcal{F}$.

Assume that these firms share a common Cobb-Douglas-type technology combining labour services $N_{f,t}$ and private capital $K_{f,t}$ rented from the mutual fund with freely available, non-excludable non-rival effective public capital $\tilde{K}_{P,t}$ according to:

$$Y_{f,t} = A_t^{1-\alpha-\alpha_p} N_{f,t}^{1-\alpha} K_{f,t}^{\alpha} K_{P,t}^{\alpha_p} \tag{4.122}$$

Here, as introduced before, A_t is a measure of technological progress, $\alpha \in (0,1)$ is the capital share in output and $\alpha_p \geq 0$ governs the marginal productivity of public capital. Letting W_t be the real wage rate for f's labour services and $r_{K,t}$ the rental fee for private capital, respectively, the *real private marginal cost* X_t resulting from cost-minimisation is given, using standard results as for all $f \in \mathcal{F}$:

$$X_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{1}{A_t^{1-\alpha-\alpha_p} K_{P,t}^{\alpha_p}} W_t^{1-\alpha} r_{K,t}^{\alpha}, \tag{4.123}$$

which also gives factor demands

$$N_{f,t} = (1-\alpha) \frac{X_t Y_{f,t}}{\tilde{W}_t} \tag{4.124}$$

$$K_{f,t} = \alpha \frac{X_t Y_{f,t}}{r_{K,t}} \tag{4.125}$$

Given (4.123), in each period f generates a real profit of

$$\begin{aligned}
\Pi_{f,t} &= \frac{P_{f,t}}{P_t}(1-\tau) Y_{f,t} - X_t Y_{f,t}, \\
&= \left[\left(\frac{P_{f,t}}{P_t} \right)^{1-\epsilon} (1-\tau) - X_t \left(\frac{P_{f,t}}{P_t} \right)^{-\epsilon} \right] Y_t \qquad (4.126) \\
&= h(P_{f,t}/P_t, X_t) Y_t \\
\text{with } h(p,X) &:= p^{1-\epsilon}(1-\tau) - X p^{-\epsilon} \qquad (4.127) \\
\text{and } h_p(p,X) &:= \frac{\partial}{\partial p} h(p,X) = (1-\epsilon) p^{-\epsilon}(1-\tau) + \epsilon X p^{-\epsilon-1} \qquad (4.128)
\end{aligned}$$

where τ is a possible revenue tax (or subsidy if $\tau < 0$) and the second line follows from equation (4.120).

Next, we assume that each firm can only reoptimise its price with a probability $1-\theta$ in each period, where the opportunity to reoptimise is an i.i.d. event for each firm. Denote the set of reoptimising firms as $\mathcal{F}_t^o \subset \mathcal{F}$ with $|\mathcal{F}_t^o| = (1-\theta)|\mathcal{F}|$.Such a reoptimising firm chooses $P_{f,t} = P_{f,t}^*$. With probability θ, the firm cannot reset its price actively; however, the price is updated by the steady-state inflation rate $\bar{\pi}$, i.e. then $P_{f,t} = (1+\bar{\pi})P_{f,t-1}$. Taking this into account and using the risky stochastic discount factor $\mathcal{F}_{t+1}^r$ to discount future real profits, the firm's manager sets prices to maximise the discounted stream of profits. Thus, for $\mathcal{F}^o$ reoptimising manager's period-t_0 optimisation problem is recursively given by

$$\begin{aligned}
V_{f,t}^* = \max_{P_{f,t}>0} \Big\{ & h(P_{f,t}/P_t, X_t) Y_t + \mathcal{E}_t^f \left[\mathcal{F}_t^r \left(\theta V_{f,t+1}(P_{f,t+1}) + (1-\theta) V_{f,t+1}^* \right) \right] \\
\text{s.t. } \forall s > t : V_{f,s}(P) = & \, h(P(1+\bar{\pi})/P_s, X_s) Y_s \\
& + \mathcal{E}_s^f \left[\mathcal{F}_{s+1}^r \left(\theta V_{f,s+1}(P(1+\bar{\pi})) + (1-\theta) V_{f,s+1}^* \right) \right] \Big\}
\end{aligned}$$

This gives a first-order condition for $P^*_{f,t}$

$$\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\mathcal{F}^r_z\right)\frac{(1+\bar{\pi})^s}{P_{t+s}}h_p((1+\bar{\pi})^s P^*_{f,t}/P_{t+s}, X_{t+s})Y_{t+s}\right]\right]=0,$$

which as usual, we can rewrite as

$$\mathcal{M}=\frac{\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\mathcal{F}^r_z\right)\frac{(1+\bar{\pi})^s}{P_{t+s}}\left(\frac{(1+\bar{\pi})^s P^*_{f,t}}{P_{t+s}}\right)^{-\epsilon}Y_{t+s}\right]\right]}{\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\mathcal{F}^r_z\right)\frac{(1+\bar{\pi})^s}{P_{t+s}}\left(\frac{(1+\bar{\pi})^s P^*_{f,t}}{P_{t+s}}\right)^{-1-\epsilon}Y_{t+s}\right]\right]}, \tag{4.129}$$

where $\mathcal{M} := \dfrac{\epsilon}{(\epsilon-1)(1-\tau)}$.

Using $P_{t+s}=\sum_{z=t+1}^{t+s}(1+\pi_{t+z})P_t$ for $s\geq 0$, we can simplify (4.129) this further to

$$\begin{aligned}\mathcal{M}&=\frac{\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\theta\mathcal{F}^r_z\right)\left(P^*_{f,t}\right)^{-\epsilon}\left(\frac{(1+\bar{\pi})^s}{P_{t+s}}\right)^{1-\epsilon}Y_{t+s}\right]\right]}{\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\theta\mathcal{F}^r_z\right)\left(P^*_{f,t}\right)^{1-\epsilon}\left(\frac{(1+\bar{\pi})^s}{P_{t+s}}\right)^{-\epsilon}X_{t+s}Y_{t+s}\right]\right]}\\
&=P^*_{f,t}\frac{\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\theta\left(\frac{1+\bar{\pi}}{1+\pi_z}\right)^{1-\epsilon}\mathcal{F}^r_z\right)\left(\frac{1}{P_t}\right)^{1-\epsilon}Y_{t+s}\right]\right]}{\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\theta\left(\frac{1+\bar{\pi}}{1+\pi_z}\right)^{-\epsilon}\mathcal{F}^r_z\right)\left(\frac{1}{P_t}\right)^{-\epsilon}X_{t+s}Y_{t+s}\right]\right]}\\
&=\frac{P^*_{f,t}}{P_t}\frac{\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\theta\left(\frac{1+\bar{\pi}}{1+\pi_z}\right)^{1-\epsilon}\mathcal{F}^r_z\right)Y_{t+s}\right]\right]}{\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\theta\left(\frac{1+\bar{\pi}}{1+\pi_z}\right)^{-\epsilon}\mathcal{F}^r_z\right)X_{t+s}Y_{t+s}\right]\right]},\end{aligned} \tag{4.130}$$

where we have assumed that agents perfectly observe contemporaneous variables at their individual level and at the aggregate level, respectively. We can express (4.130) as

$$\frac{P^*_{f,t}}{P_t}=\mathcal{M}\frac{\tilde{\zeta}_{1,f,t}}{\tilde{\zeta}_{2,f,t}}, \tag{4.131}$$

where we define auxiliary variables

$$\tilde{\zeta}_{1,f,t}:=\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\theta\left(\frac{1+\bar{\pi}}{1+\pi_z}\right)^{-\epsilon}\mathcal{F}^r_z\right)X_{t+s}Y_{t+s}\right]\right]\quad\text{and} \tag{4.132}$$

$$\tilde{\zeta}_{2,f,t}:=\mathcal{E}^f_t\left[\sum_{s\geq 0}\left[\prod_{z=t+1}^{s}\left(\theta\left(\frac{1+\bar{\pi}}{1+\pi_z}\right)^{1-\epsilon}\mathcal{F}^r_z\right)Y_{t+s}\right]\right]. \tag{4.133}$$

Assuming the law of iterated expectations holds for subjective expectations, we express the two auxiliary variables recursively as

$$\tilde{\zeta}_{1,f,t}=X_tY_t+\theta\mathcal{E}^f_t\left[\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}}\right)^{-\epsilon}\mathcal{F}^r_{t+1}\tilde{\zeta}_{1,f,t}\right]\quad\text{and} \tag{4.134}$$

$$\tilde{\zeta}_{2,f,t}=Y_t+\theta\mathcal{E}^f_t\left[\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}}\right)^{1-\epsilon}\mathcal{F}^r_{t+1}\tilde{\zeta}_{2,f,t}\right]. \tag{4.135}$$

With possible TFP growth $A_t = (1+g_t)A_{t-1}$, we can also express this with stationary versions of the auxiliary variables. In particular, let

$$\begin{aligned}
\zeta_{1,f,t} := \frac{\tilde{\zeta}_{1,f,t}}{A_t} &= X_t \frac{Y_t}{A_t} + \theta \mathcal{E}_t^f \left[\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}} \right)^{-\epsilon} \mathcal{F}_{t+1}^r \frac{\tilde{\zeta}_{1,f,t}}{A_t} \right] \\
&= X_t \frac{Y_t}{A_t} + \theta \mathcal{E}_t^f \left[\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}} \right)^{-\epsilon} \mathcal{F}_{t+1}^r (1+g_{t+1}) \frac{\tilde{\zeta}_{1,f,t}}{A_{t+1}} \right] \\
&= X_t y_t + \theta \mathcal{E}_t^f \left[\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}} \right)^{-\epsilon} \mathcal{F}_{t+1}^r (1+g_{t+1}) \zeta_{1,f,t} \right] \qquad (4.136)
\end{aligned}$$

where we use $y_t := \frac{Y_t}{A_t}$ Similarly, we can define

$$\zeta_{2,f,t} := \theta \frac{\tilde{\zeta}_{2,f,t}}{A_t} = y_t + \mathcal{E}_t^f \left[\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}} \right)^{1-\epsilon} \mathcal{F}_{t+1}^r (1+g_{t+1}) \zeta_{2,f,t} \right], \qquad (4.137)$$

such that (4.131) can be equivalently expressed as

$$\frac{P_{f,t}^*}{P_t} = \mathcal{M} \frac{\zeta_{1,f,t}}{\zeta_{2,f,t}} \qquad (4.138)$$

Price-level determination

We can use the results from the previous two subsections to determine the evolution of the aggregate price level. In particular, recall from (4.121):

$$\begin{aligned}
P_t &= \left[\int_{f\in\mathcal{F}} P_{f,t}^{1-\epsilon} d\omega_{\mathcal{F}} \right]^{\frac{1}{1-\epsilon}}. \\
\Leftrightarrow 1 &= \left[\int_{f\in\mathcal{F}} \left(\frac{P_{f,t}}{P_t} \right)^{1-\epsilon} d\omega_{\mathcal{F}} \right]^{\frac{1}{1-\epsilon}} \\
\Leftrightarrow 1 &= \int_{f\in\mathcal{F}} \left(\frac{P_{f,t}}{P_t} \right)^{1-\epsilon} d\omega_{\mathcal{F}} \qquad (4.139)
\end{aligned}$$

Noting that from (4.138) and by the assumption of indexation for non reoptimisers, we have

$$\begin{aligned}
1 &= \int_{f\in\mathcal{F}} \left(\frac{P_{f,t}}{P_t} \right)^{1-\epsilon} d\omega_{\mathcal{F}_t^o} + \int_{f\in\mathcal{F}} \left(\frac{P_{f,t}}{P_t} \right)^{1-\epsilon} d\omega_{\mathcal{F}\setminus\mathcal{F}_t^o} \\
&= \int_{f\in\mathcal{F}} \left(\frac{P_{f,t}^*}{P_t} \right)^{1-\epsilon} d\omega_{\mathcal{F}_t^o} + \int_{f\in\mathcal{F}} \left((1+\bar{\pi}) \frac{P_{f,t-1}}{P_t} \right)^{1-\epsilon} d\omega_{\mathcal{F}\setminus\mathcal{F}_t^o} \qquad (4.140)
\end{aligned}$$

If we assume that all firms share the same expectation formation $\mathcal{E}_t^f[\cdot] = \mathcal{E}_t^F[\cdot]\,\forall f \in \mathcal{F}$, from (4.136) and (4.137) the auxiliary variables from above are also identical:

$$\zeta_{i,f,t} = \zeta_{i,t} \quad \text{for } i \in \{1,2\}, f \in \mathcal{F}$$

As a result, all optimising firms $f \in \mathcal{F}_t^o$ choose the same real price from (4.138):

$$\frac{P_{f,t}^*}{P_t} = p_t^* := \mathcal{M}\frac{\zeta_{1,t}}{\zeta_{2,t}}, \tag{4.141}$$

with

$$\zeta_{1,t} := X_t y_t + \theta \mathcal{E}_t^F \left[\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}} \right)^{-\epsilon} \mathcal{F}_{t+1}^r (1+g_{t+1}) \zeta_{1,t+1} \right] \tag{4.142}$$

$$\zeta_{2,t} := y_t + \theta \mathcal{E}_t^F \left[\left(\frac{1+\bar{\pi}}{1+\pi_{t+1}} \right)^{1-\epsilon} \mathcal{F}_{t+1}^r (1+g_{t+1}) \zeta_{2,t+1} \right], \tag{4.143}$$

giving

$$\int_{f\in\mathcal{F}} \left(\frac{P_{f,t}^*}{P_t} \right)^{1-\epsilon} d\omega_{\mathcal{F}_t^o} = (1-\theta) \left(\frac{P_t^*}{P_t} \right)^{1-\epsilon}.$$

Also, realise that

$$\int_{f\in\mathcal{F}} \left((1+\bar{\pi}) \frac{P_{f,t-1}}{P_t} \right)^{1-\epsilon} d\omega_{\mathcal{F}\setminus\mathcal{F}_t^o} = \left(\frac{1+\bar{\pi}}{1+\pi_t} \right)^{1-\epsilon} \int_{f\in\mathcal{F}} \left(\frac{P_{f,t-1}}{P_{t-1}} \right)^{1-\epsilon} d\omega_{\mathcal{F}\setminus\mathcal{F}_t^o}$$

since we assume that the reoptimisation opportunity is randomly assigned to individual firms and by the law of large numbers, we have

$$\int_{f\in\mathcal{F}} \left(\frac{P_{f,t-1}}{P_{t-1}} \right)^{1-\epsilon} d\omega_{\mathcal{F}\setminus\mathcal{F}_t^o} = \theta \int_{f\in\mathcal{F}} \left(\frac{P_{f,t-1}}{P_{t-1}} \right)^{1-\epsilon} = \theta$$

Therefore, we can write (4.139) as

$$1 = (1-\theta) \left(p_t^* \right)^{1-\epsilon} + \theta \left(\frac{1+\bar{\pi}}{1+\pi_t} \right)^{1-\epsilon}. \tag{4.144}$$

Price dispersion, aggregate factor demands and profits

Furthermore, we can aggregate the factor demands curves $N_{D,t}, K_{D,t}$ for labour and capital, respectively, by adding (*here integrating*) all firms' demand schedules:

$$N_{D,t} = \int_{f\in\mathcal{F}} N_{f,t} d\omega_{\mathcal{F}}, \qquad K_{D,t} = \int_{f\in\mathcal{F}} K_{f,t} d\omega_{\mathcal{F}}$$

From (4.124) and (4.125) , we can rewrite this as

$$N_{D,t} = \int_{f\in\mathcal{F}} (1-\alpha) \frac{X_t Y_{f,t}}{\tilde{W}_t} d\omega_{\mathcal{F}} = (1-\alpha) \frac{\tilde{Y}_t}{\tilde{W}_t}, \tag{4.145}$$

$$K_{D,t} = \int_{f\in\mathcal{F}} \alpha \frac{X_t Y_{f,t}}{r_{k,t}} d\omega_{\mathcal{F}} = \alpha \frac{\tilde{Y}_t}{r_{k,t}}, \tag{4.146}$$

where

$$\tilde{Y}_t := X_t \int_{f\in\mathcal{F}} Y_{f,t} d\omega_{\mathcal{F}} = X_t \int_{f\in\mathcal{F}} \left(\frac{P_{f,t}}{P_t} \right)^{-\epsilon} d\omega_{\mathcal{F}} Y_t = X_t Y_t \xi_{p,t} \tag{4.147}$$

with the typical wedge due to price dispersion $\xi_{p,t}$ given by

$$\begin{aligned}\xi_{p,t} &:= \int_{f\in\mathcal{F}} \left(\frac{P_{f,t}}{P_t}\right)^{-\epsilon} d\omega_{\mathcal{F}} \\ &= \int_{f\in\mathcal{F}} (p_t^*)^{-\epsilon}\, d\omega_{\mathcal{F}_t^o} + \int_{f\in\mathcal{F}} \left((1+\bar{\pi})\frac{P_{f,t-1}}{P_t}\right)^{-\epsilon} d\omega_{\mathcal{F}\setminus\mathcal{F}_t^o} \\ &= \int_{f\in\mathcal{F}_t^o} (p_t^*)^{-\epsilon} + \left(\frac{1+\bar{\pi}}{1+\pi_t}\right)^{-\epsilon} \int_{f\in\mathcal{F}\setminus\mathcal{F}_\sqcup^\wr} \left(\frac{P_{f,t-1}}{P_{t-1}}\right)^{-\epsilon}.\end{aligned}$$

Similar to our previous discussion for the price level, with symmetry across firms apart from the reoptimisation status, we have

$$\begin{aligned}\xi_{p,t} &= (1-\theta)\,(p_t^*)^{-\epsilon} + \theta\left(\frac{1+\bar{\pi}}{1+\pi_t}\right)^{-\epsilon} \int_{f\in\mathcal{F}} \left(\frac{P_{f,t-1}}{P_{t-1}}\right)^{-\epsilon} d\omega_{\mathcal{F}} \\ &= (1-\theta)\,(p_t^*)^{-\epsilon} + \theta\left(\frac{1+\bar{\pi}}{1+\pi_t}\right)^{-\epsilon} \xi_{p,t-1} \qquad (4.148)\end{aligned}$$

As by now is quite familiar from the New Keynesian literature, this wedge $\xi_{p,t} \geq 1$ is 1 only if there is no price dispersion, otherwise it is bigger, making total factor demand larger relative to total output (i.e. generating inefficiencies).[82]

We can also derive total real profits of the intermediate-goods sector as

$$\begin{aligned}\Pi_{\mathcal{F},t} &= \int_{f\in\mathcal{F}} \left[(1-\tau)\frac{P_{f,t}}{P_t}Y_{f,t} - X_t Y_{f,t}\right] d\omega_{\mathcal{F}}, \\ &= (1-\tau)\int_{f\in\mathcal{F}} \left[\frac{P_{f,t}}{P_t}Y_{f,t}\right] d\omega_{\mathcal{F}} - X_t \int_{f\in\mathcal{F}} Y_{f,t}\, d\omega_{\mathcal{F}} \qquad (4.149)\end{aligned}$$

which with (4.120) and (4.147) can be expressed as

$$\begin{aligned}\Pi_{\mathcal{F},t} &= (1-\tau)\int_{f\in\mathcal{F}} \left[\left(\frac{P_{f,t}}{P_t}\right)^{1-\epsilon} Y_t\right] d\omega_{\mathcal{F}} - \xi_{p,t}X_tY_t \\ &= (1-\tau)\int_{f\in\mathcal{F}} \left[\left(\frac{P_{f,t}}{P_t}\right)^{1-\epsilon}\right] Y_t\, d\omega_{\mathcal{F}} - \xi_{p,t}X_tY_t \\ &= (1-\tau-\xi_{p,t}X_t)\,Y_t \qquad (4.150)\end{aligned}$$

[82]To see this, combine (4.144) and (4.148) to get

$$\begin{aligned}\xi_{p,t} &= (1-\theta)\left(\frac{1-\theta\left(\frac{1+\bar{\pi}}{1+\pi_t}\right)^{1-\epsilon}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta\left(\frac{1+\bar{\pi}}{1+\pi_t}\right)^{-\epsilon}\xi_{p,t-1} \\ &= \tilde{f}_p\left(\frac{1+\bar{\pi}}{1+\pi_t}, \xi_{p,t-1}\right) \\ \text{with } \tilde{f}_p(x,y) &:= (1-\theta)\left(\frac{1-\theta x^{1-\epsilon}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}} + \theta x^{-\epsilon} y.\end{aligned}$$

It is straightforward to see that on a balanced growth path with $\pi_t = \bar{\pi}$, the steady-state for $\xi_{p,t}$ is given by 1 since $\tilde{f}_p(1,1) = 1$. Note that the gradient of $f_p(x,y)$ at $(1,1)$ is $(0,\theta)'$ and $f_p(x,y) \geq 1 \forall y \geq 1$. So $\xi_{p,t} < 1$ is only possible if $\xi_{p,t-1} < 1$, but such a state is not attainable.

Private capital formation and mutual fund

The competitive mutual fund manages the portfolio of all unconstrained households, it holds $s_{F,t} \in [0,1]$ shares (priced at $\mathcal{Q}_{\mathcal{F},t}$ in real terms) of the intermediate goods firms and receives the corresponding share of aggregate profits $\Pi_{\mathcal{F},t+1}$ as well as the revenue from the revenue tax (or pays the expenditure for the subsidy if $\tau < 0$) τY_{t+1} in the next period.

In addition to that, it can also invest real amounts $\tilde{I}_t$ into fixed capital $\bar{K}_t$, which depreciates by δ_{t+1} during the next period $t+1$. Note however, that there are adjustment costs: a share $S_{I,t}$ of invested goods is lost, hence the capital stock evolves according to

$$\bar{K}_t = (1-\delta_t)\bar{K}_{t-1} + \tilde{I}_t(1-S_{I,t}) \tag{4.151}$$

Here, we assume that $S_{I,t}$ is given by a function $S_I(\cdot)$ which depends on the growth rate $g_{I,t}$ of investment between two adjacent periods:

$$S_{I,t} = S_I\left(g_{I,t}\right) \tag{4.152}$$

$$\text{with } S_I(x) = \frac{\kappa_I}{2}(x-g)^2 \text{ and} \tag{4.153}$$

$$g_{I,t} := \frac{\tilde{I}_t}{\tilde{I}_{t-1}} - 1 \tag{4.154}$$

where g is the long-run average rate of technological progress.

At the start of period t, the fund still has access to the entire stock of fixed capital $\bar{K}_t$ and it can rent it out to intermediate goods firms. At this point in time it can also choose a degree of capacity utilisation $U_{K,t} \geq 0$, turning $\bar{K}_t$ units of fixed capital into $K_{S,t} = U_{K,t}\bar{K}_t$ units of *effective capital* for production. This, however, does not increase the fixed capital stock per se; instead varying capacity utilisation alters the depreciation rate δ_t according to a functional relationship:

$$\delta_t = \delta(U_{K,t}), \tag{4.155}$$

which we characterise in more detail later on. Per unit of effective capital it then receives the rent $r_{k,t}$ from intermediate-goods producers.

The real period-profits of the fund are thus given by

$$\Pi_{F,t} = s_{F,t-1}[\mathcal{Q}_{\mathcal{F},t} + Y_t(1-\xi_{p,t}X_t)] - s_{F,t}\mathcal{Q}_{\mathcal{F},t} + r_{k,t}U_{K,t}\bar{K}_{t-1} - \tilde{I}_t \tag{4.156}$$

Here, the mutual fund takes $Y_t, \xi_{p,t}, X_t, r_{k,t}$ as well as equations (4.151)–(4.155) as given and chooses $\tilde{I}_t$ and $s_{F,t}$ in each period to optimise the expected value of the discounted stream of profits, where it uses the risky stochastic discount factor $\mathcal{F}^r_{t+1}$ and we denote period-t expectations by the fund managers as $\mathcal{E}^F_t(\cdot)$.

Taken together, we can express the value function of the fund, dependent on the fixed capital stock at the beginning of the period and the previous period's investment $\tilde{I}_{t-1}$, recursively as

$$V^*_{F,t}(K_{t-1},\tilde{I}_{t-1}) = \max_{(\tilde{I}_t,U_{K,t},s_{F,t},\bar{K}_t)} \Bigg\{ \tilde{\Pi}_{F,t} + \mathcal{E}^F_t \left[\mathcal{F}^r_{t+1} V^*_{F,t+1}(K_t,\tilde{I}_t)\right]$$

$$\text{s.t. } \tilde{\Pi}_{F,t} = s_{F,t-1}[\mathcal{Q}_{\mathcal{F},t} + Y_t(1-\xi_{p,t}X_t)] - s_{F,t}\mathcal{Q}_{\mathcal{F},t} + r_{k,t}U_{K,t}\bar{K}_{t-1} - \tilde{I}_t,$$

$$\bar{K}_t = (1-\delta(U_{K,t}))\,\bar{K}_{t-1} + \tilde{I}_t\left(1 - S_I\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} - 1\right)\right), \tag{4.157}$$

$$s_{F,t} \in [0,1], \tag{4.158}$$

$$\text{given } s_{F,t-1}, \mathcal{Q}_{\mathcal{F},t}, Y_t, \xi_{p,t}X_t, \tilde{I}_{t-1}, \bar{K}_{t-1}, \mathcal{E}^F_t\left[\mathcal{F}^r_{t+1}\right] \Bigg\}$$

Assuming the fund is always at an interior solution at $s_{F,t} = 1$ w.r.t. (4.158), the first-order conditions are

$$s_{F,t}: \quad \mathcal{Q}_{\mathcal{F},t} = \mathcal{E}^F_t\left[\mathcal{F}^r_{t+1}\left(\mathcal{Q}_{\mathcal{F},t+1} + Y_{t+1}(1-\xi_{p,t+1}X_{t+1})\right)\right], \tag{4.159}$$

$$U_{K,t}: \quad r_{K,t}\bar{K}_{t-1} = q_t\delta'(U_{K,t})\bar{K}_{t-1}, \tag{4.160}$$

$$\bar{K}_t: \quad q_t = \mathcal{E}^F_t\left[\mathcal{F}^r_{t+1}\left(r_{K,t+1}U_{K,t+1} + q_{t+1}(1-\delta(U_{K,t+1}))\right)\right], \tag{4.161}$$

$$\tilde{I}_t: \quad 1 = q_t\left(1 - S_I\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} - 1\right) - \frac{\tilde{I}_t}{\tilde{I}_{t-1}}S_I'\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} - 1\right)\right) + \mathcal{E}^F_t\left[\mathcal{F}^r_{t+1}q_{t+1}\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\right)^2 S_I'\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t} - 1\right)\right], \tag{4.162}$$

where q_t is the period-t Lagrange multiplier on (4.157).

It follows that then,

$$\tilde{\Pi}_{F,t} = Y_t(1-\xi_{p,t}X_t) + r_{k,t}U_{K,t}\bar{K}_{t-1} - \tilde{I}_t \tag{4.163}$$

Equation (4.159) gives the mutual fund's estimation for the total value of the intermediate-goods sector at the end of period t. Equation (4.160) determine optimal capacity utilisation, equation (4.161) and (4.162) determine q_t to be Tobin's q (the shadow value of capital) and also determine investment $\tilde{I}_t$.

Note that we can re-express equations (4.162) and (4.160), respectively, as

$$q_t\left(1 - S_I\left(g_{I,t}\right) - (1+g_{I,t})S_I'\left(g_{I,t}\right)\right) = 1 - \mathcal{E}^F_t\left[\mathcal{F}^r_{t+1}q_{t+1}\left(1+g_{I,t+1}\right)^2 S_I'\left(g_{I,t+1}\right)\right] \tag{4.164}$$

and

$$r_{K,t} = q_t\delta'(U_{K,t}). \tag{4.165}$$

Stationary equilibrium

In order to find a stationary equilibrium for potentially $g \neq 0$, we need to make equations (4.123), (4.141) – (4.147), (4.154), (4.157), (4.159) – (4.159), (4.161), and (4.163) – (4.165) stationary. This can be easily achieved by using y_t defining

$$k_{x,t} := \frac{K_{x,t}}{A_t}\,(x \in \{D,S,P\}), \quad \bar{k}_t := \frac{\bar{K}_t}{A_t}, \quad I_t := \frac{\tilde{I}_t}{A_t},$$
$$q_{\mathcal{F},t} = \frac{\mathcal{Q}_{\mathcal{F},t}}{A_t}, \quad w_t := \frac{W_t}{A_t}, \quad \tilde{y}_t := \frac{\tilde{Y}_t}{A_t}, \quad \tilde{\pi}_{F,t} := \frac{\tilde{\Pi}_{F,t}}{A_t}.$$

Equations (4.141) – (4.144), (4.161), (4.164) and (4.165) are already stationary. For the remaining ones of the above-mentioned equations, we get:

$$X_t = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} k_{P,t}^{-\alpha_p} w_t^{1-\alpha} r_{K,t}^\alpha \tag{4.166}$$
$$N_t = (1-\alpha)\frac{\tilde{y}_t}{w_t} \tag{4.167}$$
$$k_t = \alpha\frac{\tilde{y}_t}{r_{K,t}} \tag{4.168}$$
$$\tilde{y}_t = \xi_{p,t} X_t y_t \tag{4.169}$$
$$1 + g_{I,t} = \frac{(1+g_t)I_t}{I_{t-1}} \tag{4.170}$$
$$\bar{k}_t = \frac{1-\delta(U_{K,t})}{1+g_t}\bar{k}_{t-1} + I_t(1 - S(g_{I,t})) \tag{4.171}$$
$$q_{\mathcal{F},t} = \mathcal{E}_t^F\left[\mathcal{F}_{t+1}^r(1+g_{t+1})\left(q_{\mathcal{F},t+1} + y_{t+1}(1-\xi_{p,t+1}X_{t+1})\right)\right] \tag{4.172}$$
$$\tilde{\pi}_{F,t} = y_t(1-\xi_{p,t}X_t) + \frac{r_{K,t}U_{K,t}\bar{k}_{t-1}}{1+g_t} - I_t \tag{4.173}$$

From equation (4.153), $S_I\,(g) = S_I'\,(g) = 0$ such that along a balanced growth path with $g_{I,t} = g$, equation (4.164) collapses to

$$q_t = q = 1 \tag{4.174}$$

From (4.161), we can thus derive that along a deterministic balanced growth path, the risky stochastic discount factor $\mathcal{F}_t^r = \mathcal{F}^r$, the return to capital $r_{K,t} = r_K$ and capacity utilisation $U_{K,t} = U_K$ are related according to

$$q = \mathcal{F}^r\left(r_K U_K + q(1-\delta(U_K))\right)$$
$$\Leftrightarrow \quad 1 = \mathcal{F}^r\left(r_K U_K + (1-\delta(U_K))\right) \tag{4.175}$$

Also, from equation (4.165)

$$r_K = \delta'(U_K) \tag{4.176}$$

We use this to normalise $U_K = 1$ and restrict $\delta(\cdot)$ such that $\delta'(1) = r_K$ and $\delta(1) = \bar{\delta}$. From (4.175), this determines r_K as

$$r_K = \frac{1}{\mathcal{F}^r} - 1 + \bar{\delta} = (1+\rho)\,(1+g)^\sigma + \bar{\delta} - 1 \tag{4.177}$$

Also, along a balanced growth path, we have inflation at $\pi_t = \bar{\pi}$ and

$$p^* = 1 \tag{4.178}$$

$$\zeta_1 = \frac{Xy}{1 - \theta \mathcal{F}^r (1+g)} = \frac{Xy}{1 - \theta\beta(1+g)^{1-\sigma}} \tag{4.179}$$

$$\zeta_2 = \frac{y}{1 - \theta\beta(1+g)^{1-\sigma}}, \tag{4.180}$$

$$X = 1/\mathcal{M} \tag{4.181}$$

$$wN = (1-\alpha)y/\mathcal{M} \tag{4.182}$$

$$r_K K = \alpha y/\mathcal{M} \tag{4.183}$$

$$q_{\mathcal{F}} = \frac{\mathcal{M}-1}{\mathcal{M}} \frac{y}{1 - \theta\beta(1+g)^{1-\sigma}} \tag{4.184}$$

$$\tilde{\pi}_F = \frac{\mathcal{M}-1}{\mathcal{M}} y + \frac{r_K \bar{k}}{1+g} - I \tag{4.185}$$

$$\bar{k} = \frac{1-\bar{\delta}}{1+g} \bar{k} + I = \frac{1+g}{g+\delta} I \tag{4.186}$$

4.A.4 Labour Market

To finalise the production side of the model, we still need to match households' labour supply and firms' labour demand. As was already alluded to earlier, nominal wage rigidities are a central component for quantitative macro models. Here, in the spirit of Erceg *et al.* (2000), we achieve it by introducing a structure very akin to the production block: Differentiated labour unions, each supplying its own labour type, engage in monopolistic competition subject to Calvo-(1983)-type staggered wage setting. A competitive labour packer then combines these inputs in a CES basket, which they supply to intermediate goods firms.

