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Full Length Research Article

MODELING SUBSIDY TRANSFER IN COOPERATIVE ADVERTISING USING STACKELBERG GAME THEORY

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ABSTRACT
Dynamic game theoretic model approach stands out as a choice tool for considering subsidy transfer in cooperative advertising. In spite of the benefits of static models they are not known to have been used to study subsidy transfer. This work studies cooperative advertising subsidy transfer in a three-level manufacturer-distributor-retailer supply channel using Stackelberg static game. The retailer is directly involved in local advertising, while the manufacturer indirectly participates in retail advertising by providing subsidy to the retailer through the distributor. The work models the demand function using the effect of advertising on demand, and models the payoff using a revenue-expenditure formula. It considers four channel structures, and obtains the optimal advertising effort, the optimal participation rates, and the payoffs for each scenario. The work observes that the payoffs are large with distributor's intervention subsidy, but best with subsidy transfer. They are worst with non-provision and non-transfer of subsidy. Thus, the supply channel members should prioritize the distributor's participation in retail advertising either through subsidy transfer or intervention.

Keywords: cooperative advertising; supply channel; Stackelberg game; subsidy transfer; static game; intervention subsidy.

INTRODUCTION
Traditionally, cooperative advertising is an advertising agreement between a manufacturer and a retailer in which the manufacturer takes responsibility of a certain fraction of retail advertising cost. This definition was extended by Ezimadu (2016) to a relationship between a manufacturer, a distributor, and a retailer. Thus, shifting the concept of cooperative advertising from the traditional manufacturer-retailer model setting to a manufacturer-distributor-retailer setting necessitated by the fact that despite the efforts (resulting from internet activities, mass media and the likes) to bypass the middleman to deal directly with retailers and even consumers, there are still quite a number of situations and deals that cannot be sealed without him (the middleman). Furthermore, the scarcity of three level channel static cooperative advertising models makes this work necessary.
According to Jorgensen and Zaccour (2014), the work by Lyon (1932) was the first to consider the concept of cooperative advertising. However, this work did not involve a mathematical model. Mathematical cooperative advertising models are considered to have originated from Berger (1972). He defined cooperative advertising as a manufacturer's price discount to the retailer. His static model was foundational to the mathematics of cooperative advertising. This model was followed by a number of works in the cooperative advertising literature. For instance, Dant and Berger (1996) extended Berger (1972) to study cooperative advertising in a franchise where product demand is uncertain. In a study of the concept of cooperative advertising participation in conventional supply channels Bergen and John (1997) considered advertising spillovers, differentiation among retailers and differentiation among manufacturers on advertising participation rate. Considering channel coordination and pricing in cooperative advertising, Xie and Wei (2009) developed and compared a non-cooperative and a cooperative static game. Another comparison of cooperative and non-cooperative game was considered by Aust and Buscher (2014). They considered a one manufacturer-two retailers Stackelberg model. In view of increasing power and influence of retailers, and competition among manufacturers, He *et al.* (2013) considered a two-manufacturer single-retailer channel using three scenarios. To analyse, and provide insight on how the upstream and downstream players are related and better advice managers, Chen (2015) examined how price and cooperative advertising affects a dual-channel. Ezimadu (2019a) considered channel coordination in a manufacturer-retailer cooperative advertising setting in which the manufacturer has the freedom of switching between advertising subsidy and his price margin to influence retail price. Using Stackelberg game theory, Zhang *et al.* (2020) dealt with how channel power and information structures optimally decide the channel members' investment in advertising and marginal returns. Considering the idea of buying product online and eventually picking it up in the store, Li (2020) investigated how cooperative advertising can be effectively employed in a situation where there is cooperation between the manufacturer and the retailer.
Jorgensen *et al.* (2000) was probably the first to dynamically model cooperative advertising. Other dynamic models include Chutani and Sethi (2012), Chutani and Sethi (2018), Ezimadu and Nwozo (2019), Ezimadu (2020), Cao *et al.* (2020), Kennedy *et al.* (2021). These models usually employ differential games. The idea of incorporating uncertainty into cooperative advertising models through differential game was first considered by He *et al*. (2009). A special modification of He *et al.* (2009) was done by Ezimadu and Nwozo (2018) in a consideration of the involvement of both the manufacturer and the retailer in national and local advertising respectively, with the manufacturer also subsidizing retail advertising.
Unlike its classical definition involving only a manufacturer and a retailer, where the manufacturer participates in advertising by providing subsidy to the retailer, Ezimadu (2016) took the definition of cooperative advertising further by considering for the first time a manufacturer-distributor-retailer supply channel where the manufacturer sells to the distributor who in turn sells to the consumers. He considered a Stackelberg differential game model in which both the manufacturer who is the channel leader and the distributor who is the channel first follower participate in advertising by providing subsidy to the retailer. Ezimadu (2020) considered a situation where the manufacturer bypasses the distributor to

