Platform Coordination of the Sponsored Contents: A Game-Theoretical Analysis

Xu Zhang
School of Management, Fudan University
zhang_x21@m.fudan.edu.cn

Yifan Dou
School of Management, Fudan University
yfdou@fudan.edu.cn

Abstract

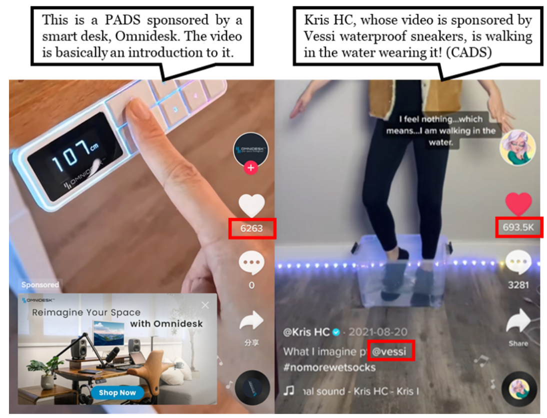
The prosperity of the content platforms (e.g., YouTube and TikTok) in the recent years has provided a novel channel of media exposure for advertisers. This paper develops a game-theoretical model to examine the role of platform advertising through both the platform itself and the content creators, i.e., the sponsored content. Interestingly, we find that the platform owner should allow both advertising channels to coexist, even they will affect the viewership negatively. We also extend our model in multiple ways, and discover that platform owner should prohibit sponsored content if the content quality would compromised with the sponsored content. We also study the roles of the creator contract and platform competition.

Keywords: Multi-sided platform, Dual mode, Creator economy, On-platform advertising, Game theory

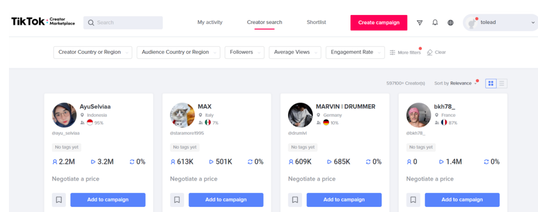
1. Introduction

Content platforms (e.g., TikTok and Bilibili) are ubiquitous these days and play an increasingly important role in the blossom of creator economy. These platforms operate in a similar fashion as the traditional multi-sided markets by connecting creators, viewers, and advertisers. However, unlike the classic setup in which the advertisers use the platform as their marketing channel, in content platforms, advertisers may leverage the popularity of content creators with sponsored contents. Therefore, we consider two ad services for content platforms: platform ads (PADS, such as traditional banner ads and pre-rolls) and creator ads (CADS), which is subject to the creator's creation process (The Economist, 2022). See Figure 1 for examples of two types of ad services.

More and more platforms are adopting the dual mode. For example, TikTok encourages CADS and operates a marketplace, but the creators must follow the platform's guidance to set their prices. Bilibili supports CADS with a much open and free trading



(a) PADS and CADS on TikTok



(b) CADS marketplace on TikTok

Figure 1: Examples of PADS and CADS.

market. Youtube once did not support CADS but recently switched lanes to allow CADS. Therefore, we are curious why the dual mode is so popular considering sponsored contents may undermine the platform's own ad revenue. In addition, we wonder why the platform does not adopt the CADS-only strategy if CADS is so efficient.

A small but booming literature has been investigating the challenges faced by the creator platforms. Hagiu and Spulber (2013) have discussed the coordination problem between the first-party investment and the third-party participation on a retailing platform given their exogenous strategic relationship. However, we find that the strategic relationship above-mentioned on the content platform is endogenously determined by

URI: https://hdl.handle.net/10125/103332
978-0-9981331-6-4

the platform. Specifically speaking, PADS and CADS may be complements, substitutes or independent from each other, depending on the platform's strategy, which is similar to the literature where mutually exclusive products are not necessarily substitutes (Mantin et al., 2014). Hagiu et al. (2020) also have done a pioneering work, but their model only assumes that consumers are homogeneous in willingness to pay (WTP), which is less relevant in the creator economy due to the great different advertisers' marketing targets and the creators' popularity. Little attention has been paid specifically to the platform context of content creation, but that's what we care about. According to our analysis, adopting the dual mode is always optimal for the platform when CADS quality is not too low. However, embedding sponsored contents may cause the distraction of creators, which may lead to the PADS-only strategy. What's more, we discuss the creator-signing issue where we find it is necessary to set the penalty for breach of contract, even though signing a contract will benefit both the platform and creators. In addition, we explain why some creators are signed while others are not, and which platform will a specific creator sign with.

2. Model

We first consider a simple model of 5 stages, which is inspired by Wauthy (1996) and Bhargava (2021), and we will expand it later. In stage 1, the platform invests in the quality of PADS, q_{P}, and decides the redistributive level, r. A positive r means that the platform will share its revenue to the creators, but a negative r means that the platform will take commissions from the creators. In stage 2, the platform and creators set their ad prices (denoted by p_{P} and p_{C}) simultaneously. In stage 3, the advertisers arrive and choose between PADS and CADS (the market shares are denoted by S_{P} and S_{C}, respectively). In stage 4, the content creators optimize the volume of content (Q) to produce. Finally, viewers consume the content while viewing the ads from both the platform and the creators. Next, we solve the model through backward induction. Notations employed in the model development are summarized in Table 1.

2.1. Stage 5: Demand from Viewers

We follow the literature to assume that $Views = \beta Q - \varepsilon A^{'}$ in which β and ε are both positive constants and $A^{'}$ is the real number of ads on the platform (Dewan et al., 2002). This equation suggests that $Views$ increases with the content offerings (i.e., more content, more views) while decreasing with ad exposures (i.e., more ads, fewer views).

Table 1: Declaration of main notations

Notations	Declaration
$q_{\mathrm{P}}, q_{\mathrm{C}}$	quality of PADS/ CADS
$p_{\mathrm{P}}, p_{\mathrm{C}}$	price of PADS/ CADS
$S_{\mathrm{P}}, S_{\mathrm{C}}$	market share of PADS/ CADS
$\pi_{\mathrm{P}}, \pi_{\mathrm{C}}$	profit of the platform/ creators
r	(platform's) redistributive level
$Views$	the number of total page views
Q	volume of content produced by creators
A	potential size of the ad market
s	(advertiser's) selective potential
α	scaling parameter for ad demand
δ, λ	(viewer's) disutility to CADS/ PADS
w	(creator's) production sensitivity
t	negative impact caused by the distraction
R	(creator's) profit discount rate
c_i	creator i's ex-ante signing cost

As a routine of the ad industry, the number of views plays a central role in determining the potential size of the ad market, which is given by $A = \mu Views$ (coefficient $\mu > 0$). It should be noted that an equilibrium is reached when $A^{'} = \phi A$ where $0 \leq \phi \leq 1$. Combining the two equations above gives $A = (\mu\beta/(1+\varepsilon\phi\mu))\,Q$, which reduces to $A = \alpha Q$ where $\alpha = \mu\beta/(1+\varepsilon\phi\mu)$. Considering that the platform and creators will take measures (whose costs are δ and λ that will be introduced later) to avoid the loss of page views, we could simply assume that $\varepsilon = 0$ for the sake of analysis, so that α becomes exogenous.