Labour packer

Since labour packer (in the following considered to be a competitive labour-packing firm) acts as the connection between labour unions and the firm sector, and since their behaviour is crucial for unions' wage setting, we start our explanation with them. The competitve labour-packing firm can assemble a continuum $\mathcal{U}$ (with $|\mathcal{U}| = 1$) of intermediate labour types $n_{u,t}$, $u \in \mathcal{U}$ into a uniform final labour output $N_{S,t}$ according to the CES aggregator

$$N_{S,t} = \left(\int_{u \in \mathcal{U}} n_{u,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, d\omega_{\mathcal{U}} \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{4.187}$$

where $\epsilon_w > 1$ is the elasticity of substitution across labour types.

The labour packer then sells this aggregated labour service to intermediate-goods producers at a price ('wage') $\tilde{W}_t$. From the receipts, it pays respective wage rate for each type of intermediate labour input $\tilde{W}_{u,t}$. This gives per-period profits for the labour packer as

$$\tilde{\Pi}_{LP,t} = \tilde{W}_t N_t^S - \int_{u \in \mathcal{U}} \left(\tilde{W}_{u,t} n_{u,t} \right) d\omega_{\mathcal{U}}. \tag{4.188}$$

The labour packer's cost-minimisation then implies the choice of $(n_{u,t})_{u \in \mathcal{U}}$ for the optimisation of (4.188) subject to (4.187), which delivers standard results similar to those discussed above for the final-goods firm: The relative demand for two intermediate labour types $u_1, u_1 \in \mathcal{U}$ is related to their relative prices according to

$$\frac{n_{u_1,t}}{n_{u_2,t}} = \left(\frac{\tilde{W}_{u_1,t}}{\tilde{W}_{u_2,t}} \right)^{-\epsilon_w},$$

which also yields the demand for one intermediate labour type u as

$$\frac{n_{u,t}}{N_{S,t}} = \left(\frac{\tilde{W}_{u_1,t}}{\tilde{W}_t} \right)^{-\epsilon_w}, \tag{4.189}$$

and which determines the cost-optimal aggregate wage rate $\tilde{W}_t$ to be

$$\tilde{W}_t = \left(\int_{u \in \mathcal{U}} \tilde{W}_{u,t}^{1-\epsilon_w} d\omega_{\mathcal{U}} \right)^{\frac{1}{1-\epsilon_w}}. \tag{4.190}$$

From this, we can also define the aggregate wage inflation rate $\pi_{w,t}$ of the economy as the growth rate of the nominal wage $P_t\tilde{W}_t$ received by the labour packer:

$$\pi_{w,t} := \frac{P_t \tilde{W}_t}{P_{t-1}\tilde{W}_{t-1}} - 1 = \frac{(1+\pi_t)\tilde{W}_t}{\tilde{W}_{t-1}} - 1;$$

which we rearrange to get the law of motion for the real wage as

$$\tilde{W}_t = \frac{1+\pi_{w,t}}{1+\pi_t} \tilde{W}_{t-1}. \tag{4.191}$$

As is standard in the literature, the labour packer does not generate any profits. This completes the demand side on the labour market, allowing us to now derive the supply schedules for $n_{u,t}$, $u \in \mathcal{U}$.

Labour unions

To start off, we assume that each intermediate labour type $u \in \mathcal{U}$ is supplied by one labour union also indexed by u. As mentioned before, all households $\hbar \in \mathcal{H}$ are members of differentiated labour unions. In particular, each household is assigned to exactly one union, but each union contains a representative mixture of households – in particular, a share s_u of the members of any union are unconstrained, a share $s_c = 1 - s_u$ are constrained. For further reference, let $\mathcal{H}_u \subset \mathcal{H}$ be the set of all members of union u. For simplicity, let $|\mathcal{H}_u| = 1$.

As discussed before, we borrow the structure from Erceg *et al.* (2000): Each union sets the wage $W_{u,t}$, which then determines $n_{u,t}$ via equation (4.189). Potentially, there is a constant tax rate τ_w (or subsidy if $\tau_w < 0$) on labour-income, which the government uses to fund lump-sum transfers $\tilde{T}_{U,t}$ to all unions. The union's receipts are given by $(1-\tau_w)W_{u,t}n_{u,t} + \tilde{T}_{U,t}$, which it splits equally across its members. Thus, each member $\hbar \in \mathcal{H}_u$ also receives from the union

$$\frac{(1-\tau_w)W_{u,t}n_{u,t} + \tilde{T}_{U,t}}{|\mathcal{H}_u|} = (1-\tau_w)W_{u,t}n_{u,t} + \tilde{T}_{U,t} \tag{4.192}$$

Similarly, the union requires all members to work identical hours:

$$N_{\hbar} = \frac{n_{u,t}}{|\mathcal{H}_u|} = n_{u,t} \quad \forall \hbar \in \mathcal{H}_u \tag{4.193}$$

Any union u's objective is to maximise its members' discounted wage income stream, valued at marginal utilities $\Lambda_{\hbar,t}$ and taking into account the disutility from labour supply. Formally, in each period, union u has a period payoff given by

$$\begin{aligned}\Pi_{u,t} &= \int_{\hbar\in\mathcal{H}} \left[\Lambda_{\hbar,t}\left((1-\tau_w)\tilde{W}_{u,t}n_{u,t} + \tilde{T}_{U,t}\right) - v(n_{u,t})\right] d\omega_{\bar{\mathcal{H}}_u} \\ &= \int_{\hbar\in\mathcal{H}} (\Lambda_{\hbar,t})\left((1-\tau_w)\tilde{W}_{u,t}n_{u,t} + \tilde{T}_{U,t}\right) d\omega_{\bar{\mathcal{H}}_u} - |\mathcal{H}_u| v(n_{u,t}) \\ &= \tilde{\Lambda}_{u,t}\left((1-\tau_w)\tilde{W}_{u,t}n_{u,t} + \tilde{T}_{U,t}\right) - v(n_{u,t}),\end{aligned} \tag{4.194}$$

where $\tilde{\Lambda}_{u,t}$ is the average marginal utility of consumption of u's members:

$$\begin{aligned}\tilde{\Lambda}_{u,t} &= \int_{\hbar\in\mathcal{H}} (\Lambda_{\hbar,t})\, d\omega_{\bar{\mathcal{H}}_u} = \int_{\hbar\in\mathcal{H}} \left(C_{\hbar,t}^{-\sigma}\right) = \left(s_u C_{u,t}^{-\sigma} + s_c C_{c,t}^{-\sigma}\right)|\mathcal{H}_u| \\ &= s_u C_{u,t}^{-\sigma} + s_c C_{c,t}^{-\sigma}\end{aligned} \tag{4.195}$$

Since (4.194) is formulated in terms of utils, the appropriate discount factor between periods t and $t+s$ is simply given by β^s for $s > 1$. To introduce nominal wage-rigidity, we assume, similar to section 4.A.3, that each union is only able to reoptimise its wage with a random probability $1-\theta_w$, where the opportunity to reoptimise is an i.i.d. event across unions. For future reference denote the set of reoptimising unions in t as $\mathcal{U}_t^o \subset \mathcal{U}$ with $|\mathcal{U}_t^o|/|\mathcal{U}| = 1-\theta_w$.

On the contrary, unions that cannot reset their wages in t adjust their nominal wages $P_t\tilde{W}_{u,t}$ according to the steady-state wage inflation rate $\bar{\pi}_w$. I.e., for $u \in \mathcal{U}\backslash\mathcal{U}_t^o$, we have:

$$P_t\tilde{W}_{u,t} = (1+\bar{\pi}_w)P_{t-1}\tilde{W}_{u,t-1},$$

from which the real-wage evolution of a non-optimising firm is given by

$$\tilde{W}_{u,t} = \frac{1+\bar{\pi}_w}{1+\pi_t}\tilde{W}_{u,t-1}. \tag{4.196}$$

A reoptimising union $u \in \mathcal{U}_t^o$ takes into account that it may not reset its price in the future. Thus, its recursively defined value function at the time of reoptimisation t_o is given by

$$\begin{aligned}V_{u,t_o}^* = \max_{\tilde{W}_{u,t_o}\geq 0} \Big\{ &\Pi_{u,t_o}(\tilde{W}_{u,t_o}) + \beta\mathcal{E}_{u,t_o}\left[\tilde{V}_{u,t}(\tilde{W}_{u,t_o})\right] \\ &\text{s.t. } \forall t \geq t_o: \\ &\tilde{V}_{u,t}(\tilde{W}) = (1-\theta)V_{u,t}^* + \theta V_{u,t}\left(\frac{1+\bar{\pi}_w}{1+\pi_t}\tilde{W}_{u,t}\right) \\ &\tilde{\Pi}_{u,t}(\tilde{W}) = \tilde{\Lambda}_{u,t}\left((1-\tau_w)\tilde{W}\tilde{n}_t\left(\tilde{W}\right) + \tilde{T}_{U,t}\right) - v\left(\tilde{n}_t\left(\tilde{W}\right)\right), \\ &\tilde{n}_t\left(\tilde{W}\right) = \left(\frac{\tilde{W}}{W_t}\right)^{-\epsilon_w} N_t \\ &V_{u,t}\left(\tilde{W}\right) = \Pi_{u,t}(\tilde{W}_{u,t}) + \beta\mathcal{E}_{u,t}\left[\tilde{V}_{u,t}(\tilde{W})\right]\Big\}\end{aligned}$$

The consequent optimality condition is given by

$$\mathcal{E}_{u,t_o}\left[\sum_{t>t_o}\left(\prod_{s=t_o+1}^{t}\left(\frac{\beta\theta_w(1+\bar{\pi}_w)}{1+\pi_s}\right)\tilde{\Pi}'_{u,t}\left(\prod_{s=t_o+1}^{t}\left(\frac{1+\bar{\pi}_w}{1+\pi_s}\right)\tilde{W}_{u,t_o}\right)\right)\right]=0,$$

which implies

$$\mathcal{E}_{u,t_o}\left[\sum_{t>t_o}\left(\prod_{s=t_o+1}^{t}\left(\beta\theta_w\left(\frac{1+\bar{\pi}_w}{1+\pi_s}\right)^{1-\epsilon_w}\right)\tilde{\Lambda}_{u,t}(1-\epsilon_w)(1-\tau_w)\left(\frac{\tilde{W}_{u,t_o}}{\tilde{W}_t}\right)^{-\epsilon_w}N_t\right.\right.$$
$$\left.\left.+\prod_{s=t_o+1}^{t}\left(\beta\theta_w\left(\frac{1+\bar{\pi}_w}{1+\pi_s}\right)^{-\epsilon_w(1+\varphi)}\right)\frac{\epsilon_w\nu}{\tilde{W}_{u,t_o}}\left[\left(\frac{\tilde{W}_{u,t_o}}{\tilde{W}_t}\right)^{-\epsilon_w}N_t\right]^{1+\varphi}\right)\right]=0.$$

Using (4.191), we can rewrite this as

$$\mathcal{E}_{u,t_o}\left[\sum_{t>t_o}\left(\prod_{s=t_o+1}^{t}\left(\frac{\beta\theta_w(1+\bar{\pi}_w)^{1-\epsilon_w}}{(1+\pi_s)(1+\pi_{w,s})^{-\epsilon_w}}\right)\tilde{\Lambda}_{u,t}(1-\epsilon_w)(1-\tau_w)\left(\frac{\tilde{W}_{u,t_o}}{\tilde{W}_{t_o}}\right)^{-\epsilon_w}N_t\right.\right.$$
$$\left.\left.+\prod_{s=t_o+1}^{t}\left(\beta\theta_w\left(\frac{1+\bar{\pi}_w}{1+\pi_{w,s}}\right)^{-\epsilon_w(1+\varphi)}\right)\frac{\epsilon_w\nu}{\tilde{W}_{u,t_o}}\left[\left(\frac{\tilde{W}_{u,t_o}}{\tilde{W}_{t_o}}\right)^{-\epsilon_w}N_t\right]^{1+\varphi}\right)\right]=0$$

or, rearranged and recursively defined as

$$\frac{\tilde{W}_{u,t_o}^{1+\epsilon_w\varphi}}{\tilde{W}_{t_o}^{\epsilon_w\varphi}}=\mathcal{M}_w\frac{\tilde{\zeta}_{1,u,t_o}}{\tilde{\zeta}_{2,u,t_o}} \tag{4.197}$$

$$\text{with}\quad \mathcal{M}_w:=\frac{\nu\epsilon_w}{(\epsilon_w-1)(1-\tau_w)} \tag{4.198}$$

$$\zeta_{1,u,t_o}:=\mathcal{E}_{u,t_o}\left[\sum_{t>t_o}\left(\prod_{s=t_o+1}^{t}\left(\beta\theta_w\left(\frac{1+\bar{\pi}_w}{1+\pi_{w,s}}\right)^{-\epsilon_w(1+\varphi)}\right)N_t^{1+\varphi}\right)\right]$$
$$=N_{t_o}^{1+\varphi}+\beta\theta_w\mathcal{E}_{u,t_o}\left[\left(\frac{1+\pi_{w,t_o+1}}{1+\bar{\pi}_w}\right)^{\epsilon_w(1+\varphi)}\zeta_{1,u,t_o+1}\right] \tag{4.199}$$

$$\zeta_{2,u,t_o}:=\mathcal{E}_{u,t_o}\left[\sum_{t>t_o}\left(\prod_{s=t_o+1}^{t}\left(\frac{\beta\theta_w(1+\bar{\pi}_w)^{1-\epsilon_w}}{(1+\pi_s)(1+\pi_{w,s})^{-\epsilon_w}}\right)\tilde{\Lambda}_{u,t}N_t\right)\right]$$
$$=\Lambda_{u,t_o}N_{t_o}+\beta\theta_w\mathcal{E}_{u,t_o}\left[\frac{(1+\bar{\pi}_w)^{1-\epsilon_w}}{(1+\pi_{t0+1})(1+\pi_{w,t0+1})^{-\epsilon_w}}\zeta_{2,u,t_o+1}\right]. \tag{4.200}$$

Assuming that all unions share the same expectation process $\mathcal{E}_{u,t}[\cdot]=\mathcal{E}_{U,t}[\cdot]$, we can obtain

$$W_{u,t}=W_t^*\quad\forall u\in\mathcal{U}_t^o, \tag{4.201}$$

$$\text{where }\tilde{W}_t^*:=\left(\mathcal{M}_w\tilde{W}_t^{\epsilon_w\varphi}\frac{\tilde{\zeta}_{1,U,t}}{\tilde{\zeta}_{2,U,t}}\right)^{\frac{1}{1+\epsilon_w\varphi}} \tag{4.202}$$

$$\tilde{\zeta}_{1,U,t}:=N_t^{1+\varphi}+\beta\theta_w\mathcal{E}_{U,t}\left[\left(\frac{1+\pi_{w,t+1}}{1+\bar{\pi}_w}\right)^{\epsilon_w(1+\varphi)}\tilde{\zeta}_{1,U,t+1}\right] \tag{4.203}$$

$$\tilde{\zeta}_{2,U,t}:=\Lambda_{U,t}N_t+\beta\theta_w\mathcal{E}_{U,t}\left[\frac{(1+\bar{\pi}_w)^{1-\epsilon_w}}{(1+\pi_{t+1})(1+\pi_{w,t+1})^{-\epsilon_w}}\tilde{\zeta}_{2,U,t+1}\right]. \tag{4.204}$$

$$\Lambda_{U,t}:=s_uC_{u,t}^{-\sigma}+s_cC_{c,t}^{-\sigma}. \tag{4.205}$$

Finally, note that the insurance scheme á la Erceg *et al.* (2000) as well as the redistribution of the returns from the τ_w tax among union eventually leads to a unified labour income for all households of

$$Y_{L,t} = \tilde{W}_t N_{S,t}. \tag{4.206}$$

Real-wage determination

Note that we with the results from subsection 4.A.4, we can rewrite (4.190) as

$$\begin{aligned}
\tilde{W}_t &= \left(\int_{u\in\mathcal{U}} \tilde{W}_{u,t}^{1-\epsilon_w} d\omega_{\mathcal{U}} \right)^{\frac{1}{1-\epsilon_w}} \\
\Leftrightarrow \quad 1 &= \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t}}{\tilde{W}_t} \right)^{1-\epsilon_w} d\omega_{\mathcal{U}} \\
&= \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t}}{\tilde{W}_t} \right)^{1-\epsilon_w} d\omega_{\mathcal{U}_t^o} + \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t}}{\tilde{W}_t} \right)^{1-\epsilon_w} d\omega_{\mathcal{U}\setminus\mathcal{U}_t^o} \\
&= (1-\theta_w) \left(\frac{\tilde{W}_t^*}{\tilde{W}_t} \right)^{1-\epsilon_w} + \theta_w \int_{u\in\mathcal{U}} \left(\frac{1+\bar{\pi}_w}{1+\pi_t} \frac{\tilde{W}_{u,t-1}}{\tilde{W}_t} \right)^{1-\epsilon_w} d\omega_{\mathcal{U}} \\
&= (1-\theta_w) \left(\frac{\tilde{W}_t^*}{\tilde{W}_t} \right)^{1-\epsilon_w} + \theta_w \int_{u\in\mathcal{U}} \left(\frac{1+\bar{\pi}_w}{1+\pi_{w,t}} \frac{\tilde{W}_{u,t-1}}{\tilde{W}_{t-1}} \right)^{1-\epsilon_w} d\omega_{\mathcal{U}} \\
&= (1-\theta_w) \left(\frac{\tilde{W}_t^*}{\tilde{W}_t} \right)^{1-\epsilon_w} + \theta_w \left(\frac{1+\bar{\pi}_w}{1+\pi_{w,t}} \right)^{1-\epsilon_w} \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t-1}}{\tilde{W}_{t-1}} \right)^{1-\epsilon_w} d\omega_{\mathcal{U}} \\
&= (1-\theta_w) \left(\frac{\tilde{W}_t^*}{\tilde{W}_t} \right)^{1-\epsilon_w} + \theta_w \left(\frac{1+\bar{\pi}_w}{1+\pi_{w,t}} \right)^{1-\epsilon_w} \qquad (4.207)
\end{aligned}$$

Also, note that akin to (4.148), we can derive wage-dispersion terms. For example, define for some real number a:

$$\begin{aligned}
\xi_{w,t}(a) &:= \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t}}{\tilde{W}_t} \right)^{a} d\omega_{\mathcal{U}} \\
&= \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t}}{\tilde{W}_t} \right)^{a} d\omega_{\mathcal{U}_t^o} + \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t}}{\tilde{W}_t} \right)^{a} d\omega_{\mathcal{U}\setminus\mathcal{U}_t^o} \\
&= (1-\theta_w) \left(\frac{\tilde{W}_t^*}{\tilde{W}_t} \right)^{a} + \theta_w \int_{u\in\mathcal{U}} \left(\frac{1+\bar{\pi}_w}{1+\pi_{w,t}} \frac{\tilde{W}_{u,t-1}}{\tilde{W}_{t-1}} \right)^{a} d\omega_{\mathcal{U}} \\
&= (1-\theta_w) \left(\frac{\tilde{W}_t^*}{\tilde{W}_t} \right)^{a} + \theta_w \left(\frac{1+\bar{\pi}_w}{1+\pi_{w,t}} \right)^{a} \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t-1}}{\tilde{W}_{t-1}} \right)^{a} d\omega_{\mathcal{U}} \\
&= (1-\theta_w) \left(\frac{\tilde{W}_t^*}{\tilde{W}_t} \right)^{a} + \theta_w \left(\frac{1+\bar{\pi}_w}{1+\pi_{w,t}} \right)^{a} \xi_{w,t-1}(a) \qquad (4.208)
\end{aligned}$$

Then, particular applications are integrals over (powers of) households' labour supplies:

$$\int_{u\in\mathcal{U}}\int_{\hbar\in\mathcal{H}} n_{u,t}^a d\omega_{\mathcal{H}_u} d\omega_{\mathcal{U}} = \int_{u\in\mathcal{U}} n_{u,t}^a \int_{\hbar\in\mathcal{H}} d\omega_{\mathcal{H}_u} d\omega_{\mathcal{U}} = \int_{u\in\mathcal{U}} n_{u,t}^a d\omega_{\mathcal{U}}$$
$$= \int_{u\in\mathcal{U}} \left(\left(\frac{\tilde{W}_{u,t}}{\tilde{W}_t}\right)^{-\epsilon_w} N_t\right)^a d\omega_{\mathcal{U}}$$
$$= \int_{u\in\mathcal{U}} \left(\frac{\tilde{W}_{u,t}}{\tilde{W}_t}\right)^{-\epsilon_w a} d\omega_{\mathcal{U}} N_t^a$$
$$= \xi_{w,t}(-a\epsilon_w)N_t^a$$

with particular applications being $a = 1$ for the actual total hours worked and $a = 1 + \varphi$ for welfare comparisons (at least under the assumption of wage setting à la Erceg *et al.* (2000)).

Stationary equilibrium

In order to express the relevant equations (4.191) and (4.202)–(4.207) of this section in stationary terms, realise that the real-wage $\tilde{W}_t$ will grow with A_t, so we need to introduce a stationary real wage per efficient unit of labour, given by $\tilde{w}_t := \frac{\tilde{W}_t}{A_t}$. Likewise, we can define

$$\lambda_{U,t} := A_t^\sigma \Lambda_{U,t} = A_t^\sigma \left(s_u C_{u,t}^{-\sigma} + s_c C_{c,t}^{-\sigma}\right) = s_u c_{u,t}^{-\sigma} + s_c c_{c,t}^{-\sigma}, \tag{4.209}$$

from which we can make (4.204) stationary:

$$\zeta_{2,U,t} := A_t^\sigma \tilde{\zeta}_{2,U,t}$$
$$= \lambda_{U,t}N_t + \beta\theta_w A_t^\sigma \mathcal{E}_{U,t}\left[\frac{(1+\bar{\pi}_w)^{1-\epsilon_w}}{(1+\pi_{t+1})(1+\pi_{w,t+1})^{-\epsilon_w}}\tilde{\zeta}_{2,U,t+1}\right]$$
$$= \lambda_{U,t}N_t + \beta\theta_w A_t^\sigma \mathcal{E}_{U,t}\left[\frac{(1+\bar{\pi}_w)^{1-\epsilon_w}}{(1+\pi_{t+1})(1+\pi_{w,t+1})^{-\epsilon_w}}A_{t+1}^{-\sigma}\zeta_{2,U,t+1}\right]$$
$$= \lambda_{U,t}N_t + \beta\theta_w \mathcal{E}_{U,t}\left[\frac{(1+g_t)^{-\sigma}(1+\bar{\pi}_w)^{1-\epsilon_w}}{(1+\pi_{t+1})(1+\pi_{w,t+1})^{-\epsilon_w}}\zeta_{2,U,t+1}\right]. \tag{4.210}$$

Note that equation (4.203) is already stationary. Also, we obtain from (4.191), (4.202) and (4.207):

$$\tilde{w}_t = \frac{1+\pi_{w,t}}{(1+\pi_t)(1+g_t)}\tilde{w}_{t-1}, \tag{4.211}$$

$$\tilde{w}_t^* := \frac{\tilde{W}_t^*}{A_t} = \frac{1}{A_t}\left(\mathcal{M}_w \tilde{W}_t^{\epsilon_w\varphi}\frac{\tilde{\zeta}_{1,U,t}}{\tilde{\zeta}_{2,U,t}}\right)^{\frac{1}{1+\epsilon_w\varphi}}$$
$$= \frac{1}{A_t}\left(\mathcal{M}_w A^{\epsilon_w\varphi}\tilde{w}_t^{\epsilon_w\varphi}\frac{\tilde{\zeta}_{1,U,t}}{A_t^{-\sigma}\zeta_{2,U,t}}\right)^{\frac{1}{1+\epsilon_w\varphi}}$$
$$= A_t^{\frac{\sigma-1}{1+\epsilon_w\varphi}}\left(\mathcal{M}_w \tilde{w}_t^{\epsilon_w\varphi}\frac{\tilde{\zeta}_{1,U,t}}{\zeta_{2,U,t}}\right)^{\frac{1}{1+\epsilon_w\varphi}} \quad \text{and} \tag{4.212}$$

$$1 = (1-\theta_w)\left(\frac{\tilde{W}_t^*}{\tilde{W}_t}\right)^{1-\epsilon_w} + \theta_w\left(\frac{1+\bar{\pi}_w}{1+\pi_{w,t}}\right)^{1-\epsilon_w}$$
$$= (1-\theta_w)\left(\frac{\tilde{w}_t^*}{\tilde{w}_t}\right)^{1-\epsilon_w} + \theta_w\left(\frac{1+\bar{\pi}_w}{1+\pi_{w,t}}\right)^{1-\epsilon_w} \quad (4.213)$$

From (4.212), it is clear that for a balanced growth path with $g \neq 0$ to exist, we must have $\sigma = 1$ in this model. In that case from (4.211), we get the relationship between stead price inflation $\bar{\pi}$, TFP growth g and wage inflation $\bar{\pi}_w$ as

$$1 + \bar{\pi}_w = (1+\bar{\pi})(1+g) \quad (4.214)$$

Also, from (4.203) and (4.204) $\zeta_{1,U,t}$ and $\zeta_{2,U,t}$ have steady long-run values

$$\zeta_{1,U} = \frac{N^{1+\varphi}}{1-\beta\theta_w} \quad (4.215)$$
$$\zeta_{2,U} = \frac{\lambda_U N}{1-\beta\theta_w(1+g)^{1-\sigma}}. \quad (4.216)$$

4.A.5 Monetary and Fiscal Policy

As in most advanced economies, the central government is assumed to be responsible for fiscal policy, whereas monetary policy is delegated to an independent central bank. Since the focus in the following will be on productive public investment, we keep the central bank as simple as possible with flexible inflation targeting. However, there are many potentially interesting interactions between this type of fiscal policy and the monetary policy scheme (see chapter 2). However, such an analysis is beyond the scope of this chapter and relegated to future research.

We assume that monetary policy sets the nominal interst rate i_t following the simple Taylor rule (4.217) with reaction parameters $\phi_\pi > 1$ and $\phi_y \geq 0$:

$$i_t = \max\{i + \phi_\pi(\pi_t - \bar{\pi}) + \phi_y(y_t/y - 1), 0\} \quad (4.217)$$

where $i = (1+r)(1+\bar{\pi}) - 1$. On the other hand, the fiscal authority has a couple more instruments at its disposal: For one, it can fund government consumption $\tilde{G}_{C,t}$ and productive government investment $\tilde{G}_{I,t}$ giving total government spending (net of interest and loan repayments) as

$$\tilde{G}_t = \tilde{G}_{C,t} + \tilde{G}_{I,t} \quad (4.218)$$

On the other hand, the government receives total tax income $\tilde{T}_t$ and, at the end of period t owes an amount B_t, which has a real interest rate r_{t+1} going into the next period. As such, the government budget constraint reads as

$$B_t = \tilde{G}_t - \tilde{T}_t + (1+r_t)B_{t-1}. \quad (4.219)$$

Government spending

Next, note as in Leeper *et al.* (2010) or Ramey (2021), we assume that both types of government expenditure may suffer from two types of frictions, which introduce implementation lags:

1. A time-to-complete, in the case of government investment better called a *time-to-build*: Any given decision about a spending increase will also increase future spending, and eventually intended benefits may only materialise after quite some time.

2. A time-to-spend: Due to inefficiencies in governance (e.,g., decision-making, procurement), inter alia, the decision to implement some spending may only effectively result in additional spending after a couple of periods.

In addition, as in Gallen and Winston (2021), we assume that both types of spending (in particular government investment) may also disrupt the services provided by public capital.

To formalise this, assume that appropriations $AP_{x,t}$ to public spending in category $x \in \{G, C\}$ follow an AR(1) process

$$AP_{x,t} := A_t \left(\overline{ap}_x + \rho_{AP,x} \left(\frac{AP_{x,t-1}}{A_{t-1}} - \overline{ap}_x \right) + \varepsilon_{ap,x,t} \right) \tag{4.220}$$

However, only after $\mathcal{T}_1 \geq 0$ periods, the appropriated amount is starting to be implemented, i.e. the amount $G_{x,new,t}$ of newly implemented projects of type x is given by

$$\tilde{G}_{x,new,t} = AP_{x,t-\mathcal{T}_1}. \tag{4.221}$$

This amount will lead to increased public spending across the next $\mathcal{T}_{2,x}$ periods according to some (fixed for the purposes of this model) schedule

$$\tilde{G}_{x,s,t} = \chi_{x,s} \tilde{G}_{x,new,t-s}, \quad s = 0, \dots, \mathcal{T}_{2,x} - 1. \tag{4.222}$$

Overall spending in category x is then given by

$$\tilde{G}_{x,t} = \sum_{s=0}^{\mathcal{T}_{2,x}-1} \tilde{G}_{x,s,t} = \sum_{s=0}^{\mathcal{T}_{2,x}-1} \chi_{x,s} \tilde{G}_{x,new,t-s}. \tag{4.223}$$

If the spending is in the investment category, it becomes productive capital in the period $t + \mathcal{T}_{2,I}$, giving a law of motion for public capital $\bar{K}_{P,t}$ of

$$\begin{aligned} \bar{K}_{P,t} &= (1 - \delta_P) \bar{K}_{P,t-1} + I_{P,final,t} \\ &= (1 - \delta_P) \bar{K}_{P,t-1} + \tilde{G}_{I,new,t-\mathcal{T}_{2,I}} \end{aligned} \tag{4.224}$$

For computations, it is easier to work with auxiliary variables for $s = 0, \dots, \mathcal{T}_{2,x}$

$$\bar{G}_{x,s,t} = \begin{cases} \tilde{G}_{x,new,t} & \text{if } s = 0, \\ \bar{G}_{x,s-1,t-1} & \text{otherwise.} \end{cases} \tag{4.225}$$

This allows us to write

$$\tilde{G}_{x,t} = \sum_{s=0}^{\mathcal{T}_{2,x}-1} \chi_{x,s} \bar{G}_{x,s,t} \quad \text{for } x \in \{C, I\} \text{ and} \tag{4.226}$$

$$\bar{K}_{P,t} = (1 - \delta_P) \bar{K}_{P,t-1} + \bar{G}_{I,\mathcal{T}_{2,I},t}. \tag{4.227}$$

and allows us to model the disruption of services in a quite straightforward way: Assume that during the implementation phase, spending in category $x \in \{C, I\}$ affects the (from the point of view of the private agents) effective stock of capital according to

$$K_{P,t} = \bar{K}_{P,t} \left[1 - \psi \left(\frac{\bar{G}_{I,0,t}}{\bar{K}_{P,t}}, \ldots, \frac{\bar{G}_{I,\mathcal{T}_{2,I}-1,t}}{\bar{K}_{P,t}}, \frac{\bar{G}_{C,0,t} - 1}{\bar{K}_{P,t}}, \ldots, \frac{\bar{G}_{C,\mathcal{T}_{2,C},t}}{\bar{K}_{P,t}} \right) \right], \tag{4.228}$$

where $\psi(gi_0, gi_1, gi_2, \ldots, gi_{\mathcal{T}_{2,I}-1}, gc_0, gc_1, gc_2, \ldots, gc_{\mathcal{T}_{2,I}-1})$ is assumed to be a weakly increasing function in all of its argument that is assumed to be 0 on the balanced growth path. I.e. with

$$G_{x,s,t} := \tilde{G}_{x,s,t}/A_t \text{ for } s = 0, \ldots, \mathcal{T}_{2,x} - 1 \text{ for } x \in \{G, C\}$$

and

$$\bar{k}_{P,t} = \frac{\bar{K}_{P,t}}{A_t}$$

as well as corresponding balanced-growth path values $G_{x,s,BGP}$ and $\bar{k}_P$, we have

$$\psi \left(\frac{G_{I,0,BGP}}{\bar{k}_P}, \ldots, \frac{G_{I,\mathcal{T}_{2,I}-1,BGP}}{\bar{k}_P}, \frac{G_{C,0,BGP}}{\bar{k}_P}, \ldots, \frac{G_{C,\mathcal{T}_{2,C}-1,BGP}}{\bar{k}_P} \right) = 0$$

and

$$\tilde{\psi}_{I,s} := \frac{\partial \psi(\cdot)}{\partial gi_s} \geq 0 \ \forall s = 0, \ldots, \mathcal{T}_{2,I} - 1, \tag{4.229}$$

$$\tilde{\psi}_{C,s} := \frac{\partial \psi(\cdot)}{\partial gc_s} \geq 0 \ \forall s = 0, \ldots, \mathcal{T}_{2,C} - 1 \tag{4.230}$$

and which will be specified in more detail later.