subsidise retail advertising, thus bringing into question whether it is feasible for the distributor to be an integral part of a cooperative advertising supply channel. This was addressed by Ezimadu (2019b) in a static game-theoretic setting.
We model a manufacturer-distributor-retailer bilateral monopolistic market setting with the distributor as the middleman, positioned to transfer the manufacturer's provided advertising subsidy to the retailer in a static setting. Thus, we consider the manufacturer as the channel leader, the distributor as first follower, and the retailer as second/last follower.
This uses a static model setting to compare individual supply channel members' performances and the channel performance when there is the possibility of subsidy transfer. Further, it provides insight into interactions between price margins, participation (subsidy) rates and payoffs in cooperative advertising. To achieve this, we consider four equilibrium situations which include a situation where:

a) there is no advertising support from neither the distributor nor the manufacturer;
b) the distributor does not release the advertising subsidy provided by the manufacturer for retail advertising;
c) the distributor provides advertising support to the retailer in the absence of subsidy from the manufacturer;
d) the distributor releases the manufacturer's provided subsidy to the retailer.

MATERIALS AND METHODS

Model Formulation

The retailer is directly involved in advertising while the manufacturer indirectly participates in advertising by providing subsidy. The distributor engages in transferring the provided subsidy to the retailer, and may provide intervention subsidy in the absence of subsidy from the manufacturer. The retailer's decision variable is his advertising effort φ_R, the manufacturer's decision variable is his subsidy rate ψ_M, while the distributor's decision variable is his participation rate ψ_D.
Advertising is characterised by diminishing returns resulting from saturation. To incorporate this fact into our model we appeal to a version of the concave function employed by Xie and Wei (2009):

$$D(\varphi_R) = \rho\varphi_R^{\frac{1}{2}}, \tag{1}$$

where the parameter ρ is the retail advertising effectiveness, representing the effect of retail advertising on demand. Similar concave functions were employed by Kim and Stealin (1999) and Karray and Zaccour (2006).
We observe that

$$Profit = Price\ Margin\ \times Demand - Expenditure \tag{2}$$

Thus, from (1) and (2) we have that the retailer's payoff (profit) is

$$\Pi_R = M_R D(\varphi_R) - (\varphi_R - \psi_D\varphi_R)$$
$$= M_R\rho\varphi_R^{\frac{1}{2}} - (1-\psi_D)\varphi_R, \tag{3}$$

where M_R is the retail price margin and $(1-\psi_D)\varphi_R$ is the retailer's expenditure. Similarly the distributor's profit is

$$\Pi_D = M_D D(\varphi_R) - (\psi_D - \psi_M)\varphi_R$$
$$= M_D\rho\varphi_R^{\frac{1}{2}} - (\psi_D - \psi_M)\varphi_R, \tag{4}$$

where M_D is the distributor's price margin, and $(\psi_D - \psi_M)\varphi_R$ represents his expenditure. Also, the manufacturer's profit is

$$\Pi_M = M_M D(\varphi_R) - \psi_M\varphi_R$$
$$= M_M\alpha\rho\varphi_R^{\frac{1}{2}} - \psi_M\varphi_R, \tag{5}$$

where M_M is the manufacturer's price margin, and $\psi_M\varphi_R$ is his expenditure.

RESULTS

The Players' Strategies

Being the channel leader, the manufacturer first informs the distributor of his price margin M_M and advertising subsidy rate ψ_M for retail advertising. In reaction the distributor in informs the retailer of his price margin M_D and participation rate ψ_D. Based on these the retailer decides his advertising effort φ_R and price margin M_R. Thus from (3) we deduce that the retailer's objective function is

$$\max \Pi_R = M_R\rho\varphi_R^{\frac{1}{2}} - (1-\psi_D)\varphi_R \tag{6}$$
$$\text{s. t.} \quad \varphi_R \geq 0.$$

That is, he maximizes his payoff (3) using his effort φ_R. Similarly, the distributor's objective function is given by

$$\max \Pi_D = M_D\rho\varphi_R^{\frac{1}{2}} - (\psi_D - \psi_M)\varphi_R \tag{7}$$
$$\text{s. t.} \quad \psi_D \in [0, 1].$$

That is, the distributor maximizes his payoff (4) using his participation rate ψ_D. The manufacturer's objective function is given by

$$\max \Pi_M = M_M\rho\varphi_R^{\frac{1}{2}} - \psi_M\varphi_R \tag{8}$$
$$\text{s. t.} \quad \psi_M \in [0, 1].$$

That is, he maximizes his payoff (5) using his subsidy rate ψ_M.