2.2. Stage 4: Creators' Content Production Decision

We follow the literature (e.g., Gupta, 2009) to assume that the creators' profit depends on three parts: 1) CADS revenue (i.e., $p_{\mathrm{C}}S_{\mathrm{C}}A$), 2) revenue sharing from the platform or the commissions paid to the platform (depending on the sign of r), and 3) the loss due to embedding more ads in their contents (in which δ represents the viewers' disutility against the creators' ads). Collectively, a representative creator's profit is given as Equation (1) below:

$$\pi_{\mathrm{C}}(Q) = p_{\mathrm{C}}S_{\mathrm{C}}A + rQ - \delta S_{\mathrm{C}}A. \tag{1}$$

We assume that the creators' content production increases with the profit per unit of content (π_{C}/Q) (Li et al., 2021). We consider a simplified linear form to characterize the overall participation of creators, $Q = w(\pi_C/Q) = w(r + \alpha(p_C - \delta) S_C)$, in which w is the sensitivity of creators' content production to the profit per content unit. We here assume that the provision of CADS will not affect the creators' content creation, but this is not realistic. In fact, providing CADS can lead to a distraction for creators that can decrease the quality of content production. We will talk about it in 4.1.

2.3. Stage 3: Advertisers' Decision

For a specific advertiser, we assume that the demand for the advertiser's product is $D = \gamma e$, where e is the marketing effort, and γ represents the inverse of the selling difficulty. We assume that the product margin is ρ and the advertiser's revenue is $u = \rho\gamma e$. We can set $\rho\gamma = 1$ — the higher the margin, the harder it is to sell, which gives $u = e$. We further assume selective potential, s, as the limit of demand such that for a specific product, the advertiser's revenue is $u = \min\{e, s\}$.

Now the advertiser faces two advertising services: the PADS with quality q_P and the CADS with quality q_C, and their prices are p_P and p_C, respectively. The advertisers' decisions are subject to individual rationality (IR) and incentive compatibility (IC) constraints. If an advertiser chooses the PADS and the PADS happens to be of superior quality (compared with the CADS), we have:

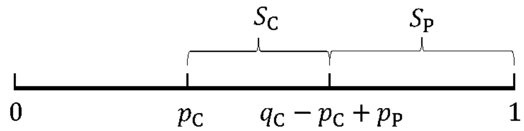
$$\min\{s, q_P\} - p_P \geq 0, \ (IR)$$

$$\min\{s, q_P\} - p_P \geq \min\{s, q_C\} - p_C. \ (IC)$$

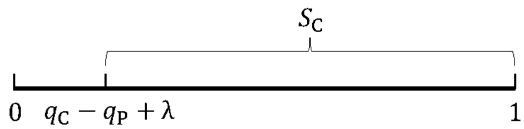
We normalize the potential size of ad market with $A = 1$. The demand functions are then $S_P = 1 - (q_C - p_C + p_P)$, $S_C = q_C - 2p_C + p_P$, as Figure 2(a) shows. The above demand functions are bound to hold only when $q_P - p_P > q_C - p_C$. We can similarly derive the demand functions in the opposite case (the PADS is of inferior quality) as Figure 2(c) shows, which are bound to hold only when $p_P < p_C$. That is to say that the PADS might be squeezed out by CADS.

2.4. Stage 2: Platform and Creators' Pricing Decision

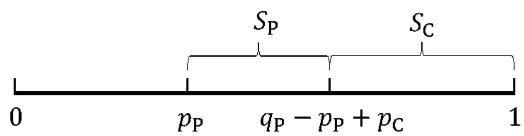
Similar to stage 4, we assume that the platform's payoff consists of three parts: 1) ads revenue from PADS (denoted by $p_P S_P A$), 2) revenue shared to creators or the commission gained from creators (depending on the sign of r again), and 3) the loss due to viewers' aversion to the PADS (denoted by $\lambda S_P A$ where λ represents



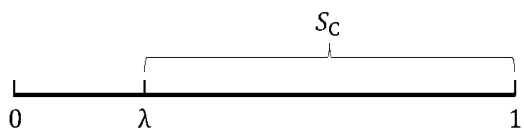
(a) Advertisers' demand ($q_P > q_C, q_P - p_P > q_C - p_C$).



(b) Advertisers' demand ($q_P > q_C, q_P - p_P < q_C - p_C$).



(c) Advertisers' demand ($q_P < q_C, p_P < p_C$).



(d) Advertisers' demand ($q_P < q_C, p_P > p_C$).

Figure 2: Advertisers' demand

the degree of viewer's disutility against PADS). The platform's profit function is given as Equation (2) below:

$$\pi_P = p_P S_P A - rQ - \lambda S_P A. \tag{2}$$

Based on Equations (1) and (2), the creators and platform choose the prices simultaneously.

The key conflict between CADS and PADS comes from their selfish pricing strategies. A natural way to resolve this conflict is to get the platform to sign up with creators. We will talk about this issue in 4.3.

2.5. Stage 1: Platform's Quality and Redistributive Level Decision

Then the platform decides the redistributive level and simultaneously chooses the investment level on PADS. In the simple model, we assume that the CADS quality is given exogenously, which is not fair and unrealistic. We will discuss the situation where the creators have the right to set their qualities of CADS at will in 4.2.

