



## DEVELOPMENT AND EMPIRICAL VALIDATION OF ALGORITHM ON DIVISION OF POLYNOMIALS FOR TENTH GRADE STUDENTS

Divya Pande<sup>1</sup>, Mr. Rajvir Singh<sup>2</sup>

### Abstract

An important task in the field of education is the improvement in the teaching learning process. Algorithmic instruction has been widely used with advantage not only in classroom study but also in teaching, in medicine industry and public services in advanced countries. Algorithms as a self – instructional technique needs to be tried out in India. Algorithm not only helps to master the new skills but, also lessens the time gap in achieving those skills. With this view and using mathetic style of programming this study was designed for tenth grade students.

**Keywords:** Development, Empirical Validation, Algorithm, Polynomial, Division of polynomial

### INTRODUCTION

Now a days, the crucial task in the field of education is the improvement in the teaching learning process. For this purpose educational researchers have tried to experiment with the innovative methods of instructions. Inspired by the scientific invention and technological development in every sphere of life, creative teachers try to replace the traditional methods of teaching with novel emerging instructional paradigms. So the nature of modern innovations is a sort of departure from the conventional mode of delivering lessons and evolving new strategies for effective delivery of curricula contents. Algorithmic instruction has been widely used with advantage not only in classroom study but also in the teaching, in medicine industry and public services in advanced countries.

Algorithms as a self –instructional technique needs to be tried out in India. Algorithms not only help to master the new skills but also, lessens the time gap in achieving those skills. The investigator, having interest in dealing with mathematical computations and having a strong will to help the school children in solving mathematical problems, has selected this topic.

### Review of Related Literature

**Hussain(1971)** developed a programme on “Factors affecting the air pressure ” in linear as well as branching formats. He studied the effectiveness under supervised and non supervised conditions in rural and urban areas . He found that learning through linear programming was found better than branching style.

<sup>1</sup> Research Scholar, Kurukshetra University, Kurukshetra, **Email:** divya\_pande1980@yahoo.com,

**Orcid:** <https://orcid.org/0000-0001-8532-3981>, **DOI:** <https://doi.org/10.53724/jmsg/v2n2.52>

<sup>2</sup> Assistant Professor, Kurukshetra University, Kurukshetra, [433]



**Hussain, M.S. (1984)** developed a mathematical programme on “Mensuration” and found that by developing mathematical exercises in demonstrated, prompted and released form to be effective and valid.

**Kumar, R. (2005)** evolved algorithm on “Quadratic equations for XI grade student” and the result shows that with the help of these type of algorithms, students’ achievement and retention power developed more as compared to conventional methods.

**Sangwan(2006)** developed and empirically validated algorithms on “Linear equations for tenth grade students” and found that 90 percent learners achieved the target in algorithmic skills.

**Mukesh (2008)** designed a programme on “Development and empirical validation of algorithm on linear equation for tenth grade students of Government Senior Secondary School(Mirzapur)” and found it to be very useful.

**Kaur, P. (2010)** developed and empirically validated mathematical programme on “Area of cone” for ninth grade students and found that the programme was effective.

## DEFINITION OF THE KEY TERMS USED

### *Development:*

Development as regard to the present study means that the programme will be constructed keeping in mind the various stages of programme preparation, writing, tryout and evaluation.

### *Empirical Validation:*

The term empirical validation with regard to the present study means that the programme will be tried out in a sample population and on the basis of the observation made, the validation of the programme will be done.

### *Algorithm:*

An algorithm is an instruction or action guide, which lays down the exact sequence of procedures within a system so as to provide recipe for solution of problems of a given type within that system.

### *Polynomial:*

A polynomial  $p(x)$  in one variable  $x$  is an algebraic expression in  $x$  of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x^1 + a_0$$

where  $a_0, a_1, a_2, \dots, a_n$  are constants and  $a_n \neq 0$

### *Division of polynomial:*

Polynomial division allows for a polynomial to be written in a divisor–quotient form which is often advantageous. Consider polynomials  $P(x)$ ,  $D(x)$ , then, for some quotient polynomial  $Q(x)$  and remainder polynomial  $R(x)$  with  $\text{degree}(R) < \text{degree}(D)$ ,



$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)} \implies P(x) = D(x)Q(x) + R(x).$$

This rearrangement is known as the **division transformation**, and derives from the arithmetical identity

$$\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$$

## OBJECTIVES

The objectives of the study were:

- Developing algorithms on division of polynomials for tenth grade students.
- Empirical validation of the algorithm on division of polynomial for tenth grade students.