The Retail Advertising Effort

Now, maximizing (6) with respect to φ_R we have $\frac{\partial\Pi_R}{\partial\varphi_R} = \frac{M_R\rho}{2\varphi_R^{\frac{1}{2}}} - 1 + \psi_D = 0$
so that

$$\varphi_R = \left(\frac{\rho M_R}{2(1-\psi_D)}\right)^2 \tag{9}$$

3.1.2. The Distributor's Participation/Subsidy Rate

Substituting (9) in (7) we have

$$\max \Pi_D = \frac{\rho^2 M_R M_D}{2(1-\psi_D)} + \left(\frac{\rho M_R}{2(1-\psi_D)}\right)^2 (\psi_M - \psi_D) \tag{10}$$
$$\text{s. t.} \quad \psi_D \in [0,1].$$

Maximizing (10) with respect to ψ_D we have

$$\frac{\partial\Pi_D}{\partial\psi_D} = \left(\frac{\rho}{1-\psi_D}\right)^2\left(\frac{M_R M_D}{2}\right) + \left(\frac{\rho M_R}{2}\right)^2\left[\frac{2}{(1-\psi_D)^3}(\psi_M - \psi_D) - \frac{1}{(1-\psi_D)^2}\right] = 0,$$

so that

$$\psi_D = \frac{2(M_D + M_R\psi_M) - M_R}{2(M_D + M_R) - M_R}. \tag{11}$$

The Manufacturer's Subsidy Rate

Substituting (11) in (9) we have

$$\varphi_R = \left(\frac{\rho M_R}{2\left(1 - \frac{2(M_D+M_R\psi_M)-M_R}{2(M_D+M_R)-M_R}\right)}\right)^2 = \left(\frac{\rho(2M_D + M_R)}{4(1-\psi_M)}\right)^2$$

Using (9) in (8) we have

$$\max \Pi_M = \frac{\rho^2 M_R M_M}{2(1-\psi_D)} - \left(\frac{\rho M_R}{2(1-\psi_D)}\right)^2 \psi_M \qquad (12)$$

s.t $\psi_M \in [0,1]$.

Using (11) in (12) we have

$$\max \Pi_M = \frac{\rho^2 M_R M_M}{2\left(1-\frac{2(M_D+M_R\psi_M)-M_R}{2(M_D+M_R)-M_R}\right)} - \left(\frac{\rho M_R}{2\left(1-\frac{2(M_D+M_R\psi_M)-M_R}{2(M_D+M_R)-M_R}\right)}\right)^2 \psi_M \qquad (13)$$

s.t $\psi_M \in [0,1]$.

Maximizing (13) with respect to ψ_M we have

$$\frac{\partial \Pi_M}{\partial \psi_M} = \frac{2\rho^2 M_R^2 M_M (2M_D+M_R)}{2(2M_R-2M_R\psi_M)^2} - \frac{\rho^2 M_R^2 (2M_D+M_R)^2}{4} \times \left[\frac{(2M_R-2M_R\psi_M)^2 + 4\psi_M(2M_R-2M_R\psi_M)^2 M_R}{(2M_R-2M_R\psi_M)^4}\right] = 0$$

so that

$$\psi_M = \frac{4M_M - M_R(2M_D+M_R)}{4M_M + M_R(2M_D+M_R)}. \qquad (14)$$

Substituting (14) in (11) we have

$$\psi_D = \frac{2\left(M_D + M_R\frac{4M_M-M_R(2M_D+M_R)}{4M_M+M_R(2M_D+M_R)}\right) - M_R}{2(M_D+M_R)-M_R}$$

$$= \frac{2M_D - M_R}{2M_D + M_R} + \left(\frac{2M_R}{2M_D+M_R}\right)\left(\frac{4M_M - M_R(2M_D+M_R)}{4M_M + M_R(2M_D+M_R)}\right). \qquad (15)$$

Thus, we obtain:

Proposition 3.1. In the game given by (6) – (8) the retail advertising effort, the distributor's participation (subsidy) rate, and the manufacturer's subsidy rate are given by (9), (15) and (14) respectively.

Game Equilibriums

Non-Provision of Subsidy Equilibrium

In this section we consider a situation where the manufacturer does not provide subsidy for the distributor to transfer to the retailer, neither does the distributor support retail advertising. Since neither of them supports retail advertising it follows that $\psi_D = \psi_M = 0$, so that (9) becomes

$$\varphi_R = \left(\frac{\rho M_R}{2}\right)^2. \qquad (16)$$

Thus, based on (16) we have that (3), (4) and (5) become

$$\Pi_R = \rho M_R \frac{\rho M_R}{2} - \left(\frac{\rho M_R}{2}\right)^2 = \left(\frac{\rho M_R}{2}\right)^2, \qquad (17)$$

$$\Pi_D = \rho M_D \frac{\rho M_R}{2} = \frac{\rho^2 M_D M_R}{2} \qquad (18)$$

and

$$\Pi_M = \rho M_M \frac{\rho M_R}{2} = \frac{\rho^2 M_M M_R}{2} \qquad (19)$$

respectively.

