A Non-cooperative Game Approach for RAN and Spectrum Sharing in Mobile Radio Networks

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***Abstract*—Mobile Network Operators (MNOs) are nowadays forced to continuously invest in their network infrastructure to keep up with the increasing bandwidth demand and traffic load coming from mobile users. In this context, MNOs have to face the strategic problem of whether to invest on their own or deploy shared networks. We address here the problem of Radio Access Network (RAN) and spectrum sharing in 4G mobile networks. Namely, we consider the case in which multiple MNOs are planning to deploy small cell Base Stations to improve their current network infrastructure; the deployment investment may be shared with other MNOs, thus giving rise to shared RANs. The RAN and spectrum sharing problem is formalized as a Generalized Nash Equilibrium Problem, where the strategy of each MNO in the game is twofold: selecting a coalition (whom to cooperate) and the fraction of the coalition cost to pay, with the goal of maximizing the individual return on investment. The proposed approach is leveraged to characterize the stable coalitions and their respective cost division policies for various network and economic conditions.**

I. INTRODUCTION

The exponential growth of mobile data and the increasing diffusion of “bandwidth-eager” user applications [1] is pushing the migration to more spectrally efficient mobile technologies such as LTE-A and eventually 5G. However, individual network roll-outs represent large sunk costs for Mobile Network Operators (MNOs) and particularly for new-entrants[1] [3] especially given the currently unaligned growth of data demand from MNO revenues. Sharing agreements for greenfield network deployments are an attractive alternative to cut down on the upfront infrastructure cost but also, when spectrum sharing is allowed, to benefit from aggregating spectrum resources, where the latter is essential to boost network capacity.

There are several alternatives for network sharing depending on the “depth” of the network architecture affected by sharing and/or its “scale”, that is, geographical footprint [4,5]. While passive sharing (site and mast sharing) is nowadays either mandated by regulators or voluntary adopted due to restrictions on the number of available sites [5,6], there are also several examples of 50:50 joint ventures for 3G/4G greenfield network deployments in which MNOs share the Radio Access Network (RAN) and, in some cases, also the spectrum [7].

In this work, we investigate a network sharing scenario which is referred to as *common spectrum network sharing* in the Third Generation Partnership Project (3GPP) specifications for network sharing [8]. Namely, we consider the case in which several MNOs operate through a single (shared) RAN but keep separate core networks; “MNOs share the total spectrum obtained from pooling together their respective allocated spectrum portions while it is also possible for MNOs with no allocated spectrum to use the pooled spectrum” [8]. Implementation-wise, a candidate enabler of such scenarios for 4G networks is Carrier Aggregation (CA), a standardized feature of LTE-A [9] that allows to boost the network capacity by pooling together the spectrum allocated in different bands[2].

We consider a set of MNOs with consolidated market shares and fixed allocated spectrum, which have plans to upgrade their RAN by investing in the deployment of small cell Base Stations (BSs) so as to improve the service and increase the revenues. Each MNO has to face the strategic problem of whether to upgrade its RAN by itself or to sign a sharing agreement with other MNOs. In case sharing agreements are put in place, we assume that each coalition of MNOs sharing the RAN will use all the aggregated spectrum resources of its members. We resort to a non-cooperative game theory approach and model the RAN/spectrum sharing problem as a Generalized Nash Equilibrium Problem[3] (GNEP) [11], where the strategy of each MNO consists of selecting a coalition and a fraction of the coalition cost to pay so that its individual return on investment is maximized. We characterize the stable coalitions and their respective cost division policies for different networks (user throughput, market and spectrum shares) and economic settings (coalition cost, mobile data pricing model).

The manuscript is organized as follows: A short literature review is presented in Section II; the problem and the GNEP formulation are stated in Section III; Sections IV and V

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[1] Despite some countries regulator efforts to encourage competition in mobile networks, by introducing spectrum set-asides during auctions and relaxing their coverage requirements, new entrants do not always succeed in deploying a network which may lead to inefficient spectrum allocations or eventually with the set-aside spectrum ending up in the hands of incumbent MNOs [2].

[2] According to [10], 116 operators have commercially launched LTE-A with CA. Moreover, given the throughput targets of 5G, CA will most likely be an enabler for future generation networks.

