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A Generalized Approach to Contingency Screening with System Islanding

Xiangtian Zheng
Texas A&M University
zxt0515@tamu.edu

Sujatha Kotamarty
ERCOT
Sujatha.Kotamarty@ercot.com

Kashia Thuerwaechter
ERCOT
Kashia.Thuerwaechter@ercot.com

Alex Lee
ERCOT
Alex.Lee@ercot.com

Shun Hsien (Fred) Huang
ERCOT
Shun-Hsien.Huang@ercot.com

Le Xie
Texas A&M University
le.xie@tamu.edu

Abstract

This paper introduces a generalized contingency analysis approach that can identify critical pairs of contingencies (N-2 contingencies) that result in severe reliability violations or large numbers of islanded system components, bridging the gap between academic research and practical applications. We formulate the practical problem of N-2 contingency analysis in a clear mathematical format. We propose a generalized contingency screening approach compatible with all types of contingencies, including generator failure, load failure, open branches, and their mixtures that can lead to islanding. The proposed approach is demonstrated in a real Texas system, showing its effectiveness in critical contingency identification and its ability to enable possible practical contingency analysis applications.

Keywords: Power grid security, contingency analysis, contingency screening.

1. Introduction

The normal functioning of large-scale infrastructure systems depends not only on the normal interconnected topologies but also on the normal equilibrium states throughout the system. Critical disturbances, namely the removal of critical components, can affect functionalities and even cause cascading failures by altering system topology and resulting in abnormal states. Therefore, identifying critical contingencies in real time is imperative for online security analysis and decision making for possible preventive measures. Today's real power systems can maintain normal operation given an arbitrary single contingency. With the increasing penetration of intermittent renewable generation, it is imperative to identify critical N-2 contingencies (two contingencies occur simultaneously) that lead to security violations to ensure system reliability.

Existing methods of critical contingency screening, ranking, and selection can be categorized into four classes, including (i) (meta) heuristic approaches based on line outage distribution factors (LODFs) and power transfer distribution factors (PTDFs), such as Davis and Overbye (2010) and Kaplunovich and Turitsyn (2016), (ii) nonlinear optimization, such as Bienstock and Verma (2010) and Donde et al. (2008), (iii) network topology analysis, such as Biswas et al. (2020), Narimani et al. (2021), and Poudel et al. (2016), and (iv) randomized algorithms, such as Eppstein and Hines (2012). However, practical approaches to rapidly identify critical N-2 contingencies for online operation are still in a nascent stage, with several gaps in the existing literature. First, the lack of a practical problem formulation with clear definition of single contingency that meets online operational needs may mislead the task objective and algorithm design. Second, existing methods are not compatible with contingencies that lead to system islanding, which hinders practical significance. Third, most of the existing methods cannot guarantee identification of all or most critical contingencies while pursuing efficiency, with a few exceptions such as Kaplunovich and Turitsyn (2016).

In this paper, we formulate the practical problem of critical N-2 contingency identification in a direct current (DC) power flow-based format. We propose a generalized contingency screening approach that can identify all critical pairs of contingencies that lead to severe thermal violation. Compared to existing DC power flow-based methods, it is compatible with all types of contingencies, making it suitable for

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practical contingency analysis problems and enabling the development of practical fast contingency screening applications. We demonstrate the proposed approach in a real Texas system, showing the effectiveness and efficiency for critical N-2 contingency identification and the ability to enable possible practical contingency analysis applications. The results of contingency screening also provide guides and insights for algorithm design.

The rest of the paper is organized as follows. Section 2 formulates the problem of contingency screening in a DC power flow-based form. Section 3 introduces a post-contingency power flow calculation method compatible with system islanding. Section 4 proposes a generalized contingency screening method. Section 5 presents numerical results in the real Texas system. Section 6 draws the conclusion and discusses future efforts in fast screening.

2. Problem Formulation

2.1. Power Grid Modeling

We consider an electric power grid $G = (V, E)$. Here, V is a set of N_{V} nodes that contain N_{g} generators, N_{l} loads and substations; E is a set of N_{E} edges that represent transmission lines and transformers. We denote the node index of generator i by n_i^{g}, the node index of load i by n_i^{l}, and the node indices of ends of branch α by n_α^{f} and n_α^{t}. In particular, we define node 0 as the slack bus.

In this paper, we formulate the power grid using a DC power flow model that has three types of equilibrium states $\boldsymbol{\theta}$, $\boldsymbol{p}$ and $\boldsymbol{f}$, which are vectors representing voltage angle at nodes, net injected power at nodes, and power flow in branches, respectively. Here, $\boldsymbol{\theta}$ and $\boldsymbol{p}$ follow the DC power flow equations

$$\boldsymbol{p} = -\boldsymbol{B}\boldsymbol{\theta}, \tag{1}$$

where $\boldsymbol{B}$ is a Jacobian matrix. And $\boldsymbol{\theta}$ and $\boldsymbol{f}$ have the relation derived from Kirchhoff laws,

$$\boldsymbol{f}_\alpha = \boldsymbol{B}_{n_\alpha^{\text{f}} n_\alpha^{\text{t}}}(\boldsymbol{\theta}_{n_\alpha^{\text{f}}} - \boldsymbol{\theta}_{n_\alpha^{\text{t}}}), \tag{2}$$

where n_α^{f} and n_α^{t} represent the indices of 'from' and 'to' nodes of branch α, respectively.

2.2. Formulation of Contingency Analysis

In this paper, a single contingency refers to outages of single or multiple system components, including generators, load, branches, and their mixtures. Therefore, we define a single contingency $\mathcal{C}$ as a set of multiple branches to be removed and multiple nodes to set net power injection to 0.