Thus, we obtain:

Proposition 3.2 Suppose neither the manufacturer nor the distributor support retail advertising, then the retail advertising effort is given by (16), and the players' payoffs are as obtained in (17), (18) and (19).

Non-Transfer of Manufacturer's Subsidy Equilibrium

By non-transfer of manufacturer's subsidy we mean a situation where the manufacturer provides advertising subsidy which he gives to the distributor in hope that it will be transmitted to the retailer, but unfortunately this does not get to the intended recipient. In this situation the distributor's advertising participation rate $\psi_D = 0$. Thus (11) becomes

$$0 = \frac{2M_D + 2M_R\psi_M - M_R}{2M_D + 2M_R - M_R}$$

so that

$$\psi_M = \frac{M_R - 2M_D}{2M_R}. \qquad (20)$$

That is $\psi_M > 0$ only if

$$M_R > 2M_D \qquad (21)$$

otherwise $\psi_M = 0$.

Thus, we deduce that if the manufacturer observes that the distributor may withhold the provided subsidy, then he would only provide subsidy if (21) holds. That is, if the retailer's margin is larger than twice that of the distributor. In the nutshell, the manufacturer may only overlook the withholding of the subsidy if the retailer's margin becomes too large for the consumers, thereby negatively affecting the players' payoffs, and consequently the channel payoff. As such any additional subsidy will amount to waste! In this case the retailer is made to solely finance advertising. As the first follower the distributor is endowed with the first follower's advantage thereby making $M_D > M_R$. As such, (20) implies that $\psi_M = 0$. Thus from (9) we obtain

$$\varphi_R = \left(\frac{\rho M_R}{2}\right)^2. \qquad (22)$$

Thus, based on (22) we have that (3), (4) and (5) become

$$\Pi_R = \rho M_R \frac{\rho M_R}{2} - \left(\frac{\rho M_R}{2}\right)^2 = \left(\frac{\rho M_R}{2}\right)^2, \qquad (23)$$

$$\Pi_D = \frac{\rho^2 M_D M_R}{2} \qquad (24)$$

and

$$\Pi_M = \frac{\rho^2 M_M M_R}{2} \qquad (25)$$

respectively.

Hence, we obtain:

Proposition 3.3 Assuming the distributor withholds the subsidy provided by the manufacturer for retail advertising, then, the manufacturer's subsidy rate is given by (20) conditioned on (21), and the retail advertising effort is given by (22), while the players' payoffs are as obtained in (23), (24) and (25).

Intervention Subsidy Equilibrium

Suppose the manufacturer decides not to provide advertising support to aid retail advertising. Considering the importance of retail advertising, being that the retailer is closer to the consumers than the top hierarchies, the distributor can decide to support the retailer by providing the needed subsidy. In such a case, $\psi_M = 0$, while $\psi_D > 0$. Thus from (11) we have

$$\psi_D = \frac{2M_D - M_R}{2M_D + M_R}. \qquad (26)$$

Clearly, $\psi_D > 0$ only if $2M_D > M_R$. We note that this is in consonance with the First Mover Advantage. Thus from (9) and (26) we have

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$$\varphi_R = \left(\frac{\rho M_R}{2\left(1 - \frac{2M_D - M_R}{2M_D + M_R}\right)}\right)^2 = \left(\frac{\rho(2M_D + M_R)}{4}\right)^2. \qquad (27)$$

And from (26) and (27) we have that (3) becomes

$$\Pi_R = M_R\rho\left(\frac{\rho(2M_D + M_R)}{4}\right) - \left(\left(\frac{\rho(2M_D + M_R)}{4}\right)^2 - \frac{2M_D - M_R}{2M_D + M_R}\left(\frac{\rho(2M_D + M_R)}{4}\right)^2\right) = \frac{\rho^2 M_R(2M_D + M_R)}{8}. \qquad (28)$$

Similarly, using (26), (27) and $\psi_M = 0$ in (4) we have

$$\Pi_D = M_D\rho\left(\frac{\rho(2M_D + M_R)}{4}\right) - \frac{2M_D - M_R}{2M_D + M_R}\left(\frac{\rho(2M_D + M_R)}{4}\right)^2 = \left(\frac{\rho(2M_D + M_R)}{4}\right)^2 \qquad (29)$$

Further, using (27) and $\psi_M = 0$ in (7) we have

$$\Pi_M = \rho M_M\left(\frac{\rho(2M_D + M_R)}{4}\right). \qquad (30)$$

Thus, we obtain:

Proposition 3.4 Suppose that the distributor provides intervention subsidy to aid retail advertising in the absence of the manufacturer's subsidy, then, his subsidy rate is given by (26), the retail advertising effort is given by (27), while the players' payoffs are given by (28), (29) and (30).