## HYPOTHESIS

85% of the students will be able to perform on 85% of the algorithmic exercises correctly and will be able to acquire 85% mastery on the criterion test.

## METHODOLOGY

For developing the algorithm on the proposed unit, the researcher passed through the following four phases:

- **PREPARATORY ANALYSIS**

Following steps involved in the preparatory stage of the algorithms, were systematically covered.

- *Step 1: Selection of the topic to be algorithmised.*
- *Step 2: Writing assumptions about learner.*

At this stage, the characteristics of the learners were written quite accurately. The assumptions about the learners or the algorithms were developed by the researcher and are as given below:

- Age : 14+
- Grade: X graders
- Sex: Boys and Girls
- Interest: The learners are keenly interested in Mathematics.
- Socio- cultural background: The students belong to middle class and upper middle class socio-economic standard and have diverse cultural background.
- Intellectual level : Average and above average
- Aptitude: As Mathematics is a compulsory subject, most of the students have got specific aptitude in computation.



- Skills: The learners possess observational, computational, discriminative, verbal skills and sketching skills.
- Scholastic Abilities: Mostly second and first divisions. The information was gathered by the investigator while she was dealing with the intended group of learners, their teachers and their report books maintained by the school office.
- **Step 3: Concept analysis or task analysis.**

Keeping in view the mental level of tenth grade students, it was decided to delineate the range of concepts to be developed in algorithm for the learners. While delineating the content, procedure termed as “information mapping” was developed.

- **Step 4: Writing objectives in behavioural terms.**

After going through the algorithm the learner will be able to:

- Write polynomial in standard form.
- Divide the polynomial efficiently with accuracy.
- Learn where to stop the division process.
- Write the algorithm on division of polynomials.
- **Step 5: Writing the pre-requisite knowledge and skills in behavioural terms.**

So before working on algorithm the learner is able to:

- Point out the importance of Mathematics in curriculum as well as in life.
- Read effectively the textbooks of Mathematics written in English language.
- Writes the formulae for different mathematical operations.
- Solve the various Mathematical problems.
- Describes the basic facts, concepts and principles of Mathematics.
- Define polynomial.
- Classify polynomial.
- Add and subtract polynomial.

The learner for whom the algorithm has been developed, possesses these pre-requisite knowledge and skills. These pre-requisite knowledge and skills assumed on the part of the learner were assessed before the students were presented with the self-instructional material by the researcher.

- **Step 6: Developing criterion test items for assessing terminal performance of the learners.**

The criterion test was developed in order to assess the competency attained by the learners at the end of completing the self instructional units. These tests were administered even in individual tryouts and small



group try out sessions and were revised accordingly. The criterion test developed by researcher contained released type of exercise in which learners were to do all the steps in the exercise. It was improved later after the small group tryout and the criterion test administered in the field tryout included ten released type exercises.

- **Step 7: Developing the core material.**

The researcher amends keeping in view that content outline, assumptions about the learner, behavioural objective and criterion test items of a particular topic of the core material. Every care was taken to keep the material up to date, accurate brief and simple as well. Some of the concepts and items of information were cross validated through the text book prescribed by N.C.E.R.T. with this background material, it became easy to develop the exercise for algorithmization.

- **WRITING THE ALGORITHMS**

In the present study the algorithm exercise are arranged stepwise and in a sequence proceeding from easy to difficult as is done in the mathematical exercise. These algorithms include three types of exercises namely,

1. Demonstrated Exercises
2. Prompted Exercises
3. Released Exercises

### 1. Demonstrated Exercises

In this type of exercise, explanation of formula, procedures with examples are provided to learner. The learner is asked to go through these exercises and get his basics cleared about the topic under study. He checks the examples and procedures of solving the problems.

**For example, Divide  $2x + 3x^2 - 1$  by  $x - 2$**

Sol :

#### **Step I: Writing the polynomial in standard form:**

1. Arrange the terms of dividend ( $2x + 3x^2 - 1$ ) in the decreasing order of their degrees, hence we get ( $3x^2 + 2x - 1$ ).
2. Arrange the terms of divisor ( $x - 2$ ) in the decreasing order of their degrees.