Given an electric power grid with initial system states, our goal is to design a contingency prioritization strategy that can efficiently identify all critical pairs of single contingencies $\mathcal{C}_i \cup \mathcal{C}_j$ from a predefined contingency set $\{\mathcal{C}_i\}_{i=1}^{N_{\text{C}}}$, which maximize or near-maximize the severity of post-contingency states in Eq. 3.

$$\max_{i,\, j} \quad \delta(\boldsymbol{f}', \boldsymbol{p}', \boldsymbol{\theta}') \tag{3a}$$

$$\text{s.t.} \quad \boldsymbol{B}' = T_{\text{B}}(\boldsymbol{B}, \mathcal{C}_i \cup \mathcal{C}_j) \tag{3b}$$

$$\boldsymbol{p}' = T_{\text{p}}(\boldsymbol{p}, \mathcal{C}_i \cup \mathcal{C}_j) \tag{3c}$$

$$\boldsymbol{\theta}' = -\boldsymbol{B'}^{-1}\boldsymbol{p'} \tag{3d}$$

$$\boldsymbol{f}' = \boldsymbol{X}^{-1}\boldsymbol{A}\boldsymbol{\theta}' \tag{3e}$$

where function δ evaluates the degree to which the post-contingency state violates the security criteria, functions T_{B} and T_{p} convert Jacobian matrix and net power injection according to contingencies, $\boldsymbol{X}^{-1}$ is a $N_{\text{E}} \times N_{\text{E}}$ diagnal matrix of branch susceptance $\text{diag}\left(\boldsymbol{B}_{n_1^{\text{f}} n_1^{\text{t}}}, \cdots, \boldsymbol{B}_{n_{N_{\text{E}}}^{\text{f}} n_{N_{\text{E}}}^{\text{t}}}\right)$, and $\boldsymbol{A}$ is a $N_{\text{E}} \times N_{\text{V}}$ incidence matrix where row i has only two nonzero elements with 1 on column n_i^{f} and -1 on column n_i^{t}.

In a DC power flow-based format, common efforts on contingency screening can be classified into two parts: (i) accelerating post-contingency power flow estimation that is commonly implemented by leveraging LODFs and PTDFs and (ii) greatly compressing large-dimensional searching space of N-2 contingencies in an efficient manner. As the main contribution of this paper, we focus on improving the feasibility of power flow estimation in practical applications and therefore propose a generalized contingency screening approach that can identify all types of critical N-2 contingencies, with the objective of bridging the gap between academic research and practical applications.

3. Generalized DC Power Flow-based Contingency Analysis

In this section, leveraging LODFs and PTDFs, we introduce a post-contingency power flow calculation method compatible with system islanding.

For convenience in the introduction of algorithms, we first introduce the common variables used in DC power flow-based contingency analysis. We define the matrix of PTDFs as $\boldsymbol{P}$ whose entry $\boldsymbol{P}_{i\alpha}$ means the proportion of point-to-point power transfer between node 0 (slack bus) and node i that is distributed to branch

α, as defined in

$$\boldsymbol{P}_{i\alpha} \coloneqq \Delta \boldsymbol{f}_\alpha / T_i, \tag{4}$$

where T_i means power injection into node 0 (slack bus) with power absorption by node i and $\Delta \boldsymbol{f}_\alpha$ means the change in power flow in branch α.

We also define the matrix of LODFs as $\boldsymbol{L}$ whose entry $\boldsymbol{L}_{\alpha\beta}$ means the proportion of pre-outage real power flow on branch α that is redistributed to branch β as a result of the outage of branch α, as defined in

$$\boldsymbol{L}_{\alpha\beta} \coloneqq \Delta \boldsymbol{f}_\beta / \boldsymbol{f}_\alpha, \tag{5}$$

where $\Delta \boldsymbol{f}_\beta$ means the change in the power flow of branch β while $\boldsymbol{f}_\alpha$ means the pre-outage power flow of branch α. It is worth noting that LODFs and PTDFs have the relation

$$\boldsymbol{L}_{\alpha\beta} = \left(\boldsymbol{P}_{n^{\mathrm{t}}_\alpha \beta} - \boldsymbol{P}_{n^{\mathrm{f}}_\alpha \beta}\right) / \left(1 - \boldsymbol{P}_{n^{\mathrm{t}}_\alpha \alpha} + \boldsymbol{P}_{n^{\mathrm{f}}_\alpha \alpha}\right), \tag{6}$$

where the denominator $1 - \boldsymbol{P}_{n^{\mathrm{t}}_\alpha \alpha} + \boldsymbol{P}_{n^{\mathrm{f}}_\alpha \alpha}$ is derived from the impacts of virtual power injection and withdrawal on the branch α (see Fig. 1-c).

We also define the vector of power distribution factors as $\boldsymbol{\gamma}$ whose entry $\boldsymbol{\gamma}_i$ represents the proportion of power imbalance resulting from contingencies that is accommodated by generator i, which can be used for automatic re-dispatch. For a common special case that the power imbalance is accommodated by only the slack bus, all $\boldsymbol{\gamma}$ are 0.

3.1. Power Flow Calculation after Single Component Outage

Inspired by existing LODF-based power flow calculation methods, including Davis and Overbye (2010), Guo et al. (2009), and Kaplunovich and Turitsyn (2016), we introduce a post-contingency power flow calculation method given different types of single component outage. As shown in Fig. 1, the key idea is *to exploit superposition by adding virtual loads and generators, making the system the equivalent of removing components.* In such a way, we can leverage LODFs and PTDFs rather than re-calculating full DC power flow equations that involve large-dimensional inverse matrices.

Single generator outage For an outage of generator i (Fig. 1-a), the post-contingency power flow can be calculated by

$$\Delta \boldsymbol{f} = \begin{bmatrix} \boldsymbol{P}_{n^{\mathrm{g}}_1} \\ \vdots \\ \boldsymbol{P}_{n^{\mathrm{g}}_{i-1}} \\ \boldsymbol{P}_{n^{\mathrm{g}}_i} \\ \boldsymbol{P}_{n^{\mathrm{g}}_{i+1}} \\ \vdots \\ \boldsymbol{P}_{n^{\mathrm{g}}_{N_{\mathrm{g}}}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\gamma_1 \\ \vdots \\ -\gamma_{i-1} \\ 1 \\ -\gamma_{i+1} \\ \vdots \\ -\gamma_{N_{\mathrm{g}}} \end{bmatrix} \boldsymbol{p}^{\mathrm{g}}_i, \tag{7a}$$

$$\boldsymbol{P}_{n^{\mathrm{g}}_k} = [\boldsymbol{P}_{n^{\mathrm{g}}_k 1}, \cdots, \boldsymbol{P}_{n^{\mathrm{g}}_k N_{\mathrm{E}}}], \quad 1 \le k \le N_{\mathrm{g}}, \tag{7b}$$

where $\boldsymbol{p}^{\mathrm{g}}_i$ is the power of generator i.