[3] In GNEPs the strategy of each player affects not only the other players' payoff, as in Nash Equilibrium Problems, but also their strategy domains.

describe the considered scenarios and analyze the equilibria. Concluding remarks are drawn in Section VI.

II. RELATED WORK

Game theory has been largely used for addressing several problems arising in HetNets [12] and in particular to model resource and network sharing problems. Non-cooperative games have been adopted in [13] and [14]: [13] models spectrum sharing in unlicensed bands among selfish MNOs, whereas [14] models problems of network selection in a heterogeneous wireless access network scenario. Instead, [15] and [16] resort to cooperative game theory. Hew *et al.* ([15]) model the problem of resource allocation in a shared network in two steps: the resource sharing among the operators, and the resource bargaining among the users and Mobile Virtual Network Operators of each operator. The sharing of different wireless access technologies among operators is considered in [16]. Along the same lines, [17] investigates the sharing between LTE access network femtocells and a Wi–Fi access network.

In the context of infrastructure and spectrum sharing, coalition formation is addressed in [18]–[20] through non-cooperative games. These works bear similarities with ours as they also aim at determining stable coalitions for a finite set of MNOs with fixed market shares and pre-allocated spectrum. However, players (MNOs) payoffs are expressed only in terms of network cost estimates and in a coalition such costs are split either uniformly among its members [18,19] or according to the Shapley value [20]. Differently, we propose here more refined payoff models for the MNOs: in fact, we account for both the MNO revenue (as a function of the average user rate perceived by users) and cost in determining the MNO's payoff for a given strategy profile where, in particular, the share of cost paid by an MNO in a coalition is part of its strategy and not set a priori. In our work, the coalition formation is explicitly modeled as a GNEP, which has been lately used to model problems concerning players that share a common resource (e.g., in telecommunications, the power allocation problem for a Digital Subscriber Line [11]).

III. THE PROBLEM

A. Problem statement

We consider a set $\mathcal{O}$ of MNOs that coexist in a dense urban area where they provide data services through pre-4G macrocell networks but aim at upgrading their radio access technology by deploying 4G small cells. We assume MNOs inherit the share of users from their individual up and running networks, that is, being N the number of users in the given area, each MNO $i \in \mathcal{O}$ has a fixed market share σ_i. We further assume that at least one of the MNOs owns a spectrum license[4] of b_i units (possibly zero) of bandwidth which it plans to exploit for the network of small cells. Each MNO may decide to deploy its individual network of small cells or to collaborate with other MNOs to deploy a shared one. If at least two MNOs deploy a shared network, we assume they will agree on aggregating their available spectrum. We denote by $\mathcal{S}$ the set of all possible coalitions that can be created, that is, the set of all the possible subsets of MNOs agreeing to deploy a shared network and by $\mathcal{S}_i$ the set of the coalitions MNO i can join. If all the MNOs in $s \in \mathcal{S}$ decide to join coalition s, then s deploys a shared network infrastructure of total cost $\widetilde{c}_s$ which has to be split among its members, that is, each MNO $i \in s$ has to pay a fraction of $\widetilde{c}_s$ while incurring revenues $\widetilde{r}_s^i$ (see Section III-C). The strategy of MNO i consists of selecting a coalition from $\mathcal{S}_i$ and the fraction of its cost to pay that maximizes its return on investment, that is, the difference between its revenues $\widetilde{r}_s^i$ and the fraction of $\widetilde{c}_s$ it is accounted for. Since a coalition cannot be created unless selected by all its members and the total cost of a coalition has to be split among them, it is clear that the strategy of each MNO depends on the strategies of the other MNOs: we formulate this problem as a GNEP where the set of players coincides with the set of MNOs.