Subsidy Transfer Equilibrium

In this section we consider a situation where there is commitment from all the players towards advertising the product. In this situation the manufacturer's provided advertising support is transferred to the retailer. That is $\psi_D > 0$ and $\psi_M > 0$. Thus using (15) in (9) we have

$$\varphi_R = \frac{M_R^2\rho^2}{4\left(1 - \left[\frac{2M_D - M_R}{2M_D + M_R} + \left(\frac{2M_R}{2M_D + M_R}\right)\left(\frac{4M_M - M_R(2M_D + M_R)}{4M_M + M_R(2M_D + M_R)}\right)\right]\right)^2} = \frac{M_R^2\rho^2}{4(2)^2 M_R^2\left(\frac{2M_R(2M_D + M_R)}{(2M_D + M_R)(4M_M + M_R(2M_D + M_R))}\right)^2}$$

$$= \left(\frac{\rho\left(4M_M + M_R(2M_D + M_R)\right)}{8M_R}\right)^2. \qquad (31)$$

From (15) and (31) we have that (6) becomes

$$\Pi_R = M_R\rho\left(\frac{\rho\left(4M_M + M_R(2M_D + M_R)\right)}{8M_R}\right) - \left\{1 - \left[\frac{2M_D - M_R}{2M_D + M_R} + \left(\frac{2M_R}{2M_D + M_R}\right)\left(\frac{4M_M - M_R(2M_D + M_R)}{4M_M + M_R(2M_D + M_R)}\right)\right]\right\} \times \left(\frac{\rho\left(4M_M + M_R(2M_D + M_R)\right)}{8M_R}\right)^2$$

$$= \frac{\rho^2\left(4M_M + M_R(2M_D + M_R)\right)}{8} - \left\{\frac{8M_R^2}{8M_M + 2M_R(2M_D + M_R)}\right\}\left(\frac{\rho\left(4M_M + M_R(2M_D + M_R)\right)}{8M_R}\right)^2 = \frac{\rho^2\left(4M_M + M_R(2M_D + M_R)\right)}{16}. \qquad (32)$$

Now, considering (14) and (15) we have

$$\psi_M - \psi_D = \frac{4M_M - M_R(2M_D + M_R)}{4M_M + M_R(2M_D + M_R)} - \left[\frac{2M_D - M_R}{2M_D + M_R} + \left(\frac{2M_R}{2M_D + M_R}\right)\left(\frac{4M_M - M_R(2M_D + M_R)}{4M_M + M_R(2M_D + M_R)}\right)\right] = \frac{M_R(M_R - 2M_D)}{4M_M + M_R(2M_D + M_R)}. \qquad (33)$$

Thus, from (31) and (33) we have that (7) becomes

$$\Pi_D = M_D\rho\left(\frac{\rho\left(4M_M + M_R(2M_D + M_R)\right)}{8M_R}\right) - \frac{M_R(M_R - 2M_D)}{4M_M + M_R(2M_D + M_R)}\left(\frac{\rho\left(4M_M + M_R(2M_D + M_R)\right)}{8M_R}\right)^2 = \frac{\rho^2\left(4M_M + M_R(2M_D + M_R)\right)(10M_D - M_R)}{8^2 M_R}. \qquad (34)$$

Further, substituting (14) and (31) into (8) we have

$$\Pi_M = \rho M_M\left(\frac{\rho\left(4M_M + M_R(2M_D + M_R)\right)}{8M_R}\right) - \frac{4M_M - M_R(2M_D + M_R)}{4M_M + M_R(2M_D + M_R)}\left(\frac{\rho\left(4M_M + M_R(2M_D + M_R)\right)}{8M_R}\right)^2 = \frac{\rho^2\left(4M_M + M_R(2M_D + M_R)\right)[4M_M(2M_R - 1) + M_R(2M_D + M_R)]}{64M_R^2}.$$

Thus, we obtain:

Proposition 3.5 Suppose that the distributor transmits the manufacturer's provided subsidy to the retailer, then the distributor's participation rate, the manufacturer's subsidy rate and retail advertising effort are given by (15), (14) and (31), and the players' payoffs are as obtained in (32), (34) and (35).

DISCUSSION

Since the advertising effectiveness $\rho \in [0,1]$, we let $\rho = 0.3$. Further, we recall that the manufacturer enjoys first mover's advantage, and is followed by the distributor. It is therefore rational to have that $M_M > M_D > M_R > 0$. Thus, we let $M_M = 2.0$,

$M_D = 1.5,\ M_R = 1.0$. To simplify the discussion in this section we will use the following subscripts:

1) $\psi_D = \psi_M = 0$: The manufacturer and the distributor do not participate in retail advertising;
2) $\psi_D = 0, \psi_M > 0$: The manufacturer's provided advertising support is not transmitted to the retailer;
3) $\psi_D > 0, \psi_M = 0$: The distributor provides special intervention subsidy when manufacturer does not provide;
4) $\psi_D > 0, \psi_M > 0$: The distributor transmits the manufacturer's advertising support to the retailer.