Single load outage Similarly, for an outage of load i (Fig. 1-b), the post-contingency power flow can be calculated by

$$\Delta \boldsymbol{f} = \begin{bmatrix} \boldsymbol{P}_{n^{\mathrm{g}}_1} \\ \vdots \\ \boldsymbol{P}_{n^{\mathrm{g}}_{N_{\mathrm{g}}}} \\ \boldsymbol{P}_{n^{\mathrm{l}}_i} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_{N_{\mathrm{g}}} \\ -1 \end{bmatrix} \boldsymbol{p}^{\mathrm{l}}_i, \tag{8a}$$

$$\boldsymbol{P}_{n^{\mathrm{g}}_k} = [\boldsymbol{P}_{n^{\mathrm{g}}_k 1}, \cdots, \boldsymbol{P}_{n^{\mathrm{g}}_k N_{\mathrm{E}}}], \quad 1 \le k \le N_{\mathrm{g}}, \tag{8b}$$

$$\boldsymbol{P}_{n^{\mathrm{l}}_i} = [\boldsymbol{P}_{n^{\mathrm{l}}_i 1}, \cdots, \boldsymbol{P}_{n^{\mathrm{l}}_i N_{\mathrm{E}}}], \tag{8c}$$

where $\boldsymbol{p}^{\mathrm{l}}_i$ is the power of load i.

Single branch outage We classify single branch outage into two types, depending on whether it leads to network separation, namely, islanding of nodes. Here, we define islanded nodes as those that lose electrical connection to node 0 (slack bus).

For branch outages, to make the target branch equivalent to an open branch using superposition, it must be guaranteed that the virtual power injection and withdrawal at ends of the target branch equal to its post-contingency power flow (see Figs. 1-c and 1-d). Therefore, the key is to estimate the amount of virtual power injection and withdrawal.

For an outage of branch α that does not result in network separation (Fig. 1-c), post-contingency power flow can be calculated in a PTDF-based form,

$$\Delta \boldsymbol{f} = \left(\boldsymbol{P}^{\mathrm{T}}_{n^{\mathrm{t}}_\alpha} - \boldsymbol{P}^{\mathrm{T}}_{n^{\mathrm{f}}_\alpha}\right) \lambda \boldsymbol{f}_\alpha, \tag{9a}$$

$$\boldsymbol{P}_{n^{\mathrm{t}}_\alpha} = [\boldsymbol{P}_{n^{\mathrm{t}}_\alpha 1}, \cdots, \boldsymbol{P}_{n^{\mathrm{t}}_\alpha N_{\mathrm{E}}}], \tag{9b}$$

$$\boldsymbol{P}_{n^{\mathrm{f}}_\alpha} = [\boldsymbol{P}_{n^{\mathrm{f}}_\alpha 1}, \cdots, \boldsymbol{P}_{n^{\mathrm{f}}_\alpha N_{\mathrm{E}}}], \tag{9c}$$

$$\lambda = 1 / \left(1 - \boldsymbol{P}_{n^{\mathrm{t}}_\alpha \alpha} + \boldsymbol{P}_{n^{\mathrm{f}}_\alpha \alpha}\right), \tag{9d}$$

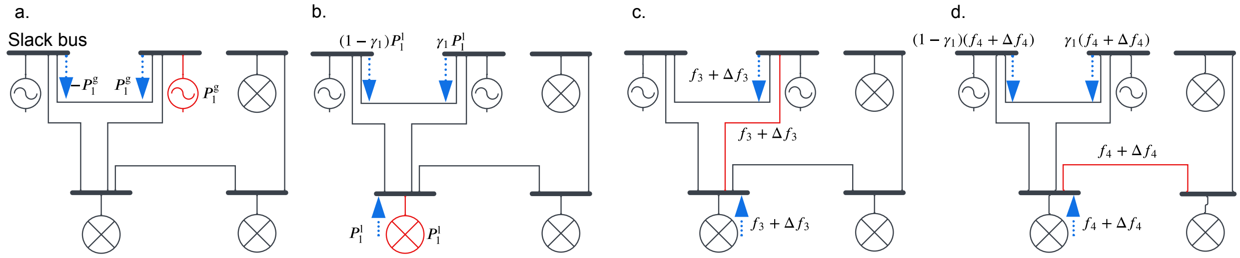

Figure 1. Conceptual diagram of how to calculate post-contingency power flow given different types of single component outage by exploiting superposition. a. Single generator outage. b. Single load outage. c. Single branch outage that does not result in islanding. d. Single branch outage that results in islanding. Here, red components represent failed components while blue arrows represent virtual power injection and withdrawal.

where the coefficient $1/\left(1-\boldsymbol{P}_{n^{\mathrm{t}}_{\alpha}\alpha}+\boldsymbol{P}_{n^{\mathrm{f}}_{\alpha}\alpha}\right)$ is derived from the impact of virtual power injection and withdrawal on the power flow of branch α.

With Eq. 6, post-contingency power flow can alternatively be calculated in an LODF-based form, which is more common as introduced in existing literature such as Davis and Overbye (2010), Guo et al. (2009), and Kaplunovich and Turitsyn (2016),

$$\Delta \boldsymbol{f} = \boldsymbol{L}_{\alpha}^{\mathrm{T}} \boldsymbol{f}_{\alpha}, \tag{10a}$$
$$\boldsymbol{L}_{\alpha} = [\boldsymbol{L}_{\alpha 1}, \cdots, \boldsymbol{L}_{\alpha N_{\mathrm{E}}}], \tag{10b}$$

where $\boldsymbol{f}_{\alpha}$ is the pre-contingency power flow of branch α.

For an outage of branch α that results in network separation (Fig. 1-d), we propose the following method to calculate post-contingency power flow. The first step is to identify islanded nodes and then find boundary branches that connect the main system and islands. For the case of single branch outage that results in islanding, branch α must be a a boundary branch, which means that it must have an islanded node and another nonislanded node that connects to the main system. The second step is to add virtual power injection at the nonislanded node, of which the amount must be equal to the power flow of branch α. The final step is to add virtual loads to generator nodes (re-disptach) to maintain supply-demand balance. Denoting the nonislanded node as $n_{\alpha}^{\mathrm{main}}$, post-contingency power flow can be calculated by

$$\Delta \boldsymbol{f} = \begin{bmatrix} \boldsymbol{P}_{n_1^{\mathrm{g}}} \\ \vdots \\ \boldsymbol{P}_{n_{N_{\mathrm{g}}}^{\mathrm{g}}} \\ \boldsymbol{P}_{n_{\alpha}^{\mathrm{main}}} \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_{N_{\mathrm{g}}} \\ -1 \end{bmatrix} \lambda \boldsymbol{f}_{\alpha}, \tag{11a}$$

$$\boldsymbol{P}_{n_k^{\mathrm{g}}} = [\boldsymbol{P}_{n_k^{\mathrm{g}}1}, \cdots, \boldsymbol{P}_{n_k^{\mathrm{g}}N_{\mathrm{E}}}],\ 1 \le k \le N_{\mathrm{g}}, \tag{11b}$$