Social security system

A potential complication in this model arises from the fact that households that go from being unconstrained to being constrained take with them all of the liquid funds that they have, which they want to consume completely. If one wants to calibrate a THANK model in the spirit of Bilbiie (2021) to an economy with positive government debt along the balanced growth path (i.e. $B_t/A_t = b > 0$), a situation may arise that has $c_c > c_u$, i.e. in terms of the model without other frictions, $r > \rho > r_K$. In order to avoid such a situation, one needs to reduce the amount of bond holdings that the unconstrained can freely choose/take with them to the constrained island. A social security system that holds a fixed amount is one such way:

Assume that for reasons beyond the scope of the model, a social security fund may exist which implements all of the insurance services discussed with respect to the household sector, in particular when it comes to the labour income. This fund's only task is to provide liquidity services for all of the members and for that it needs to (and will always) hold real bond totalling

$$\tilde{B}_t = \gamma b A_t, \tag{4.231}$$

where b is the balanced-growth-path value of total debt and $\gamma \in [0, 1]$. Its only sources of revenue/expenditures relevant for our discussion are interest received on its bond holdings and a lump-sum transfer from its members $\tilde{T}_{B,t}$ (or a transfer to them if $T_{B,t} < 0$). This gives a law of motion

$$\tilde{B}_t = \tilde{T}_{B,t} + (1 + r_t)\tilde{B}_{t-1}. \tag{4.232}$$

Taxation

As introduced in the description of the household sector, both constrained and unconstrained agents pay lump-sum transfers $\tilde{T}_{x,t},\ x \in \{u,c\}$, an amount $\tilde{T}_{B,t}$ of which is transferred to the social security system. That is, the government receives a net amount of

$$\tilde{T}_t = \sum_{x\in\{u,c\}} s_x \left(\tilde{T}_{x,t} - \tilde{T}_{B,t}\right). \tag{4.233}$$

Then, to complete the description of the fiscal block of the economy, we need to specify two additional equations, one linking $\tilde{T}_t$ to the rest of the model and one determining the two $\tilde{T}_{x,t}$ (or two equations determining the two $\tilde{T}_{x,t}$. However, in line with, e.g., Ramey (2021), we pursue the other way.).

For the first one, we consider two variants:

1. Either, the fiscal authority keeps government debt fixed at its steady state value, i.e.

$$B_t = bA_t \quad \forall t$$

which from (4.219) implies

$$\tilde{T}_t = \tilde{G}_t + (1+r_t)bA_{t-1} - bA_t. \tag{4.234}$$

2. Otherwise, it follows a fiscal rule, e.g. one of the form

$$\begin{aligned}\tilde{T}_t - A_tT = (1-\rho_T)\Big[&\phi_G(\tilde{G}_t - A_tG) + \phi_B(1+g)\left(B_{t-1} - bA_{t-1}\right)\Big] \\ &+ \rho_T(1+g)(\tilde{T}_{t-1} - A_{t-1}T)\end{aligned} \tag{4.235}$$

with reaction parameters $\phi_G, \phi_B \geq 0$ (such that $\phi_G + \phi_B > 0$) and persistence $\rho_T \in [0,1)$.[83]

Regarding the determination of the distribution of taxes across agents, let the taxation of unconstrained agents $\tilde{T}_{u,t}$ be given by

$$\tilde{T}_{u,t} = \bar{T}_uA_t + \vartheta_u(\tilde{T}_t - A_tT) + \vartheta_\pi\tilde{\Pi}_{F,t} + \tilde{T}_{B,t}, \tag{4.236}$$

where, necessarily, $\vartheta_u > 0$. Also, $\vartheta_\pi \in [0, s_c/s_u]$ is a potential redistribution of profits and capital income as in Bilbiie *et al.* (2022).

This results in taxation of the constrained

$$\begin{aligned}\tilde{T}_{c,t} &= \frac{\tilde{T}_t - s_u(\tilde{T}_{u,t} - \tilde{T}_{B,t})}{s_c} + \tilde{T}_{B,t} \\ &= \frac{\tilde{T}_t - s_u[\bar{T}_uA_t + \vartheta_u(\tilde{T}_t - A_tT) + \vartheta_\pi\tilde{\Pi}_{F,t}]}{s_c} + \tilde{T}_{B,t}\end{aligned} \tag{4.237}$$

[83]Note that here, we abstract from a reaction coefficient on Y_t since we are not interested in automatic stabilisers per se. However, the case $\frac{\partial\tilde{T}_t}{\partial Y_t} > 0$ would of course be interesting. A more detailled discussion of the funding of public investments is to be pursued in a follow-up project.

Stationary equilibrium

Note that the Taylor rule (4.217) is fully formulated in stationary variables, thus that we can focus on the fiscal variables, starting with equations (4.218) and (4.219)

$$G_t := \frac{\tilde{G}_t}{A_t} = G_{C,t} + G_{I,t}, \tag{4.238}$$

$$\text{with } G_{x,t} := \frac{\tilde{G}_{x,t}}{A_t} \quad x \in \{C, I\}, \tag{4.239}$$

$$\begin{aligned} b_t = \frac{B_t}{A_t} &= G_t - T_t + (1+r_t)\frac{B_{t-1}}{A_t} \\ &= G_t - T_t + \frac{1+r_t}{1+g_t} b_{t-1}. \end{aligned} \tag{4.240}$$

From equation (4.220), we directly see that appropriations in both categories $x \in \{G, I\}$ grow with the TFP growth factor such that we can easily find a stationary

$$ap_{x,t} := \frac{AP_{x,t}}{A_t} = \left(\overline{ap}_x + \rho_{AP,x}\left(ap_{x,t} - \overline{ap}_x\right) + \varepsilon_{ap,x,t}\right) \tag{4.241}$$

This also means that $G_{x,s,t} = \frac{\bar{G}_{x,s,t}}{A_t}$ is stationary:

$$G_{x,s,t} = \begin{cases} G_{x,new,t} & \text{if } s = 0, \\ \frac{G_{x,s-1,t-1}}{1+g_t} & \text{otherwise,} \end{cases} \tag{4.242}$$

where

$$G_{x,new,t} = \frac{\tilde{G}_{x,new,t}}{A_t} = \frac{AP_{t-\mathcal{T}_1}}{A_t} = \frac{ap_{x,t-\mathcal{T}_1}}{\prod_{s=0}^{\mathcal{T}_1-1}(1+g_{t-s})} \tag{4.243}$$

such that

$$G_{x,s,t} = \frac{ap_{x,t-\mathcal{T}_1-s}}{\prod_{j=0}^{\mathcal{T}_1-1+s}(1+g_{t-j})}$$

As a result, we have that overall spending in each category is stationary at

$$G_{x,t} := \frac{\tilde{G}_{x,t}}{A_t} = \sum_{s=0}^{\mathcal{T}_{2,x}-1} \chi_{x,s} G_{x,s,t} = \sum_{s=0}^{\mathcal{T}_{2,x}-1} \chi_{x,s} \frac{ap_{x,t-\mathcal{T}_1-s}}{\prod_{j=0}^{\mathcal{T}_1-1+s}(1+g_{t-j})} \tag{4.244}$$

and the stock of public capital can be made stationary as

$$\begin{aligned} \bar{k}_{P,t} &= \frac{1-\delta_P}{1+g_t}\bar{k}_{P,t-1} + G_{I,\mathcal{T}_{2,I},t} \\ &= \frac{1-\delta_P}{1+g_t}\bar{k}_{P,t-1} + \frac{ap_{I,t-\mathcal{T}_1-\mathcal{T}_{2,I}}}{\prod_{j=0}^{\mathcal{T}_1+\mathcal{T}_{2,I}-1}(1+g_{t-j})}. \end{aligned} \tag{4.245}$$

Finally note that

$$k_{P,t} := \frac{K_t^P}{A_t} \tag{4.246}$$

$$= \bar{k}_{P,t}\left[1 - \psi\left(\frac{G_{I,0,t}}{\bar{k}_{P,t}}, \dots, \frac{G_{I,\mathcal{T}_{2,I}-1,t}}{\bar{k}_{P,t}}, \frac{G_{C,0,t}}{\bar{k}_{P,t}}, \dots, \frac{G_{C,\mathcal{T}_{2,C}-1,t}}{\bar{k}_{P,t}}\right)\right]. \tag{4.247}$$

In the main text, we set $\psi(\cdot) = 0$. However, future work should consider the case $\psi(\cdot) \geq 0$ to analyse disruptions from investment on existing infrastructure as in Gallen and Winston (2021). For the THANK-type model with positive government debt $b > 0$ along the balanced growth path, we can find stationary versions of (4.231) and (4.231):

$$\tilde{b}_t := \frac{\tilde{B}_t}{A_t} = \gamma b \tag{4.248}$$

$$\begin{aligned} \tilde{b}_t &= T_{B,t} + (1 + r_t)\frac{\tilde{B}_{t-1}}{A_t} \\ &= T_{B,t} + \frac{1 + r_t}{1 + g_t}\tilde{b}_{t-1}, \end{aligned} \tag{4.249}$$

where $T_{B,t} = \tilde{T}_{B,t}/A_t$. This also implies

$$T_{B,t} = \gamma b - \frac{1 + r_t}{1 + g_t}\gamma b = \frac{g_t - r_t}{1 + g_t}\gamma b. \tag{4.250}$$

Next, we linearise the taxation block, starting with (4.233):

$$\begin{aligned} T_t := \frac{\tilde{T}_t}{A_t} &= \sum_{x \in \{u,c\}} s_x \left(\frac{\tilde{T}_{x,t}}{A_t} - \frac{\tilde{T}_{B,t}}{A_t} \right) \\ &= \sum_{x \in \{u,c\}} s_x T_{x,t} - T_{B,t}. \end{aligned} \tag{4.251}$$

Considering the governance of overall taxation T_t,

1. if the fiscal authority keeps government debt fixed at its balanced-growth-path value, we immediately have

$$b_t = b \quad \forall t \tag{4.252}$$

$$T_t = G_t + \frac{1 + r_t}{1 + g_t}b - b = G_t + \frac{r_t - g_t}{1 + g_t}b, \tag{4.253}$$

2. conversely, if it follows the fiscal rule (4.235), it is made stationary as

$$\begin{aligned} T_t - T = (1 - \rho_T) &\left[\phi_G(G_t - G) + \phi_B \frac{1 + g}{1 + g_t}(b_{t-1} - b) \right] \\ &+ \rho_T \frac{1 + g}{1 + g_t}(T_{t-1} - T) \end{aligned} \tag{4.254}$$

Regarding the determination of the distribution of taxes across agents, we get from (4.236) and (4.237)

$$T_{u,t} = \bar{T}_u + \vartheta_u(T_t - T) + \vartheta_\pi \tilde{\pi}_{F,t} + T_{B,t}, \tag{4.255}$$

$$T_{c,t} = \frac{T_t - s_u[\bar{T}_u + \vartheta_u(T_t - T) + \vartheta_\pi \tilde{\pi}_{F,t}]}{s_c} + T_{B,t} \tag{4.256}$$

This implies that the relationships among the variables of this subsection along a balanced growth path are given by

$$\frac{g-r}{1+g}b = G - T, \tag{4.257}$$

$$G = G_C + G_I, \tag{4.258}$$

$$G_{x,0,BGP} = \frac{\overline{ap}_x}{(1+g)^{\mathcal{T}_1}} \quad \text{for } x \in \{G, C\} \tag{4.259}$$

$$G_{x,s,BGP} = \frac{G_{x,s-1,BGP}}{1+g} = \frac{\overline{ap}_x}{(1+g)^{\mathcal{T}_1+s}} \quad \text{for } s = 1, \dots \mathcal{T}_{2,x}, x \in \{G, C\} \tag{4.260}$$

$$G_x = \sum_{s=0}^{\mathcal{T}_{2,x}-1} \chi_{x,s} G_{x,s,BGP} = \frac{\overline{ap}_x}{(1+g)^{\mathcal{T}_1}} \sum_{s=0}^{\mathcal{T}_{2,x}-1} \chi_{x,s}(1+g)^{-s} \tag{4.261}$$

$$\frac{g+\delta_P}{1+g}\bar{k}_P = G_{I,\mathcal{T}_{2,I},BGP} = \frac{\overline{ap}_I}{(1+g)^{\mathcal{T}_1+\mathcal{T}_{2,I}}} \tag{4.262}$$

$$k_P = \bar{k}_P \tag{4.263}$$

$$\tilde{b} = \gamma b \tag{4.264}$$

$$T_B = \frac{g-r}{r+g}\gamma b \tag{4.265}$$

$$T_u = \bar{T}_u + \vartheta_\pi \tilde{\pi}_F + T_B, \tag{4.266}$$

$$T_c = \frac{T - s_u \bar{T}_u}{s_c} - \frac{s_u}{s_c}\vartheta_\pi \tilde{\pi}_F + T_B \tag{4.267}$$

Feedback into consumption

Plugging (4.256) into (4.108) gives

$$c_{c,t} = y_{L,t} - \left(\frac{T_t - s_u[\bar{T}_u + \vartheta_u(T_t - T) + \vartheta_\pi \tilde{\pi}_{F,t}]}{s_c} + T_{B,t} \right) + \frac{1+r_t}{1+g_t}\left(p_{c|c}\bar{b}_t + \frac{p_{c|u}s_u}{s_c} b_{u,t-1} \right) - \bar{b}_t \tag{4.268}$$

Similarly, consumption of the unconstrained is given by

$$c_{u,t} = y_{L,t} - \left(\bar{T}_u + \vartheta_u(T_t - T) + T_{B,t} \right) + \tilde{\pi}_{F,t}\left(\frac{1}{s_u} - \vartheta_\pi \right) + \frac{1+r_t}{1+g_t}\left(\frac{p_{u|c}s_c}{s_u}\bar{b}_{t-1} + p_{u|u}b_{u,t-1} \right) - b_{u,t} \tag{4.269}$$

These imply steady state consumptions of

$$c_c = y_L - \frac{T - s_u\bar{T}_u}{s_c} + \frac{s_u\vartheta_\pi}{s_c}\tilde{\pi}_F + \frac{r-g}{1+g}\gamma b + \frac{1+r}{1+g}\left(p_{c|c}\bar{b} + \frac{p_{c|u}s_u}{s_c}b_u \right) - \bar{b} \tag{4.270}$$

$$c_u = y_L - \bar{T}_u + \tilde{\pi}_F\left(\frac{1}{s_u} - \vartheta_\pi \right) + \frac{r-g}{1+g}\gamma b + \frac{1+r}{1+g}\left(\frac{p_{u|c}s_c}{s_u}\bar{b} + p_{u|u}b_u \right) - b_u \tag{4.271}$$

4.A.6 Market Clearing

Finally, we require goods markets to clear, in particular for the final-goods firm, this gives

$$Y_t = C_t + \tilde{I}_t + \tilde{G}_t,$$

where total consumption C_t is given as

$$C_t = s_u C_{u,t} + s_c C_{c,t},$$

or in stationary terms

$$y_t = c_t + I_t + G_t, \tag{4.272}$$
$$c_t = s_u c_{u,t} + s_c c_{c,t}. \tag{4.273}$$

Moreover, clearing on the labour market requires

$$N_t = N_{S,t} = N_{D,T}$$
$$\tilde{w}_t = w_t,$$

on the capital market

$$k_t = U_{K,t} \frac{\bar{k}_{t-1}}{1+g_t},$$

and on the bond market

$$B_t = s_u B_{u,t} + s_c B_{c,t} + \tilde{B}_t,$$

or expressed in a stationary manner

$$b_t = s_u b_{u,t} + s_c b_{c,t} + \tilde{b}_t = s_u b_{u,t} + s_c \bar{b}_t + \gamma b, \tag{4.274}$$

implying steady-state values

$$k = \frac{\bar{k}}{1+g} \tag{4.275}$$
$$b_u = \frac{(1-\gamma)b - s_c \bar{b}}{s_u} \tag{4.276}$$

4.B Recursive Linearisation

In order to linearise the model, we perform a first-order Taylor approximation of the model equations around the balanced growth path. Both the methods used and most of the steps are common to the literature. See, for instance, the textbook treatment by Galí (2015) for a detailed explanation. With respect to the household sector, the reader is advised to look at the derivations in Bilbiie (2021).

4.B.1 Household Sector

From the household block, we linearise equations (4.103) – (4.109)

$$\begin{aligned}
\hat{b}^{In}_{x,t} &:= b^{In}_{x,t} - b^{In}_{x} \\
&\approx \frac{1}{1+g_t}\left(p_{x|x}(b_{x,t-1}-b_x) + \frac{p_{x|y}s_y}{s_x}(b_{y,t-1}-b_y)\right) \\
&\quad - \frac{g_t - g}{1+g}\left(p_{x|x}b_x + \frac{p_{x|y}s_y}{s_x}b_y\right) \\
&= \frac{1}{1+g_t}\left(p_{x|x}\hat{b}_{x,t-1} + \frac{p_{x|y}s_y}{s_x}\hat{b}_{y,t-1}\right) \\
&\quad - \frac{\hat{g}_t}{(1+g)^2}\left(p_{x|x}b_x + \frac{p_{x|y}s_y}{s_x}b_y\right), \quad x,y \in \{u,c\}, x \neq y \qquad (4.277)
\end{aligned}$$

with $\hat{g}_t = g_t - g$, $\hat{b}_{x,t} = b_{x,t} - b_x$, $x \in \{u,c\}$

$$\begin{aligned}
\hat{\mathcal{F}}^r_{t+1} &:= \mathcal{F}^r_{t+1} - \mathcal{F}^r \\
&\approx -\sigma\beta\left((1+g)\frac{c_u}{c_u}\right)^{-\sigma-1}\left[\frac{c_u}{c_u}\hat{g}_t + (1+g)\left(\frac{\hat{c}_{u,t+1}}{c_u} - \frac{c_u\hat{c}_{u,t}}{c_u^2}\right)\right] \\
&= -\sigma\beta\,(1+g)^{-\sigma-1}\left[\hat{g}_t + (1+g)\left(\frac{\hat{c}_{u,t+1}}{c_u} - \frac{\hat{c}_{u,t}}{c_u}\right)\right] \\
&= -\sigma\beta\,(1+g)^{-\sigma}\left[\frac{\hat{g}_t}{1+g} + \left(\frac{\hat{c}_{u,t+1}}{c_u} - \frac{\hat{c}_{u,t}}{c_u}\right)\right] \\
&= -\sigma\mathcal{F}^r\left[\frac{\hat{g}_{t+1}}{1+g} + \left(\frac{\hat{c}_{u,t+1}}{c_u} - \frac{\hat{c}_{u,t}}{c_u}\right)\right] \qquad (4.278)
\end{aligned}$$

with $\hat{c}_{x,t} = c_{x,t} - b_x$, $x \in \{u,c\}$

$$\begin{aligned}
\hat{q}^S_t &:= q^S_t - q^S \\
&\approx \mathcal{F}^r(1+g)\cdot\mathcal{E}^{HH}_t\left[\frac{\hat{\mathcal{F}}^r_{t+1}}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g} + \hat{q}^S_{t+1} + \hat{\pi}_{\mathcal{F},t+1}\right] \\
&= \beta(1+g)^{1-\sigma}\cdot\mathcal{E}^{HH}_t\left[-\sigma\left(\frac{\hat{c}_{u,t+1}}{c_u} - \frac{\hat{c}_{u,t}}{c_u}\right)\right. \\
&\qquad\qquad \left. +(1-\sigma)\frac{\hat{g}_{t+1}}{1+g} + \hat{q}^S_{t+1} + \hat{\pi}_{F,t+1}\right] \qquad (4.279)
\end{aligned}$$

with $\hat{\pi}_{F,t} := \tilde{\pi}_{F,t} - \tilde{\pi}_F$

$$\hat{b}_{c,t} := b_{c,t} - b_c = \bar{b}_t - \bar{b} = 0 \qquad (4.280)$$

$$\hat{c}_{c,t} \approx \hat{y}_{L,t} - \hat{T}_{c,t} + \frac{1+r}{1+g}\left(p_{c|c}\tilde{b}_{t-1} + \frac{p_{c|u}s_u}{s_c}\hat{b}_{u,t-1}\right.$$

$$+\left(\frac{\hat{r}_t}{1+r}-\frac{\hat{g}_t}{1+g}\right)\left(p_{c|c}\bar{b}+\frac{p_{c|u}s_u}{s_c}b_u\right)\Bigg)-\tilde{b}_t \tag{4.281}$$

with $\hat{y}_{L,t} := y_{L,t} - y_L, \quad \hat{T}_{c,t} = T_{c,t} - T_c,$

$$\begin{aligned}
\hat{\mathcal{F}}^s_{t+1} &:= \mathcal{F}^s_{t+1} - \mathcal{F}^s \\
&\approx -\sigma\beta \sum_{x\in\{u,c\}} p_{x|u}\Bigg[\left((1+g)\frac{c_x}{c_u}\right)^{-\sigma-1}\Bigg(\hat{g}_{t+1}\frac{c_x}{c_u} \\
&\qquad +(1+g)\left[\frac{\hat{c}_{x,t}}{c_u}-\frac{c_x\hat{c}_{u,t}}{c_u^2}\right]\Bigg)\Bigg] \\
&= -\sigma\beta \sum_{x\in\{u,c\}} p_{x|u}\left[\left((1+g)\frac{c_x}{c_u}\right)^{-\sigma}\left(\frac{\hat{g}_{t+1}}{1+g}+\frac{\hat{c}_{x,t}}{c_x}-\frac{\hat{c}_{u,t}}{c_u}\right)\right] \\
&= -\sigma\mathcal{F}^s \sum_{x\in\{u,c\}} \bar{\eta}_x\left(\frac{\hat{g}_{t+1}}{1+g}+\frac{\hat{c}_{x,t+1}}{c_x}-\frac{\hat{c}_{u,t}}{c_u}\right) \\
&= -\sigma\mathcal{F}^s\left[\frac{\hat{g}_{t+1}}{1+g}-\frac{\hat{c}_{u,t}}{c_u}+\sum_{x\in\{u,c\}}\bar{\eta}_x\left(\frac{\hat{c}_{x,t+1}}{c_x}\right)\right]
\end{aligned} \tag{4.282}$$

$$\begin{aligned}
\text{with } \bar{\eta}_x &:= \frac{\beta p_{x|u}\left((1+g)\frac{c_x}{c_u}\right)^{-\sigma}}{\mathcal{F}^s} = \frac{\beta p_{x|u}\left((1+g)\frac{c_x}{c_u}\right)^{-\sigma}}{\beta\sum_{y\in\{c,u\}}p_{y|u}\left((1+g)\frac{c_y}{c_u}\right)^{-\sigma}} \\
&= \frac{p_{x|u}c_x^{-\sigma}}{\sum_{y\in\{c,u\}}p_{y|u}c_y^{-\sigma}} \quad \text{and} \quad \sum_{x\in\{u,c\}}\eta_x = 1,
\end{aligned} \tag{4.283}$$

$$\begin{aligned}
0 &\approx \mathcal{E}^{HH}_t\left[\hat{\mathcal{F}}^s_{t+1}\right](1+r)+\mathcal{F}^s\hat{r}_{pre,t}+\sigma\hat{c}_{u,t}c_u^{\sigma-1}\bar{\xi}+c_u^\sigma\xi_t \\
&= \left(\mathcal{E}^{HH}_t\left[\frac{\hat{\mathcal{F}}^s_{t+1}}{\mathcal{F}^s}\right]+\frac{\hat{r}_{pre,t}}{1+r}\right)(1+r)\mathcal{F}^s+\sigma c_u^\sigma\left(\frac{\hat{c}_{u,t}}{c_u}\bar{\xi}+\xi_t\right) \\
&= (1+r)\mathcal{F}^s\Bigg[\frac{\sigma c_u^\sigma}{(1+r)\mathcal{F}^s}\left(\frac{\hat{c}_{u,t}}{c_u}\bar{\xi}+\frac{\xi_t}{\sigma}\right) \\
&\qquad +\mathcal{E}^{HH}_t\left[-\sigma\left[\frac{\hat{g}_{t+1}}{1+g}-\frac{\hat{c}_{u,t}}{c_u}+\sum_{x\in\{u,c\}}\bar{\eta}_x\left(\frac{\hat{c}_{x,t+1}}{c_x}\right)\right]\right]+\frac{\hat{r}_{pre,t}}{1+r}\Bigg] \\
&= (1+r)\mathcal{F}^s\Bigg[\frac{\sigma}{(1+r)\mathcal{F}^s}\left(\frac{\hat{c}_{u,t}}{c_u}\left(\bar{\xi}c_u^\sigma+\mathcal{F}^s(1+r)\right)+\frac{\xi_t}{\sigma}c_u^\sigma\right) \\
&\qquad -\sigma\,\mathcal{E}^{HH}_t\left[\frac{\hat{g}_{t+1}}{1+g}+\sum_{x\in\{u,c\}}\bar{\eta}_x\left(\frac{\hat{c}_{x,t+1}}{c_x}\right)\right]+\frac{\hat{r}_{pre,t}}{1+r}\Bigg] \\
&= \sigma\frac{\hat{c}_{u,t}}{c_u}+(1+r)\mathcal{F}^s\Bigg[\frac{\xi_t c_u^\sigma}{(1+r)\mathcal{F}^s}+\frac{\hat{r}_{pre,t}}{1+r}
\end{aligned}$$

$$
\begin{aligned}
&\quad - \sigma\, \mathcal{E}_t^{HH}\left[\frac{\hat{g}_{t+1}}{1+g} + \sum_{x\in\{u,c\}}\left(\frac{\bar{\eta}_x \hat{c}_{x,t+1}}{c_x}\right)\right]\Bigg]\\
&= \sigma\frac{\hat{c}_{u,t}}{c_u} + \Bigg[\xi_t c_u^{\sigma} + (1+r)\mathcal{F}^s\frac{\hat{r}_{pre,t}}{1+r}\\
&\quad - \sigma(1+r)\mathcal{F}^s\, \mathcal{E}_t^{HH}\left[\frac{\hat{g}_{t+1}}{1+g} + \sum_{x\in\{u,c\}}\left(\frac{\bar{\eta}_x \hat{c}_{x,t+1}}{c_x}\right)\right]\Bigg] \qquad (4.284)
\end{aligned}
$$

$$
\begin{aligned}
r_{pre,t} &:= r_{pre,t} - r\\
&\approx \frac{1+i}{1+\bar{\pi}}\left(\frac{i_t - i}{1+i} - \frac{\mathbb{E}_t[\pi_{t+1}-\bar{\pi}]}{1+\bar{\pi}}\right)\\
&= (1+r)\left(\frac{\hat{i}_t}{1+i} - \frac{\mathbb{E}_t[\hat{\pi}_{t+1}]}{1+\bar{\pi}}\right), \qquad (4.285)
\end{aligned}
$$

with $\hat{i}_t := i_t - i$ and $\hat{\pi}_t := \pi_t - \bar{\pi}$.