The graphs in this work were plotted using Mathematica software.

The Effect of the Players' Margins on Participation

From (11) and (14) we recall that the manufacturer and the distributor's participation rates are related to the price margins. In this subsection we illustrate how the manufacturer and distributor react in terms of these rates to the price margins.

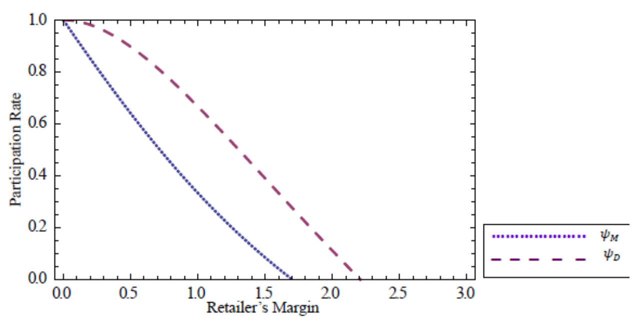

Figure 1. Effect of the retailer's price margin on the players' participation rates

Figure 1 shows that as the retailer's margin increases, both the manufacturer and the distributor's participation rates reduce. However, while the distributor is more "hesitant" in the reduction, the manufacturer is more rapid in reducing the subsidy as the retailer's margin increases. Clearly, the distributor's hesitation is understandable being that he is the immediate beneficiary of the retailer's sales, coupled with the fact that his participation is not actually his expenditure, but the manufacturer's expenditure.

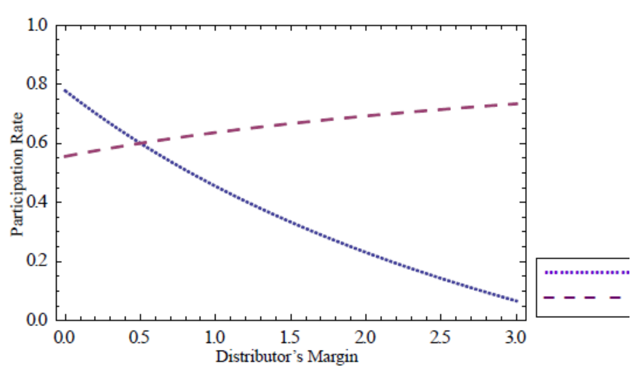

Figure 2. Effect of the distributor's price margin on the players' participation rates

On the other hand, Figure 2 shows that as the distributor's margin increases, the manufacturer rapidly reduces the intended subsidy for retail advertising, while the distributor increases his participation which tends to 1 as his margin becomes very large. That is, the distributor can potentially provide total subsidy as a motivation from large margin. The manufacturer's reaction is quite understandable since he is the retailer's actual motivator. Thus, he may not be comfortable with the distributor's large price margin to the retailer. To express his reservation to the distributor, he may reduce his subsidy rate with increasing distributor's margin.

We recall that (11) implies that when the manufacturer does not support the retailer, the distributor can personally provide subsidy for retail advertising as given in (26). This is the situation portrayed in Figure 3.

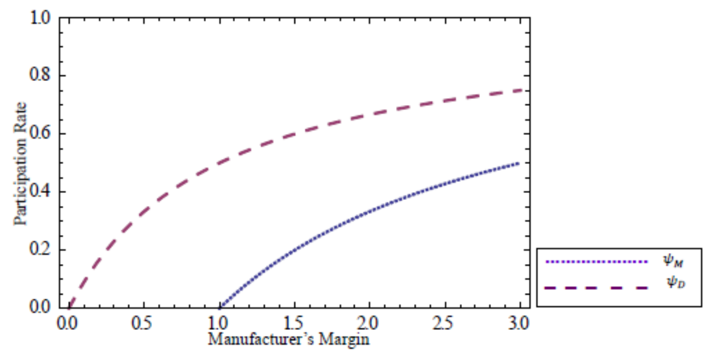

Figure 3. Effect of the manufacturer's margin on the participation rates

It shows that the distributor provides subsidy in the absence of subsidy from the manufacturer, which comes only after a certain level of manufacturer's margin is achieved. That is, the manufacturer only starts providing subsidy only after this margin level. Clearly, with increasing margin comes increasing manufacturer's subsidy, which subsequently leads to increase in the distributor's participation rate. Thus, as the manufacturer's margin increases, his subsidy to the distributor also increases, leading to increase in distributor's participation. This implies that the manufacturer can use his margin to coordinate the channel since it determines both participation rates which can influence the payoffs.