$$\boldsymbol{P}_{n_{\alpha}^{\mathrm{main}}} = [\boldsymbol{P}_{n_{\alpha}^{\mathrm{main}}1}, \cdots, \boldsymbol{P}_{n_{\alpha}^{\mathrm{main}}N_{\mathrm{E}}}], \tag{11c}$$

$$\lambda = 1/\left(\mathbb{1}_{\alpha} - \sum_{i=1}^{N_{\mathrm{g}}} \gamma_i \boldsymbol{P}_{n_i^{\mathrm{g}}\alpha} + \boldsymbol{P}_{n_{\alpha}^{\mathrm{main}}\alpha}\right), \tag{11d}$$

$$\mathbb{1}_{\alpha} = \begin{cases} 1, & \text{if } n_{\alpha}^{\mathrm{main}} \text{ is a “from” node} \\ -1, & \text{if } n_{\alpha}^{\mathrm{main}} \text{ is a “to” node} \end{cases}, \tag{11e}$$

where $\boldsymbol{f}_{\alpha}$ is the pre-contingency power flow of branch α, and $\mathbb{1}_{\alpha}$ is a sign function that depends on whether the node of boundary branch α in the main system is a “from” or “to” node.

3.2. Power Flow Calculation after Multi-component Outage

For a multi-component outage that involves outages of multiple generators, loads, and branches, without loss of generality, we denote the indices of m_{g} failed generators by $\{i\}_{i=1}^{m_{\mathrm{g}}}$, the indices of m_{l} failed loads by $\{j\}_{j=1}^{m_{\mathrm{l}}}$, and the indices of m_{b} open branches by $\{k\}_{k=1}^{m_{\mathrm{b}}}$.

Omitting the process of identifying islanded nodes, we denote the set of the indices of islanded nodes by V_{isl} and that of non-islanded nodes by $V_{\mathrm{main}} := V \setminus V_{\mathrm{isl}}$. Open branches can be classified into three types, main-system branches Ω, boundary branches Φ, and islanded branches Ψ, as defined by

$$\Omega = \{i \in \{k\}_{k=1}^{m_\text{b}} | n_i^\text{f} \in V_\text{main},\, n_i^\text{t} \in V_\text{main}\}, \tag{12a}$$

$$\begin{aligned}\Phi = &\{i \in \{k\}_{k=1}^{m_\text{b}} | n_i^\text{f} \in V_\text{isl},\, n_i^\text{t} \in V_\text{main}\} \\ &+ \{i \in \{k\}_{k=1}^{m_\text{b}} | n_i^\text{f} \in V_\text{main},\, n_i^\text{t} \in V_\text{isl}\},\end{aligned} \tag{12b}$$

$$\Psi = \{i \in \{k\}_{k=1}^{m_\text{b}} | n_i^\text{f} \in V_\text{isl},\, n_i^\text{t} \in V_\text{isl}\}. \tag{12c}$$

Without loss of generality, we assume $\Omega = \{k\}_{k=1}^{\omega}$ and $\Phi = \{k\}_{k=m_\text{bm}+1}^{m_\text{bm}+m_\text{bb}}$. We also assume nodes of boundary branches that are in the main system are $\{k\}_{k=1}^{m_\text{bm}}$.

Intuitively, post-contingency power flow in the main system is related to only generators, loads, main-system branches, and boundary branches, except islanded branches. Therefore, the calculation of post-contingency power flow can be summarized into following three steps.

Impact of Multi-Generator Outage Based on Eq. 7, the vector $\Delta \boldsymbol{f}^\text{g}$ of power flow changes derived from multi-generator outage can be calculated by

$$\Delta \boldsymbol{f}^\text{g} = \sum_{i=1}^{m_\text{g}} \left(\begin{bmatrix} \boldsymbol{P}_{n_i^\text{g}} \\ \boldsymbol{P}_{n_{m_\text{g}+1}^\text{g}} \\ \vdots \\ \boldsymbol{P}_{n_{N_\text{g}}^\text{g}} \end{bmatrix}^\text{T} \begin{bmatrix} 1 \\ -\gamma_{m_\text{g}+1} \\ \vdots \\ -\gamma_{N_\text{g}} \end{bmatrix} \boldsymbol{p}_i^\text{g} \right). \tag{13}$$

Impact of Multi-Load Outage Based on Eq. 8, the vector $\Delta \boldsymbol{f}^\text{l}$ of power flow changes derived from multi-load outage can be calculated by

$$\Delta \boldsymbol{f}^\text{l} = \sum_{i=1}^{m_\text{l}} \left(\begin{bmatrix} \boldsymbol{P}_{n_{m_\text{g}+1}^\text{g}} \\ \vdots \\ \boldsymbol{P}_{n_{N_\text{g}}^\text{g}} \\ \boldsymbol{P}_{n_i^\text{l}} \end{bmatrix}^\text{T} \begin{bmatrix} \gamma_{m_\text{g}+1} \\ \vdots \\ \gamma_{N_\text{g}} \\ -1 \end{bmatrix} \boldsymbol{p}_i^\text{l} \right). \tag{14}$$

Impact of Multi-Branch Outage Based on Eqs. 9 and 11, the vector $\Delta \boldsymbol{f}^\text{b}$ of power flow changes derived from multi-load outage can be calculated by two steps.

The first step is to estimate the amount of power injection and withdrawal $\boldsymbol{t}^\text{b}$ that incorporates interactions between multiple outages by

$$\boldsymbol{f}' = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{f}_{\Omega_1} + \Delta \boldsymbol{f}_{\Omega_1}^\text{g} + \Delta \boldsymbol{f}_{\Omega_1}^\text{l} \\ \vdots \\ \boldsymbol{f}_{\Omega_\omega} + \Delta \boldsymbol{f}_{\Omega_\omega}^\text{g} + \Delta \boldsymbol{f}_{\Omega_\omega}^\text{l} \\ \boldsymbol{f}_{\Phi_1} + \Delta \boldsymbol{f}_{\Phi_1}^\text{g} + \Delta \boldsymbol{f}_{\Phi_1}^\text{l} \\ \vdots \\ \boldsymbol{f}_{\Phi_\phi} + \Delta \boldsymbol{f}_{\Phi_\phi}^\text{g} + \Delta \boldsymbol{f}_{\Phi_\phi}^\text{l} \end{bmatrix}^\text{T}, \tag{15a}$$

$$\boldsymbol{M}_{11} = \begin{bmatrix} 1 & \cdots & -\boldsymbol{L}_{\Omega_\omega \Omega_1} \\ \vdots & \ddots & \vdots \\ -\boldsymbol{L}_{\Omega_1 \Omega_\omega} & \cdots & 1 \end{bmatrix}, \tag{15b}$$