**Step II:** To obtain the first term of the quotient, divide the highest degree term of the dividend (i.e.  $3x^2$ ) by the highest degree term of the divisor (i.e.  $x$ ). This is  $3x$ . Then carry out the division process. What remains is ( $8x - 1$ ).

**Step III:** Now obtain the second term of the quotient, divide the highest degree term of the new dividend ( $8x$ ) by the highest degree term of the divisor ( $x$ ).

This gives ( $8$ ). Again carry out division process with ( $8x - 1$ )



**Step IV:** What remains is (15). Now, the degree of (15) is less than the degree of the divisor (x-2). So we can't continue the division any further.

**Step V:** The quotient is (3x+8). The remainder is (15)

## 2. Prompted Exercises

The second type of exercises are given through prompting the learning. Demonstrated exercise is produced through another example leaving one or two blanks to be filled by the learners. The learner goes through such problems and fills the blanks as they proceed.

**For example,** Divide  $2x^2 + 1 + 3x$  by  $2 + x$

Sol :

**Step I: Writing the polynomial in standard form:**

1. Arrange the terms of dividend ( $2x^2 + 1 + 3x$ ) in the decreasing order of their degrees, hence we get ( $2x^2 + 3x + 1$ ).
2. Arrange the terms of divisor ( $2 + x$ ) in the decreasing order of their degrees, hence we get ( $x+2$ ).

**Step II:** To obtain the first term of the quotient, divide the highest degree term of the dividend ( $2x^2$ ) by the highest degree term of the divisor ( $x$ ). This is ( $2x$ ).

Then carry out the division process. What remains is ( $-x + 1$ )

**Step III:** Now obtain the second term of the quotient, divide the highest degree term of the new dividend ( $-x$ ) by the highest degree term of the divisor ( $x$ ).

This gives ( $-1$ ). Again carry out the division process with ( $-x+1$ )

**Step IV:** What remains is 3

Now, the degree of 3 is less than the degree of the divisor ( $x+2$ ). So we can't continue the division any further.

**Step V:** The quotient is \_\_\_\_\_

The remainder is \_\_\_\_\_

## 3 Released Exercises

Released exercises are the third type of exercises provided in writing mathematical algorithm. This type of exercise is more or less the same as before but the only difference is that no guidance is provided in it and solving of such an exercise depends on the skills developed in learner. He has to work out all the steps involved in the completion of task himself.



For example,

Divide  $x^2 + x + 1$  by  $x - 1$

Sol :

Step I : \_\_\_\_\_

Step II : \_\_\_\_\_

Step III : \_\_\_\_\_

Step IV : \_\_\_\_\_

Step V : \_\_\_\_\_

## Sequencing the exercise

In writing the algorithm, steps in exercises have to be constructed and put in a specific order so as to ensure the establishment of the terminal behaviour. The procedure of arranging the exercises in a set systematic and serial order is called “sequencing”. The following steps were used by the investigator:

### 1. Logical Sequencing

Here the programmer thinks of a logical order in which the subject matter is to be presented in a generated sequence. The approaches followed by the expert are:

#### (a) *The Ruleg Approach and Equal Approach*

The Ruleg (Rule followed by example) system of sequencing the exercises was devised by Glass and Home. The programmer employs deductive reasoning in such situations.

#### (b) *Matrix Approach*

Davis has enunciated this approach. In this approach, the concept matrixes are assigned to the exercise developed on this concept. The investigator followed on this approach while sequencing the exercises of the algorithm developed in Arithmetic.

#### (c) *Methodical Approach*

The methodical approach **D-P-R** was followed while sequencing the steps in the algorithms by the researcher.

### 2. Empirical Sequencing

After having tried out the algorithms on a small group of learners some alteration were made in the Mathematical sequences. The conceptual steps /procedure arrived at through empirical testing are termed as “empirical sequences”. These empirical sequences are always given importance over logical sequences. It is said to be the rule that ideally the sequence of exercises is ultimately to be decided by the learners and not by the expert who is designing the algorithm.