The Effect of Participation on Payoff

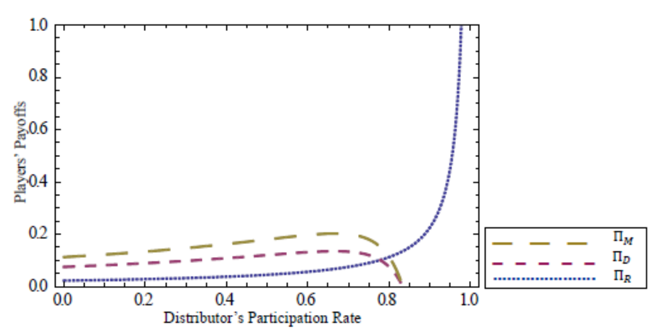

Figure 4. Effect of the distributor's participation on the players' payoffs

From Figure 4 it is clear that as the distributor's participation rate

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increases, the retailer's payoff increases. Similarly, both the distributor and the manufacturer's payoffs increase but eventually exhibit reduction. That is, both players' optimal benefits from the distributor's participation are possible only up to certain levels. Exceeding these levels will result in rapid marginal decline in the payoffs. Further, it is natural for them to ensure that their individual payoffs are larger than that of the retailer. Thus, if he must participate in providing subsidy above the level where his payoff starts exhibiting reduction, then it must not go beyond where his payoff is larger than that of the retailer.

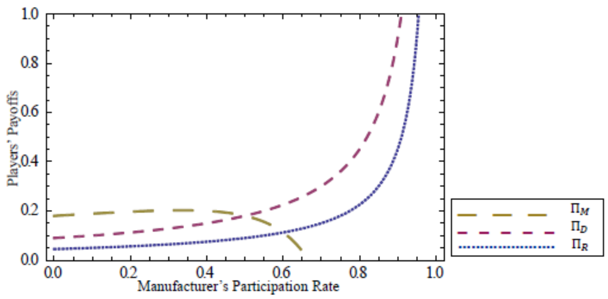

Figure 5. Effect of the manufacturer's participation on the players' payoffs

From Figure 5 the distributor and the retailer's payoffs continuously increase with the manufacturer's participation. On the other hand, the manufacturer's payoff exhibits continuous increase with his participation, but reduces and eventually becomes zero for very large participation. We note that for lower subsidy levels, the manufacturer's payoff is larger than the distributor and retailer's payoffs. This eventually reduces with increasing subsidy. Being the channel leader, the manufacturer would want to ensure that his payoff is the largest. To achieve this, he should provide subsidy only up to the level where his payoff is the largest. This will eventually starve the distributor of subsidy supplies, thereby constraining him to maintain participation only up to the level allowed/provided by the manufacturer. By so doing, the manufacturer can ensure large payoff, co-ordinate the channel activities and the payoffs.