$$\boldsymbol{M}_{12} = \begin{bmatrix} -\boldsymbol{P}_{n_{\Phi_1}^\text{main} \Omega_1} & \cdots & -\boldsymbol{P}_{n_{\Phi_\phi}^\text{main} \Omega_1} \\ \vdots & \ddots & \vdots \\ -\boldsymbol{P}_{n_{\Phi_1}^\text{main} \Omega_\omega} & \cdots & -\boldsymbol{P}_{n_{\Phi_\phi}^\text{main} \Omega_\omega} \end{bmatrix} + \begin{bmatrix} \sum_{i=m_\text{g}+1}^{N_\text{g}} \gamma_i \boldsymbol{P}_{n_i^\text{g} \Omega_1} \\ \vdots \\ \sum_{i=m_\text{g}+1}^{N_\text{g}} \gamma_i \boldsymbol{P}_{n_i^\text{g} \Omega_\omega} \end{bmatrix}, \tag{15c}$$

$$\boldsymbol{M}_{21} = \begin{bmatrix} -\boldsymbol{L}_{1(m_\text{bm}+1)} & \cdots & -\boldsymbol{L}_{m_\text{bm}(m_\text{bm}+1)} \\ \vdots & \ddots & \vdots \\ -\boldsymbol{L}_{1(m_\text{bm}+m_\text{bb})} & \cdots & -\boldsymbol{L}_{m_\text{bm}(m_\text{bm}+m_\text{bb})} \end{bmatrix}, \tag{15d}$$

$$\boldsymbol{M}_{22} = \begin{bmatrix} \mathbb{1}_{\Phi_1} - \boldsymbol{P}_{n_{\Phi_1}^\text{main} \Phi_1} & \cdots & -\boldsymbol{P}_{n_{\Phi_\phi}^\text{main} \Phi_1} \\ \vdots & \ddots & \vdots \\ -\boldsymbol{P}_{n_{\Phi_1}^\text{main} \Phi_\phi} & \cdots & \mathbb{1}_{\Phi_\phi} - \boldsymbol{P}_{n_{\Phi_\phi}^\text{main} \Phi_\phi} \end{bmatrix} + \begin{bmatrix} \sum_{i=m_\text{g}+1}^{N_\text{g}} \gamma_i \boldsymbol{P}_{n_i^\text{g} \Phi_1} \\ \vdots \\ \sum_{i=m_\text{g}+1}^{N_\text{g}} \gamma_i \boldsymbol{P}_{n_i^\text{g} \Phi \phi} \end{bmatrix}. \tag{15e}$$

The second step is to calculate post-contingency power flow $\tilde{\boldsymbol{f}}$ by

$$\tilde{\boldsymbol{f}} = \boldsymbol{f} + \Delta \boldsymbol{f}^\text{g} + \Delta \boldsymbol{f}^\text{l} + \Delta \boldsymbol{f}^\text{bm} + \Delta \boldsymbol{f}^\text{bb}, \tag{16a}$$

$$\Delta \boldsymbol{f}^\text{bm} = \sum_{i=1}^{\omega} \boldsymbol{L}_{\Omega_i}^\text{T} \boldsymbol{f}_i', \tag{16b}$$

$$\Delta \boldsymbol{f}^\text{bb} = \sum_{i=1}^{\phi} \left(\begin{bmatrix} \boldsymbol{P}_{n_{m_\text{g}+1}^\text{g}} \\ \vdots \\ \boldsymbol{P}_{n_{N_\text{g}}^\text{g}} \\ \boldsymbol{P}_{n_{\Phi_i}^\text{main}} \end{bmatrix}^\text{T} \begin{bmatrix} \gamma_{m_\text{g}+1} \\ \vdots \\ \gamma_{N_\text{g}} \\ -1 \end{bmatrix} \boldsymbol{f}_{\omega+i}' \right). \tag{16c}$$

In summary, post-contingency power flow due to multi-component outages can be calculated by Eqs. 13-16. It is worth noting that operations on matrices and vectors are computationally efficient. Even for operations in Eqs. 13, 14, and 16c that are derived from generation re-dispatching, $\sum_{i=1}^{N_{\mathrm{g}}} \boldsymbol{P}_{n_i^{\mathrm{g}}}^{\mathrm{T}} \gamma_{n_i^{\mathrm{g}}}$ can be calculated in advance and repeatedly used because $\boldsymbol{P}$ and $\boldsymbol{\gamma}$ are constant.

4. N-2 Contingency Screening

In this section, we present an N-2 contingency screening method in Algorithm 1 based on the proposed DC power flow-based post-contingency power flow calculation, which consists of offline preprocessing, online fast selection, and online screening.[1]

4.1. Offline Preprocessing

Offline preprocessing refers to calculations that are independent of scenarios, including the calculation of the PTDF $\boldsymbol{P}$ and LODF $\boldsymbol{L}$ matrices and the identification of islanded nodes $\{\mathcal{N}_i\}_{i=1}^{N_{\mathrm{C}}}$. Specifically, for a single contingency $\mathcal{C}_i \in \{\mathcal{C}_i\}_{i=1}^{N_{\mathrm{C}}}$ that represents a set of components to be removed, we use a depth-first search to identify the resulting islanded nodes $\mathcal{N}_i$. See more details in Appendix.

4.2. Online Fast Selection

Online fast selection is inspired by the method proposed in Kaplunovich and Turitsyn (2016). We define $\overline{\xi}$ and $\underline{\xi}$ that represent how close the post-contingency power flow of each branch is to the upper/lower limits,

$$\overline{\xi}_{j,\alpha}^{i} := \Delta \boldsymbol{f}_{j,\alpha}^{\mathcal{C}_i} / |\overline{\boldsymbol{f}}_{\alpha} - \boldsymbol{f}_{\alpha}|, \tag{17a}$$

$$\underline{\xi}_{j,\alpha}^{i} := \Delta \boldsymbol{f}_{j,\alpha}^{\mathcal{C}_i} / |\overline{\boldsymbol{f}}_{\alpha} + \boldsymbol{f}_{\alpha}|, \tag{17b}$$

where $\Delta \boldsymbol{f}_{j,\alpha}^{\mathcal{C}_i}$ refers to the α_{th} entry of the corresponding single terms in the summation in Eqs. 13, 14, 16b, and 16c, which can be interpreted as the impact of j_{th} component of contingency $\mathcal{C}_i$ on the power flow of branch α, $\overline{\boldsymbol{f}}_{\alpha}$ is the thermal limit of branch α, $\boldsymbol{f}_{\alpha}$ is the pre-contingency power flow of branch α.