## Editing and review of the algorithms

The first draft of the algorithm was written by the investigator to evolve whatever is desired for the learners for attaining mastery of computational skills and procedures. After completion of the first draft before going for tryout, the algorithms were checked by several specialists for editing namely, subject matter expert,



technique expert and language expert. After completion of three versions of editing operations, the final algorithm was ready for tryout experiments.

## • TRY OUT OF ALGORITHM

The algorithm were subjected to experimental tryout three times, namely, individual tryout (one individual at a time), small group tryout (ten pupils simultaneously) and final field tryout (thirty students of tenth grade).

### Individual tryout

In the individual tryout arrangement, each exercise was administered on a single learner in a very informal situation. The neatly hand written exercises written on plain paper of the algorithm on division of polynomials was presented to the learner one by one. The confirmatory responses were written on the back page of the plain paper.

- (i) This is an algorithm .You are to help in developing successful algorithms on division of polynomials.
- (ii) Work on the exercises step by step. Write your calculated response on a separate sheet of paper.
- (iii) Then tally your responses with the correct answer of the exercise which is written on the back side of the plain paper, but don't see the answers while writing your own answer.
- (iv) Tell frankly and freely about any difficulty or inadequacy regarding the languages, exercises and concepts etc. in a particular step. Here the investigator was face to face with a single learner. The learner worked through the algorithms step by step and the exercises were corrected, modified and simplified. It gave an opportunity to the researcher to study the reaction of the learner immediately after the learner had completed the exercise. The algorithm was administered on randomly selected four learners individually and simultaneously.

### Small group tryout and final field tryout

After having made the changes on the basis of individual tryout the algorithms were re-written in a definite form separately on cards and again tried on a sample group of ten students of class tenth in **Geeta Niketan Awasiya Vidyalaya, Kurukshetra**. The students after reading the instructions on the first page understand the procedure of working by themselves. The researcher guided those who had any doubts for easy understanding of the exercises for them. The confirmatory responses were inserted at the back page of the cards. The learners were asked to underline difficult words, sentences and tough segments in the exercise.

The field tryout experiments proved to be very fruitful for calculating error rates and for preparing sequence progression chart. The attainment scores of the learners on criterion test gave an idea about the efficiency of the programming and the attainment of innumeracy skills by the students. Tryout tests not only revealed the efficacy of the algorithms, but also helped the researcher in modifying the algorithms.





## • EVALUATION OF ALGORITHM

Evaluation of algorithm is the final stage in the development of an algorithm. Evaluation is done to improve the quality of instructional material. The small group tryout results and field tryout results are often analysed in terms of error-rates, sequence progression and criterion test scores. Evaluation of a programme is always done on the basis of: Internal Criteria; External Criteria

### Internal Criteria

The programmed material is tested during its development process in terms of :

- (a) Error Rate (b) Sequence Progression

#### (a) Error Rate

The analysis of error rate is done on the basis of responses obtained for each frame. If on a particular frame, the learner is not able to respond in accordance with the stipulated programme, it is given the name of “error”. There may be one or more than one response in the exercise. For the algorithmic programmes such errors made by all individuals on all steps are counted and added and then error rate can be calculated by using the formula

$$\text{Error Rate} = \frac{\text{Total number of errors committed} \times 100}{\text{Total number of responses} \times \text{Total number of students}}$$

#### Error Rate on The Basis of Algorithm

Table1

S. No.	Exercise	Error rate %	Percentage of success
1	1	2.66%	97.33%
2	2	2.22%	97.78%

#### Interpretation of Error Rate

From table 1, we can see that the error rates of the first exercise do not exceed 2.66 percent. This implies that 97.33 percent of the learners were able to complete 97.33 percent of the questions of the exercise correctly. Similarly in second exercise, error rates do not exceed 2.22 percent. This implies that 97.78 percent of the learners were able to complete 97.78 percent of the questions of exercise correctly. By calculating error rates of both the exercises and after interpreting the result, researcher found that 85 percent of the students were able to perform on 85 percent of the questions of both the exercises correctly.