3. Analysis

We start with the definition of strategic relationships between PADS and CADS, which are determined

Table 2: Equilibrium prices of CADS and PADS $(q_{\mathrm{C}} > q_{\mathrm{P}})$

q_{P} and q_{C}	p_{P}	p_{C}
$\lambda < q_{\mathrm{P}} \leq (1+\delta+4\lambda)/6$ $\max\{q_{\mathrm{P}}, (1+\delta)/2\} < q_{\mathrm{C}} \leq 1$	q_{P}	$(1+\delta)/2$
$\lambda < q_{\mathrm{P}} \leq (1+\delta+4\lambda)/6$ $\max\{q_{\mathrm{P}}, 3q_{\mathrm{P}}-2\lambda\} < q_{\mathrm{C}} \leq (1+\delta)/2$	q_{P}	q_{C}
$\lambda < q_{\mathrm{P}} \leq (1+\delta+4\lambda)/6$ $q_{\mathrm{P}} < q_{\mathrm{C}} \leq 3q_{\mathrm{P}}-2\lambda$	$(q_{\mathrm{C}}+2\lambda)/3$	$(4q_{\mathrm{C}}+2\lambda)/3-q_{\mathrm{P}}$
$(1+\delta+4\lambda)/6 < q_{\mathrm{P}} \leq (3+3\delta-2\lambda)/4$ $q_{\mathrm{P}} < q_{\mathrm{C}} \leq (3q_{\mathrm{P}}+3+3\delta-2\lambda)/7$	$(q_{\mathrm{C}}+2\lambda)/3$	$(4q_{\mathrm{C}}+2\lambda)/3-q_{\mathrm{P}}$
$(1+\delta+4\lambda)/6 < q_{\mathrm{P}} \leq (3+3\delta-2\lambda)/4$ $\max\{q_{\mathrm{P}}, (3q_{\mathrm{P}}+3+3\delta-2\lambda)/7\} < q_{\mathrm{C}} \leq 1$	$(q_{\mathrm{P}}+1+\delta+4\lambda)/7$	$(4-3q_{\mathrm{P}}+4\delta+2\lambda)/7$
$(3+3\delta-2\lambda)/4 < q_{\mathrm{P}} \leq 1-\lambda$ $q_{\mathrm{P}} < q_{\mathrm{C}} \leq 1$	$1-q_{\mathrm{P}}+\delta$	$1-q_{\mathrm{P}}+\delta$

Table 3: Equilibrium prices of CADS and PADS $(q_{\mathrm{C}} < q_{\mathrm{P}})$

q_{C} and q_{P}	p_{C}	p_{P}
$\delta < q_{\mathrm{C}} \leq (1+\lambda+4\delta)/6$ $\max\{q_{\mathrm{C}}, (1+\lambda)/2\} < q_{\mathrm{P}} \leq 1$	q_{C}	$(1+\lambda)/2$
$\delta < q_{\mathrm{C}} \leq (1+\lambda+4\delta)/6$ $\max\{q_{\mathrm{C}}, 3q_{\mathrm{C}}-2\delta\} < q_{\mathrm{P}} \leq (1+\lambda)/2$	q_{C}	q_{P}
$\delta < q_{\mathrm{C}} \leq (1+\lambda+4\delta)/6$ $q_{\mathrm{C}} < q_{\mathrm{P}} \leq 3q_{\mathrm{C}}-2\delta$	$(q_{\mathrm{P}}+2\delta)/3$	$(4q_{\mathrm{P}}+2\delta)/3-q_{\mathrm{C}}$
$(1+\lambda+4\delta)/6 < q_{\mathrm{C}} \leq (3+3\lambda-2\delta)/4$ $q_{\mathrm{C}} < q_{\mathrm{P}} \leq (3q_{\mathrm{C}}+3+3\lambda-2\delta)/7$	$(q_{\mathrm{P}}+2\delta)/3$	$(4q_{\mathrm{P}}+2\delta)/3-q_{\mathrm{C}}$
$(1+\lambda+4\delta)/6 < q_{\mathrm{C}} \leq (3+3\lambda-2\delta)/4$ $\max\{q_{\mathrm{C}}, (3q_{\mathrm{C}}+3+3\lambda-2\delta)/7\} < q_{\mathrm{P}} \leq 1$	$(q_{\mathrm{C}}+1+\lambda+4\delta)/7$	$(4-3q_{\mathrm{C}}+4\lambda+2\delta)/7$
$(3+3\lambda-2\delta)/4 < q_{\mathrm{C}} \leq 1-\delta$ $q_{\mathrm{C}} < q_{\mathrm{P}} \leq 1$	$1-q_{\mathrm{C}}+\lambda$	$1-q_{\mathrm{C}}+\lambda$

endogenously by the platform. We then conduct the analysis through backward induction according to the 5-stages model above-mentioned.

Definition 1 *If $\partial S_{\mathrm{C}}/\partial q_P = 0$ and $\partial p_{\mathrm{C}}/\partial q_{\mathrm{P}} = 0$, we identify that PADS and CADS are independent. Otherwise, they are complements (substitutes) if $\partial S_{\mathrm{C}}/\partial q_{\mathrm{P}} \geq 0$ and $\partial p_{\mathrm{C}}/\partial q_{\mathrm{P}} \geq 0$ (respectively, if $\partial S_{\mathrm{C}}/\partial q_{\mathrm{P}} \leq 0$ and $\partial p_{\mathrm{C}}/\partial q_{\mathrm{P}} \leq 0$. Note that q_{P} is endogenously determined by the platform.*

3.1. Setting Ads Price

We benchmark our analysis with the PADS-only strategy under which the platform forbids the creators to embed ads in the content creation, and the CADS-only strategy where the platform gives up its own advertising service. When the platform and creators set prices for PADS and CADS, they usually assume that the potential size of the advertising market remains constant. We here assume that $A = 1$.

3.1.1. PADS-only Strategy and CADS-only Strategy When PADS-only strategy is adopted with price p_{P} and quality q_{P}, the demand reduces to $S_{\mathrm{P}} = 1 - p_{\mathrm{P}}$, and the platform's profit function is $\pi_{\mathrm{P}} = S_{\mathrm{P}}(p_{\mathrm{P}} - \lambda)$. When $(1+\lambda)/2 \leq q_{\mathrm{P}}$, the optimal price is $p_{\mathrm{P}}{}^* = (1+\lambda)/2$ and the equilibrium profit is then $\pi_{\mathrm{P}}^* = (1-\lambda)^2/4$. If $q_{\mathrm{P}} < (1+\lambda)/2$, we have $p_{\mathrm{P}}{}^* = q_{\mathrm{P}}$ and $\pi_{\mathrm{P}}^* = -q_{\mathrm{P}}{}^2 + q_{\mathrm{P}}(1+\lambda) - \lambda$.

When CADS-only strategy is adopted with price p_{C} and quality q_{C}, the demand reduces to $S_{\mathrm{C}} = 1 - p_{\mathrm{C}}$, and the platform's profit function is $\pi_{\mathrm{C}} = S_{\mathrm{C}}(p_{\mathrm{C}} - \delta)$. When $(1+\delta)/2 \leq q_{\mathrm{C}}$, the optimal price is $p_{\mathrm{C}}{}^* = (1+\delta)/2$ and the equilibrium profit is then $\pi_{\mathrm{C}}^* = (1-\delta)^2/4$. If $q_{\mathrm{C}} < (1+\delta)/2$, we have $p_{\mathrm{C}}{}^* = q_{\mathrm{C}}$ and $\pi_{\mathrm{C}}^* = -q_{\mathrm{C}}{}^2 + q_{\mathrm{C}}(1+\delta) - \delta$.