[1] Code is published in a Github repository that can be found at https://github.com/tamu-engineering-research/N2_CTG_screening.

We also define Γ as follows,

$$\Gamma_{i,k}^{j} := \begin{cases} \boldsymbol{f'}_k^{\mathcal{C}_i \cup \mathcal{C}_j} / \boldsymbol{f'}_k^{\mathcal{C}_i}, & \text{if } \mathcal{C}_{i,k} \text{ is branch outage,} \\ 1, & \text{else,} \end{cases} \tag{18}$$

where $\mathcal{C}_{i,k}$ means the k_{th} component of $\mathcal{C}_i$, $\Gamma_{i,k}^{j}$ means the impact of $\mathcal{C}_j$ on the $\mathcal{C}_{i,k}$, and $\boldsymbol{f'}_k^{\mathcal{C}_j}$ is the kth entry of $\boldsymbol{f'}$ in Eq. 15a.

According to Kaplunovich and Turitsyn (2016), the sufficient and necessary condition of a critical N-2 contingency $\mathcal{C}_i \cup \mathcal{C}_j$ can be converted to

$$\max_{\alpha} \left(\sum_{k=1}^{|\mathcal{C}_i|} \overline{\xi}_{k,\alpha}^{i} \Gamma_{i,k}^{j} + \sum_{k=1}^{|\mathcal{C}_j|} \overline{\xi}_{k,\alpha}^{j} \Gamma_{j,k}^{i} \right) > 1, \tag{19a}$$

$$\text{or } \max_{\alpha} \left(\sum_{k=1}^{|\mathcal{C}_i|} \underline{\xi}_{k,\alpha}^{i} \Gamma_{i,k}^{j} + \sum_{k=1}^{|\mathcal{C}_j|} \underline{\xi}_{k,\alpha}^{j} \Gamma_{j,k}^{i} \right) > 1, \tag{19b}$$

Fast selection can be achieved by the worst-case analysis that is a necessary condition of a critical N-2 contingency.

$$\sum_{k=1}^{|\mathcal{C}_i|} \max_{\alpha} \left(\overline{\xi}_{k,\alpha}^{i} \right) \Gamma_{i,k}^{j} + \sum_{k=1}^{|\mathcal{C}_j|} \max_{\alpha} \left(\overline{\xi}_{k,\alpha}^{j} \right) \Gamma_{j,k}^{i} > 1 \tag{20a}$$

$$\text{or } \sum_{k=1}^{|\mathcal{C}_i|} \max_{\alpha} \left(\underline{\xi}_{k,\alpha}^{i} \right) \Gamma_{i,k}^{j} + \sum_{k=1}^{|\mathcal{C}_j|} \max_{\alpha} \left(\underline{\xi}_{k,\alpha}^{j} \right) \Gamma_{j,k}^{i} > 1 \tag{20b}$$

As shown in Algorithm 1, we set $\boldsymbol{H}_{ij}$ to 1 if Eq. 20 holds, which is used to indicate whether to further calculate post-contingency power flow.

4.3. Online Screening

Given PTDF matrix $\boldsymbol{P}$, LODF $\boldsymbol{L}$, identified islanded nodes $\{\mathcal{N}_i\}_{i=1}^{N_{\mathrm{C}}}$, and fast selection results $\boldsymbol{H}$, we show an online screening method based on the generalized post-contingency power flow calculation method in Algorithm 1. Specifically, we use total thermal overload throughout the grid to quantify post-contingency severity as defined in Given a contingency $\mathcal{C}_i$,

$$\delta_{\overline{\boldsymbol{f}}} \left(\tilde{\boldsymbol{f}} \left(\mathcal{C}_i \right) \right) = \sum_{\alpha=1}^{N_{\mathrm{E}}} \max \left(|\tilde{\boldsymbol{f}}_{\alpha} \left(\mathcal{C}_i \right)| - \overline{\boldsymbol{f}}_{\alpha}, 0 \right), \tag{21}$$

where $\tilde{\boldsymbol{f}}_{\alpha}$ is a function of contingency $\mathcal{C}$, meaning the resulting post-contingency power flow of branch α, and $\overline{\boldsymbol{f}}_{\alpha}$ is the thermal limit of branch α.

Algorithm 1: N-2 contingency screening

Input $\overline{\boldsymbol{f}}$,
Initialize $\boldsymbol{H}$, $\boldsymbol{D}$,
Step 1 offline preprocessing
Calculate $\boldsymbol{P}$, $\boldsymbol{L}$, $\{\mathcal{N}_i\}_{i=1}^{N_\mathrm{C}}$,
Step 2 online fast selection
Calculate $\overline{\xi}$, $\underline{\xi}$ by Eq. 17
Calculate Γ by Eq.18
Calculate $\max_\alpha \overline{\xi}$, $\max_\alpha \underline{\xi}$
for $i = 1, \cdots, N_\mathrm{C} - 1$ **do**
  for $j = i+1, \cdots, N_\mathrm{C}$ **do**
    if Eq.20 holds **then**
      $\boldsymbol{H}_{ij} \longleftarrow 1$
    end if
  end for
end for
Step 3 online screening
for $i = 1, \cdots, N_\mathrm{C} - 1$ **do**
  for $j = i+1, \cdots, N_\mathrm{C}$ **do**
    if $\boldsymbol{H}_{ij} = 1$ **then**
      Create $\mathcal{C}_{ij} \longleftarrow \mathcal{C}_i \cup \mathcal{C}_j$
      Create $\mathcal{N}_{ij} \longleftarrow \mathcal{N}_i \cup \mathcal{N}_j$
      Calculate $\tilde{\boldsymbol{f}}\left(\mathcal{C}_{ij}\right)$ with $\mathcal{N}_{ij}$ by Eqs. 13-16
      Estimate $\boldsymbol{D}_{ij} \longleftarrow \delta_{\overline{\boldsymbol{f}}}\left(\tilde{\boldsymbol{f}}\right)$ by Eq. 21
    end if
  end for
end for
Return $\boldsymbol{D}$

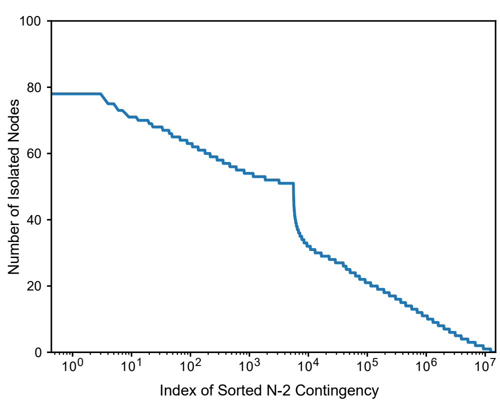

Figure 2. Number of islanded nodes of all N-2 contingencies sorted in descending order.