. **Comparison of each Player's Payoffs**

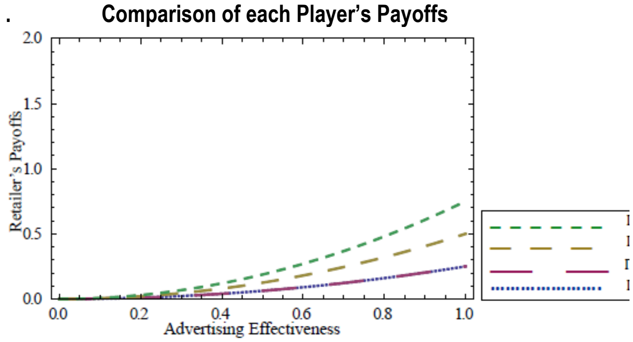

Figure 6. Retailer's payoffs for the four channel structures

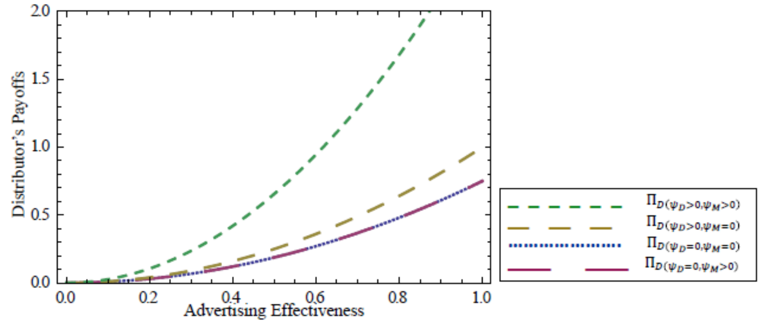

Figure 7. Distributor's payoffs for the four channel structures

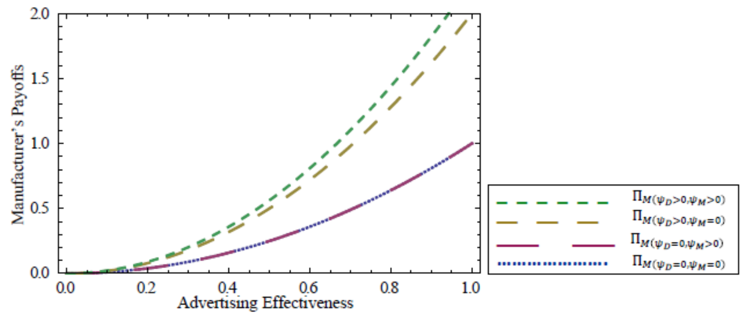

Figure 8. Manufacturer's payoffs for the four channel structures

From Figure 6, Figure 7 and Figure 8 it is obvious that the players perform better with the distributor's transmission of the subsidy. Their performances are worst with non-transfer and non-participation by both the manufacturer and the distributor. It is important to note that the distributor does not have a better payoff with non-transfer or non-provision of subsidy. This is also the case with the manufacturer! Rather, all the players perform well with the distributor's intervention subsidy; and best with transfer of subsidy.

Comparison of the Channel Payoff

In this section we let

$\Pi_{(\psi_D=\psi_M=0)} = \Pi_{R(\psi_D=\psi_M=0)} + \Pi_{D(\psi_D=\psi_M=0)} + \Pi_{M(\psi_D=\psi_M=0)}$;

$\Pi_{(\psi_D>0,\psi_M=0)} = \Pi_{R(\psi_D>0,\psi_M=0)} + \Pi_{D(\psi_D>0,\psi_M=0)} + \Pi_{M(\psi_D>0,\psi_M=0)}$;

$$\Pi_{(\psi_D=0,\psi_M>0)} = \Pi_{R(\psi_D=0,\psi_M>0)} + \Pi_{D(\psi_D=0,\psi_M>0)} + \Pi_{M(\psi_D=0,\psi_M>0)}$$

and

$\Pi_{(\psi_D>0,\psi_M>0)} = \Pi_{R(\psi_D>0,\psi_M>0)} + \Pi_{D(\psi_D>0,\psi_M>0)} + \Pi_{M(\psi_D>0,\psi_M>0)}$.

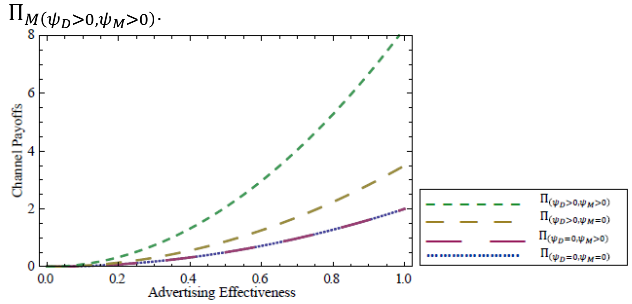

Figure 9. Channel payoffs for all-four channel structures

Figure 9 shows that the entire channel payoff is better with the transfer of the provided subsidy. Apart from this, another good scenario is the intervention scenario, where the distributor provides advertising support when the manufacturer doesn't. Clearly, the channel is worst-off with both non-provision and non-transfer of subsidy. Thus, the distributor's participation is very crucial to all the players and to the channel. It is therefore necessary that the channel, especially the manufacturer should provide incentive that would encourage the distributor to intervene when he (the manufacturer) is indisposed to provide subsidy. Also, where necessary, he should evolve a policy or policies that would constrain the distributor to transfer the provided subsidy.

Conclusion

This work used four channel settings to study the effect of participations on the individual players' payoffs and the entire channel payoff, and also looked at the influence of the players' margins on participation.

Clearly, the players' payoffs and the entire channel payoff are largest with the transfer of the subsidy provided by the manufacturer, and worst with non-participation and non-transfer of the subsidy. Although, the payoffs are large with the distributor's intervention subsidy, however, they are not as large as those obtainable with transfer of subsidy. Thus, the distributor's participation is very crucial to the players and the entire channel. As such, the channel should evolve a policy that will encourage or constrain the distributor (where necessary) to participate in advertising. Further, the manufacturer has the option of using his optimal subsidy to ensure that his payoff is the largest.

The distributor's participation rate should increase with his margin, while the manufacturer's participation should reduce with it. In addition, while both participation rates should increase with the manufacturer's margin with hesitation from the manufacturer, they should also reduce with the retailer's margin.

This work has some limitations and possible extensions. We note that apart from the retailer being the only player directly involved in advertising, both of the top hierarchies can directly participate in regional or national advertising to boost the retailer's effort. This can shed more light on cooperative advertising. Further, we observe that the retailer or the distributor can be influential or powerful enough to dictate terms to the manufacturer. In this case such a powerful player can be considered as the supply channel leader. Modelling subsidy transfer in this setting can be very insightful.

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