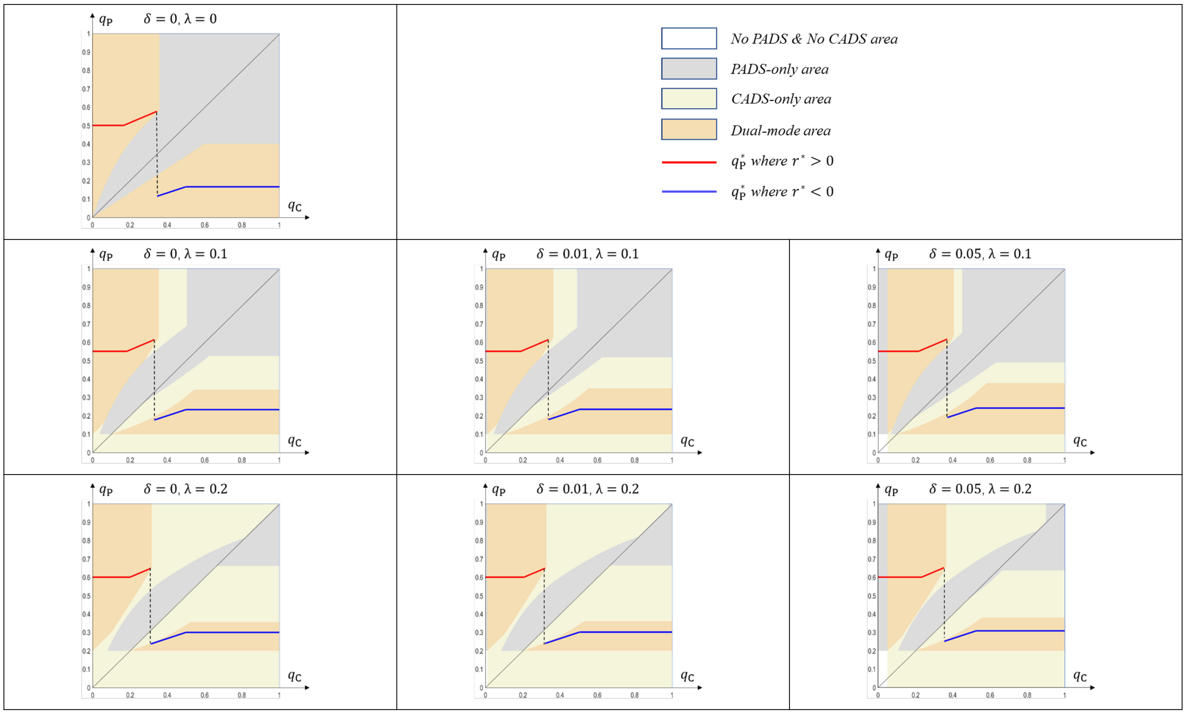


Figure 3: The adoption of CADS and PADS

3.1.2. Dual Mode First, we consider the case when the CADS is of superior quality, $q_C > q_P$. The profit functions are $\pi_C = S_C (p_C - \delta) + r/\alpha$ and $\pi_P = S_P (p_P - \lambda) - r/\alpha$, respectively. It should be noted that the demand functions may be $S_C = 1 - (q_P - p_P + p_C)$ and $S_P = (q_P - p_P + p_C) - p_P$, and the equilibrium prices are derived in Table 2. However, the demand functions may also be $S_C = 1 - \lambda$ and $S_P = 0$, where $p_C^* = \lambda$.

Similarly, we can derive the equilibrium when the PADS is of superior quality ($q_C < q_P$) in Table 3, which is symmetric to Table 2 above. In this case, the demand functions may also be $S_C = 1 - (q_C - q_P + \lambda)$ and $S_P = 0$, where $p_C^* = q_C - q_P + \lambda$.

According to the above tables and Definition 1, we have the following Proposition 1, which uncovers the strategic relationship between PADS and CADS from the platform's perspective.

Proposition 1 *PADS and CADS could be substitutes, complements, or independent from each other, depending on the qualities of PADS and CADS.*

3.2. The Optimal Quality of PADS and The Optimal Redistributive Level

The platform chooses the quality q_P and the redistributive level simultaneously. The platform is now open to three strategies: PADS-only stratgey, CADS-only strategy and the dual mode strategy. What's more, the platforms can also exit the market in extreme circumstances.

Considering that viewers might be more tolerant of CADS (e.g., die-hard fans), and sometimes even deliberately watch ads to support creators (Kim and Huh, 2021), we assume that $0 < \delta < \lambda \leq 1$.

The optimal platform strategy is given by Proposition 2. Interestingly, we find that the dual mode is almost always the optimal strategy. We illustrate the key idea in Figure 3. For a given pair of $\{q_P, q_C\}$, the warm areas represents the region in which the platform would end up allowing on-platform advertising from creators, and the deeper warm area represents the region where the dual mode should be adopted. We then optimize q_P as a function of q_C, resulting in a non-monotonic, piece-wise curve of q_P^*, which echoes the idea of Choi and Shin (1992).

Proposition 2 *The optimal q_P always falls in the dual mode area except for the case where $q_C < \delta$, suggesting*

Table 4: Decisions on revenue-sharing and commission-taking

q_C	Relationship between q_C and q_P^*	$r^* > 0$
$0 < q_C < (1/884)(621 + 657\delta - 394\lambda)$ - $(21/884)\sqrt{(225 - 62\delta + 185\delta^2 - 388\lambda - 308\delta\lambda + 348\lambda^2)}$	$q_C < q_P^*$	Yes
$(1/884)(621 + 657\delta - 394\lambda) - (21/884)$ $\sqrt{(225 - 62\delta + 185\delta^2 - 388\lambda - 308\delta\lambda + 348\lambda^2)} < q_C < 1$	$q_C > q_P^*$	No

that for all $q_C > \delta$, the platform should allow CADS, but CADS-only strategy will never be adopted.

Moving on to the optimal redistributive level. We are mainly concerned with its sign. In Figure 3, we marked the curve of q_P^* with different colors, where the red means that the r^* is positive and the blue means that the r^* is negative. We conclude the decisions on r in Table 4, where exists a cut-off point of q_C (determined by λ and δ). If q_C is smaller than the cut-off point, r^* will be positive, otherwise it will be negative. The findings are summarized in Proposition 3.