#### (b) Sequence Progression

After having completed the administration of programming the responses given by the learner were checked by the investigator and on the basis of their errors made on each exercise (either prompted or released) the flowchart of sequence progression was prepared. On the given flowchart the sign “x” indicates



the error made by the individual learner on a specific step of the exercise concerned. On the bottom of the horizontal line are entered the total number of errors made by all the thirty (class tenth) students on a particular step of a particular exercise. On the extreme right vertical line are entered the total number of errors made by individual learner on the different steps of the exercise contained in the flowchart.

### Interpretation of Sequence Progression Chart

After completing the administration of algorithms, the sequence progression chart was prepared on the basis of responses given by the learner in each exercise. A close observation of sequence progression reveals that in exercise one, step 3 of question 4, step 4 of question 5 and step 2 of question 6 were a bit difficult. In exercise two, step 4 of question 5 was a bit difficult. A few students were not able to respond to those steps correctly, cause is that exercises were in released form. Other reasons were that some of the learners were comparatively slow. However, more than 90 percent learners were able to calculate 85 percent of the exercises with almost cent –percent accuracy.

EXERCISE 1  
QUESTION NUMBER

QUESTION No.	1					2					3					4					5					6					ERRORS					
Step No.	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4		0	1	2	3	4
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ERRORS	0	1	0	0	2	0	0	0	1	0	3	0	0	0	3	1	1	1	0	0	12															



EXERCISE 2  
QUESTION NUMBER

QUESTION No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	ERRORS
Step No.	0	4	3	4	2	3	4	1	2	3	4	5	6																		
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29																															
30																														1	
ERRORS	0	1	1	0	1	1	0	0	0	0	0	2	1	1	1	8															

(ii) External Criteria

The algorithm developed by the investigator has been evaluated on the basis of final field tryout in terms of:

- (a) Error Rate
- (b) Sequence progression

The error rate of all the six algorithms, evolved by the researcher were calculated by applying the formula

$$\text{Error Rate} = \frac{\text{Total number of errors committed} \times 100}{\text{Total number of responses} \times \text{Total number of students}}$$



## Error Rate of the Criterion Test

Table 2

S.No.	QUESTION	ERROR RATE%	PERCENTAGE OF SUCCESS
1	Divide $(3x^3 - 2x + 3)$ by $(2x^2 - 1)$	2%	98%
2	Divide $(9x-1+14x^3 - 5x^2)$ by $(2x-1)$	0.66%	99.34%
3	Divide $(3x^3 + x^5 + x^2)$ by $(x^3 + 1)$	2%	98%
4	Divide $(2x^3 + x + 1 + x^4)$ by $x^2$	1.33%	98.67%
5	Divide $(x + 1 + x^2)$ by $(x-1)$	1.66%	98.34%
6	Divide $(x^4 + x^6 + 2)$ by $(1 + x^4)$	10.83%	89.17%

By calculating error rates of all the questions and after interpreting the result, researcher found that 85 percent of the students were able to perform on 85 percent of the questions correctly.

### (b) SEQUENCE PROGRESSION

Sequence progression charts for the criterion test are given below:

#### Sequence Progression Chart for Question 1 of Criterion Test

Table 3

#### Steps of Algorithm for Question 1

S.No.	I	II	III	IV	V	ERRORS
1						0
2						0
3						0
4						0
5						0
6			X			1
7						0
8						0
9						0
10						0
11						0
12						0
13						0
14						0
15						0
16						0
17						0
18						0
19						0
20						0
21						0
22		x				1
23						0
24						0
25	X					1
26						0
27						0
28						0
29						0
30						0
ERRORS	1	1	1			3



## Sequence Progression Chart For Question 2 of Criterion Test

Table 4

### Steps of Algorithm For Question 2

S.No.	I	II	III	IV	V	ERRORS
1						0
2						0
3						0
4						0
5						0
6						0
7						0
8						0
9						0
10						0
11						0
12						0
13				x		1
14						0
15						0
16						0
17						0
18						0
19						0
20						0
21						0
22						0
23						0
24						0
25						0
26						0
27						0
28						0
29						0
30						0
ERRORS	0	0	0	1	0	1

## Sequence Progression Chart For Question 3 of Criterion Test

Table 5

### Steps of Algorithm For Question 3

S.No.	I	II	III	IV	V	ERRORS
1						0
2						0
3						0
4						0
5						0
6						0
7			X			1
8						0
9						0
10						0
11						0
12						0
13						0
14						0