B. Generalized Nash game model

Since the strategy of each MNO $i \in \mathcal{O}$ consists of selecting a coalition and deciding the fraction of the coalition cost it will pay, we define two families of variables $\{x_s^i \in \{0,1\}\}_{s \in \mathcal{S}_i}$ and $\{\alpha_s^i \in [0,1]\}_{s \in \mathcal{S}_i}$. Variable x_s^i equals 1 if MNO i would like to join coalition $s \in \mathcal{S}_i$ and 0 otherwise; variable α_s^i represents the fraction of the cost $\widetilde{c}_s$ MNO i is willing to pay. However, coalition s cannot be created unless selected by all its members. Therefore we define variables $\{y_s^i \in \{0,1\}\}_{s \in \mathcal{S}_i}$, where y_s^i equals 1 if MNO i really joins the coalition s since all the members of s are willing to join s (i.e., $x_s^j = 1$ for any $j \in s$). Roughly speaking, x_s^i represent what the player would like to do, while y_s^i represent what it really does. The utility of MNO i from coalition s is then equal to $\tilde{r}_s^i y_s^i - \tilde{c}_s \alpha_s^i$. Therefore, the problem of MNO i can be formulated as follows, where the variables controlled by MNO i are typed in bold:

$$\max \sum_{s \in \mathcal{S}_i} (\widetilde{r}_s^i \boldsymbol{y}_s^i - \widetilde{c}_s \boldsymbol{\alpha}_s^i) \tag{1}$$

$$\sum_{s \in \mathcal{S}_i} \boldsymbol{x}_s^i \leq 1 \tag{2}$$

$$\boldsymbol{y}_s^i \leq x_s^j, \quad \forall s \in \mathcal{S}_i, \, \forall j \in s \tag{3}$$

$$\boldsymbol{y}_s^i = \boldsymbol{\alpha}_s^i + \sum_{j \in s \setminus \{i\}} \alpha_s^j, \quad \forall s \in \mathcal{S}_i \tag{4}$$

$$\boldsymbol{x}_s^i \in \{0,1\}, \boldsymbol{y}_s^i \in \{0,1\}, \boldsymbol{\alpha}_s^i \in [0,1], \quad \forall s \in \mathcal{S}_i \tag{5}$$

The objective function (1) maximizes the sum over all coalitions of the utility of MNO i, namely its return on investment. Constraint (2) makes sure that MNO i becomes part of at most one coalition, whereas constraints (3) guarantee that MNO i selects only an active coalition, i.e., a coalition selected by all of its members. Constraints (4) set to zero all the fractions of cost α_s^i for the coalitions MNO i is not part of and guarantee that the total cost of an active coalition is split

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[4] A spectrum license is usually purchased at the time spectrum auctions are organized and therefore it is not immediately associated to any network infrastructure although the license acquisition requires for services to be launched within given deadlines. Moreover, 2G/3G networks spectrum licenses are being refarmed for higher spectral efficiency technologies such as 4G [21].

among its members, that is, given the fraction of cost MNOs $j \in s\backslash\{i\}$ are willing to pay, MNO i should pay the remaining cost fraction. Finally, (5) gives the variable domains.

Notice that, for each coalition $s \in \mathcal{S}$, constraints (3) and (4) make the strategy space of each MNO $i \in s$ dependent on the strategies selected by MNOs $j \in s \setminus \{i\}$.

We look for equilibria for this problem, which are all the coalitional structures such that each MNO belonging to a coalition with at least two members cannot improve its profit by leaving the coalition.

C. *Cost and revenues definition*

Since the RAN cost is the dominant component of the total cellular network cost [22], we limit the cost analysis only to radio equipments and adopt a simplified leased line pricing model for the backhaul transmission cost as in [23]. Let D be the investment period and g_s the total cost incurred in D from a single small cell BS activated by coalition s, which accounts for the radio equipment capital (CAPEX) and operational (OPEX) expenditures, backhauling cost and the site buildout cost. Let $\widetilde{b}_s$ be the aggregated bandwidth of coalition s, that is, $\widetilde{b}_s = \sum_{i\in s} b_i$, whereas β_s denotes the number of MNOs in s that have a spectrum license, that is, $\beta_s = |\{i \in s : \; b_i > 0\}|$. The radio equipment CAPEX $g_s^{\mathrm{c,r}}$ consists of the cost $g_{\mathrm{small}}^{\mathrm{c,r}}$ of a single-carrier small cell BS and of a fixed radio equipment cost per additional carrier supported by a single BS calculated as a percentage ϕ of the cost $g_{\mathrm{macro}}^{\mathrm{c,r}}$ of a single-carrier macrocell BS (as in [24]):

$$g_s^{\mathrm{c,r}} = g_{\mathrm{small}}^{\mathrm{c,r}} + (\beta_s - 1)\phi g_{\mathrm{macro}}^{\mathrm{c,r}}. \tag{6}$$