Here, we have used the steady-state relationships at various points, in particular $\mathcal{F}^s(1+r) + c_u^{\sigma}\bar{\xi} = 1$. Also, similar to the last derivation, we can easily show that $\hat{r}_t := r_t - r$ satisfies approximately

$$
\frac{\hat{r}_t}{1+r} \approx \frac{\hat{i}_t}{1+i} - \frac{\hat{\pi}_t}{1+\bar{\pi}} \qquad (4.286)
$$

4.B.2 Firm Sector

For the firm sector, we need to linearise (4.141) – (4.144), (4.161), (4.164) – (4.173). Doing so in turn, we get:

$$
\begin{aligned}
\hat{p}_t^* &:= p_t^* - 1 \approx \mathcal{M}\left(\frac{\hat{\zeta}_{1,t}}{\zeta_2} - \frac{\zeta_1}{\zeta_2^2}\hat{\zeta}_{2,t}\right)\\
&= \frac{\hat{\zeta}_{1,t}}{\zeta_1} - \frac{\hat{\zeta}_{2,t}}{\zeta_2} \qquad (4.287)
\end{aligned}
$$

$$
\begin{aligned}
\hat{\zeta}_{1,t} &:= \zeta_{1,t} - \zeta_1\\
&\approx x_t y + X\hat{y}_t + \theta\mathcal{F}^r(1+g)\zeta_1\\
&\quad \cdot \mathcal{E}_t^F\left[\epsilon\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\hat{\mathcal{F}}_{t+1}^r}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{1,t+1}}{\zeta_1}\right]\\
&= Xy\left(\frac{x_t}{X} + \frac{\hat{y}_t}{y}\right) + \theta\mathcal{F}^r(1+g)\zeta_1\\
&\quad \cdot \mathcal{E}_t^F\left[\epsilon\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\hat{\mathcal{F}}_{t+1}^r}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{1,t+1}}{\zeta_1}\right]\\
&= \left(1 - \theta\mathcal{F}^r(1+g)\right)\zeta_1\left(\frac{x_t}{X} + \frac{\hat{y}_t}{y}\right) + \theta\mathcal{F}^r(1+g)\zeta_1\\
&\quad \cdot \mathcal{E}_t^F\left[\epsilon\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\hat{\mathcal{F}}_{t+1}^r}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{1,t+1}}{\zeta_1}\right] \qquad (4.288)
\end{aligned}
$$

$$
\begin{aligned}
\text{with } x_t &:= X_t - X \\
\hat{\zeta}_{2,t} &:= \zeta_{2,t} - \zeta_2 \\
&\approx \hat{y}_t + \theta \mathcal{F}^r (1+g) \zeta_2 \\
&\quad \cdot \mathcal{E}_t^F \left[(\epsilon - 1) \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\mathcal{F}^r_{t+1}}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{2,t+1}}{\zeta_2} \right] \\
&= \left(1 - \theta \mathcal{F}^r (1+g)\right) \zeta_2 \frac{\hat{y}_t}{y} + \theta \mathcal{F}^r (1+g) \zeta_2 \\
&\quad \cdot \mathcal{E}_t^F \left[(\epsilon - 1) \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\mathcal{F}^r_{t+1}}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{2,t+1}}{\zeta_2} \right] && (4.289) \\
0 &\approx (1-\theta)(1-\epsilon) \hat{p}_t^* + \theta (\epsilon - 1) \frac{\hat{\pi}_t}{1+\bar{\pi}} && (4.290) \\
\hat{q}_t &:= q_t - 1 \\
&= \mathcal{F}^r \mathcal{E}_t^F \left[\frac{\hat{\mathcal{F}}^r_{t+1}}{\mathcal{F}^r} \left(1 + r_K - \bar{\delta}\right) + r_K \left(\frac{\hat{r}_{K,t+1}}{r_K} + \hat{u}_{t+1} \right) \right. \\
&\quad \left. + \hat{q}_{t+1} (1 - \bar{\delta}) - \delta'(1) \hat{u}_{t+1} \right] \\
&= \mathcal{F}^r \mathcal{E}_t^F \left[\frac{\hat{\mathcal{F}}^r_{t+1}}{\mathcal{F}^r} \left(r_K + (1 - \bar{\delta})\right) + \hat{r}_{K,t+1} + \hat{q}_{t+1} (1 - \bar{\delta}) \right] && (4.291) \\
0 &\approx \hat{q}_t \left(1 - S_I(g) - S_I'(g)\right) - \left(S_I'(g) + (1+g) S_I''(g)\right) \hat{g}_{I,t} \\
&\quad + \mathcal{F}^r (1+g)^2 \mathcal{E}_t^F \left[S_I'(g) \left(\frac{\hat{\mathcal{F}}^r_{t+1}}{\mathcal{F}^r} + \hat{q}_{t+1} + \frac{2 g_{I,t+1}}{1+g} \right) \right. \\
&\quad \left. + S_I''(g) \hat{g}_{I,t+1} \right] \\
&= \hat{q}_t - \kappa_I (1+g) \hat{g}_{I,t} + \mathcal{F}^r (1+g)^2 \kappa_I \mathcal{E}_t^F \left[\hat{g}_{I,t+1} \right] && (4.292) \\
\hat{r}_{K,t} &\approx \hat{q}_t \delta'(1) + \delta''(1) \hat{u}_t && (4.293) \\
x_t &\approx X \left[(1-\alpha) \frac{\hat{w}_t}{w} + \alpha \frac{\hat{r}_{K,t}}{r_K} - \alpha_p \frac{\hat{k}_{P,t}}{k_P} \right] && (4.294)
\end{aligned}
$$

where $\hat{w}_t := w_t - w, \quad \hat{r}_{K,t} := r_{K,t} - r_K, \quad \hat{k}_{P,t} := k_{P,t} - k_P,$

$$
\begin{aligned}
n_t &:= N_t - N \approx N \left(\frac{\tilde{y}_t - \tilde{y}}{\tilde{y}} - \frac{\hat{w}_t}{w} \right) && (4.295) \\
\hat{k}_t &= k_t - k \approx k \left(\frac{\tilde{y}_t - \tilde{y}}{\tilde{y}} - \frac{\hat{r}_{K,t}}{r_K} \right) && (4.296) \\
\tilde{y}_t - \tilde{y} &\approx \hat{\xi}_{p,t} X y + \xi_p \hat{X}_t y + \xi_p X \hat{y}_t \\
&= \tilde{y} \left(\hat{\xi}_{p,t} + \frac{\hat{X}_t}{X} + \frac{\hat{y}_t}{y} \right) && (4.297)
\end{aligned}
$$

with $\hat{y}_t := y_t - y$ and $\hat{\xi}_{p,t} = \xi_{p,t} - \xi_p = \xi_{p,t} - 1$,

$$
\hat{g}_{I,t} = g_{I,t} - g \approx \frac{I(1+g)}{I} \left[\frac{\hat{I}_t}{I} + \frac{\hat{g}_t}{1+g} - \frac{\hat{I}_{t-1}}{I} \right]
$$

$$= \hat{g}_t + (1+g)\left[\frac{\hat{I}_t}{I} - \frac{\hat{I}_{t-1}}{I}\right] \tag{4.298}$$

$$\begin{aligned}
\tilde{k}_t &:= \bar{k}_t - \bar{k} \\
&\approx \frac{1-\bar{\delta}}{1+g}\bar{k}\left[\frac{\tilde{k}_{t-1}}{\bar{k}} - \frac{\hat{g}_t}{1+g} - \frac{\delta'(1)}{1-\bar{\delta}}\hat{u}_t\right] \\
&\quad + \hat{I}_t(1-S(g)) - IS'(g)g\hat{_{I,t}} \\
&\approx \frac{1-\bar{\delta}}{1+g}\bar{k}\left[\frac{\tilde{k}_{t-1}}{\bar{k}} - \frac{\hat{g}_t}{1+g} - \frac{\delta'(1)}{1-\bar{\delta}}\hat{u}_t\right] + I\frac{\hat{I}_t}{I} \\
&\approx \frac{\bar{k}}{1+g}\left[(1-\bar{\delta})\left[\frac{\bar{k}_{t-1}}{\bar{k}} - \frac{\hat{g}_t}{1+g} - \frac{\delta'(1)}{1-\bar{\delta}}\hat{u}_t\right] + (g+\bar{\delta})\frac{\hat{I}_t}{I}\right],
\end{aligned} \tag{4.299}$$

$$\begin{aligned}
\text{with } \hat{u}_t &= U_{K,t} - 1, \\
\hat{q}_{\mathcal{F},t} &:= q_{\mathcal{F},t} - q_{\mathcal{F}} \\
&\approx \mathcal{F}^r(1+g)\mathcal{E}_t^F\left[\left(\frac{\hat{\mathcal{F}}^r_{t+1}}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g}\right)(q_{\mathcal{F}} + y(1-\xi_p X))\right. \\
&\quad \left. + \hat{q}_{\mathcal{F},t+1} + (1-\xi_p X)\hat{y}_t - yX\hat{\xi}_{p,t} - y\xi_p x_t\right] \\
&= \mathcal{F}^r(1+g)\mathcal{E}_t^F\left[\left(\frac{\hat{\mathcal{F}}^r_{t+1}}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g}\right)\left(q_{\mathcal{F}} + y\frac{\mathcal{M}-1}{\mathcal{M}}\right)\right. \\
&\quad \left. + \hat{q}_{\mathcal{F},t+1} + \frac{y}{\mathcal{M}}\left((\mathcal{M}-1)\frac{\hat{y}_t}{y} - \hat{\xi}_{p,t} - \frac{x_t}{X}\right)\right]
\end{aligned} \tag{4.300}$$

$$\begin{aligned}
\hat{\pi}_{F,t} &= \frac{y}{\mathcal{M}}\left((\mathcal{M}-1)\frac{\hat{y}_t}{y} - \hat{\xi}_{p,t} - \frac{x_t}{X}\right) - \hat{I}_t \\
&\quad + \frac{r_K\bar{k}}{1+g}\left[\frac{\hat{r}_{K,t}}{r_K} + \hat{u}_t + \frac{\tilde{k}_{t-1}}{\bar{k}} - \frac{\hat{g}_t}{1+g}\right]
\end{aligned} \tag{4.301}$$

Also, we have

$$\hat{k}_t = \frac{\bar{k}}{1+g}\left(\frac{\tilde{k}_{t-1}}{\bar{k}} - \frac{\hat{g}_t}{1+g} + \hat{u}_t\right) \tag{4.302}$$

$$= k\left(\frac{\tilde{k}_{t-1}}{\bar{k}} - \frac{\hat{g}_t}{1+g} + \hat{u}_t\right). \tag{4.303}$$

Furthermore, we can derive a Phillips Curve according to (4.287)–(4.290):

$$\begin{aligned}
\hat{p}^*_t &:= \frac{\hat{\zeta}_{1,t}}{\zeta_1} - \frac{\hat{\zeta}_{2,t}}{\zeta_2} \\
&= (1-\theta\mathcal{F}^r(1+g))\left(\frac{x_t}{X} + \frac{\hat{y}_t}{y}\right) + \theta\mathcal{F}^r(1+g) \\
&\quad \cdot \mathcal{E}_t^F\left[\epsilon\frac{\pi_{t+1}}{1+\bar{\pi}} + \frac{\hat{\mathcal{F}}^r_{t+1}}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{1,t+1}}{\zeta_1}\right]
\end{aligned}$$

$$
\begin{aligned}
&- \left(1 - \theta \mathcal{F}^r (1+g)\right) \frac{\hat{y}_t}{y} - \theta \mathcal{F}^r (1+g) \\
&\cdot \mathcal{E}_t^F \left[(\epsilon - 1) \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\mathcal{F}^r_{t+1}}{\mathcal{F}^r} + \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{2,t+1}}{\zeta_2} \right] \\
&= \left(1 - \theta \mathcal{F}^r (1+g)\right) \frac{x_t}{X} + \theta \mathcal{F}^r (1+g) \mathcal{E}_t^F \left[\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\hat{\zeta}_{1,t+1}}{\zeta_1} - \frac{\hat{\zeta}_{2,t+1}}{\zeta_2} \right] \\
&= \left(1 - \theta \mathcal{F}^r (1+g)\right) \frac{x_t}{X} + \theta \mathcal{F}^r (1+g) \mathcal{E}_t^F \left[\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + p^*_{t+1} \right] \qquad (4.304)
\end{aligned}
$$

Combine this with

$$
\hat{p}_t^* \approx \frac{\theta}{1-\theta} \frac{\hat{\pi}_t}{1+\bar{\pi}}
$$

to obtain

$$
\begin{aligned}
\frac{\theta}{1-\theta} \frac{\hat{\pi}_t}{1+\bar{\pi}} &= \left(1 - \theta \mathcal{F}^r (1+g)\right) \frac{x_t}{X} \\
&\quad + \theta \mathcal{F}^r (1+g) \mathcal{E}_t^F \left[\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\theta}{1-\theta} \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} \right] \\
&= \left(1 - \theta \mathcal{F}^r (1+g)\right) \frac{x_t}{X} \\
&\quad + \frac{\theta}{1-\theta} \mathcal{F}^r (1+g) \mathcal{E}_t^F \left[\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} \right],
\end{aligned}
$$

which delivers a variant of the New Keynesian Phillips curve

$$
\begin{aligned}
\frac{\hat{\pi}_t}{1+\bar{\pi}} &= \frac{1-\theta}{\theta} \left(1 - \theta \mathcal{F}^r (1+g)\right) \frac{x_t}{X} + \mathcal{F}^r (1+g) \mathcal{E}_t^F \left[\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} \right] \\
&= \kappa_p \frac{x_t}{X} + \beta (1+g)^{1-\sigma} \mathcal{E}_t^F \left[\frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} \right] \qquad (4.305) \\
\text{with } \kappa_p &:= \frac{(1-\theta)\left(1 - \theta \beta (1+g)^{1-\sigma}\right)}{\theta}
\end{aligned}
$$

This New Keynesian Phillips curve (4.305) also makes $\hat{\zeta}_{1,t}$, $\hat{\zeta}_{2,t}$ and p_t^* redundant. Finally, we can linearise (4.148) to obtain the familiar result from the New Keynesian literature that price dispersion is almost constant around a balanced growth path as long as non-optimising firms' adjustment follows steady-state inflation

$$
\begin{aligned}
\hat{\xi}_{p,t} &\approx -\epsilon (1-\theta) \hat{p}_t^* + \theta \epsilon \frac{\hat{\pi}_t}{1+\bar{\pi}} + \theta \hat{\xi}_{p,t-1} \\
&= \theta \hat{\xi}_{p,t-1}, \qquad (4.306)
\end{aligned}
$$

which is zero as long as $\hat{\xi}_{p,t-1} = 0$.

4.B.3 Labour Market

For the labour market, we need to linearise (4.203), (4.210) – (4.213).

$$\begin{aligned}
\hat{\zeta}_{1,U,t} &:= \tilde{\zeta}_{1,U,t} - \tilde{\zeta}_{1,U} \\
&\approx (1+\varphi)\frac{n_t}{N}N^{1+\varphi} + \beta\theta_w \mathcal{E}_{U,t}\left[\epsilon_w(1+\varphi)\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}}\zeta_{1,U} + \hat{\zeta}_{1,U,t+1}\right] \\
&= (1+\varphi)\zeta_{1,U}\frac{n_t}{N}N^{1+\varphi} + \beta\theta_w\zeta_{1,U}\mathcal{E}_{U,t}\left[\epsilon_w(1+\varphi)\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}} + \frac{\hat{\zeta}_{1,U,t+1}}{\zeta_{1,U}}\right] \\
&= \zeta_{1,U}\left[(1-\beta\theta)(1+\varphi)\frac{n_t}{N}\right. \\
&\qquad \left. +\beta\theta_w\mathcal{E}_{U,t}\left[\epsilon_w(1+\varphi)\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}} + \frac{\hat{\zeta}_{1,U,t+1}}{\zeta_{1,U}}\right]\right]
\end{aligned} \tag{4.307}$$

$$\begin{aligned}
\hat{\zeta}_{2,U,t} &:= \zeta_{2,U,t} - \zeta_{2,U,t} \\
&\approx \hat{\lambda}_{U,t}N + \lambda_U n_t + \beta\theta_w(1+g)^{1-\sigma}\zeta_{2,U} \\
&\qquad \cdot \mathcal{E}_{U,t}\left[\epsilon_w\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w} - \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} - \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{2,U,t+1}}{\zeta_{2,U}}\right] \\
&\approx \zeta_{2,U}\left[(1-\beta\theta(1+g)^{1-\sigma})\left(\frac{\hat{\lambda}_{U,t}}{\lambda_U} + \frac{n_t}{N}\right) + \beta\theta_w(1+g)^{1-\sigma}\right. \\
&\qquad \left. \cdot\mathcal{E}_{U,t}\left[\epsilon_w\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w} - \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} - \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{2,U,t+1}}{\zeta_{2,U}}\right]\right]
\end{aligned} \tag{4.308}$$

$$\text{with } \hat{\lambda}_{U,t} := \lambda_{U,t} - \lambda_U \approx -\sigma \sum_{x\in\{u,c\}} s_x c_x^{-\sigma}\frac{\hat{c}_{x,t}}{c_t} \tag{4.309}$$

$$\frac{\hat{w}_t}{w} \approx \frac{\hat{\pi}_{w,t}}{1+\bar{\pi}_w} - \frac{\hat{\pi}_t}{1+\bar{\pi}} - \frac{\hat{g}_t}{(1+g)} + \frac{\hat{w}_{t-1}}{w} \tag{4.310}$$

$$\begin{aligned}
\hat{w}_t^* &:= w_t^* - w^* \\
&\approx \tilde{w}^*\frac{1}{1+\epsilon_w\varphi}\left[\epsilon_w\varphi\frac{\hat{w}_t}{w} + \frac{\hat{\zeta}_{1,U,t}}{\zeta_{1,U}} - \frac{\hat{\zeta}_{2,U,t}}{\zeta_{2,U}}\right]
\end{aligned} \tag{4.311}$$

$$0 \approx (1-\theta_w)(1-\epsilon_w)\left(\frac{\hat{w}_t^*}{w^*} - \frac{\hat{w}_t}{w}\right) + \theta_w(\epsilon_w - 1)\frac{\hat{\pi}_{w,t}}{1+\bar{\pi}_w} \tag{4.312}$$

Note that using (4.311), we can rewrite (4.312) as

$$\begin{aligned}
\frac{\hat{\pi}_{w,t}}{1+\bar{\pi}_w} &= \frac{1-\theta_w}{\theta_w}\left(\frac{\hat{w}_t^*}{w^*} - \frac{\hat{w}_t}{w}\right) \\
&= \frac{1-\theta_w}{\theta_w}\left(\frac{1}{1+\epsilon_w\varphi}\left[\epsilon_w\varphi\frac{\hat{w}_t}{w} + \frac{\hat{\zeta}_{1,U,t}}{\zeta_{1,U}} - \frac{\hat{\zeta}_{2,U,t}}{\zeta_{2,U}}\right] - \frac{\hat{w}_t}{w}\right) \\
&= \frac{1-\theta_w}{\theta_w(1+\epsilon_w\varphi)}\left(\frac{\hat{\zeta}_{1,U,t}}{\zeta_{1,U}} - \frac{\hat{\zeta}_{2,U,t}}{\zeta_{2,U}} - \frac{\hat{w}_t}{w}\right)
\end{aligned} \tag{4.313}$$

Next, with $\sigma = 1$, for the recursive formulation, we can write

$$\begin{aligned}
\frac{\hat{\zeta}_{1,U,t}}{\zeta_{1,U}} - \frac{\hat{\zeta}_{2,U,t}}{\zeta_{2,U}} &= \Bigg[(1-\beta\theta)(1+\varphi)\frac{n_t}{N} \\
&\qquad +\beta\theta_w \mathcal{E}_{U,t}\left[\epsilon_w(1+\varphi)\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}} + \frac{\hat{\zeta}_{1,U,t+1}}{\zeta_{1,U}}\right]\Bigg] \\
&\quad - \Bigg[(1-\beta\theta(1+g)^{1-\sigma})\left(\frac{\hat{\lambda}_{U,t}}{\lambda_U} + \frac{n_t}{N}\right) + \beta\theta_w(1+g)^{1-\sigma} \\
&\qquad \cdot\mathcal{E}_{U,t}\left[\epsilon_w\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w} - \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} - \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{2,U,t+1}}{\zeta_{2,U}}\right]\Bigg] \\
&= (1-\beta\theta)(1+\varphi)\frac{n_t}{N} \\
&\qquad + \beta\theta_w \mathcal{E}_{U,t}\left[\epsilon_w(1+\varphi)\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}} + \frac{\hat{\zeta}_{1,U,t+1}}{\zeta_{1,U}}\right] \\
&\quad - (1-\beta\theta)\left(\frac{\hat{\lambda}_{U,t}}{\lambda_U} - \frac{n_t}{N}\right) + \beta\theta_w \\
&\qquad \cdot\mathcal{E}_{U,t}\left[\epsilon_w\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w} - \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} - \frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{2,U,t+1}}{\zeta_{2,U}}\right] \\
&= (1-\beta\theta)\left(\varphi\frac{n_t}{N} - \frac{\hat{\lambda}_{U,t}}{\lambda_U}\right) + \beta\theta\mathcal{E}_{U,t}\Bigg[\epsilon_w\varphi\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w} + \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} \\
&\qquad +\frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{\zeta}_{1,U,t+1}}{\zeta_{1,U}} - \frac{\hat{\zeta}_{2,U,t+1}}{\zeta_{2,U}}\Bigg] \qquad (4.314)
\end{aligned}$$

Note that we can rearrange (4.313) as

$$\frac{\hat{\zeta}_{1,U,t}}{\zeta_{1,U}} - \frac{\hat{\zeta}_{2,U,t}}{\zeta_{2,U}} = \frac{\hat{w}_t}{w} + \frac{\theta_w(1+\epsilon_w\varphi)}{1-\theta_w}\frac{\hat{\pi}_{w,t}}{1+\bar{\pi}_w}, \qquad (4.315)$$

so that we can rewrite (4.314) as

$$\begin{aligned}
&\frac{\hat{w}_t}{w} + \frac{\theta_w(1+\epsilon_w\varphi)}{1-\theta_w}\frac{\hat{\pi}_{w,t}}{1+\bar{\pi}_w} \\
&= (1-\beta\theta)\left(\varphi\frac{n_t}{N} - \frac{\hat{\lambda}_{U,t}}{\lambda_U}\right) + \beta\theta\mathcal{E}_{U,t}\Bigg[\epsilon_w\varphi\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w} + \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} \\
&\qquad +\frac{\hat{g}_{t+1}}{1+g} + \frac{\hat{w}_{t+1}}{w} + \frac{\theta_w(1+\epsilon_w\varphi)}{1-\theta_w}\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w}\Bigg] \\
&= (1-\beta\theta)\left(\varphi\frac{n_t}{N} - \frac{\hat{\lambda}_{U,t}}{\lambda_U}\right) + \beta\theta\mathcal{E}_{U,t}\Bigg[\epsilon_w\varphi\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w} + \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} + \frac{\hat{g}_{t+1}}{1+g} \\
&\qquad +\frac{\hat{w}_t}{w} + \frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w} - \frac{\hat{\pi}_{t+1}}{1+\bar{\pi}} - \frac{\hat{g}_{t+1}}{1+g} + \frac{\theta_w(1+\epsilon_w\varphi)}{1-\theta_w}\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w}\Bigg] \\
&= (1-\beta\theta)\left(\varphi\frac{n_t}{N} - \frac{\hat{\lambda}_{U,t}}{\lambda_U}\right) + \beta\theta\mathcal{E}_{U,t}\left[\frac{\hat{w}_t}{w} + \frac{1+\epsilon_w\varphi}{1-\theta_w}\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w}\right]
\end{aligned}$$

$$\Leftrightarrow \quad \frac{\theta_w(1+\epsilon_w\varphi)}{1-\theta_w}\frac{\hat{\pi}_{w,t}}{1+\bar{\pi}_w}$$
$$= (1-\beta\theta)\left(\varphi\frac{n_t}{N} - \frac{\hat{\lambda}_{U,t}}{\lambda_U} - \frac{\hat{w}_t}{w}\right) + \beta\frac{\theta(1+\epsilon_w\varphi)}{1-\theta_w}\mathcal{E}_{U,t}\left[\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w}\right] \tag{4.316}$$

Note that we can rearrange (4.316) to obtain a standard New Keynesian Wage Phillips curve

$$\frac{\hat{\pi}_{w,t}}{1+\bar{\pi}_w} = \kappa_w\left(\varphi\frac{n_t}{N} - \frac{\hat{\lambda}_{U,t}}{\lambda_U} - \frac{\hat{w}_t}{w}\right) + \beta\mathcal{E}_{U,t}\left[\frac{\hat{\pi}_{w,t+1}}{1+\bar{\pi}_w}\right] \tag{4.317}$$
$$\text{with } \kappa_w := \frac{(1-\theta_w)(1-\beta\theta)}{\theta(1+\epsilon_w\varphi)}.$$

This also makes $\hat{\zeta}_{1,U,t}$, $\hat{\zeta}_{2,U,t}$ and $\hat{w}_t^*$ redundant.

4.B.4 Government Sector

The Taylor rule (4.217) is already (piecewise) linearised, it will be the only non-linear equation we consider.

$$\hat{i}_t = i_t - i = \max\{\phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t/y, -i\}; \tag{4.318}$$

For the fiscal variables, we have

$$\hat{G}_t := G_t - G = \hat{G}_{C,t} + \hat{G}_{I,t}, \tag{4.319}$$
$$\text{with } \hat{G}_{x,t} := G_{x,t} - G_x \quad x \in \{C, I\}, \tag{4.320}$$
$$\hat{b}_t := B_t - b \tag{4.321}$$
$$\approx \hat{G}_t - \hat{T}_t + \frac{1+r}{1+g}\hat{b}_{t-1} + \left[\frac{\hat{r}_t}{1+r} - \frac{\hat{g}_t}{1+g}\right]\frac{1+r}{1+g}b, \tag{4.322}$$
$$\hat{a}p_{x,t} := ap_{x,t} - \overline{ap}_x = \rho_{AP,x}\hat{a}p_{x,t-1} + \varepsilon_{ap,x,t} \tag{4.323}$$
$$\hat{G}_{x,s,t} = G_{x,s,t} - G_{x,s,BGP} \tag{4.324}$$
$$\approx \begin{cases} G_{x,0,BGP}\frac{ap_x}{(1+g)^{\mathcal{T}_1}}\left(\hat{a}p_{x,t}/ap - \sum_{j=0}^{\mathcal{T}_1}\frac{\hat{g}_{t-j}}{1+g}\right) & \text{if } s=0 \\ G_{x,s,BGP}\left(\frac{\hat{G}_{x,s-1,t-1}}{G_{x,s-1,BGP}} - \frac{\hat{g}_t}{1+g}\right) & \text{if } s=1,\ldots,\mathcal{T}_{2,x} \end{cases} \tag{4.325}$$
$$\hat{G}_{x,t} = \sum_{s=0}^{\mathcal{T}_{2,x}-1}\chi_{x,s}\hat{G}_{x,s,t} \tag{4.326}$$
$$\tilde{k}_{P,t} := \bar{k}_{P,t} - \bar{k}_P$$
$$\approx \frac{1-\delta_P}{1+g}\tilde{k}_{P,t-1} - \frac{1-\delta_P}{1+g}\bar{k}_P\frac{\hat{g}_t}{1+g} + \hat{G}_{I,\mathcal{T}_{2,I},t}$$
$$\Leftrightarrow \frac{\tilde{k}_{P,t}}{\bar{k}_P} = \frac{1-\delta_P}{1+g_t}\left(\frac{\bar{k}_{P,t-1}}{\bar{k}_P} - \frac{\hat{g}_t}{1+g}\right) + \frac{g+\delta_P}{1+g}\frac{\hat{G}_{I,\mathcal{T}_{2,I},t}}{G_{I,\mathcal{T}_{2,I},BGP}}. \tag{4.327}$$

Finally, effective capital and actual fixed capital are linked via

$$\begin{aligned} k_{P,t} &:= k_{P,t} - k_P \\ &\approx k_P \left(\frac{\tilde{k}_{P,t}}{\bar{k}} - \sum_{s=0}^{\mathcal{T}_{2,I}-1} \tilde{\psi}_{I,s} \frac{G_{I,s,BGP}}{k_P} \left(\frac{\hat{G}_{I,s,t}}{G_{I,s,BGP}} - \frac{\tilde{k}_{P,t}}{\bar{k}_P} \right) \right. \\ &\quad \left. - \sum_{s=0}^{\mathcal{T}_{2,C}-1} \tilde{\psi}_{C,s} \frac{G_{C,s,BGP}}{k_P} \left(\frac{\hat{G}_{C,s,t}}{G_{C,s,BGP}} - \frac{\tilde{k}_{P,t}}{\bar{k}_P} \right) \right). \end{aligned} \tag{4.328}$$

For the THANK-type model with positive government debt $b > 0$ along the balanced growth path, we also have:

$$\hat{T}_{B,t} = T_{B,t} - T_B = \left[\frac{\hat{g}_t}{1+g} - \frac{\hat{r}_t}{1+r} \right] \frac{1+r}{1+g} \gamma b. \tag{4.329}$$

Next, we linearise the taxation block, starting with (4.233):

$$\hat{T}_t = \sum_{x \in \{u,c\}} s_x \hat{T}_{x,t} - \hat{T}_{B,t}. \tag{4.330}$$

Considering the governance of overall taxation T_t,

1. if the fiscal authority keeps government debt fixed at its balanced-growth-path value, we immediately have

$$\hat{b}_t = 0 \quad \forall t \tag{4.331}$$

$$\hat{T}_t = \hat{G}_t + \left[\frac{\hat{r}_t}{1+r} - \frac{\hat{g}_t}{1+g} \right] \frac{1+r}{1+g} b, \tag{4.332}$$

2. conversely, if it follows the fiscal rule (4.235), it is made stationary as

$$\hat{T}_t = (1 - \rho_T) \left[\phi_G \hat{G}_t + \phi_B \hat{b}_{t-1} \right] + \rho_T \frac{1+g}{1+g_t} \hat{T}_{t-1} \tag{4.333}$$

Regarding the determination of the distribution of taxes across agents, we get from (4.255) and (4.256)

$$\hat{T}_{u,t} = \vartheta_u \hat{T}_t + \vartheta_\pi \hat{\pi}_{F,t} + \hat{T}_{B,t} \tag{4.334}$$

$$\hat{T}_{c,t} = \frac{\hat{T}_t (1 - s_u \vartheta_u) - s_u [\vartheta_\pi \hat{\pi}_{F,t}]}{s_c} + \hat{T}_{B,t} \tag{4.335}$$

4.B.5 Market Clearing Conditions

For the market clearing conditions, we need to linearise (4.272) – (4.274), which is trivial, given that they are already linear.

$$\hat{y}_t \approx \hat{c}_t + \hat{I}_t + \hat{G}_t, \tag{4.336}$$

$$\hat{c}_t \approx s_u \hat{c}_{u,t} + s_c \hat{c}_{c,t}. \tag{4.337}$$

$$\hat{b}_t \approx s_u \hat{b}_{u,t} \tag{4.338}$$

4.C Additional Results

4.C.1 More on IRFs and Multipliers

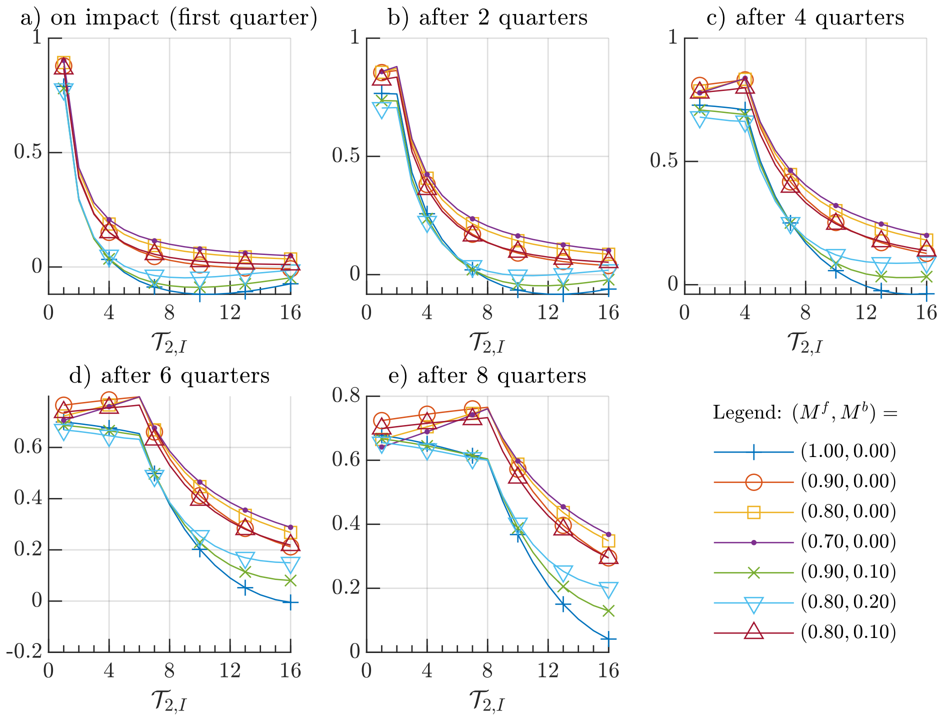

Figure 4.C.1: Impulse response of output for various types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded

As discussed in the main text, here, we briefly cover some additional aspects on multipliers. First, Figure 4.C.1 depicts the IRFs to the baseline experiment with $p_{c|u} = \xi_b ar = g = b_t = 0$ at selected points in time over the course of the first two years. Here, as before we normalize $\hat{ap}_{I,t_0} = 1$. In the figure, you clearly see how the IRF 'moves' as time progresses: In any period, as additional public investment is started, this raises output up to the point when public investment peaks (after $\mathcal{T}_{2,I} - 1$ periods, after which the first batch of investment is finished).

Note however the key difference between overall myopic expectations (with $M^f + M^b < 1$) and those with $M^f + M^b = 1$: As public investment peaks, the IRF is highest for overall myopic agents: they mostly react to current events and less to anticipated ones, pushing the immediate effects up. Note, however, that after this, (to the left of the peak in the panels), the IRFs decline. In Figures 4.C.2 and 4.C.3, the responses of private investment and consumption are shown. Observe that in each case, there is a marked decline in consumption in the initial period, which is driven by the increase in taxes, which mostly affects constrained agents. Due to adjustment costs, the fall in investment is muted (however, utilisation also adjusts). Figure 4.C.4 Shows the resulting short-run multipliers (discounted present value, up to 2 years), where one can clearly see how the forward-looking multipliers start to increase. Figures 4.C.5 and 4.C.6 present versions of Figure 4.8 with the discounted present value multipliers of government investment on private consumption and investment depicted. It is clear that in the long-run public investment leads to crowding-in of investment, whereas in the short-run the responses to the shock are negative. Figure 4.C.7 shows the cumulated discounted multipliers on output around the time of completion of projects. Note that the multiplier is positive throughout. Also note that with overall myopia, the multiplier is generally still larger than with rational expectations. The gap is increasing in time to build. Panels b) and c) then depict how forward-looking expectations lead to a catching up in terms of the cumulative output response.