Proposition 3 *When $q_C > q_P^*$, the platform charges creators for their on-platform advertising ($r^* < 0$, the blue curve in Figure 3); Otherwise, the platform pays creators for the content contribution ($r^* > 0$, the red curve in Figure 3). What's more, the bigger the λ, the lower the value of cut-off point; the bigger the δ, the higher the value of cut-off point.*

We can get some practical implications from Proposition 3. If the platform hosts super-popular contributors, it is unwise to forbid them from uploading sponsored contents; Besides, it is not optimal, either, to start charging the grass-root creators from posting sponsored contents. Interestingly, at the end of the analysis, we find that PADS and CADS are always not substitutes on the way toward equilibrium.

4. Extensions

4.1. Compromised Content Quality

Embedding advertising services may distract creators and thus decrease the quality of creator content. In this extension, we consider another content production function:

$$Q = w\left(r + \alpha\left(p_C - \delta\right)S_C - t\left(\alpha\left(p_C - \delta\right)S_C\right)^2\right).$$

Note that w is the sensitivity of creators' content production to the profit per unit of content, and t is the strength of the negative impact caused by the distraction. To make sure that $\partial Q/\partial\left(\pi_C/Q\right) > 0$, we have $0 \le t \le 1/2$.

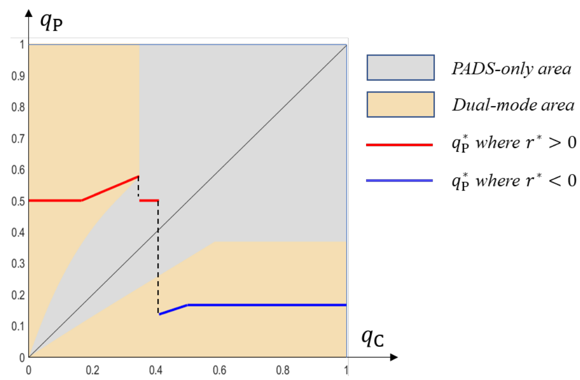


Figure 4: The adoption of PADS and CADS ($\delta = \lambda = 0$, $\alpha = 1$, $w = 1/2$, and $t = 1/2$).

Figure 4 above illustrates the optimal platform strategies in a form that is similar to Figure 3. While the changes from Figure 3 to Figure 4 seem minor, it introduces a curve segment in-between where the platform's optimal strategy is to forbid CADS. This is summarized by Proposition 4.

Proposition 4 *The platform's optimal strategy is to forbid CADS under a mediate level of q_C if the quality of creator content can be compromised by embedding ads.*

4.2. Endogenous CADS Quality

We have assumed that the quality of CADS, q_C, is given exogenously in the baseline model. However, it is unfair and unrealistic for creators to be banned from setting the quality of CADS. If we make the CADS Quality (q_C) decided by creators, Stage 1 in our analysis model should be changed. In new Stage 1, the creators and platform choose their qualities of CADS and PADS simultaneously, and then the platform sets its redistributive level. It should be noted that we assume that the quality choice is costless.

Figure 5 above shows the equilibrium of the quality pair — $\{q_P^*, q_C^*\}$, and the corresponding redistributive level, r^*. Compared with Figure 3, not all q_P^* mentioned in Figure 3 exists in equilibrium considering that the creators can decide q_C endogenously. The equilibrium

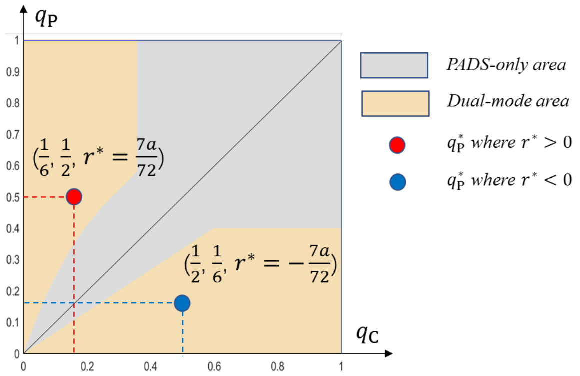


Figure 5: The equilibrium of the quality pair and the adoption PADS and CADS ($\delta = \lambda = 0$).

quality pairs are $\{q_{\mathrm{C}}^* = 1/6, q_{\mathrm{P}}^* = 1/2\}$ and $\{q_{\mathrm{C}}^* = 1/2, q_{\mathrm{P}}^* = 1/6\}$. We represent the equilibrium quality pairs in solid spots. In fact, the above two equilibrium points bring the same profits to the platform and creators, although their redistributive policies are totally different. In addition, we care about the prices in equilibrium. Interestingly, we find that $p_{\mathrm{C}}^* = q_{\mathrm{C}}^*$ and $p_{\mathrm{P}}^* = q_{\mathrm{P}}^*$ in equilibrium.

Proposition 5 *The equilibrium quality pairs always fall in the dual mode area, suggesting that for all equilibrium, the platform should adopt the dual mode considering endogenous CADS quality.*

4.3. Contracted Creators

The platform can choose to sign the contract with a specific creator to achieve the operation right of CADS, which means that the platform can set the quality and price of CADS at will. The creators can also decide whether to accept the above contract or not according to their profit functions.

4.3.1. No Contract We take the situation that the platform sign no contract with the creator as the benchmark. At this point, according to the discussion in 4.2, we need to discuss two cases separately where PADS is of superior quality or CADS is of superior quality. When PADS is of superior quality, we have $p_{\mathrm{P}}^* = q_{\mathrm{P}}^* = 1/2$, and $p_{\mathrm{C}}^* = q_{\mathrm{C}}^* = 1/6$, where $r^* = 7\alpha/72$. It is easy to get that $\pi_{\mathrm{P0}} = \pi_{\mathrm{C0}} = w(11\alpha/72)^2$. Similarly, we can also get the same profits when CADS is of superior quality.

4.3.2. Signing a Contract If the platform and the creator agree to sign the above contract, their profit functions are expected to change as follows, $\pi_{\mathrm{Ps}} = (p_{\mathrm{P}}S_{\mathrm{P}} + p_{\mathrm{C}}S_{\mathrm{C}})\,A - rQ$ and $\pi_{\mathrm{Cs}} = rQ$. If PADS is of superior quality, we have $q_{\mathrm{P}}^* = p_{\mathrm{P}}^* = 3/4$ and $q_{\mathrm{C}}^* = p_{\mathrm{C}}^* = 1/2$, where $r^* = 5\alpha/32$. It is easy to get that $\pi_{\mathrm{Ps}} = \pi_{\mathrm{Cs}} = w(5\alpha/32)^2$. Similarly, we can also get the same profits when CADS is of superior quality. So far, we have found that for a single-period game, both the platform and the creator's profits have been improved after signing the contract. However, the contract might not be self-enforcing, and it is necessary to discuss the motives of the creator for unilateral breach of contract.