15						0
16						0
17						0
18						0
19						0
20						0
21						0
22						0
23					x	1
24						0
25						0
26						0
27						0
28						0
29		x				1
30						0
ERRORS	0	1	1	0	1	3

## Sequence Progression Chart For Question 4 of Criterion Test

Table 6

### Steps of Algorithm For Question 4

S.No.	I	II	III	IV	V	ERRORS
1						0
2						0
3						0
4						0
5						0
6						0
7						0
8						0
9						0
10				x		1
11						0
12						0
13						0
14						0
15						0
16						0
17						0
18						0
19						0
20						0
21						0
22						0
23						0
24						0
25						0
26						0
27						0
28						0
29					x	1
30						0
ERRORS	0	0	0	1	1	2



## Sequence Progression Chart For Question 5 of Criterion Test

Table 7

### Steps of Algorithm For Question 5

S.No.	I	II	III	IV	V	VI	ERRORS
1							0
2							0
3							0
4							0
5							0
6							0
7							0
8							0
9							0
10							0
11							0
12							0
13							0
14							0
15							0
16							0
17			X				1
18							0
19							0
20							0
21				x			1
22							0
23							0
24							0
25		X					1
26							0
27							0
28							0
29							0
30							0
ERRORS	0	1	1	1	0	0	3

## Sequence Progression Chart For Question 6 of Criterion Test

Table 8

### Steps of Algorithm For Question 6

S.No.	I	II	III	IV	ERRORS
1		X			1
2					0
3					0
4					0
5					0
6		X			1
7		X			1
8		X			1
9					0
10					0
11					0
12					0
13					0



14					0
15			x		1
16		X			1
17					0
18					0
19				x	1
20					0
21		X			1
22					0
23					0
24			x		1
25	X				1
26			x		1
27					0
28		X			1
29		X			1
30					0
ERRORS	1	8	3	1	13

## Interpretation of Sequence Progression Chart

More than 90 percent learners were able to calculate 85 percent of the exercises with almost cent – percent accuracy.

## FINDINGS

The programme was evaluated and empirically validated by the investigator on the basis of final field tryout data in terms of:

### (i) Findings on Algorithm

The programmed material is tested during its development process in terms of :

- **Error Rate**

By calculating error rates of both the exercises and after interpreting the result, researcher found that 85 percent of the students were able to perform on 85 percent of the questions of both the exercises correctly.

- **Sequence Progression**

A close observation of sequence progression reveals that in exercise one, step 3 of question 4 , step 4 of question 5 and step 2 of question 6 were a bit difficult. In exercise two, step 4 of question 5 was a bit difficult. A few students were not able to respond to those steps correctly, cause is that exercises were in released form. Another reason was that some of the learners were comparatively slow. More than 90 percent learners were able to calculate 85 percent of the exercises with almost cent –percent accuracy.

### (ii) Findings on Criterion Test

The algorithm developed by the investigator has been evaluated on the basis of final field tryout in terms of:





## ▪ *Error Rate*

From table 2, the error rates of the first question do not exceed 2 percent. This implies that 98 percent of the learners were able to complete 98 percent of the questions correctly. Similarly in second question error rates do not exceed 0.66 percent. This implies that 99.34 percent of the learners were able to complete 99.34 percent of the question correctly. In third question error rates do not exceed 2 percent. This implies that 98 percent of the learners were able to complete 98 percent of the question correctly. In fourth question error rates do not exceed 1.33 percent. This implies that 98.67 percent of the learners were able to complete 98.67 percent of the question correctly. In fifth question error rates do not exceed 1.66 percent. This implies that 98.34 percent of the learners were able to complete 98.34 percent of the question correctly. In sixth question error rates do not exceed 10.83 percent. This implies that 89.17 percent of the learners were able to complete 89.17 percent of the question correctly. By calculating error rates of all the questions and after interpreting the result, researcher found that 85 percent of the students were able to perform on 85 percent of the questions correctly.

## ▪ *Sequence progression*

A *close observation* of sequence progression reveals that step 2 and 3 of question 6, were a bit difficult. A few students were not able to respond to those steps correctly, cause is that exercises were in released form. Other reasons were that some of the learners were comparatively slow. However, more than 90 percent learners were able to calculate 85 percent of the exercises with almost cent –percent accuracy.

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