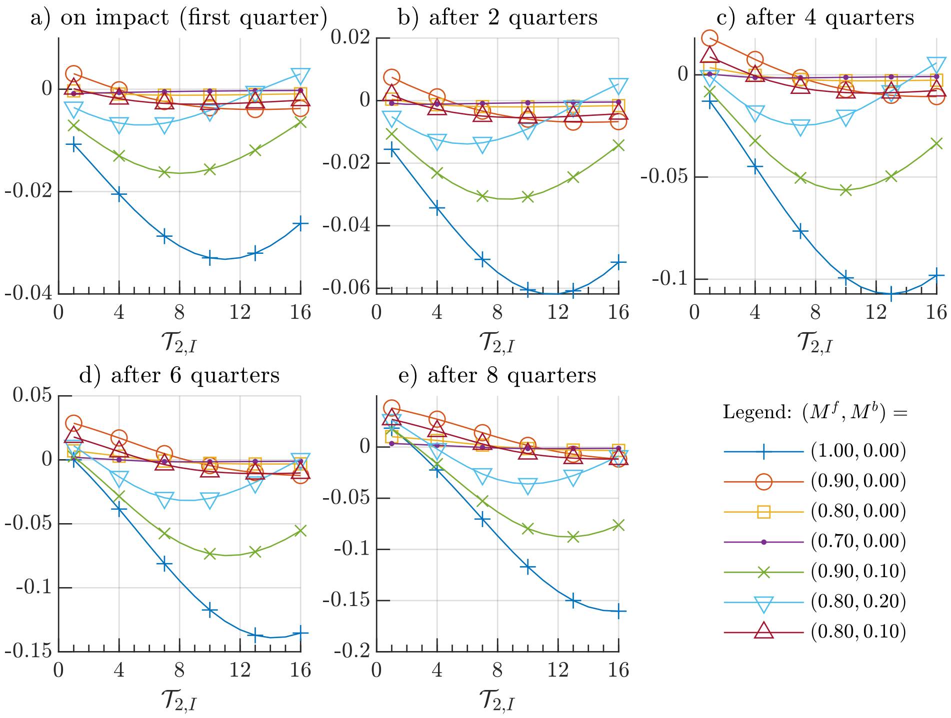

Figure 4.C.2: Impulse response of investment for various types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$ in the short run, baseline calibration, fully tax-funded

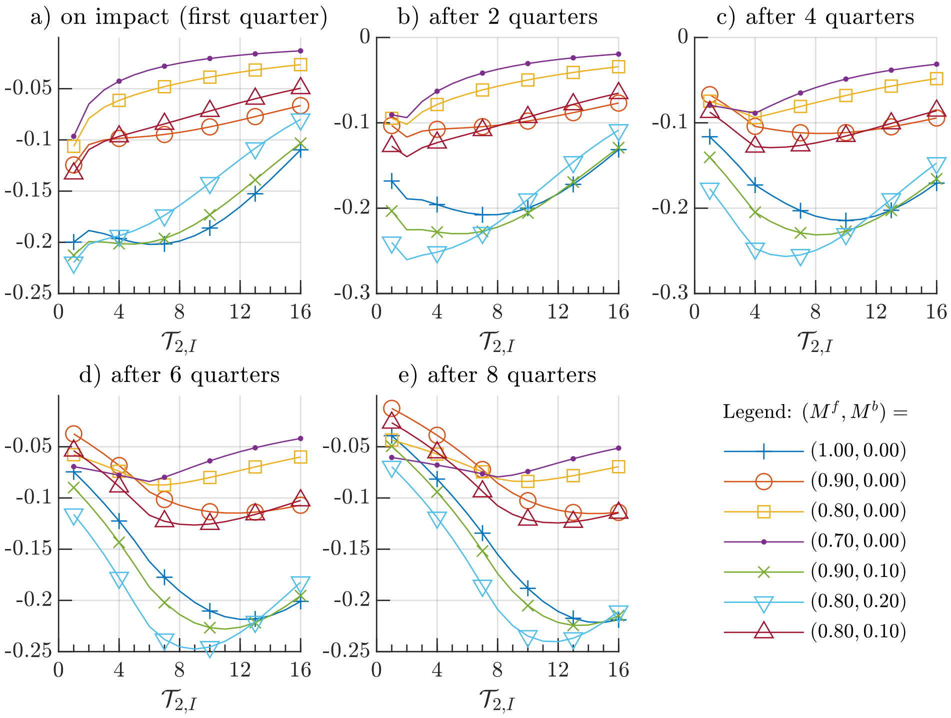

Figure 4.C.3: Impulse response of consumption for various types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$ in the short run, baseline calibration, fully tax-funded

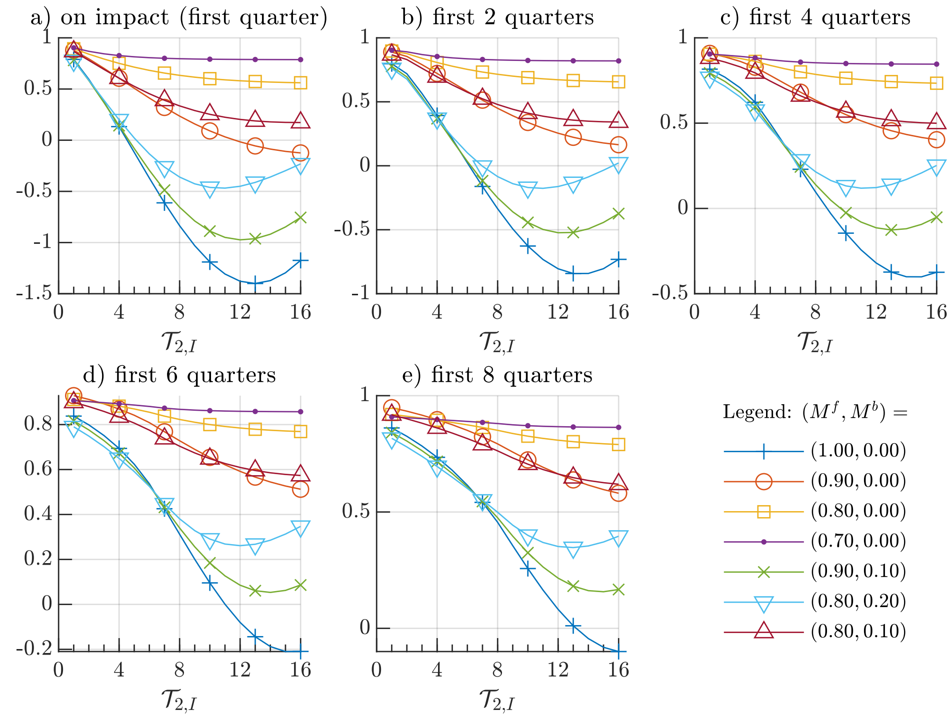

Figure 4.C.4: Cumulative discounted multipliers on output for various types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$ in the short run, baseline calibration, fully tax-funded

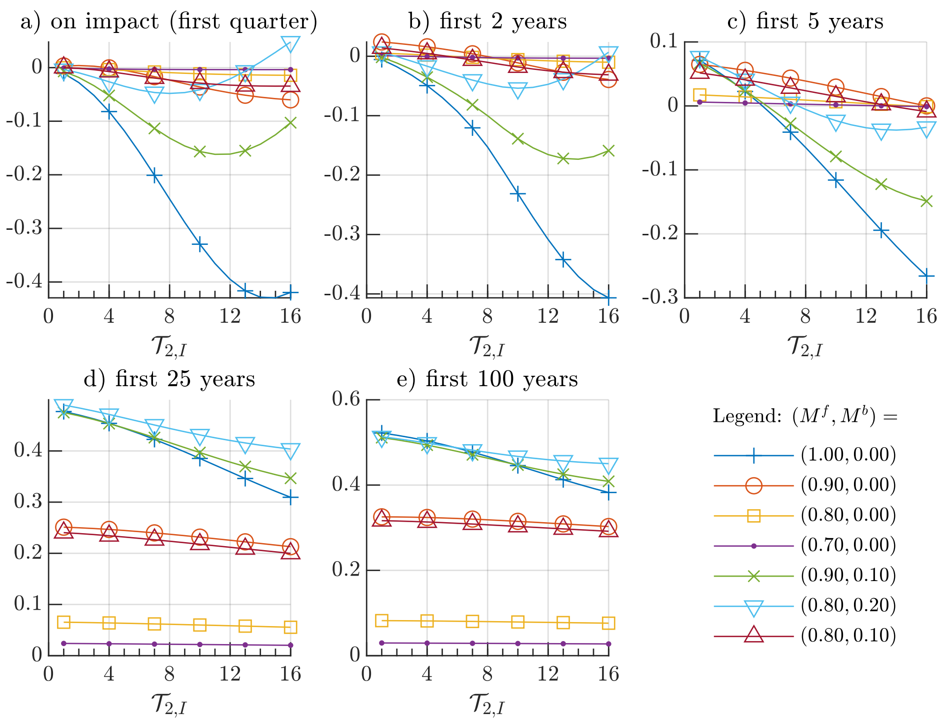

Figure 4.C.6: Government investment multipliers (priv. inv.) for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded

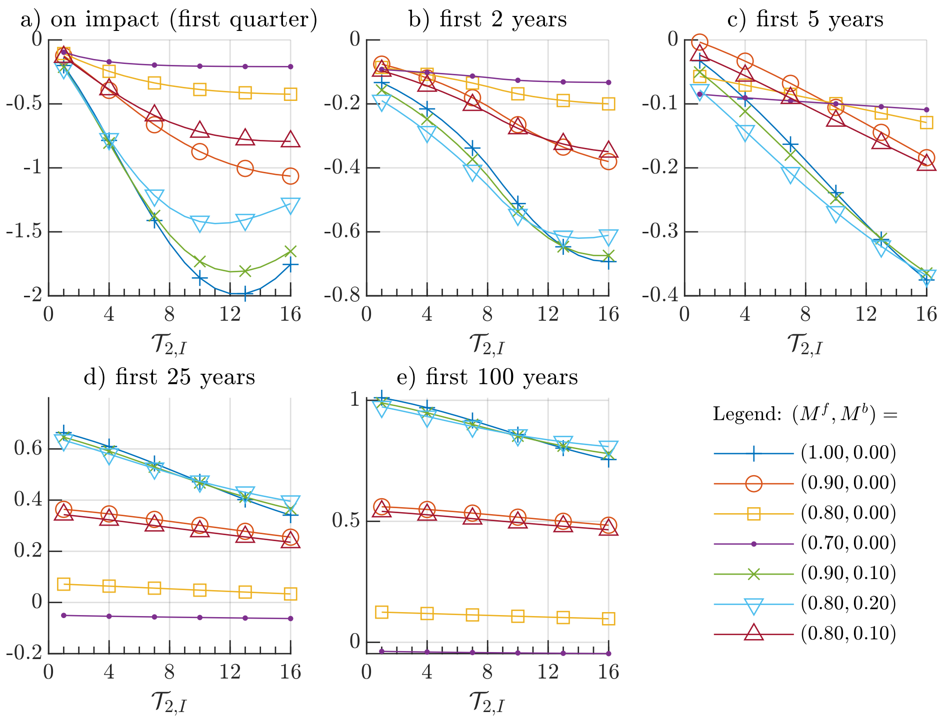

Figure 4.C.5: Government investment multipliers (consumption) for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded

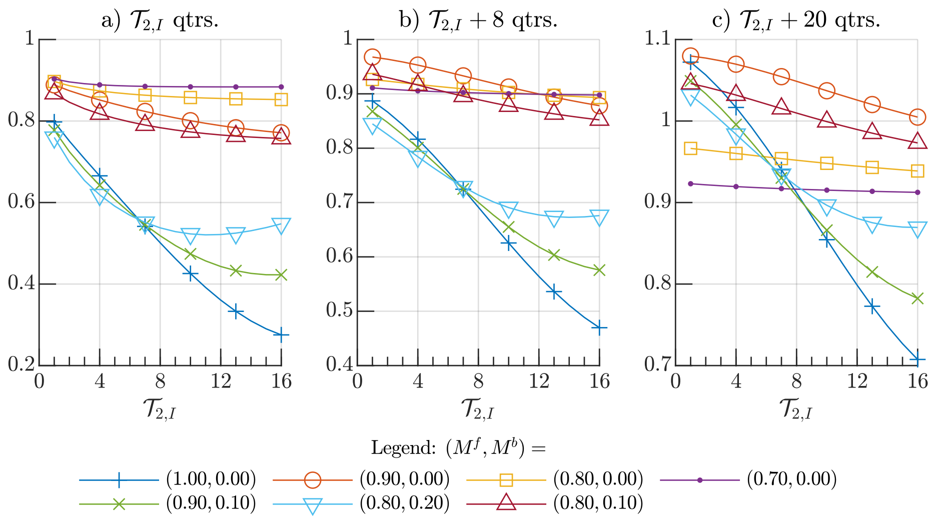

Figure 4.C.7: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I}=0$, baseline calibration, fully tax-funded, around the time of completion, discounted at β

4.C.2 A Note on the Mechanism

Next, we briefly want to take a look at the mechanism and also differentiate our results from those of public consumption. Note again that public investment has several components: it raised current aggregate demand per se, it raises taxation (at least in the medium run) and it raises productivity in the long run. As such, a shock to public investment acts as contmporaneous demand shock, but also as a news shock regarding future demand and supply. The latter part, regarding aggregate suppl is the main difference between public investment and consumption. We can easily see this if we rerun our simulations from the main text with government consumption – or if we directly redo the simulations, but we assume that public capital does not change (or that this change does not rasie productivity). Here, for simplicity, we replace the linearised law of motion for public capital given by equation (4.53) with a simple

$$\hat{k}_{p,t} = 0.$$

We then consider the same shock to public investment, but since there will not be an effect on public capital, this is more akin to a government consumption shock with the same time structure. Hence, we call it a (quasi-)'government consumption multiplier'. Figure 4.C.8 shows the results from such a simulation in a manner akin to the analysis in the main text in Figure 4.8: Note that as with real public investment, the impact multiplier is declining in the 'time to build' (or, more precisely, since there is nothing that is really being built here, the duration of the spending programme). This is due to the fact that in the intial periods, there is an upward-sloping time profile of government spending and taxation ahead of the private agents. Anticipating lower consumption due to increased taxation actually also lowers consumption today, which is then amplified via constrained agents' high MPC. In the end, this effect is clearly present in all the figures shown in this chapter. Here, for very long $\mathcal{T}_{2,I}$, the impact multiplier also becomes negative and quite a lot so.

Quite like with public investment, this decline in multipliers is largest for fully rational expectations and it is significantly dampened with myopic agents. As was the case before, over time, the multiplier increases. But crucially, since there is not productivity-enhancing effect, it does not increas much. in fact, with rational expectations, the long-run discounted multiplier is only slightly larger than 0.7 for $\mathcal{T}_{2,I}$, which is the case with the most front-loading possible. But more generally, the multiplier never exceeds 1. And different from the analysis in the main text, rational (or quasi-rational hybrid) expectations always have a lower multiplier than more myopic ones. I.e., the reversal from the main text never happens.

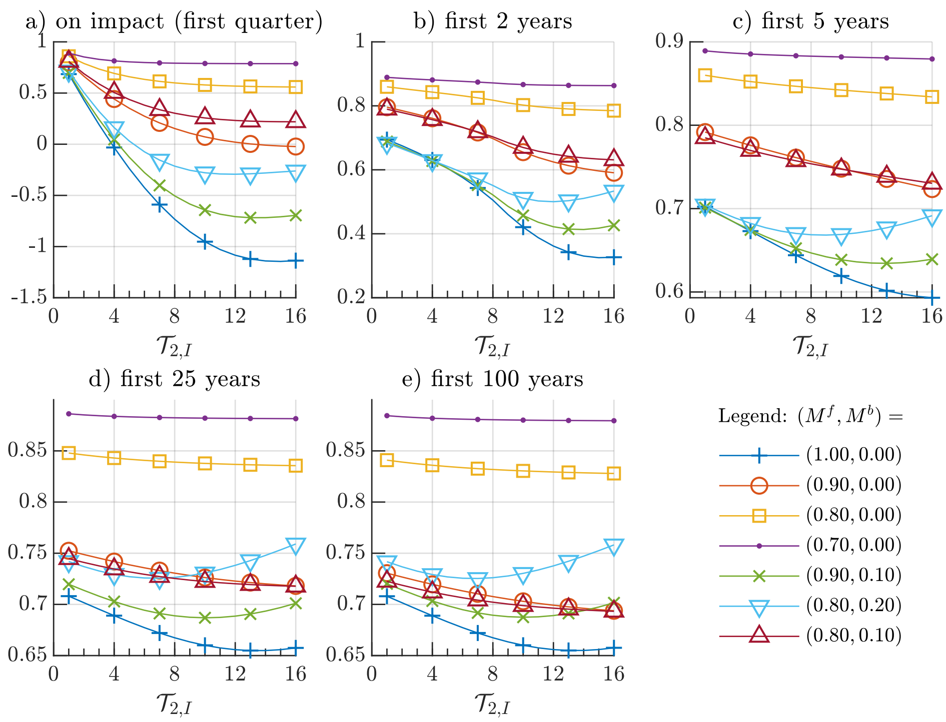

Figure 4.C.8: Government consumption multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded, discounted at β

Likewise, if we redo the analysis with the ELB enabled (Figure 4.C.9), we can still observe that the maximum impact is achieved with a project duration of the same length as the ELB spell. Ans again, hybrid expectations imply relatively large multipliers. Note however, that compared to the analysis in the main text, multipliers are still a lot smaller. These results can be easily rationalised: (1) With a higher $\mathcal{T}_{2,I}$, the income profile becomes upward-sloping across the project horizon; which implies an increase in the (implicit) natural interest rate, this makes the ELB less 'bad' (according to standard logic). (2) With productivity gains due to public investment the former effect in reinforced, but at the same time there can be deflationary pressures in the future due to increased TFP. Moving the end of investment further into the future remedies the effect of deflationary pressures during the ELB episode. Overall, this second effect also raises multipliers. In Figure 4.C.9, as with public consumption, these effects are absent. (3) Here, we can also give a reason why the backward-looking term in hybrid expectations raises the multiplier at the ELB so much: There is, in general a positive multiplier effect on consumption. Since expectations are partially backward-looking, higher consumption in one period implies in the next period a higher expected consumption (for the second period after the next one). I.e., a current increase in consumption raises the entire path of future expected consumptions per se. As such, the multiplier effect becomes reinforced (since agents anticipate this to some extent.)

Note, however, that even in this case the long-run multiplier, albeit larger, remains lower than the government-investment multiplier from the main text.

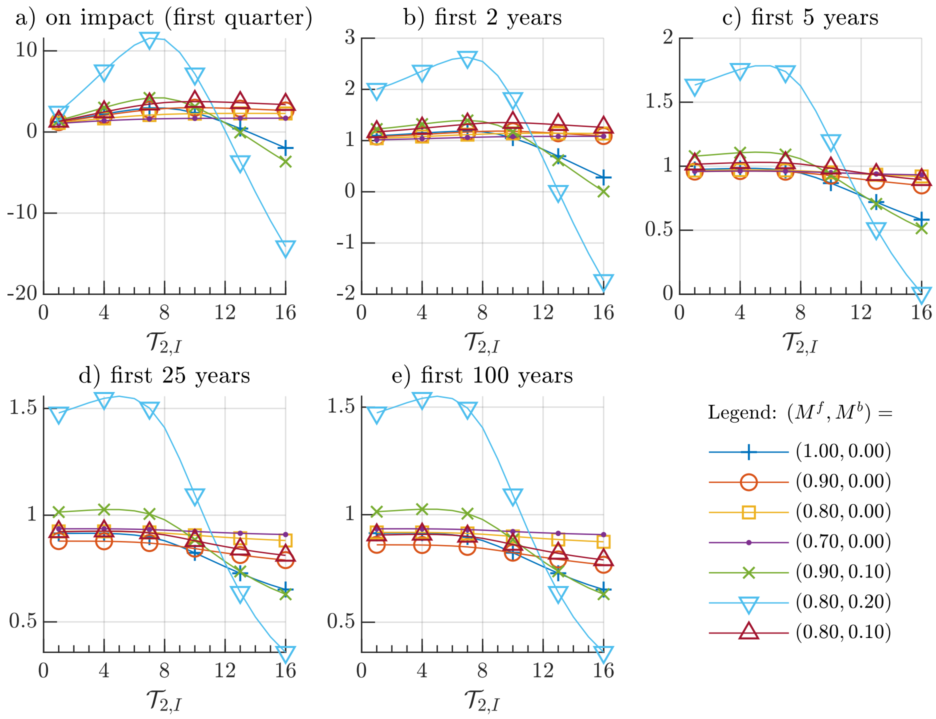

Figure 4.C.9: Government consumption multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded, discounted at β, at the ELB

4.C.3 An On-Off Shock

Next, we also consider an on-off shock to public investment, i.e. $\hat{ap}_{I,t}$ is raised in one period only. The results are depicted in Figure 4.C.10. Here, the impact multiplier is always positive, which is due to the lack of an upward-sloping tax schedule. However, apart from this, the general observations from the main text carry over: With myopic expectations, short-run multipliers for longer time to build are still higher, long-run multipliers are lower. Note, however, some shifts in details: E.g., multipliers with rational expectations 'overtake' myopic ones earlier, hybrid expectations actually have a lower multiplier than rational ones for low value of $\mathcal{T}_{2,I}$.

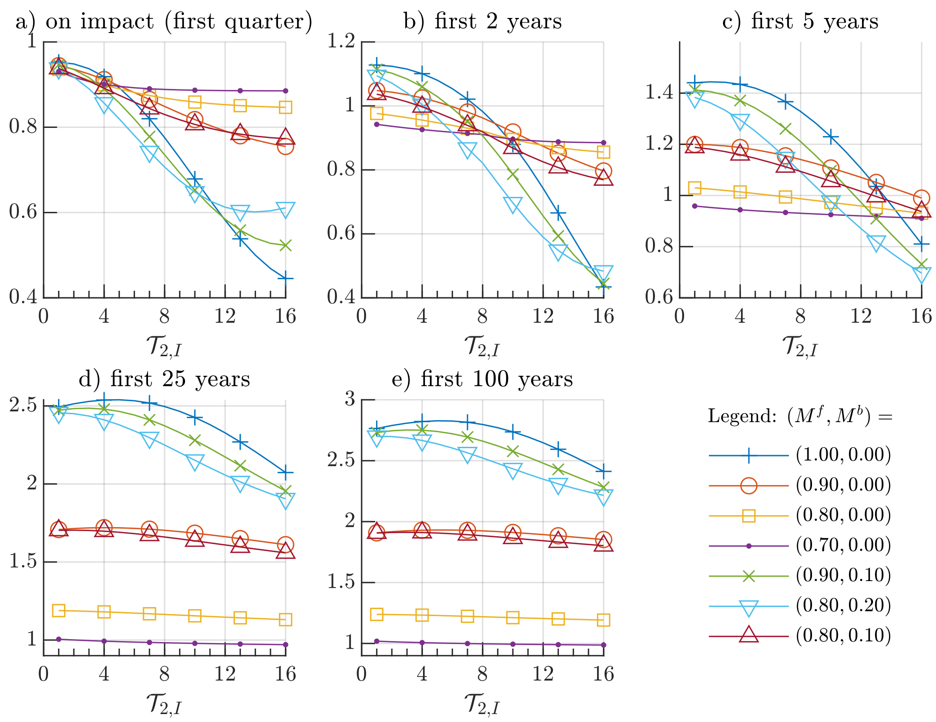

Figure 4.C.10: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded, discounted at β, no persistence of shock

4.C.4 Imperfect Expectations Only in Some Groups of Variables

In addition, we also considered imperfect expectations only in some group of forward-looking variables. In particular, the groupings from the main text (Subsections 4.2.2 and 4.2.3) are considered as separate sets of M_a^f and M_a^b here for $a = c, p, w, q$.

For purely myopic expectations, Figure 4.C.11 shows the impact multipliers – each subplot represents a combination of variable groups that are affected by myopic expectations, whereas the others are rational. Notably, M_c (only the forward-looking part on consumption) can hardly be varied without M_q^f without the model losing determinacy (which is due to the effect on $\hat{q}_t$ we discuss in the main text). Apart from this, one can note that almost all subplots confirm the results obtained for myopia in all equations for the case with partial myopia: Impact multipliers tend to be larger for higher degrees of myopia (lower M^f). Exceptions exist for high M^f (close to 1) and relatively long time to build; but crucially, only if myopia affects investment decisions.

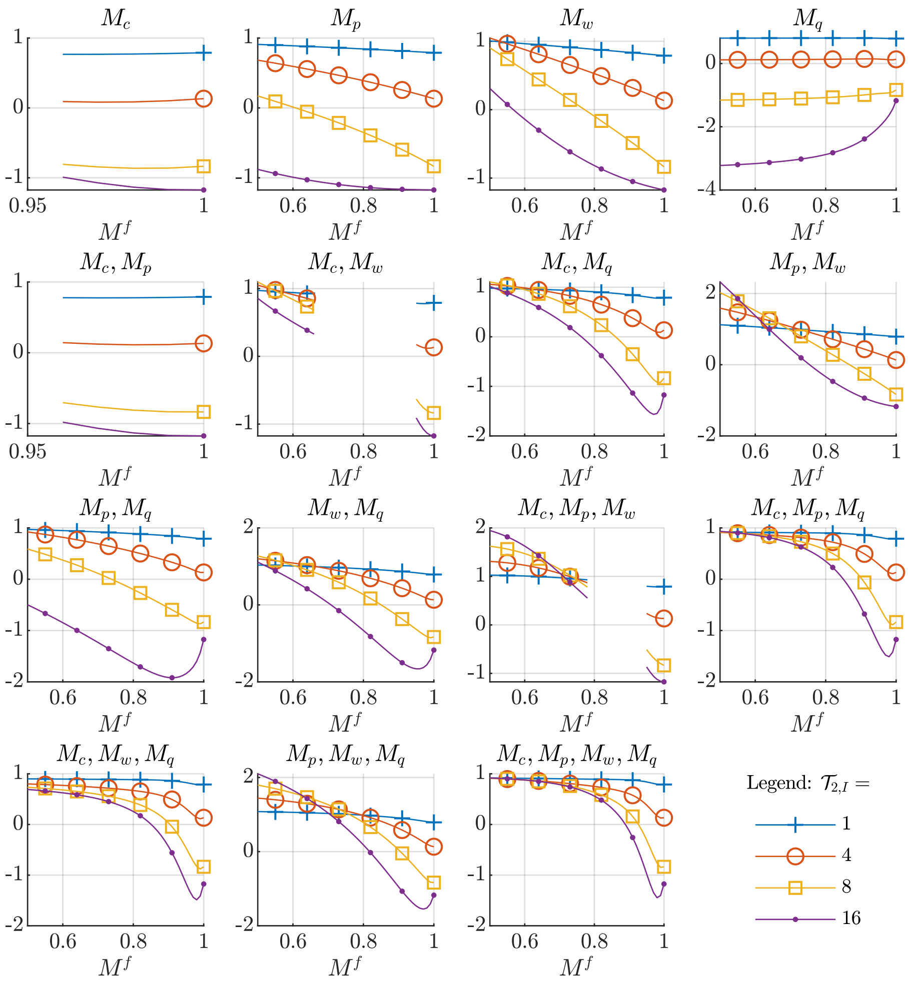

Figure 4.C.11: Impact multipliers for partial myopia, baseline experiment, no time to spend

Figure 4.C.12 shows the long-run multipliers (discounted sum across 400 years, discount factor β). Here, we see a difference, as myopia only in wage- or price-setting implies a stronger long-run multiplier as well. This is intuitive, because the deflationary tenden-

cies of higher future productivity (which affect output in a negative way) are lessened. On the other hand, it becomes clear that the reduction in long-run multipliers is mainly due to the effects via M_q: If it is lower, either alone or with any Phillips-curve expectational parameter, the multiplier quickly becomes smaller. On the other hand, if we vary M_c and M_q jointly, the non-linear results from the main text can be restored. (Also note that, as before, M_c cannot be varied much, without M_q.)

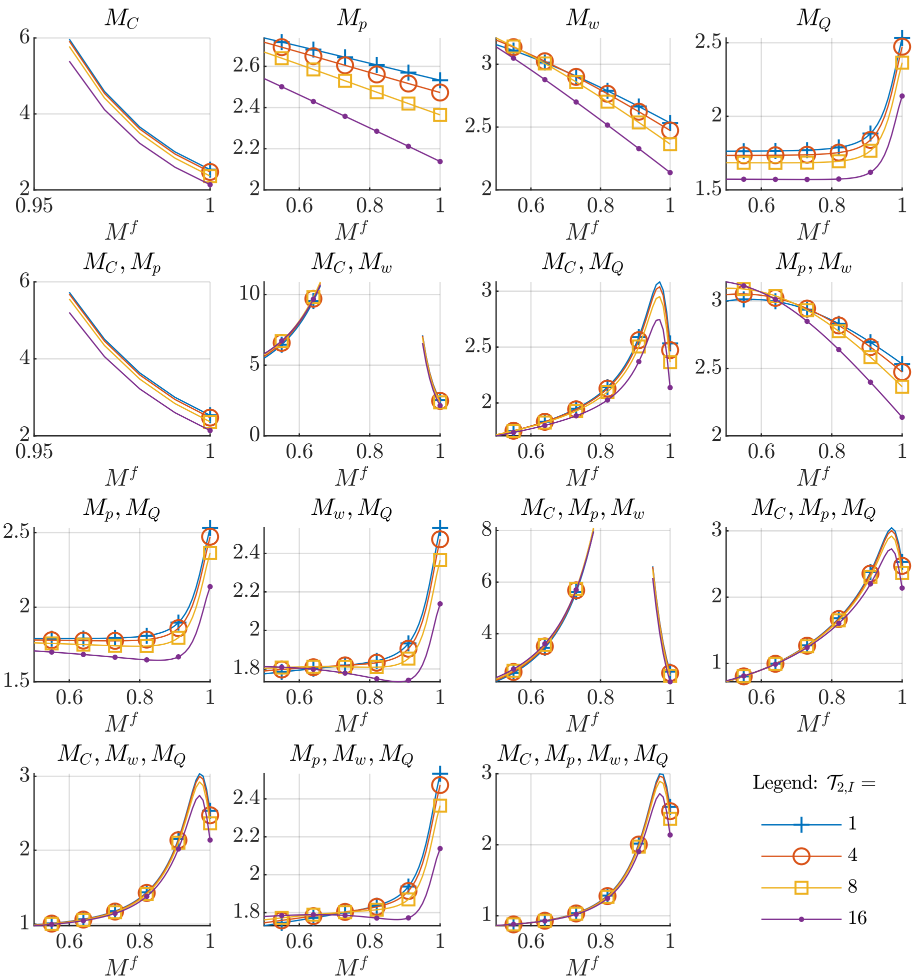

Figure 4.C.12: Discounted long-run multipliers (100 years) for partial myopia, baseline experiment, no time to spend

Next, turning to hybrid expectations with $M_a^b = 1 - M_a^f$, we perform a similar analysis. Figure 4.C.13 again shows the impact multupliers. Interestingly, here, it becomes a lot clearer that the decrease in forward-lookingness (lowering M^f) generally 'biases' the multiplier towards 1 (which here holds for almost all combinations of affected expectations). Also note that since $M_c^f + M_c^b = 1$, here, we can also vary M_c^f relatively freely.