4.3.3. Breach of Contract We are concerned about the unilateral breach of contract by the creator. When PADS is of superior quality, the platform set that $q_{\mathrm{P}}^* = p_{\mathrm{P}}^* = 3/4$ and $r^* = 5\alpha/32$ according to the contract. If the creator decide to breach, they will provide CADS with a quality of $3/4$ and give it a price of $1/2$, to get a profit of $\pi_{\mathrm{Cd}} = w(13\alpha/32)^2$. When CADS is of superior quality, we can similarly get that the same profit. At this point, the creator monopolize the advertising market and defraud the first phase of redistribution from the platform.

Consider an infinite game, and make the creator's profit discount rate as R. The conditions to make sure the contract is self-enforcing can be written as (Bull, 1987):

$$\pi_{\mathrm{Cs}} \geq 0, \ \ (IR)$$

$$(\pi_{\mathrm{Cs}} - \pi_{\mathrm{C0}})\,/(1 - R) \geq \pi_{\mathrm{Cd}} - \pi_{\mathrm{C0}}. \ (IC)$$

By solving the above inequalities, to make sure that the contract is self-enforcing, $\mathrm{R} \geq 0.992$ is necessary whether PADS is of superior quality or not.

Proposition 6 *Although signing a contract can always bring greater profits to both parties, the creator may still unilaterally breach the contract. When the discount rate is not high enough, it is necessary to set the penalty for breach of contract.*

4.4. Platform Competition

The above discussions are based on the assumption that a single platform monopolizes the market. However, platform competitors exist widely in reality, and we are concerned about the platform's strategy of signing contracts with creators after introducing competitors. If the platform signs a contract with a specific creator, the signed creator can only upload content on this platform, and the price and quality of CADS are determined by the platform, too. We first assume that signing the contract will bring the platform a fixed ex-ante cost, c. Then we can write the game as Table 5 shows.

Table 5: The simultaneous game between platform 1 and 2 ($c_1 > c_2$)

		Platform 2	
		Sign	Not Sign
Platform 1	Sign	$(-c_1, c_1 - c_2)$	$\left(w\,(5\alpha/32)^2 - c_1, 0\right)$
	Not Sign	$\left(0, w\,(5\alpha/32)^2 - c_2\right)$	$\left(w\,(11\alpha/72)^2/2, w\,(11\alpha/72)^2/2\right)$

In the game above, if both platforms choose not to sign with the creator, then the creator will upload the same content on both platforms. Therefore, for either platform, the creator's content becomes less attractive to advertisers, and each platform will receive half of the monopoly profit in equilibrium. If one platform opts to sign with the creator and the other opts not to, the signed platform will get a monopoly profit minus the fixed ex-ante signing cost, and the unsigned platform will get nothing. If both platforms decide to sign the contract with the creator, they will start a Bertrand competition. As a result, the platform with the cost advantage monopolizes the market and gains profit equal to its cost advantage, while the platform with the cost disadvantage loses the fixed ex-ante signing cost.

When $c_1 > c_2$, it is easy to get that there's a $\overline{c_2} = 1057\alpha^2 w/82944$ making {platform 1 does not sign, platform 2 signs} the only Nash equilibrium if $c_2 \leq \overline{c_2}$. If $c_2 > \overline{c_2}$, {platform 1 does not sign, platform 2 does not sign} is the only Nash equilibrium. Similarly, when $c_1 < c_2$, there is also a $\overline{c_1} = 1057\alpha^2 w/8294444$ that makes {platform 1 signs, platform 2 does not sign} the only Nash equilibrium if $c_1 \leq \overline{c_1}$. If $c_1 > \overline{c_1}$, {platform 1 does not sign, platform 2 does not sign} is the only Nash equilibrium.

Proposition 7 *The platform with the cost advantage will sign with the creator. However, when the signing costs are too high relative to $\alpha^2 w$, neither platform will sign the creator (the proof is omitted due to page limit).*

5. Conclusion

Allowing content creators to provide ad service might seem detrimental to the platform's profit for content platforms. However, our paper suggests that the optimality is much less intuitive and depends on the qualities of PADS and CADS. Our analytical results suggest that the dual mode is almost always the optimal strategy for the content platform, since PADS and CADS are not substitutes on the way toward equilibrium. However, PADS-only strategy might be dominant when the CADS is of mediate quality considering embedding sponsored ads may distract creators and thus decrease the content quality. These findings shed light on the growing industry of content platforms.

References

Bhargava, H. K. (2021). The creator economy: Managing ecosystem supply, revenue sharing, and platform design. *Management Science*.

Bull, C. (1987). The existence of self-enforcing implicit contracts. *The Quarterly Journal of Economics*, *102*(1), 147–159.

Choi, C. J., & Shin, H. S. (1992). A comment on a model of vertical product differentiation. *The Journal of Industrial Economics*, 229–231.

Dewan, R., Freimer, M., & Zhang, J. (2002). Managing web sites for profitability: Balancing content and advertising. *Proceedings of the 35th Annual Hawaii International Conference on System Sciences*, 2340–2347.

Gupta, S. (2009). Customer-based valuation. *Journal of Interactive Marketing*, *23*(2), 169–178.

Hagiu, A., & Spulber, D. (2013). First-party content and coordination in two-sided markets. *Management Science*, *59*(4), 933–949.

Hagiu, A., Teh, T.-H., & Wright, J. (2020). Should platforms be allowed to sell on their own marketplaces? *Available at SSRN 3606055*.

Kim, E., & Huh, J. (2021). "i'm watching this ad so you can make more money that you deserve!": Voluntary ad-viewing on youtube. *American Academy of Advertising. Conference. Proceedings (Online)*, 8–8.

Li, X., Shi, M., & Zhao, C. Y. (2021). Incentivizing mass creativity: An empirical study of the online publishing market. *Available at SSRN 3842153*.

Mantin, B., Krishnan, H., & Dhar, T. (2014). The strategic role of third-party marketplaces in retailing. *Production and Operations Management*, *23*(11), 1937–1949.

The Economist. (2022). Schumpeter: Creative seduction. *The Economist*, (JANUARY, 15TH–21ST), 57.

Wauthy, X. (1996). Quality choice in models of vertical differentiation. *The Journal of Industrial Economics*, *44*(3), 345–353.