5. Numerical Results

In this section, we demonstrate the proposed generalized DC power flow-based contingency screening approach on the real Texas system in different scenarios. The proposed approach shows effectiveness in the identification of critical contingencies. The results of critical contingency identification in different scenarios indicate the gap between existing approaches and practical approaches and demonstrate the importance of developing a generalized contingency screening approach.

5.1. System Configuration

The proposed approach is demonstrated on the real Texas grid that consists of about 8000 nodes, 9000 branches, 1000 generators, and 7000 loads We consider four scenarios, including high load, low load, high wind generation, and low wind generation (see more details in Appendix). Specifically, the list of contingencies of interest contains about 5500 single contingencies of different types, such as multi-generator, multi-load, multi-branch outages, and their mixtures. Due to confidentiality regulations, the detailed system model is not publicly available.

5.2. Performance of Contingency Screening

We show the performance of the proposed contingency screening method in Table 1.[2] The results demonstrate that the proposed contingency screening method can identify all critical N-2 contingencies in an effective and efficient manner, leveraging the generalized post-contingency power flow calculation and fast selection methods.

First, the proposed N-2 contingency screening method can quantify post-contingency severity for all types of N-2 contingencies within two hours, even with about 50% of single contingencies that can lead to islanding (Table 2). On the contrary, all existing DC power flow-based screening methods are incapable of quantifying post-contingency severity with system islanding. Therefore, the proposed method enables the development of future fast screening algorithms towards practical applications.

Second, the online fast selection method inspired by Kaplunovich and Turitsyn (2016) can effectively filter out most non-critical N-2 contingencies that do not lead to thermal violation (Table 1), allowing for

[2] It takes 15 minutes to finsh AC power flow-based N-1 contingency enumeration by the same computer, which can serve as a reference for the comparison between different computing devices (see computational configuration in Appendix).

Table 1. **Performance of the Proposed N-2 Contingency Screening Approach under Different Scenarios**

Scenario	High load	Low load	High wind	Low wind
Time	124 min	96 min	104 min	94 min
# N-2 CTGs	15570990	15089271	15111253	15492961
# N-2 filtered (%)	1595907 (10.2%)	929808 (6.2%)	1178564 (7.8%)	873907 (5.6%)
# N-2 VLTs (%)	1495259 (9.6%)	818796 (5.4%)	1088211 (7.2%)	779738 (5.0%)

Time means the total time consumption in minutes, assuming that all scenarios-independent matrices, such as the PTDF and LODF matrices, are prepared in advance.
N-2 CTGs means the total number of N-2 contingencies.
N-2 filtered (%) means the number of remaining N-2 contingencies after fast selection and its percentage.
N-2 VLTs (%) means the number of N-2 contingencies that actually lead to thermal violations and its percentage.

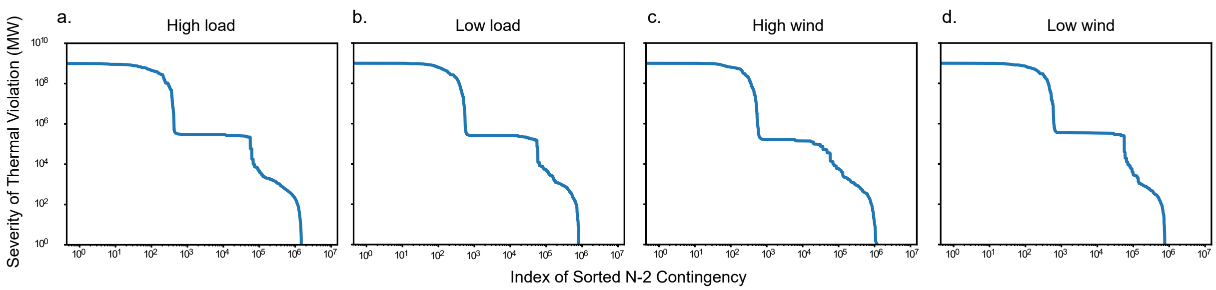

Figure 3. Severity of thermal violation of all N-2 contingencies sorted in descending order in different scenarios. a. High load. b. Low load. c. High wind. d. Low wind.

a 10 to 20-fold acceleration for the subsequent online screening step. It should be noted that the previous study Kaplunovich and Turitsyn (2016) demonstrated a 30 to 1000-fold acceleration in several IEEE case scenarios, where a single contingency is defined as a single-branch outage. The difference in empirical performance comes from the impacts of the difference between the practical and simple definition of a single contingency in two aspects.

- The share of critical N-2 multi-component contingencies (5 to 10%) in this paper is much higher than that of critical N-2 single-branch contingencies (0.1 to 0.5%) as reported in Kaplunovich and Turitsyn (2016).

- The fast screening method dedicated to single-branch contingencies becomes computationally inefficient to identify critical N-2 multi-component contingencies.

It underscores the importance of practical contingency definition and problem formulation that meet online operational needs. Therefore, it is critical for future studies to develop fast screening algorithms for practical applications based on the problem formulation and contingency screening approach introduced in this paper.

5.3. Analysis of Identified Critical N-2 Contingencies

Fig. 2 presents the number of islanded nodes of all N-2 contingencies sorted descending order, which is independent of scenarios. It shows that most N-2 contingencies result in islanding, which confirms the motivation to develop a generalized contingency screening approach compatible with system islanding.

Fig. 3 presents the estimated severity of thermal violation of all N-2 contingencies sorted in descending order in four different scenarios, which is the total thermal overloads (Eq. 21) estimated by the proposed post-contingency power flow calculation method. One key finding is that only a small fraction of N-2 contingencies cause much more severe thermal violation than others, which should be considered as the (near) most critical N-2 contingencies for further investigation. Another key finding is that the sorted severity curve consistently presents a staircase-like shape in all scenarios. This is because the most severe and near-most severe N-2 contingencies, such as the top 10,000 N-2 contingencies, are strongly related to only a few single contingencies that play a dominant role in thermal violation. These two findings can potentially provide more insights for online security analysis and decision making. For example, the problem of critical N-2 contingency identification can potentially be converted into a problem of dominant single contingency identification.