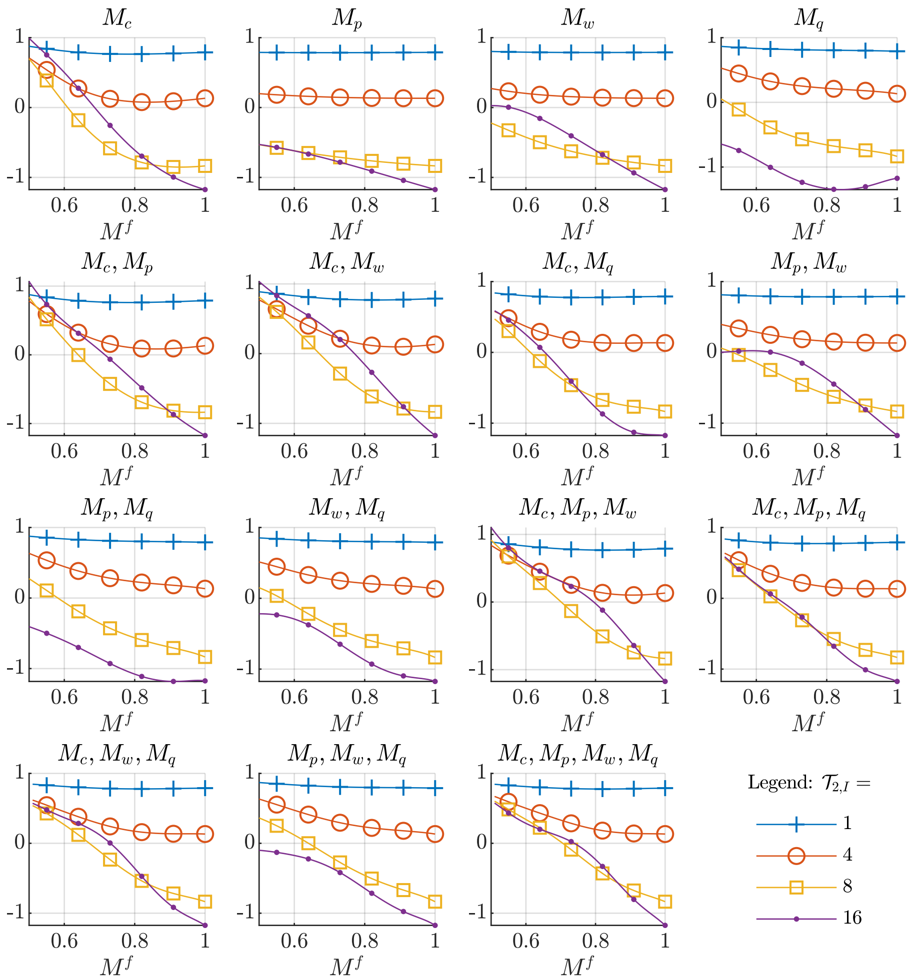

Figure 4.C.13: Impact multipliers for hybrid expectations in a subset of variables, baseline experiment, no time to spend

Lastly, Figure 4.C.14 shows the results for hybrid expectations with $M_a^b = 1 - M_a^f$ only in a subset of variables for the long-run discounted cumulative multiplier. Notably, for all variable groups except c, the long-run multiplier is increasing in M_a^f, meaning that decreased forward-lookingness decreases the long-run multiplier. At the same time, if we also vary M_c^f, the long-run multipliers tend to increase at least for $M_c^f \leq 0.8$. The reason is that as expectations become very backward-looking, past increases in consumption etc. raise expectations beyond the level consistent with rational expectations. Expectations thus overshoot eventually (mirroring results from, inter alia, Angeletos *et al.* (2021)), which endogenously prolong the initial response. [84]

[84]However, with hybrid expectations, the economy can also be prone to cycling (which we do not discuss further).

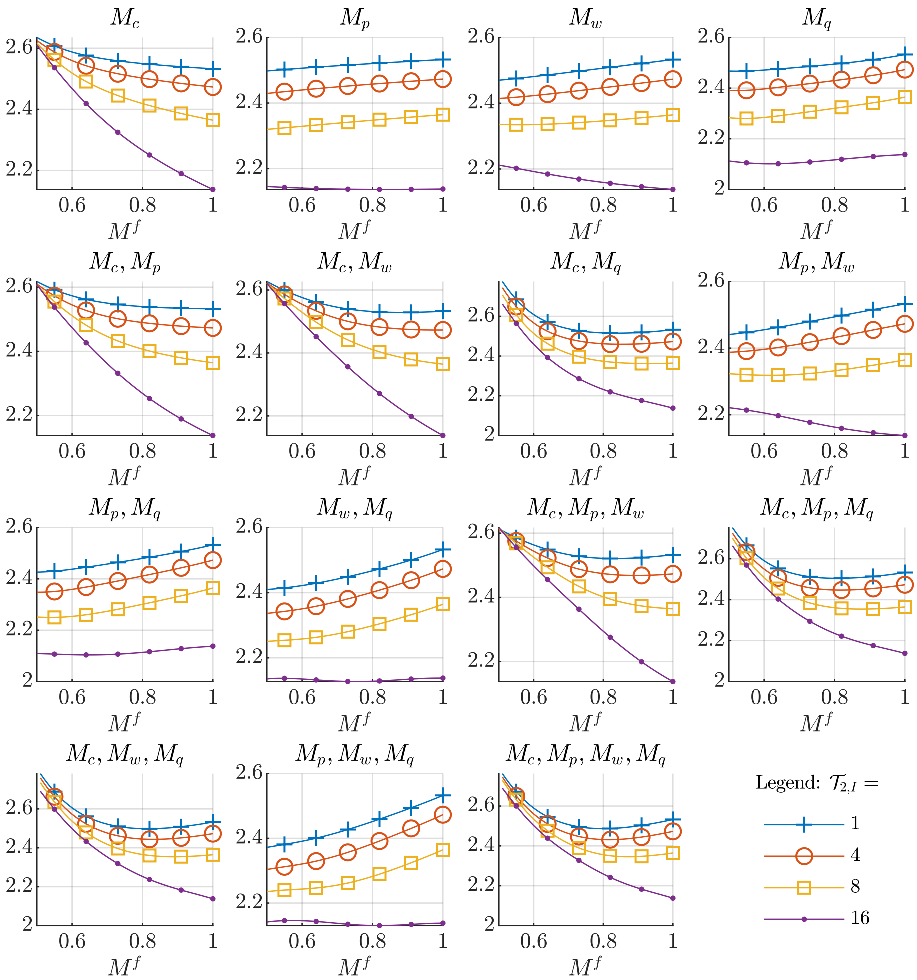

Figure 4.C.14: Discounted long-run multipliers (100 years) for hybrid expectations in a subset of variables, baseline experiment, no time to spend

4.C.5 Additional Figures for Section 4.4

Additional IRF Charts

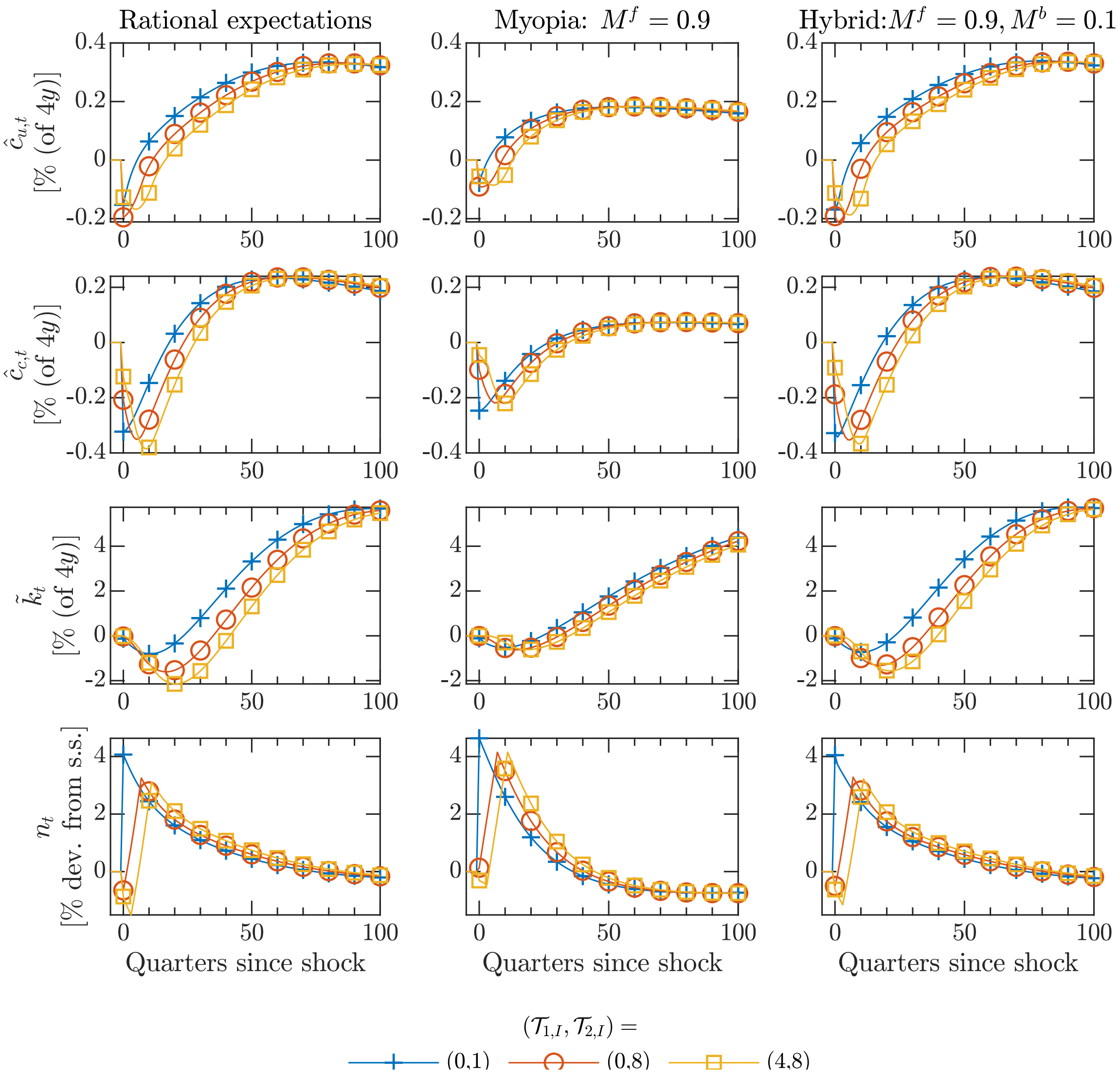

Figure 4.C.15: Additional impulse responses for the baseline experiment, pt. 1

Note: The figure shows the impulse responses of unconstrained agents' and constrained agents' consumption, private capital and labor supply in response to a shock to appropriations to government investment of size 1 in period t_0 (Figures 4.2, 4.5) for different forms of forming expectations. The response variables are ordered along the rows of the figure, different expectational formulations along the columns. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. All variables except labour supply are expressed in terms of annual output, labour supply is measured as a percentage deviation from steady state.

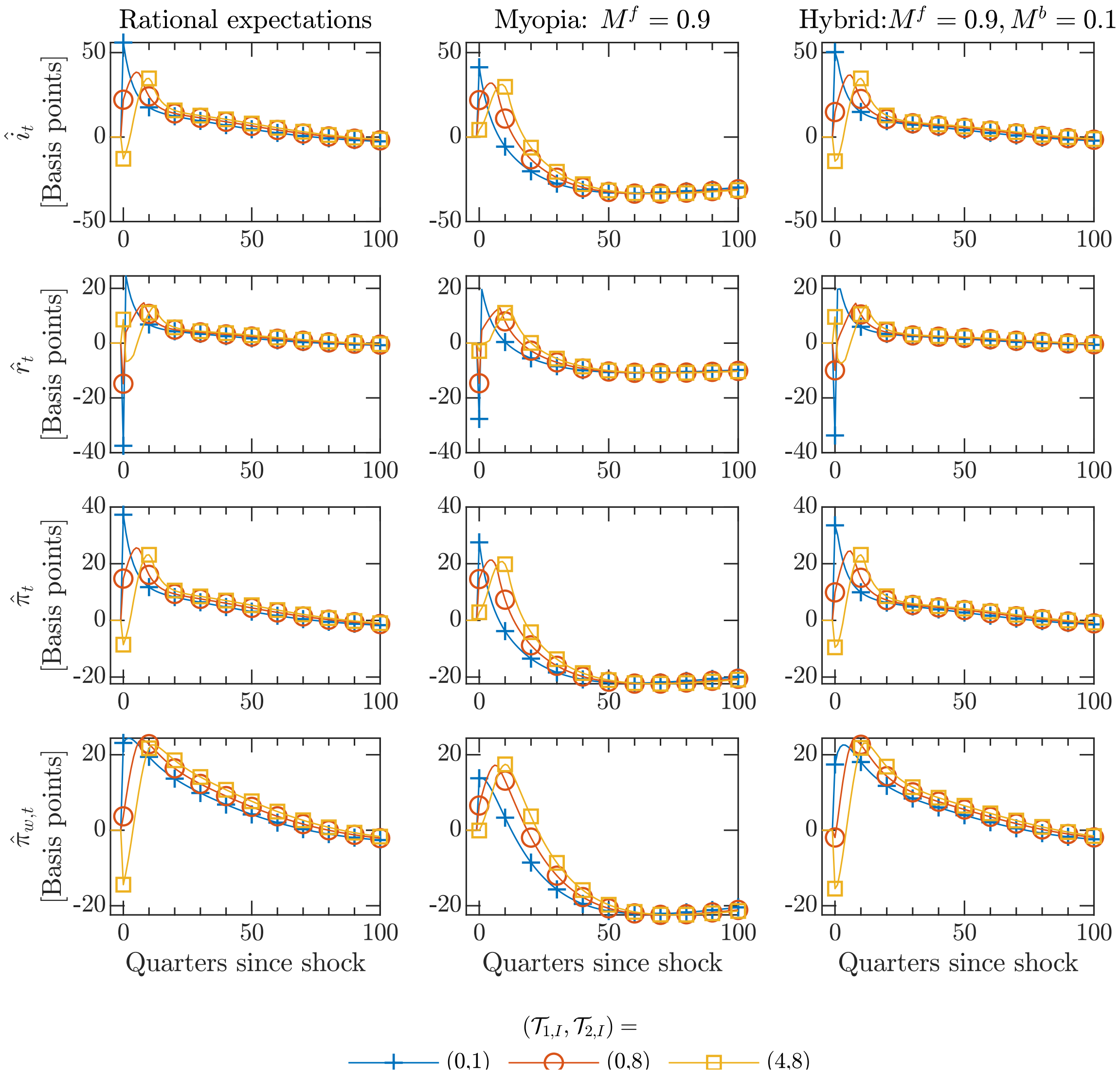

Figure 4.C.16: Additional impulse responses for the baseline experiment, pt. 2

Note: The figure shows the impulse responses of nominal interest and real interest rates as well as price and wage inflation in response to a shock to appropriations to government investment of size 1 in period t_0 (Figures 4.2, 4.5) for different forms of forming expectations. The response variables are ordered along the rows of the figure, different expectational formulations along the columns. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. All variables are expressed in basis points (quarterly).

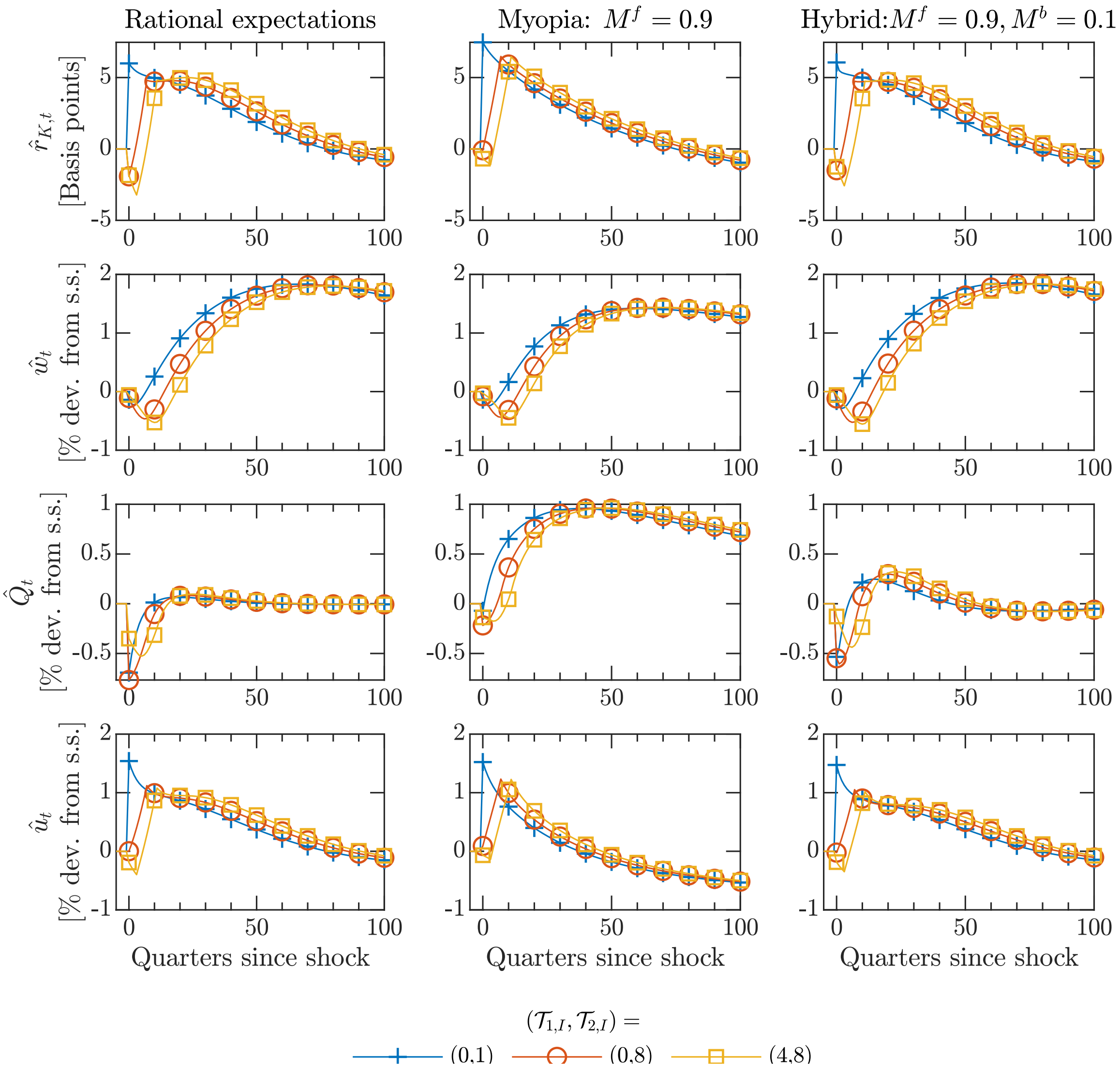

Figure 4.C.17: Additional impulse responses for the baseline experiment, pt. 3

Note: The figure shows the impulse responses of the real rental rate of capital, real wages (per efficiency unit of labour), Tobin's q and capacity utilisation in response to a shock to appropriations to government investment of size 1 in period t_0 (Figures 4.2, 4.5) for different forms of forming expectations. The response variables are ordered along the rows of the figure, different expectational formulations along the columns. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. All variables are expressed as a percentage deviation from steady state, except for the rental rate of capital which is expressed in terms of basis points at a quarterly frequency.

More IRFs, by variable

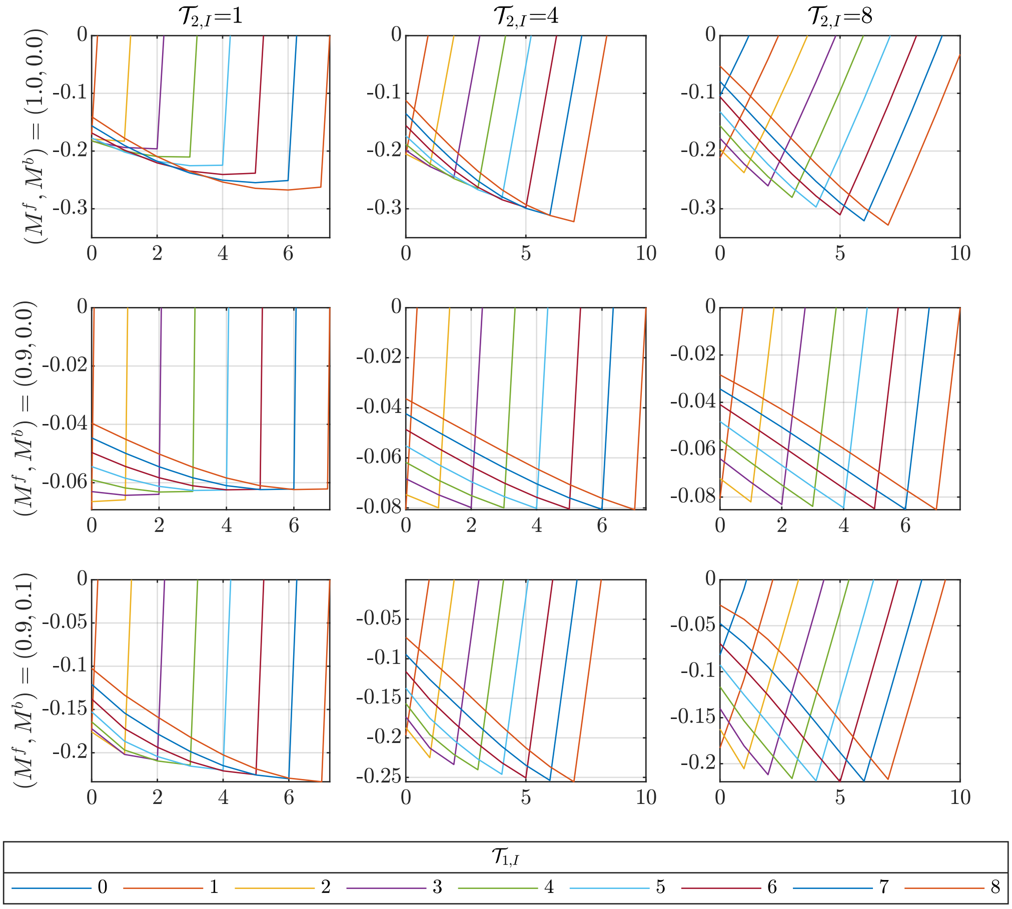

Figure 4.C.18: Initial impulse response of output with time to spend, last period before first payment

Note: The figure depicts the impulse response function for output for a shock to appropriations in period t_0 with time to spend $\mathcal{T}_{1,I} > 0$. Each panel represents a different combination time to build $\mathcal{T}_{2,I}$ (varied along columns) and expectations (varied along the rows). Simulations are based on the linear TANK model without government debt ($b_t = 0$) and without growth ($g = 0$). The shock of appropriations in t_0 is normalised to 1.

IRFs for different degrees of myopia

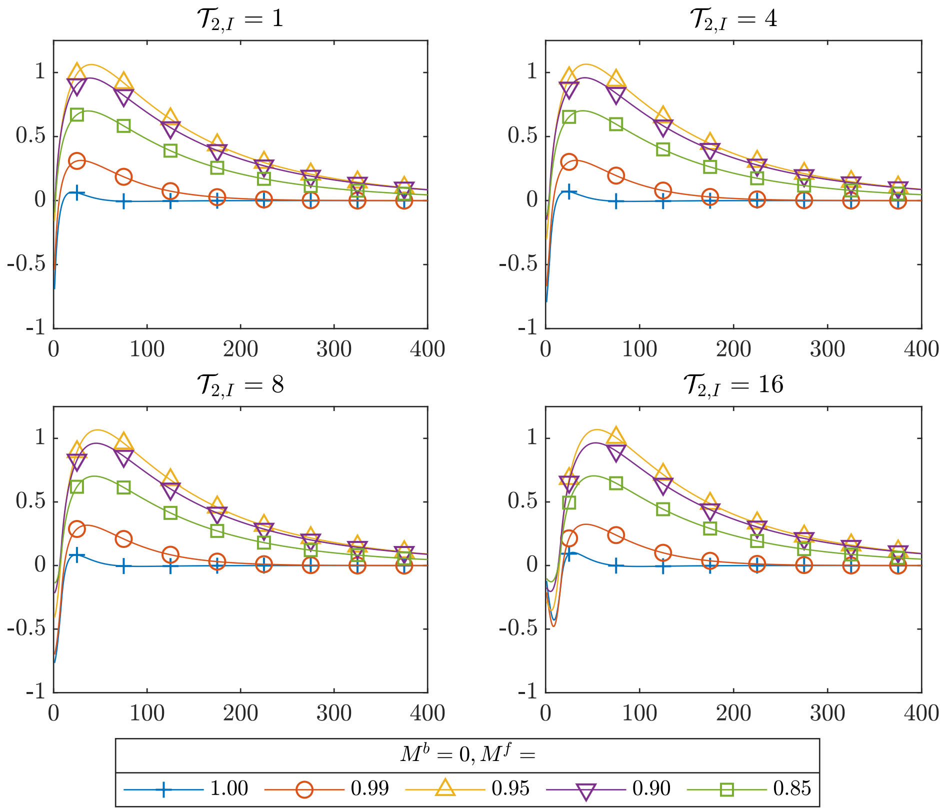

Figure 4.C.19: IRFs for $\hat{q}_t$ for different values of M with myopic expectations, baseline, no debt, no time to spend.

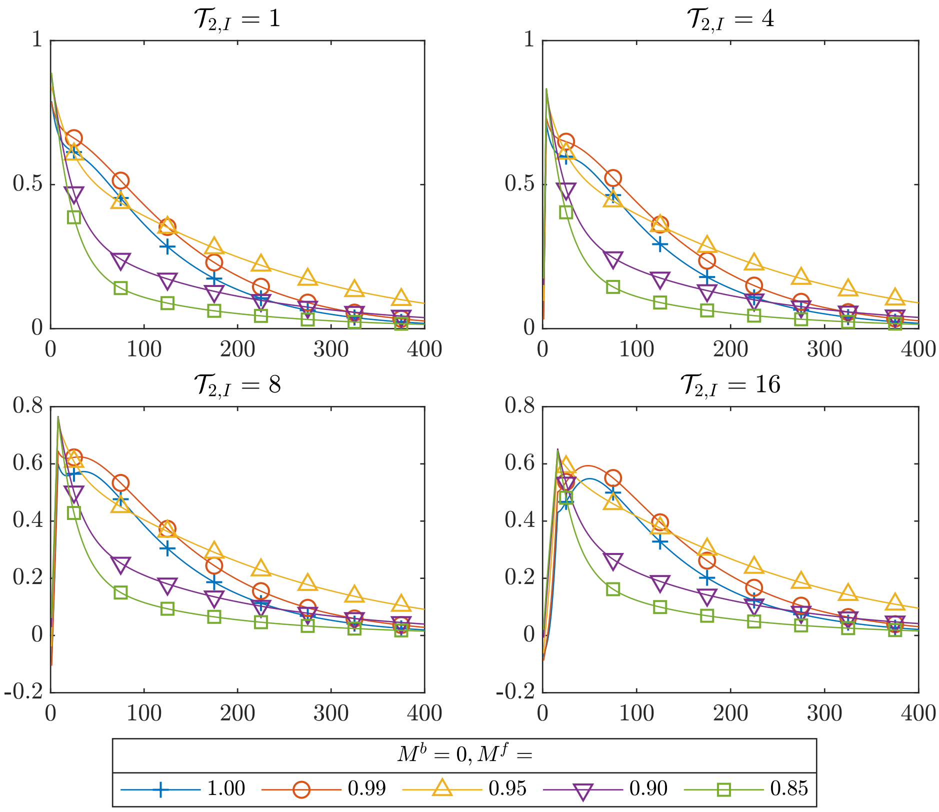

Figure 4.C.20: IRFs for $\hat{y}_t$ for different values of M with myopic expectations, baseline, no debt, no time to spend.

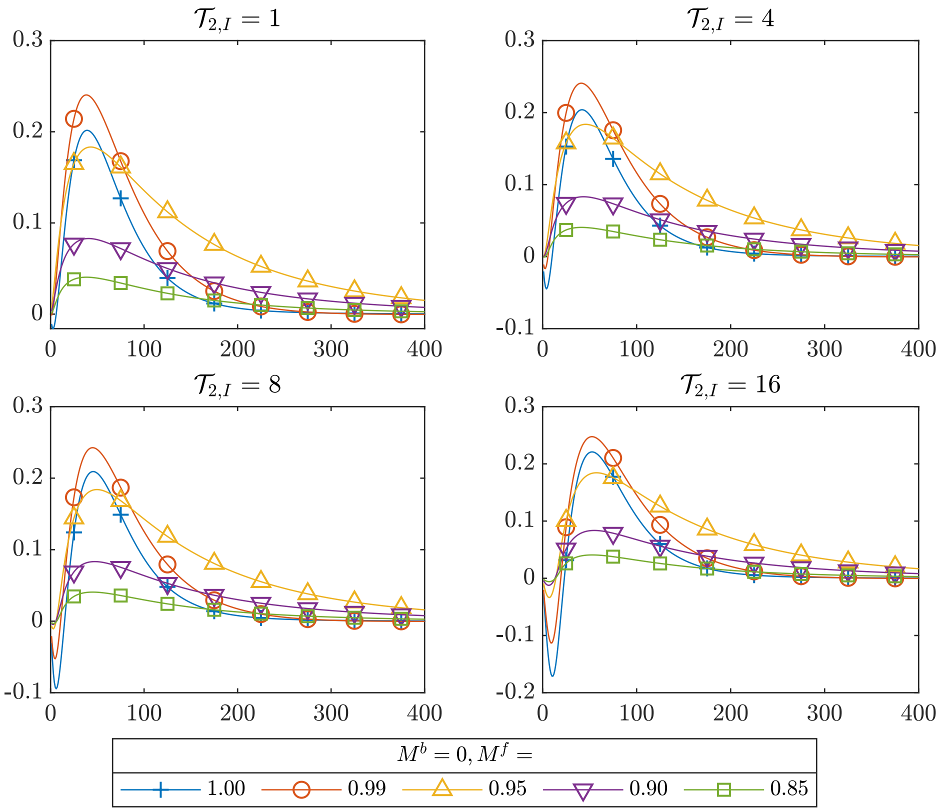

Figure 4.C.21: IRFs for $\hat{I}_t$ for different values of M wit myopic expectations, baseline, no debt, no time to spend.

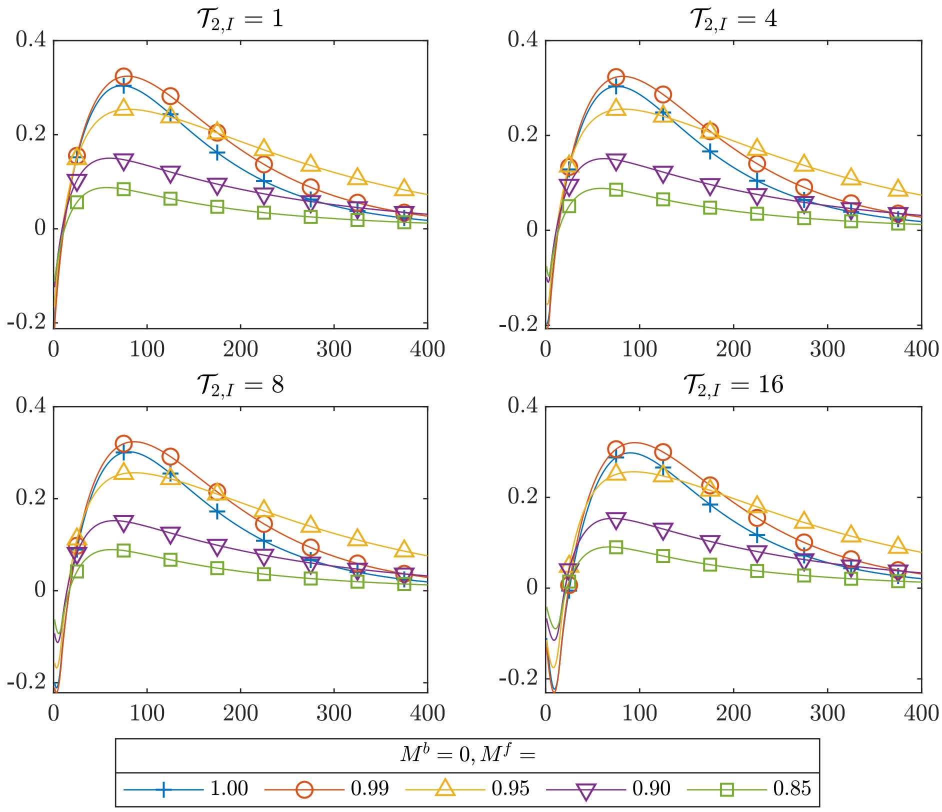

Figure 4.C.22: IRFs for $\hat{c}_t$ for different values of M with myopic expectations, baseline, no debt, no time to spend.