A. Appendix

A.1. Proof of Proposition 1

Given that $0 < \delta < \lambda \leq 1$ and $A = 1$, the platform and creators firstly set their prices. When $q_{\mathrm{C}} > q_{\mathrm{P}}$, according to profit functions, we get:

$$p_{\mathrm{C}}{}^* = \min\left\{(1 - q_{\mathrm{P}} + p_{\mathrm{P}} + \delta)/2, q_{\mathrm{C}} - q_{\mathrm{P}} + p_{\mathrm{P}}\right\},$$

$$p_{\mathrm{P}}{}^* = \min\left\{(q_{\mathrm{P}} + p_{\mathrm{C}} + 2\lambda)/4, q_{\mathrm{P}}, p_{\mathrm{C}}\right\}.$$

Thus, there are six situations to be considered. 1) $p_{\mathrm{C}}{}^* = (1 - q_{\mathrm{P}} + p_{\mathrm{P}} + \delta)/2$ and $p_{\mathrm{P}}{}^* = (q_{\mathrm{P}} + p_{\mathrm{C}} + 2\lambda)/4$. In this situation, by solving the above equations, we can get that $p_{\mathrm{P}}{}^* = (q_{\mathrm{P}} + 1 + \delta + 4\lambda)/7$ and $p_{\mathrm{C}}{}^* = (4 - 3q_{\mathrm{P}} + 4\delta + 2\lambda)/7$. To make this situation hold, we have to make sure that $(q_{\mathrm{P}} + p_{\mathrm{C}} + 2\lambda)/4 \leq q_{\mathrm{P}}$, $(q_{\mathrm{P}} + p_{\mathrm{C}} + 2\lambda)/4 \leq p_{\mathrm{C}}$ and $(1 - q_{\mathrm{P}} + p_{\mathrm{P}} + \delta)/2 \leq q_{\mathrm{C}} - q_{\mathrm{P}} + p_{\mathrm{P}}$, which are equal to $(1 + \delta + 4\lambda)/6 \leq q_{\mathrm{P}} \leq (3 + 3\delta - 2\lambda)/4$ and $(3q_{\mathrm{P}} + 3 + 3\delta - 2\lambda)/7 \leq q_{\mathrm{C}} \leq 1$.

Similarly, we can derive the equilibrium prices and corresponding conditions for the following situations: 2) $p_{\mathrm{C}}{}^* = (1 - q_{\mathrm{P}} + p_{\mathrm{P}} + \delta)/2$ and $p_{\mathrm{P}}{}^* = q_{\mathrm{P}}$; 3) $p_{\mathrm{C}}{}^* = (1 - q_{\mathrm{P}} + p_{\mathrm{P}} + \delta)/2$ and $p_{\mathrm{P}}{}^* = p_{\mathrm{C}}$; 4) $p_{\mathrm{C}}{}^* = q_{\mathrm{C}} - q_{\mathrm{P}} + p_{\mathrm{P}}$ and $p_{\mathrm{P}}{}^* = (q_{\mathrm{P}} + p_{\mathrm{C}} + 2\lambda)/4$; 5) $p_{\mathrm{C}}{}^* = q_{\mathrm{C}} - q_{\mathrm{P}} + p_{\mathrm{P}}$ and $p_{\mathrm{P}}{}^* = q_{\mathrm{P}}$; 6) $p_{\mathrm{C}}{}^* = q_{\mathrm{C}} - q_{\mathrm{P}} + p_{\mathrm{P}}$ and $p_{\mathrm{P}}{}^* = p_{\mathrm{C}}$.

Based on the equilibrium and Definition 1, the strategic relationship between PADS and CADS can be easily obtained.

A.2. Proof of Proposition 2

We take the situation where $\delta = \lambda = 0$ as an example. When the dual mode is adopted, given that $1/2 < q_{\mathrm{C}}$, we can easily get $q_{\mathrm{P}}^* = 1/6$. If $(1/22)(20 - 7sqrt(3)) < q_{\mathrm{C}} \leq 1/2$, we have $q_{\mathrm{P}}^* = q_{\mathrm{C}}/3$.

Given that $0 < q_{\mathrm{C}} \leq (1/22)(20 - 7sqrt(3))$, we find that there might be two possible values of q_{P}^*, so we have to compare them to find which one is dominating. The lower q_{P}^* is $q_{\mathrm{C}}/3$, and the corresponding profit can be solved as

$$\pi_{\mathrm{PL}}^* = (1/324)\,\alpha^2 w\left(q_{\mathrm{C}}\left(-9 + 7q_{\mathrm{C}}\right)\right)^2.$$

The higher q_{P}^* has two possible values. If $0 < q_{\mathrm{C}} \leq 1/6$, the higher q_{P}^* is 1/2, and the profit now is solved as

$$\pi_{\mathrm{PH}}^* = (1/64)\,\alpha^2 w\left(-4q_{\mathrm{C}}{}^2 + 1 + 2q_{\mathrm{C}}\right)^2.$$

When $1/6 < q_{\mathrm{C}} \leq (1/22)(20 - 7sqrt(3))$, the higher q_{P}^* is $(3q_{\mathrm{C}} + 3)/7$, and the profit now can be represented as

$$\pi_{\mathrm{PH}}^* = \left(\alpha^2 w\left(18 + 11q_{\mathrm{C}}{}^2 + q_{\mathrm{C}}(-20)\right)^2\right)/9604\,.$$

We find that $\pi_{\mathrm{PH}}^* > \pi_{\mathrm{PL}}^*$ holds most of the time, unless $9/26 < q_{\mathrm{C}} \leq (1/22)(20 - 7sqrt(3))$, which is a very small interval.

When the dual mode is not adopted, $q_{\mathrm{P}}^* = 1/2$, and the profit at this time is $\pi_{\mathrm{b}}^* = \alpha^2 w/64$, which is always smaller than the profit with q_{P}^* when the dual mode is adopted.

When $0 < \delta < \lambda \leq 1$, the proof idea is similar to the above. However, there are two main differences to note. First, the introduction of λ and δ changes the price equilibrium and the quality equilibrium. Given that $0 < q_{\mathrm{C}} \leq (1 + \lambda + 4\delta)/6$, we have $q_{\mathrm{P}}^* = (1 + \lambda)/2$.

We have $q_{\mathrm{P}}^* = (3 + 3\lambda - 2\delta + 3q_{\mathrm{C}})/7$, if $(1 + \lambda + 4\delta)/6 < q_{\mathrm{C}} \leq (1/884)(621 + 657\delta - 394\lambda) - (21/884)\sqrt{(225 - 62\delta + 185\delta^2 - 388\lambda - 308\delta\lambda + 348\lambda^2)}$.

If $(1/884)(621 + 657\delta - 394\lambda) - (21/884)\sqrt{(225 - 62\delta + 185\delta^2 - 388\lambda - 308\delta\lambda + 348\lambda^2)} < q_{\mathrm{C}} \leq (1 + \delta)/2$, we have $q_{\mathrm{P}}^* = (q_{\mathrm{C}} + 2\lambda)/3$.

When $(1 + \delta)/2 < q_{\mathrm{C}} \leq 1$, we have $q_{\mathrm{P}}^* = (1 + \delta + 4\lambda)/6$.