6. Concluding Remarks

In this paper, we introduce a generalized fast contingency screening approach that can identify all critical N-2 contingencies that lead to severe reliability violation in thermal limits. We formulate the practical problem of N-2 contingency analysis in a clear mathematical format. Compared to existing DC power flow-based methods, it is compatible with all types of single-component and multi-component outages, making it suitable for practical contingency analysis problems and enabling the development of practical fast contingency screening applications.

It should be noted that the proposed approach considers only thermal violations due to the inherent limitation of DC power flow-based methods. However, voltage non-linearity under different system conditions becomes increasingly common due to deepening penetration of renewables, meaning contingencies potentially cause abnormal voltage across the grid and even further lead to cascading failures. Besides, although the proposed approach is N-2 contingency analysis, it becomes intractable for a general N-k contingency analysis problem with $k \geq 3$ to be solved within a reasonable time limit. Therefore, it is desirable that future study will develop an AC power flow-based fast contingency screening approach that can practically identify most critical N-k contingencies that lead to severest reliability violations in voltage and thermal limits within a reasonable time limit. Specifically, due to its combinatorial optimization (NP-hard) nature, it is crucial to propose a reliable, efficient heuristic method that changes the existing paradigm.

References

Bienstock, D., & Verma, A. (2010). The nk problem in power grids: New models, formulations, and numerical experiments. *SIAM Journal on Optimization*, *20*(5), 2352–2380.

Biswas, R. S., Pal, A., Werho, T., & Vittal, V. (2020). A graph theoretic approach to power system vulnerability identification. *IEEE Transactions on Power Systems*, *36*(2), 923–935.

Davis, C. M., & Overbye, T. J. (2010). Multiple element contingency screening. *IEEE Transactions on Power Systems*, *26*(3), 1294–1301.

Donde, V., López, V., Lesieutre, B., Pinar, A., Yang, C., & Meza, J. (2008). Severe multiple contingency screening in electric power systems. *IEEE Transactions on Power Systems*, *23*(2), 406–417.

Eppstein, M. J., & Hines, P. D. (2012). A "random chemistry" algorithm for identifying collections of multiple contingencies that initiate cascading failure. *IEEE Transactions on Power Systems*, *27*(3), 1698–1705.

Guo, J., Fu, Y., Li, Z., & Shahidehpour, M. (2009). Direct calculation of line outage distribution factors. *IEEE Transactions on Power Systems*, *24*(3), 1633–1634.

Kaplunovich, P., & Turitsyn, K. (2016). Fast and reliable screening of N-2 contingencies. *IEEE Transactions on Power Systems*, *31*(6), 4243–4252.

Narimani, M. R., Huang, H., Umunnakwe, A., Mao, Z., Sahu, A., Zonouz, S., & Davis, K. (2021). Generalized contingency analysis based on graph theory and line outage distribution factor. *IEEE Systems Journal*.

Poudel, S., Ni, Z., & Sun, W. (2016). Electrical distance approach for searching vulnerable branches during contingencies. *IEEE Transactions on Smart Grid*, *9*(4), 3373–3382.

Appendix

Algorithm for Identifying Islanded Nodes

Here, we assume that the subgraphs of isolated nodes are of small size, namely, the diameter is bounded. Therefore, we set an upper bound $\overline{d} = 20$ based on empirical experience with the real Texas grid. As shown in Algorithm 2, we iteratively perform the depth-first search starting from one node of open branches and label all visited nodes in the same round as "islanded nodes" if the final maximal depth is less than $\overline{d}$.

It is worth noting that although we estimate islanded nodes $\mathcal{N}_{ij}$ of an N-2 contigency by $\mathcal{N}_i \cup \mathcal{N}_j$ in Algorithm 1, the actual set of islanded nodes $\mathcal{N}_{ij}$ is not necessarily equal to the union of two sets $\mathcal{N}_i \cup \mathcal{N}_j$, namely, $\mathcal{N}_i \cup \mathcal{N}_j \subseteq \mathcal{N}_{ij}$. However, $\mathcal{N}_i \cup \mathcal{N}_j = \mathcal{N}_{ij}$ empirically holds for most cases. Furthermore, if an N-2 contingency has $\mathcal{N}_i \cup \mathcal{N}_j \subset \mathcal{N}_{ij}$, $\boldsymbol{M}$ in Eq. 16 becomes noninvertible, requiring reidentification of $\mathcal{N}_{ij}$ via Algorithm 2, which nevertheless rarely happens.

Statistics of Different Scenarios and Contingencies

Table 2 shows the information of the scenarios considered in this paper that are collected from real records, including conditions with high load, low load,

Table 2. Statistics of Different Scenarios and Contingencies

Scenario	High load	Low load	High wind	Low wind
Total load (GW)	77.94	36.67	45.44	42.01
# CTG	5581	5494	5498	5567
# ISL CTG (%)	2610 (46.8%)	2615 (47.6%)	2590 (47.1%)	2740 (49.2%)
# N-2 CTGs	15570990	15089271	15111253	15492961

CTG means the total number of predefined single contingencies.
ISL CTG (%) means the number of single contingencies that lead to system islanding and its percentage.
N-2 CTGs means the total number of N-2 contingencies.

high wind generation, and low wind generation. Since the model used in this paper does not have information on the types of generation, we cannot show the active power output of wind generation.

Computation Configuration and Code Implementation

The computational environment consists of an Intel Core i7-9700 CPU, a 1 TB SSD disk, and 32 GB memory. Codes are implemented in Python that can perform simulation by interacting with PowerWorld through a package called ESA. Note that we did not use any parallel computing technique to accelerate the contingency screening process.

Algorithm 2: Identification of islanded nodes via depth-first search

Input $\{\mathcal{C}_i\}_{i=1}^{N_\mathrm{C}}, \overline{d}$
for $i = 1, \cdots, N_\mathrm{C}$ **do**
  Initialize $\mathcal{N}_i = \{\}$
  for $j \in$ open branch nodes **do**
    Initialize $d_\mathrm{max} = 0, \mathcal{N}_{ij} = \{\}$
    Set node j as the source node
    while $d_\mathrm{max} < \overline{d}$ **do**
      $k \leftarrow$ new node via depth-first search
      if k is None **then**
        Break
      end if
      $d_\mathrm{max} \leftarrow \max(d_\mathrm{max}, d(j,k))$
      $\mathcal{N}_{ij} \leftarrow \mathcal{N}_{ij} \cup \{k\}$
    end while
    if $d_\mathrm{max} < \overline{d}$ **then**
      $\mathcal{N}_i \leftarrow \mathcal{N}_i \cup \mathcal{N}_{ij}$
    end if
  end for
end for
Return $\{\mathcal{N}_i\}_{i=1}^{N_\mathrm{C}}$