Cumulated sums and multipliers

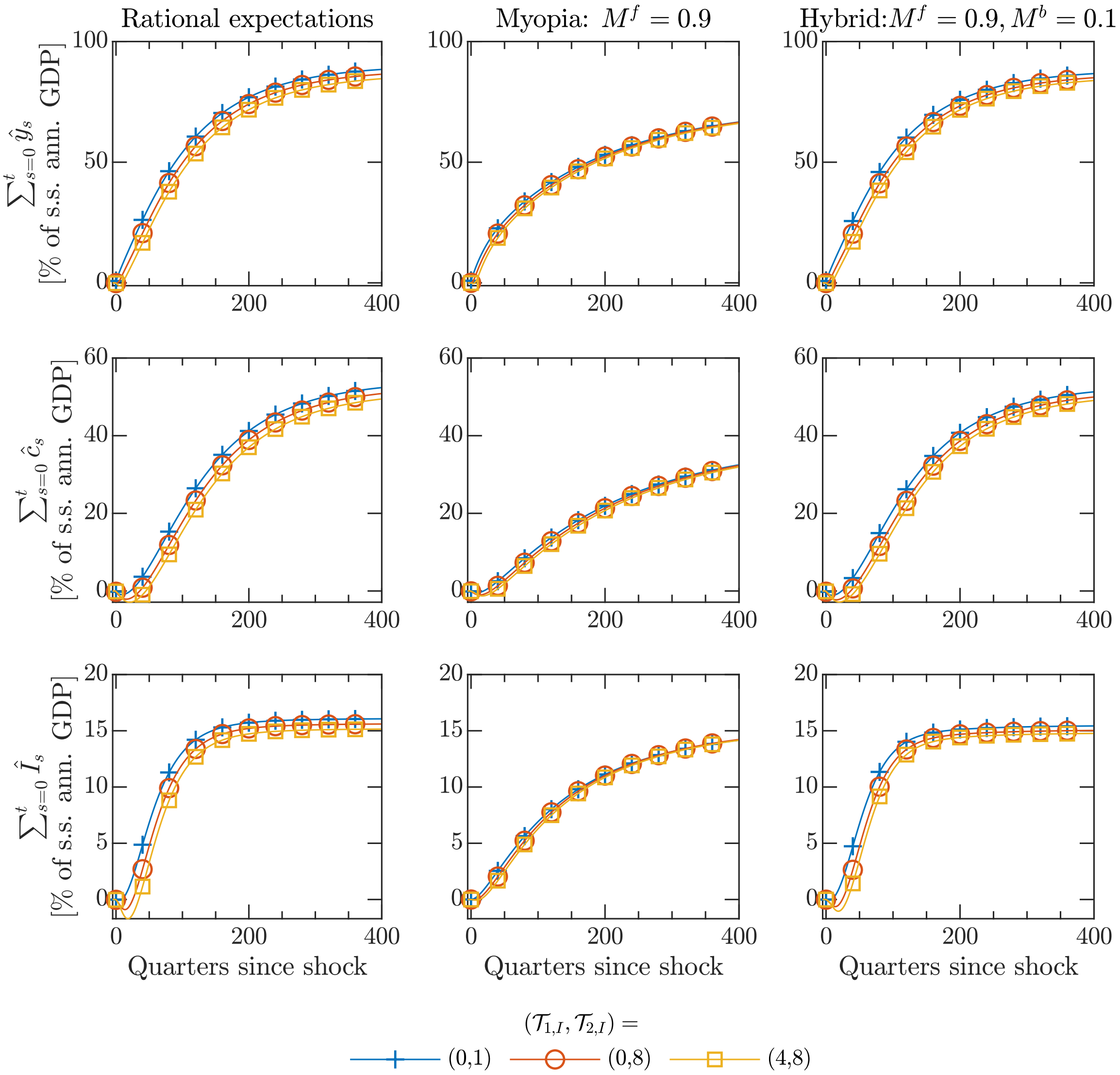

Figure 4.C.23: Cumulative sum of output, consumption and investment response, undiscounted, baseline calibration

Note: The figure shows the (undiscounted) cumulative sum of the impulse responses of the output as well as private consumption and investment in response to a shock to appropriations to government investment of size 1 in period t_0 (Figures 4.2, 4.5) for different forms of forming expectations. The response variables are ordered along the rows of the figure, different expectational formulations along the columns. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. All variables are expressed as a percentage deviation from steady state, except for the rental rate of capital which is expressed in terms of basis points at a quarterly frequency.

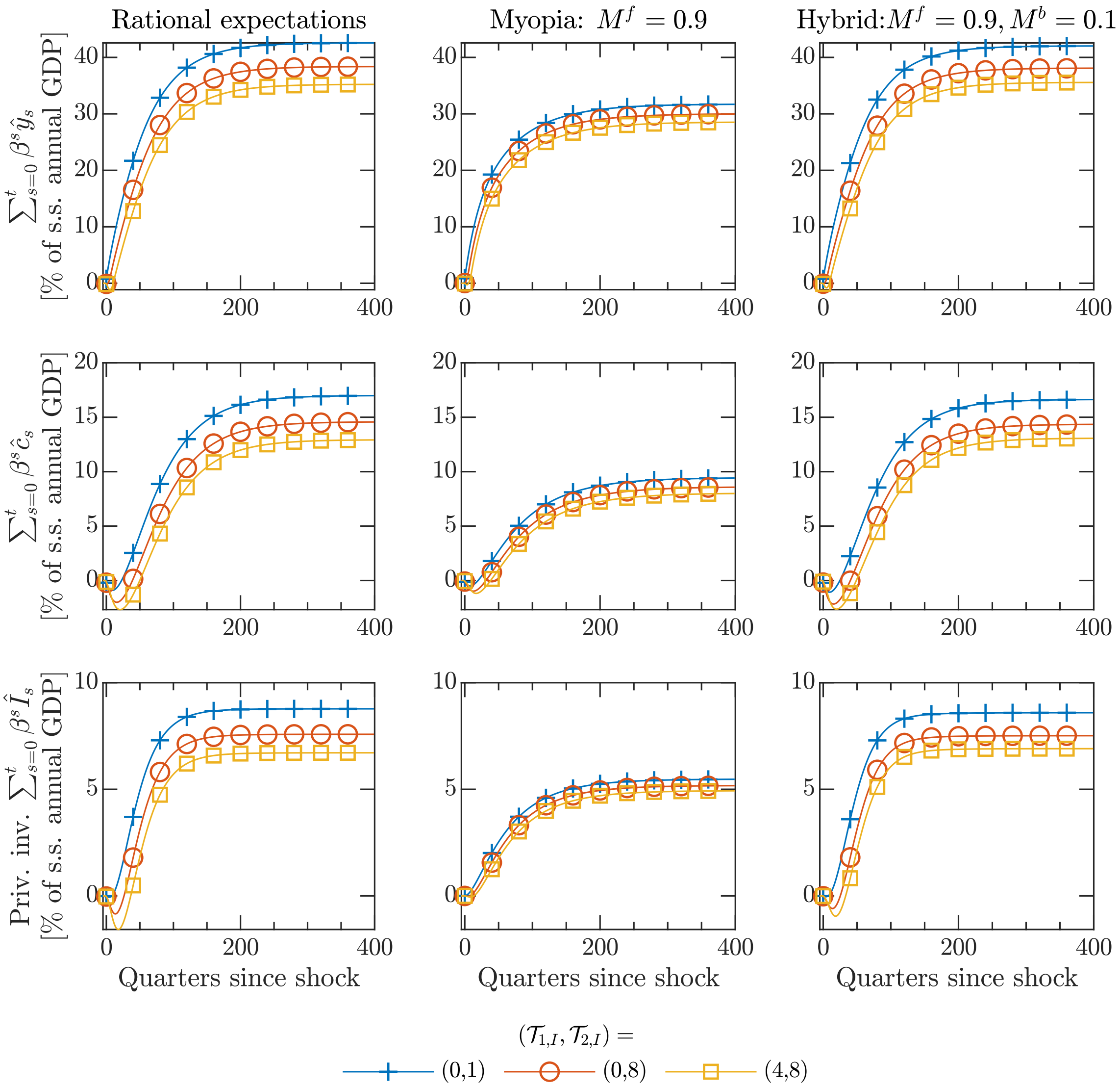

Figure 4.C.24: Cumulative discounted sum of output, consumption and investment response, discount factor: β, baseline calibration

Note: The figure shows the discounted cumulative sum of the impulse responses of the output as well as private consumption and investment in response to a shock to appropriations to government investment of size 1 in period t_0 (Figures 4.2, 4.5) for different forms of forming expectations. The response variables are ordered along the rows of the figure, different expectational formulations along the columns. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. All variables are expressed as a percentage deviation from steady state, except for the rental rate of capital which is expressed in terms of basis points at a quarterly frequency.

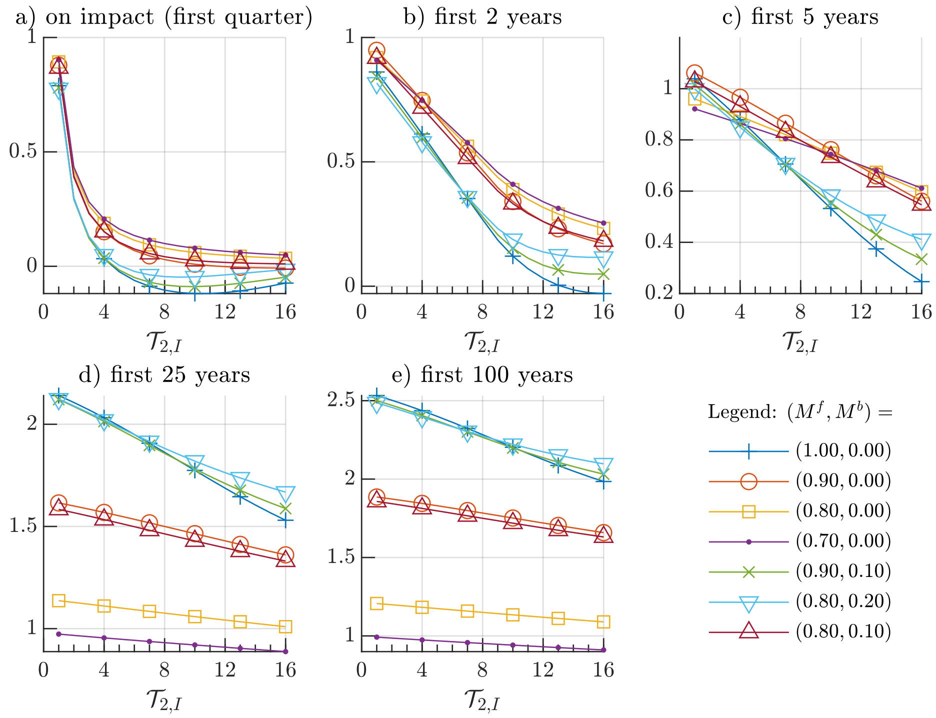

Figure 4.C.25: Appropriations multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated impulse responses of output in response to a shock to appropriations to government investment in period t_0 for different forms of forming expectations. The multiplier is based on equation (4.69). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences.

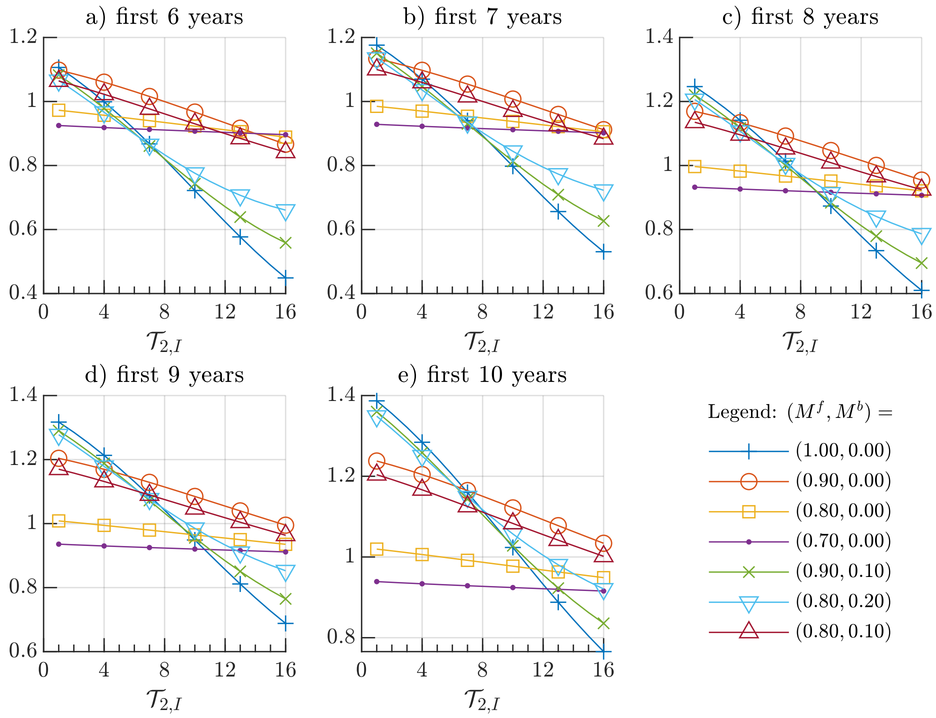

Figure 4.C.26: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$ in the medium term, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multiplier of public investment on output in response to a shock to appropriations in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences.

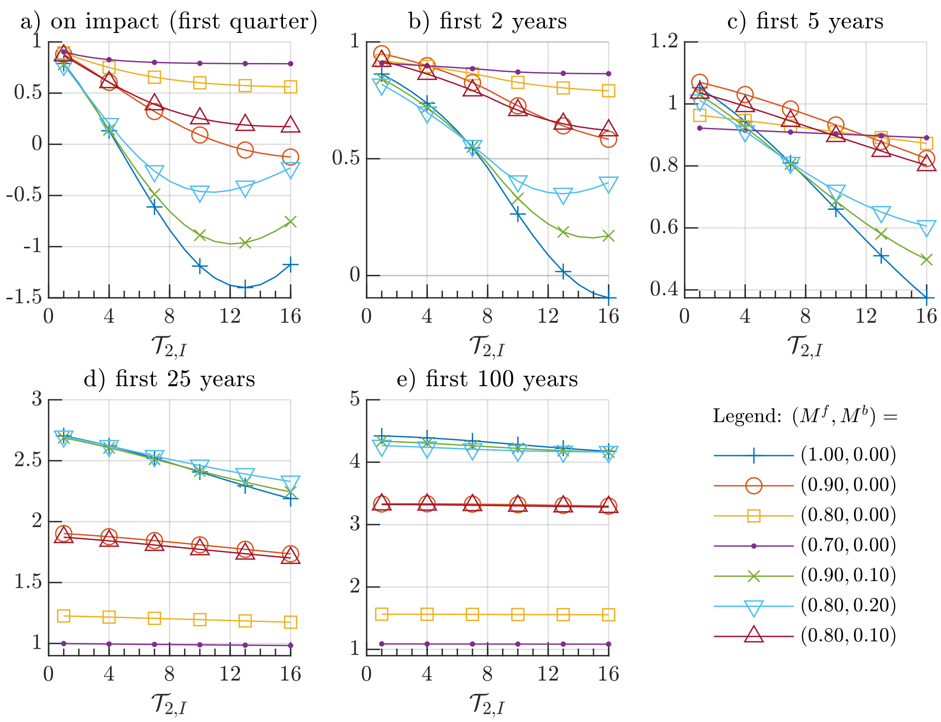

Figure 4.C.27: Government investment multipliers (undiscounted integral) for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multiplier of public investment on output in response to a shock to appropriations in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment. Hybrid expectations with $M^f + M^b = 1$ have been left out to facilitate a comparison of the remaining expectations.

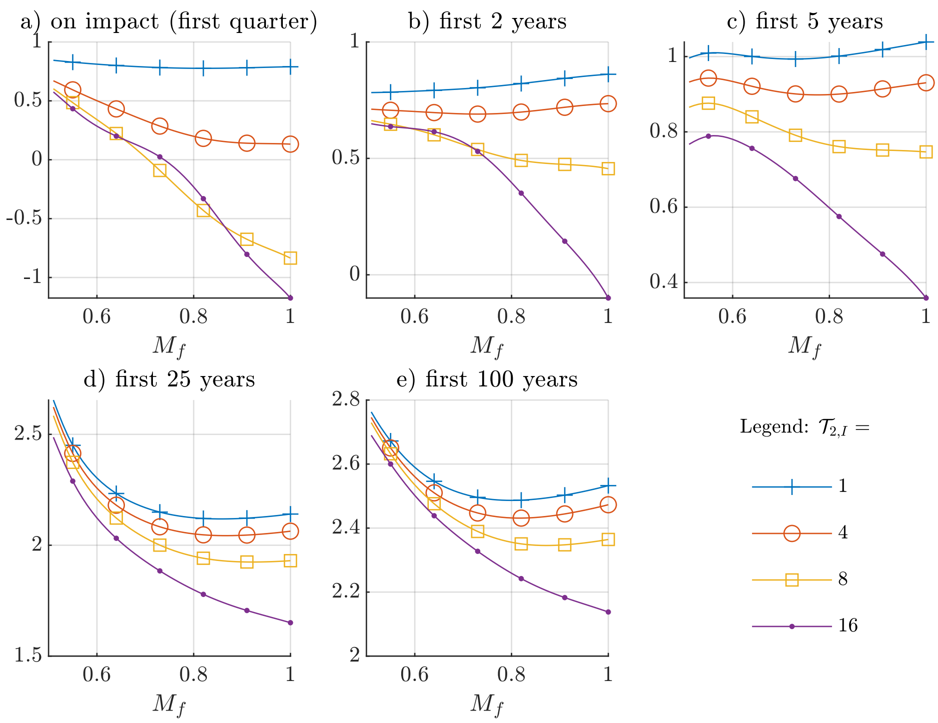

Figure 4.C.28: Government investment multipliers for different values of $\mathcal{T}_{2,I}$ across different values of M^f for hybrid expectations with $M^b = 1 - M^f$, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multiplier of public investment on output in response to a shock to appropriations in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. $\mathcal{T}_{1,I} = 0$ for all simulations.

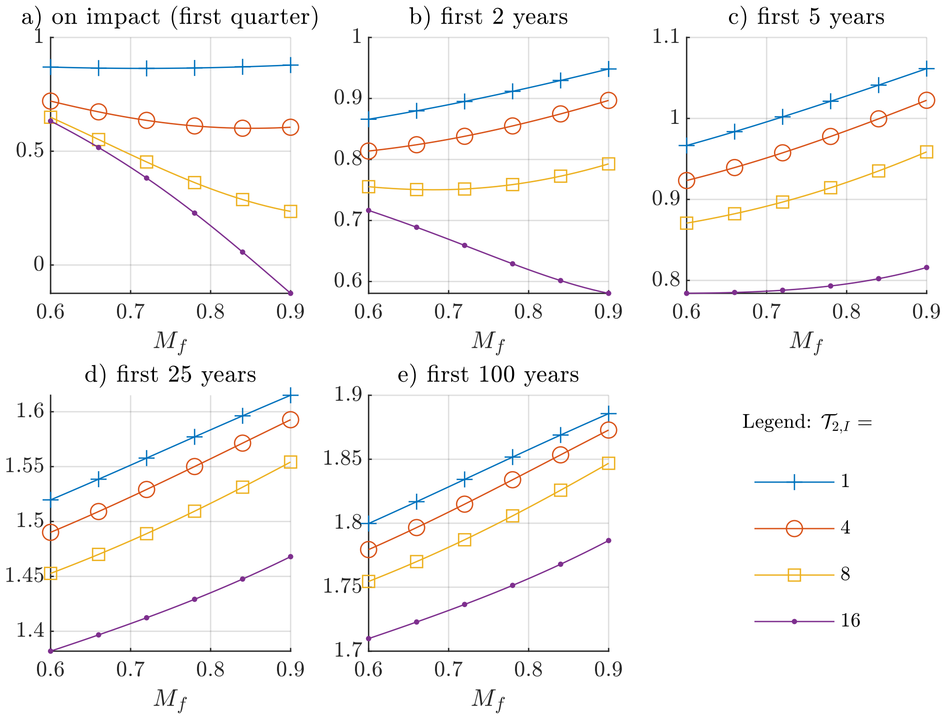

Figure 4.C.29: Government investment multipliers for different values of $\mathcal{T}_{2,I}$ across different values of M^f for hybrid expectations with $M^b = 0.9 - M^f$, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multiplier of public investment on output in response to a shock to appropriations in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. $\mathcal{T}_{1,I} = 0$ for all simulations.

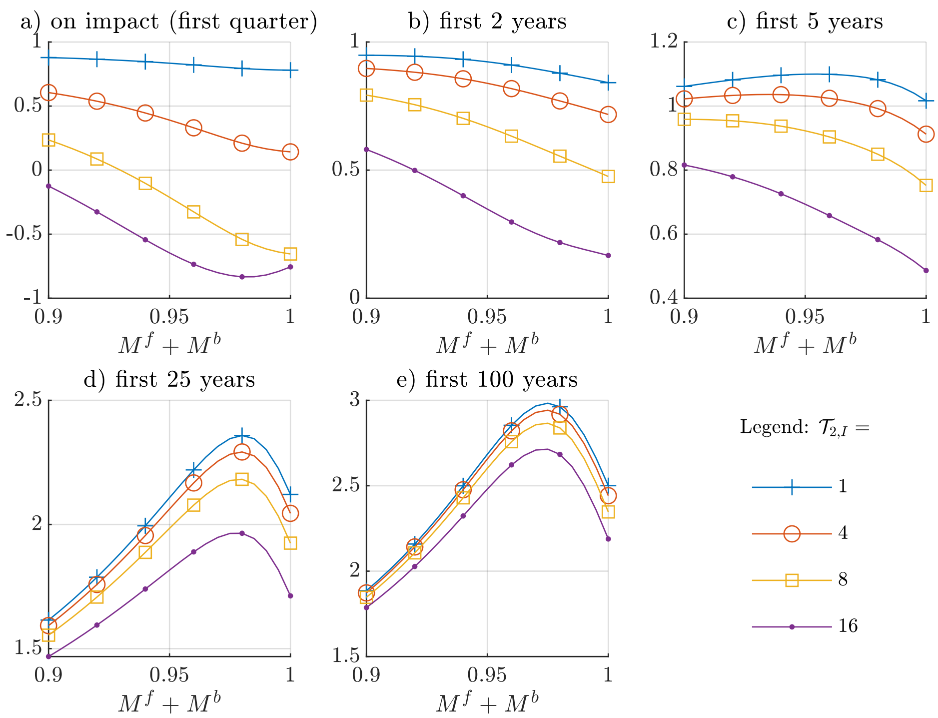

Figure 4.C.30: Government investment multipliers for different values of $\mathcal{T}_{2,I}$ across different values of $M^f + M^b$ for hybrid expectations with $M^f = 0.9$, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multiplier of public investment on output in response to a shock to appropriations in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. $\mathcal{T}_{1,I} = 0$ for all simulations.

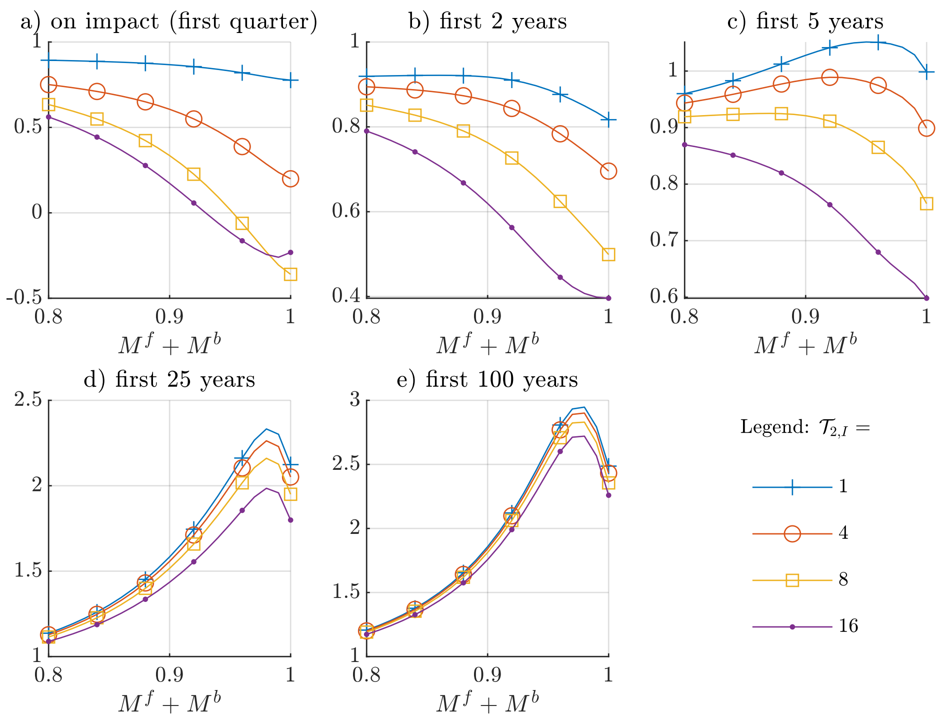

Figure 4.C.31: Government investment multipliers for different values of $\mathcal{T}_{2,I}$ across different values of $M^f + M^b$ for hybrid expectations with $M^f = 0.8$, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multiplier of public investment on output in response to a shock to appropriations in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. $\mathcal{T}_{1,I} = 0$ for all simulations.

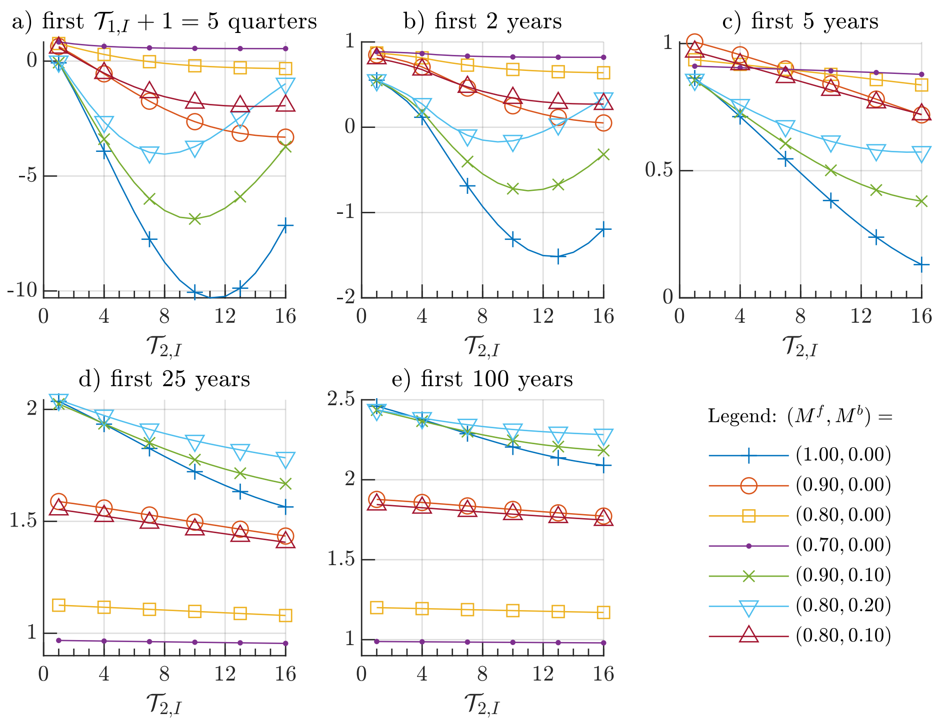

Figure 4.C.32: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I}=4$, baseline calibration, fully tax-funded

Note: The figure shows the discounted cumulated multiplier of public investment on output in response to a shock to appropriations in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g=0$), no government debt ($b_t=0$) and no wealth preferences.

A minor robustness test

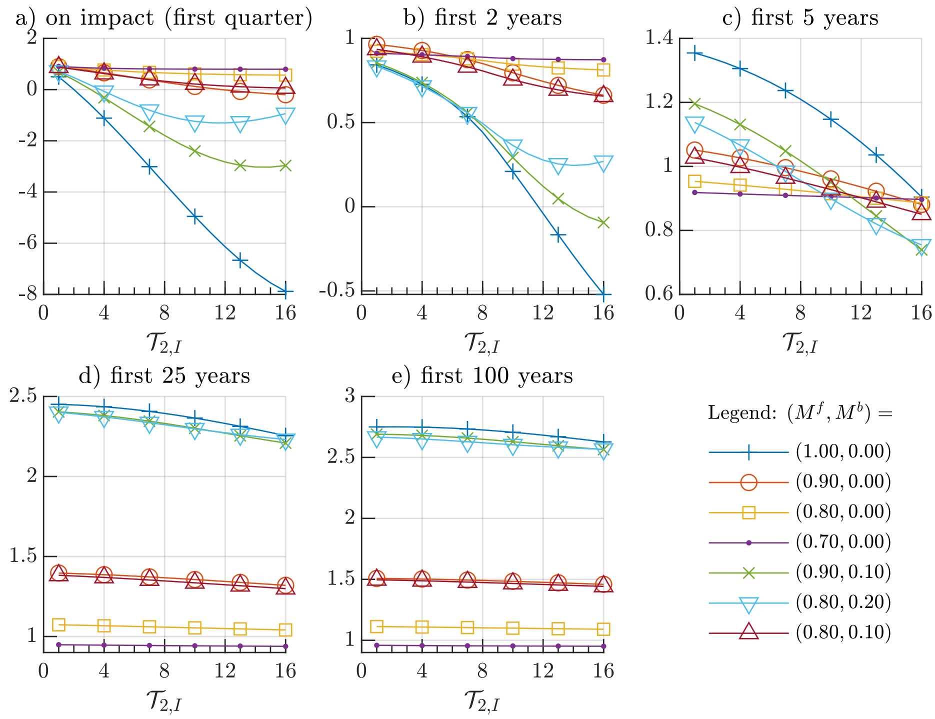

Figure 4.C.33: Robustness test with simplified pseudo-Euler equation for capital, pt. 1

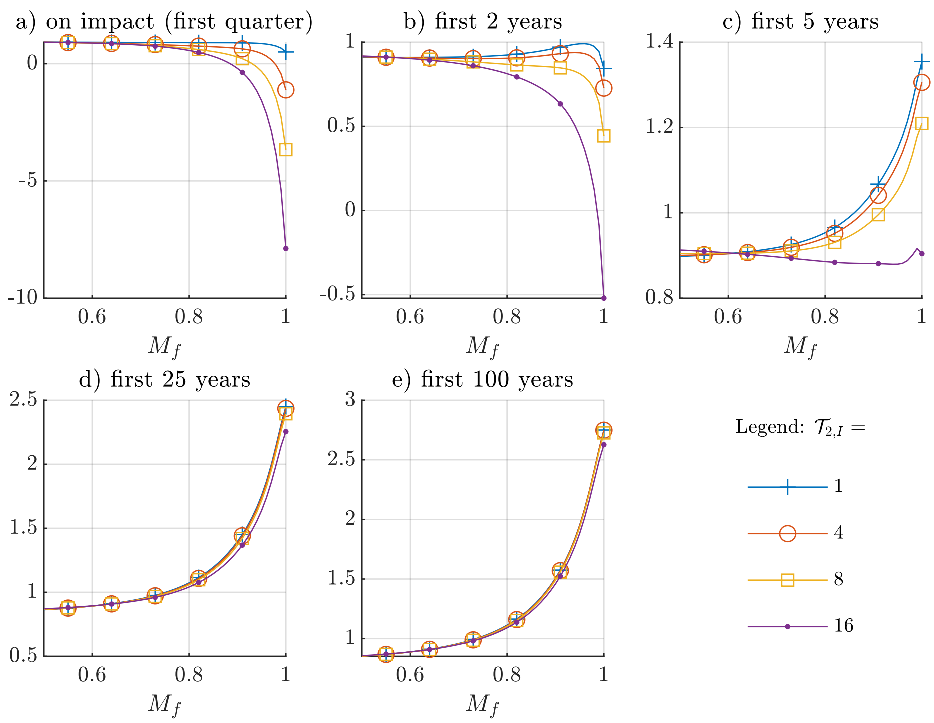

Figure 4.C.34: Robustness test with simplified pseudo-Euler equation for capital, pt. 2

Effective lower bound

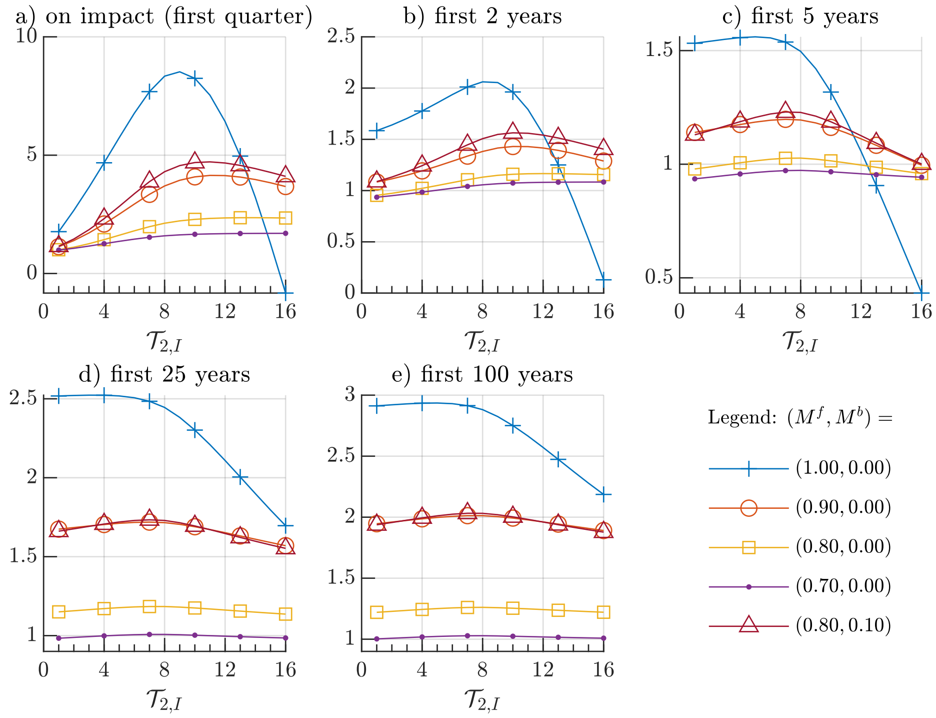

Figure 4.C.35: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, with bonds issued. Steady-state debt: $b = 0$

Note: The figure shows the discounted cumulated impulse response of output in response to a shock to appropriations to government investment in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment. Hybrid expectations with $M^f + M^b = 1$ have been left out to facilitate a comparison of the remaining expectations.