Second, the PADS could be squeezed out of the market by CADS, so we have to discuss whether creators decide to monopolize the market or not. We first calculate the creators' profit (π_{CM}^*) when they decide to monopolize the market, and then we compare it with the creators' profit (π_{C}^*) when they decide to share the market with the platform. We find that $\pi_{\mathrm{CM}}^* > \pi_{\mathrm{C}}^*$ holds only when both q_{P} and q_{C} are small. In addition, it should be noted that the platform will not provide PADS when $q_{\mathrm{P}} < \lambda$ according to its IR constraint, and the creators will not provide CADS when $q_{\mathrm{C}} < \delta$ for the same reason.

A.3. Proof of Proposition 3

We take the situation where $\delta = \lambda = 0$ as an example. To optimize the redistributive level, the platform faces a tradeoff between a larger share of the platform and a larger potential size of the whole advertising market. We first write the platform's profit function, $\pi_{\mathrm{P}} = p_{\mathrm{P}} S_{\mathrm{P}} A - rQ - \lambda S_{\mathrm{P}} A$, where $A = \alpha Q = \alpha w\left(r + \alpha\left(p_{\mathrm{C}} - \delta\right) S_{\mathrm{C}}\right)$.

When there is only PADS on the platform, and $1/2 < q_{\mathrm{P}}$, we have $\pi_{\mathrm{P}}^* = (1/4)\,rw\,(\alpha - 4r)$. It is easy to get that $r^* = \alpha/8$, which is positive. And when $q_{\mathrm{P}} \leq 1/2$, we have $\pi_{\mathrm{P}}^* = rw\left(-r - \alpha\left(q_{\mathrm{P}} - 1\right) q_{\mathrm{P}}\right)$, and $r^* = -(1/2)\,\alpha\left(-1 + q_{\mathrm{P}}\right) q_{\mathrm{P}}$ that is positive, too.

When the dual mode is adopted, and $q_{\mathrm{C}} > q_{\mathrm{P}}$, there are four situations to be considered. 1) When $0 < q_{\mathrm{P}} \leq 1/6$, and $1/2 < q_{\mathrm{C}} \leq 1$, we have $\pi_{\mathrm{P}}^* = -(1/8)(\alpha + 4r)w(2r + \alpha(-1 + 2q_{\mathrm{P}})q_{\mathrm{P}})$. Then we can get that $r^* = (1/8)\left(-\alpha + 2\alpha q_{\mathrm{P}} - 4\alpha q_{\mathrm{P}}{}^2\right)$, which is always negative.

Similarly, we can achieve the equilibrium redistributive level for the following situations: 2) When $0 < q_{\mathrm{P}} \leq 1/6$, and $3q_{\mathrm{P}} < q_{\mathrm{C}} \leq 1/2$, we have $r^* = (1/2)\left(-\alpha q_{\mathrm{C}} + \alpha q_{\mathrm{C}}{}^2 + \alpha q_{\mathrm{C}} q_{\mathrm{P}} - \alpha q_{\mathrm{P}}{}^2\right)$, which is always negative; 3) If $1/6 < q_{\mathrm{P}} \leq 3/4$, and $(3q_{\mathrm{P}} + 3)/7 < q_{\mathrm{C}} \leq 1$, we can solve the profit function to get that $r^* = -(1/14)\alpha\left(2 - 4q_{\mathrm{P}} + q_{\mathrm{P}}{}^2\right)$, which is always negative; 4) When $0 < q_{\mathrm{P}} \leq 3/4$, and $q_{\mathrm{P}} < q_{\mathrm{C}} \leq \min\{3q_{\mathrm{P}}, (3q_{\mathrm{P}} + 3)/7\}$, we have that $r^* = (1/18)\left(-12\alpha q_{\mathrm{C}} + 14\alpha q_{\mathrm{C}}{}^2 + 9\alpha q_{\mathrm{P}} - 9\alpha q_{\mathrm{C}} q_{\mathrm{P}}\right)$, which is always negative.

From the calculation mentioned above, we find that if there is only PADS on the platform, $r^* > 0$ is always true; If the dual mode is adopted and $q_{\mathrm{C}} > q_{\mathrm{P}}$, $r^* < 0$ is always true. Similarly, we can calculate the r^* when the dual mode is adopted and $q_{\mathrm{C}} < q_{\mathrm{P}}$, and we find $r^* > 0$ is always true at that time.

A.4. Proof of Proposition 4

Except for different production functions, the proof idea in this part is consistent with Proposition 2. We first calculate the q_{P}^* and the corresponding platform profit (π_{P}^*) when the dual mode is adopted, and then we compare it with the platform profit when the dual mode is not adopted. It should be noted that if $0.348 < q_{\mathrm{C}} < 0.410$, $\pi_{\mathrm{b}}^* > \pi_{\mathrm{P}}^*$. That is to say, CADS should be forbidden by the platform at this point.

A.5. Proof of Proposition 5

Considering that the platform can adjust its redistributive level at will, the optimal PADS quality of the ecosystem is optimal for the platform and the creators. Because the equilibrium prices of PADS and CADS are symmetric, the expressions of q_{P}^* and q_{C}^* should be also symmetric. Therefore, we can draw the non-monotonic, piece-wise curve of q_{C}^* according to the curve of q_{P}^* above-mentioned. The intersections of the two curves are the equilibrium points.

A.6. Proof of Proposition 6

When there's no contract between the platform and creator, the equilibrium in Proposition 5 will be reached. When the platform and the creator agree to sign the contract above-mentioned, the profit functions are expected to change. We can get the corresponding equilibrium by solving the first-order conditions of the new profit functions with respect to price, quality, etc. If the creator considers breaching the contract, he/she should optimize the price and quality of CADS according to the given information about the platform's price, quality, and redistributive level in the contract.

Considering an infinite game, if the creator decides not to breach the contract, his/her discounted profit can be expressed as (where n is an infinitely large natural number): $V = \pi_{\mathrm{Cs}} + \pi_{\mathrm{Cs}}\mathrm{R} + \ldots + \pi_{\mathrm{Cs}}\mathrm{R}^{n-1} = \pi_{\mathrm{Cs}}/(1 - \mathrm{R})$.

If the creator decides to breach the contract, his/her discounted profit can be expressed as: $V_{\mathrm{d}} = \pi_{\mathrm{Cd}} + \pi_{\mathrm{C0}}\mathrm{R} + \ldots + \pi_{\mathrm{C0}}\mathrm{R}^{n-1} = \pi_{\mathrm{C0}}/(1 - \mathrm{R}) - \pi_{\mathrm{C0}} + \pi_{\mathrm{Cd}}$.

To make sure that the contract is self-enforcing, we have to make that $V \geq 0$ and $V \geq V_{\mathrm{d}}$, which leads to the IR and IC constraints in the text.