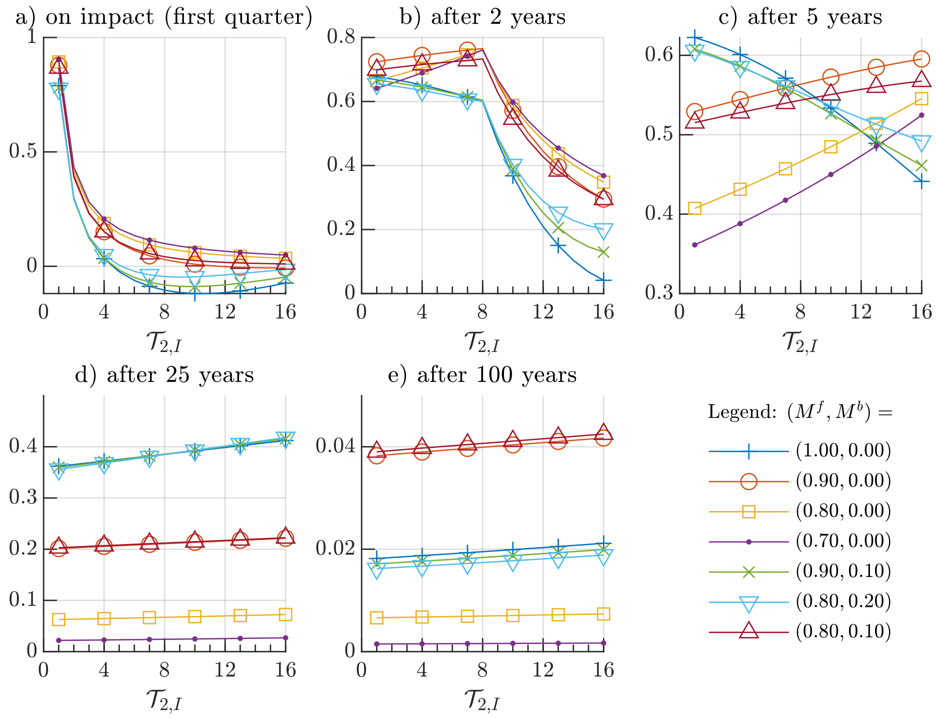

Figure 4.C.36: Impulse response of output to a shock to government investment at the ELB for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, no bonds issued. Steady-state debt: $b = 0$
Note: The figure shows the change of output in response to a shock to appropriations to government investment in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment. The shock of appropriations in t_0 is normalised to 1.

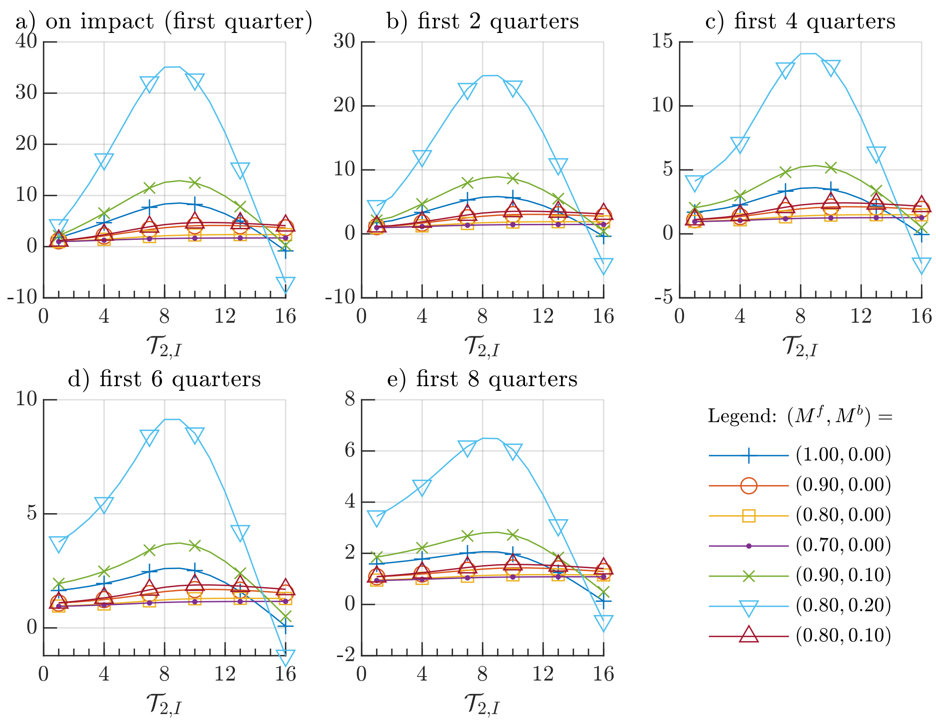

Figure 4.C.37: Short-run government investment multipliers at the ELB for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, no bonds issued. Steady-state debt: $b = 0$

Note: The figure shows the discounted cumulated impulse response of output in response to a shock to appropriations to government investment in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment. Hybrid expectations with $M^f + M^b = 1$ have been left out to facilitate a comparison of the remaining expectations.

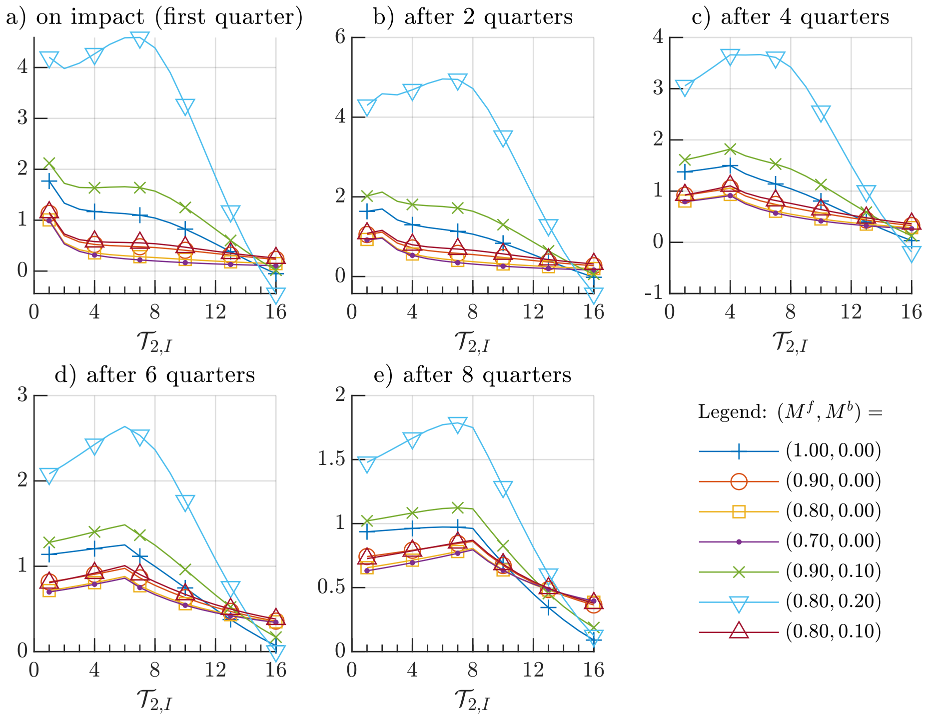

Figure 4.C.38: Short- run impulse response of output to a shock to government investment at the ELB in the short run for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, no bonds issued. Steady-state debt: $b = 0$

Note: The figure shows the impulse respnse of output, private consumption and private investment in response to a shock to appropriations to government investment in period t_0 for different forms of forming expectations. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment. Hybrid expectations with $M^f + M^b = 1$ have been left out to facilitate a comparison of the remaining expectations.

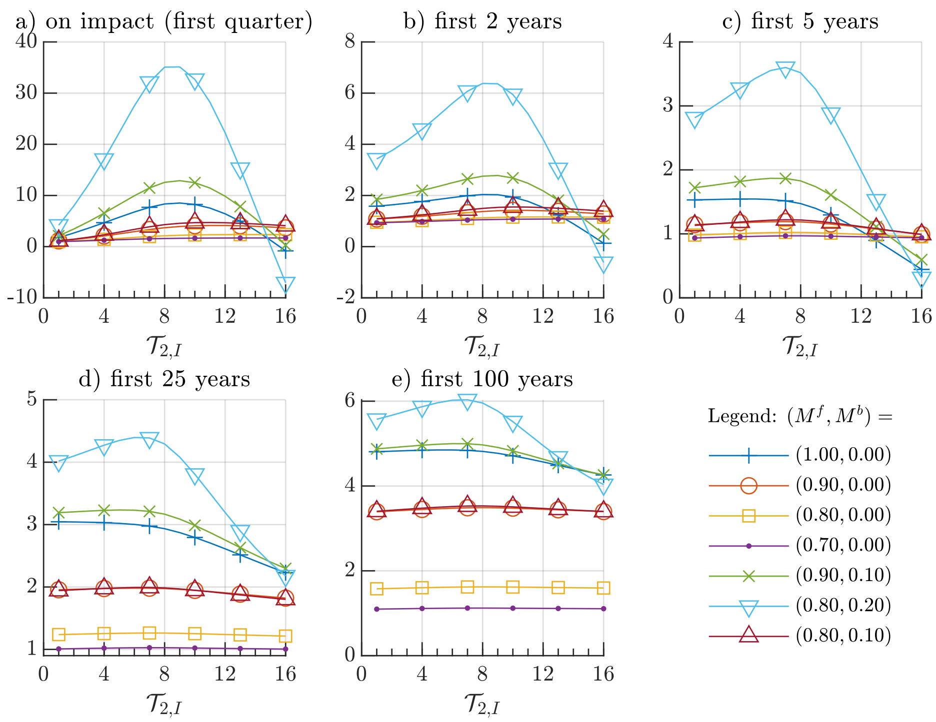

Figure 4.C.39: Government investment multipliers at the ELB for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, no bonds issued. Steady-state debt: $b = 0$, undiscounted integral

Note: The figure shows the undiscounted cumulated impulse responses of output in response to a shock to appropriations to government investment in period t_0 for different forms of forming expectations. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no steady-state government debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment.

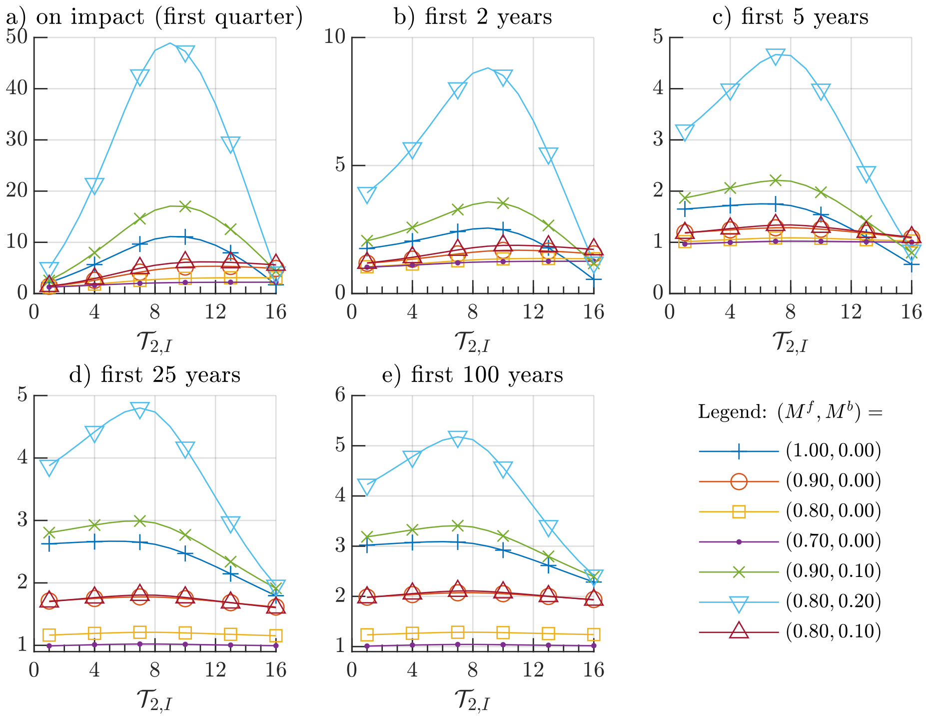

Figure 4.C.40: Government investment multipliers at the ELB for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, with bonds issued. Steady-state debt: $b = 0$

Note: The figure shows the undiscounted cumulated impulse responses of output in response to a shock to appropriations to government investment in period t_0 for different forms of forming expectations. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment. Taxation is given by (4.56).

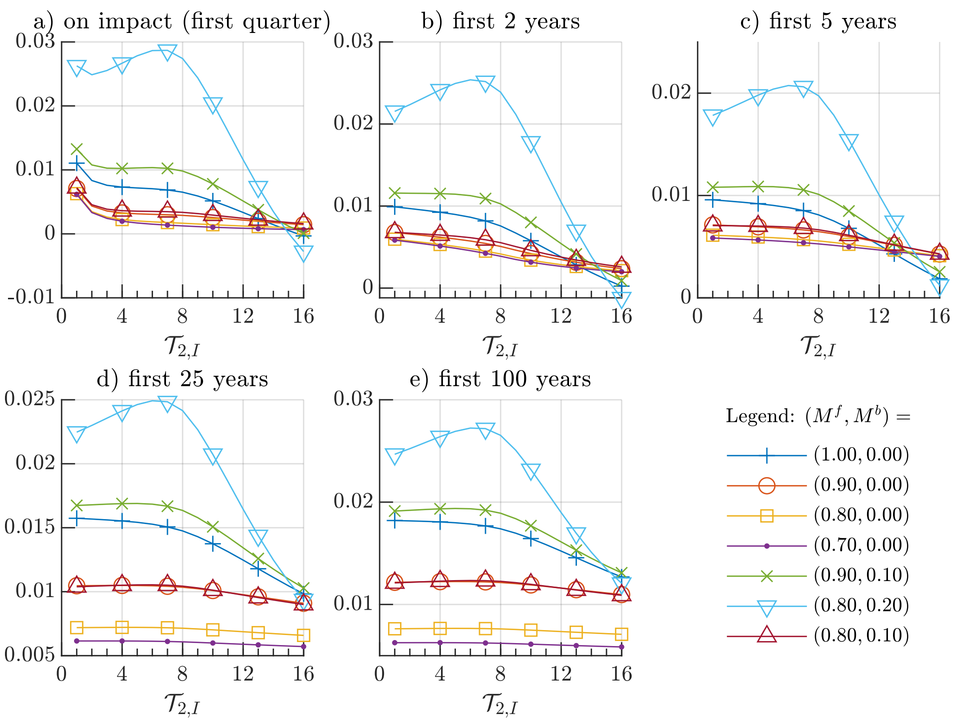

Figure 4.C.41: Appropriations multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, baseline calibration, fully tax-funded, at the ELB

Note: The figure shows the discounted cumulated impulse responses of output in response to a shock to appropriations to government investment in period t_0 for different forms of forming expectations. The multiplier is based on equation (4.69). The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$) and no wealth preferences. The economy is simultaneously pushed to the ELB for 8 periods in the period of the shock to government investment.

Agent heterogeneity

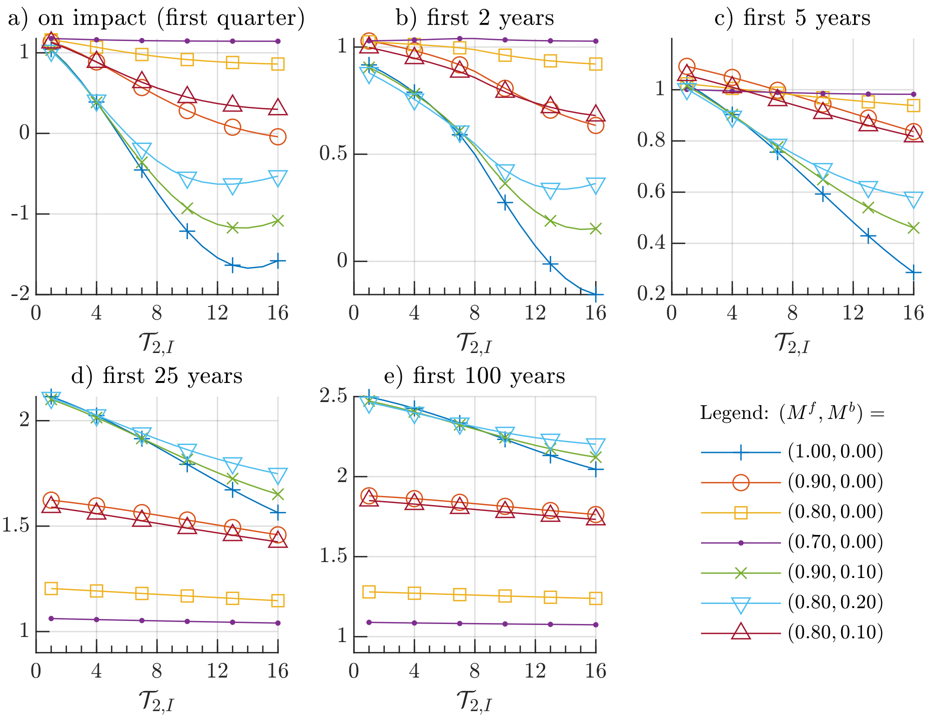

Figure 4.C.42: Government investment multipliers for different types of expectations across different values of $\mathcal{T}_{2,I}$ for $\mathcal{T}_{1,I} = 0$, THANK model, bonds issued, high stady-state debt ($b = 4y$).

Results introducing wealth preferences and growth

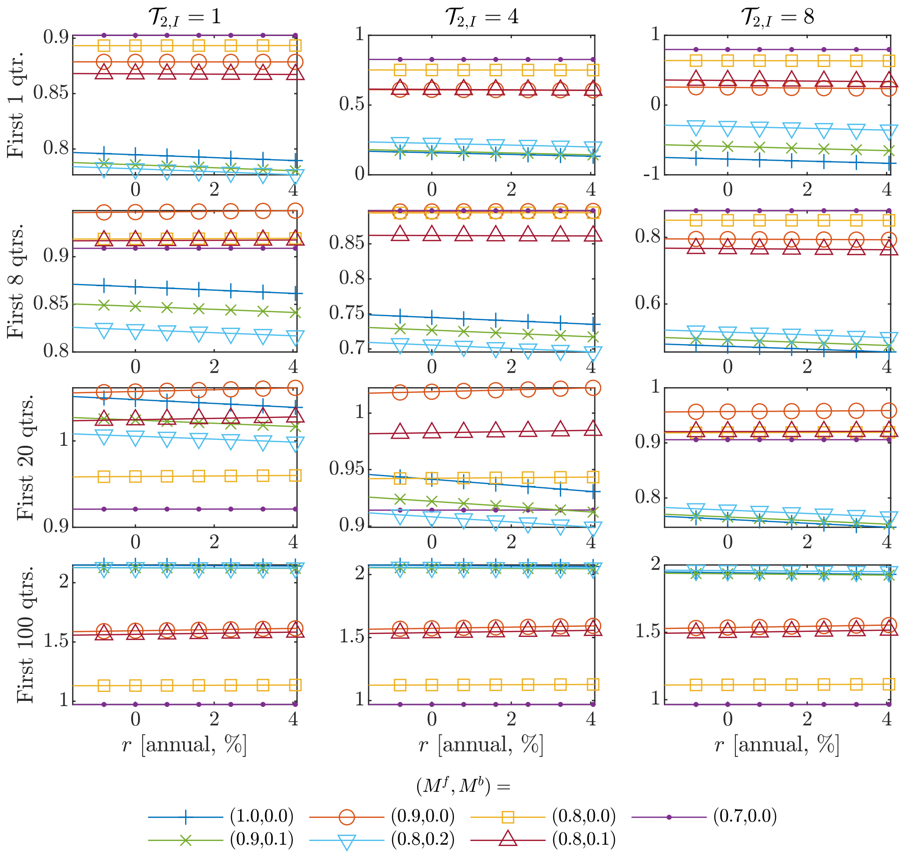

Figure 4.C.43: The effect of adding wealth preferences to the model, no debt
Note: The figure shows the discounted cumulated government investment multiplier on output in response to a shock to appropriations in period t_0 for different forms of forming expectations and different values for $\mathcal{T}_{2,I}$. $\mathcal{T}_{1,I} = 0$. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), no government debt ($b_t = 0$), but wealth preferences $\bar{\xi} > 0$. In particular, $\bar{\xi}$ is calibrated to achieve an r as given along the horizontal axis.

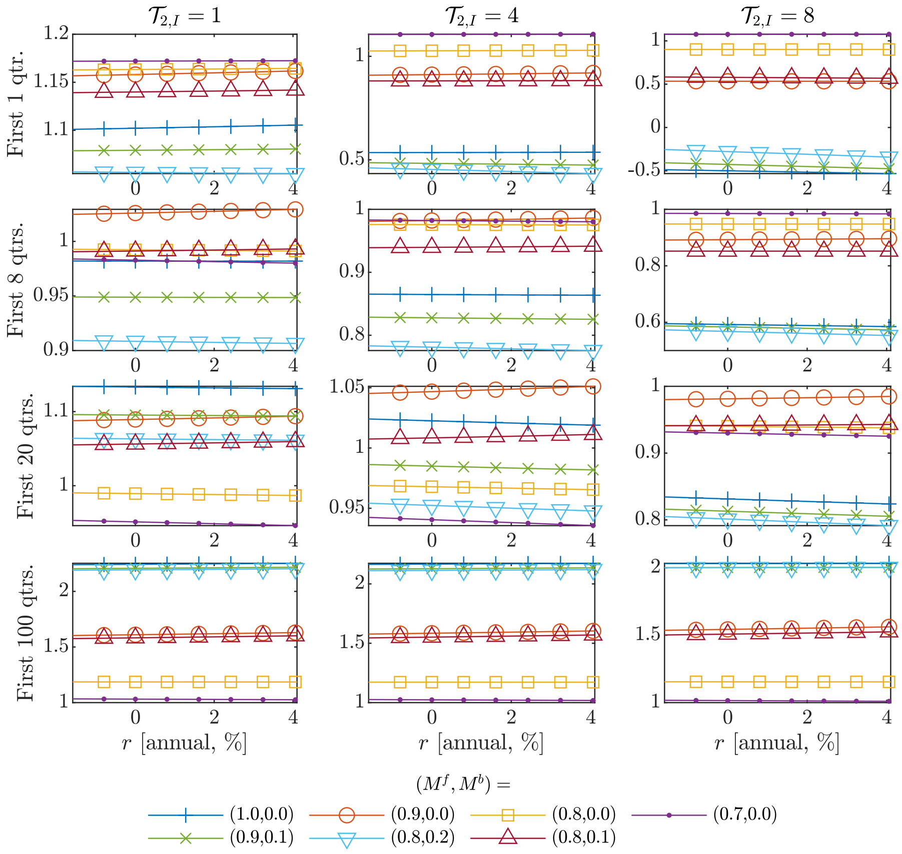

Figure 4.C.44: The effect of adding wealth preferences to the model, with debt

Note: The figure shows the discounted cumulated government investment multiplier on output in response to a shock to appropriations in period t_0 for different forms of forming expectations and different values for $\mathcal{T}_{2,I}$. $\mathcal{T}_{1,I} = 0$. The discount factor used for periods larger than t_0 is β. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), steady-state government debt $b = 4y$, and wealth preferences $\bar{\xi} > 0$. In particular, $\bar{\xi}$ is calibrated to achieve an r as given along the horizontal axis. Government taxation is given by (4.56).

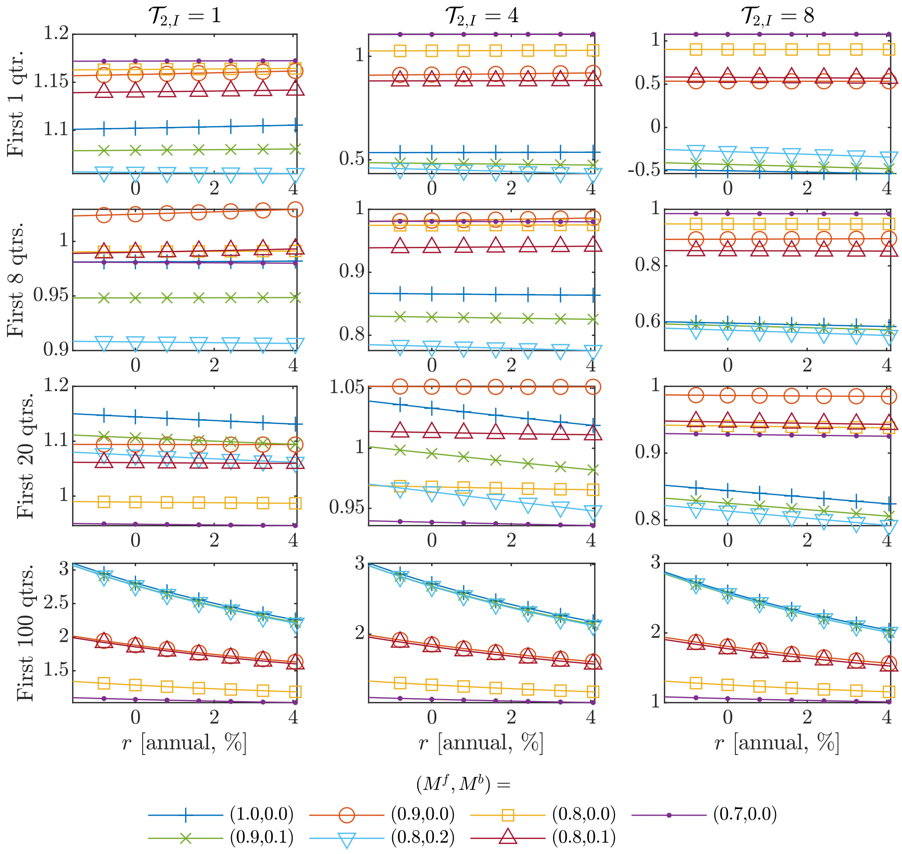

Figure 4.C.45: The effect of adding wealth preferences to the model, with debt, safe discount factor

Note: The figure shows the discounted cumulated government investment multiplier on output in response to a shock to appropriations in period t_0 for different forms of forming expectations and different values for $\mathcal{T}_{2,I}$. $\mathcal{T}_{1,I} = 0$. The discount factor used for periods larger than t_0 is $\frac{1}{1+r}$. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with zero growth ($g = 0$), steady-state government debt $b = 4y$, and wealth preferences $\bar{\xi} > 0$. In particular, $\bar{\xi}$ is calibrated to achieve an r as given along the horizontal axis. Government taxation is given by (4.56).

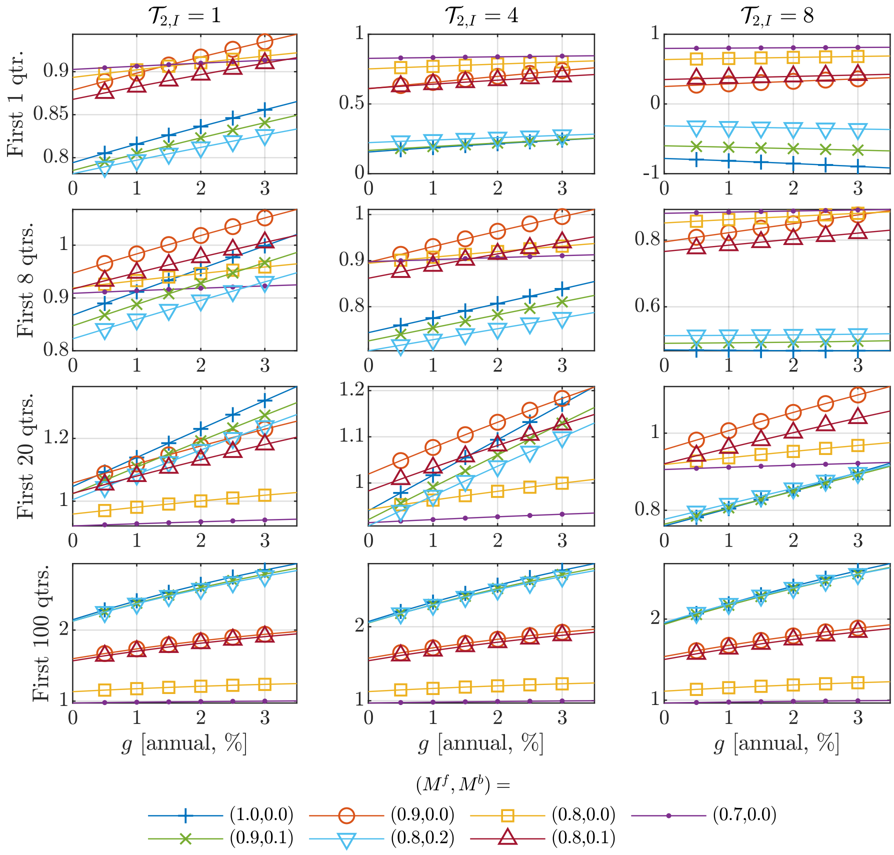

Figure 4.C.46: The effect of adding growth and wealth preferences to the model, with debt, safe discount factor

Note: The figure shows the discounted cumulated government investment multiplier on output in response to a shock to appropriations in period t_0 for different forms of forming expectations and different values for $\mathcal{T}_{2,I}$. $\mathcal{T}_{1,I} = 0$. The discount factor used for periods larger than t_0 is $\frac{1}{1+r}$. The different subplots show the cumulated response at different horizons. The figure is based on simulations of the linear TANK model with growth ($g \geq 0$), no government debt $b_t = 0$, and wealth preferences $\bar{\xi} > 0$. In particular, $\bar{\xi}$ is calibrated to achieve an $r = 1.005^{\frac{1}{4}} - 1$ for g given along the horizontal axis.

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München, 16.03.2022

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Alexander Schwemmer