The backhaul leased line pricing model consists of an upfront fee $g^{\mathrm{c,b}}$ and the annual leasing cost $g_s^{\mathrm{o,b}}$ that we assume, in the worst case, will be proportional to $\widetilde{b}_s$. Thus, being $g_0^{\mathrm{o,b}}$ the annual leased line cost for a b_0 (units of bandwidth) carrier, we set $g_s^{\mathrm{o,b}}$ equal to $\frac{\widetilde{b}_s}{b_0} g_0^{\mathrm{o,b}}$. The Operations and Maintenance (O&M) annual cost of the radio equipment is calculated as a percentage ξ of the corresponding total radio CAPEX [23, 25]. Therefore, the total cost g_s incurred by coalition s from deploying and operating a small cell BS in D is given by:

$$g_s = g_s^{\mathrm{c,r}} + g^{\mathrm{c,b}} + g^{\mathrm{c,s}} + \frac{D}{12}\left(\xi g_s^{\mathrm{c,r}} + g_s^{\mathrm{o,b}}\right). \tag{7}$$

We remark that the considered cost values refer to HSPA technology as in [23,24], given that, to the best of our knowledge, CA-enabled equipment cost are not made publicly available by any vendor. However, in [24] it is argued that costs of the physical infrastructure of new radio access technologies tend to be similar to previous ones, therefore such costs represent a good estimate at least in orders of magnitude.

The revenues $\widetilde{r}_s^i$ incurred by MNO i in coalition s are calculated based on a simple data service pricing model defined as a function of the average data rate perceived by users of s as in [26]. Let $\rho_s^{nom}(u_s)$ be the nominal user rate coalition s can obtain activating u_s BSs. In LTE, the nominal user rate is the maximum achievable rate for a certain level of Signal to Interference and Noise Ratio (SINR) and a given system bandwidth perceived by a single user when assigned all downlink LTE resource blocks from its serving BS. The downlink SINR is a function of the number of BSs activated by the coalition the user belongs to, as a larger number of BSs results in the user being on the average closer to its serving BS, and therefore receiving a stronger signal, but also closer to the interfering ones[5]. The average rate $\rho_s(u_s)$ perceived by a user in coalition s can be expressed as a function of $\rho_s^{nom}(u_s)$ and of the load of its serving BS:

$$\rho_s(u_s) = \rho_s^{nom}(u_s)(1-\eta)^{\left(\sum_{i\in s}\sigma_i N\right)/u_s}, \qquad \forall s \in \mathcal{S}, \tag{8}$$

where parameter η is the user activity factor representing the probability that a user is actually active in his/her serving BS, $\sum_{i\in s}\sigma_i N$ is the total number of users of coalition s whereas $(\sum_{i\in s}\sigma_i N)/u_s$ gives the average number of users served by one BS. The nominal rate is therefore scaled down by the factor $(1-\eta)^{\left(\sum_{i\in s}\sigma_i N\right)/u_s}$ representing the average congestion level at a serving BS.

Let δ denote the monthly price per user and per unit (Mbps) of data service. Given an average user rate $\rho_s(u_s)$ provided by coalition $s \in \mathcal{S}_i$ by activating u_s BSs (see Section IV), the revenues r_s^i incurred from MNO i when in s, at the end of the investment lifetime D are modeled as a linear function of $\rho_s(u_s)$:

$$r_s^i = \delta D \sigma_i N \rho_s(u_s), \qquad \forall i \in s. \tag{9}$$

Let $\widetilde{u}_s$ denote the number of BSs that maximizes the global return on investment of coalition s calculated according to Equation (10):

$$\widetilde{u}_s = \operatorname*{argmax}_{\substack{u_s\in\mathbb{Z}_+\\ u_s\le U_{\max}}} \left(\sum_{i\in s}\delta D\sigma_i N\rho_s(u_s) - g_s u_s\right), \tag{10}$$

where $U_{\max}$ represents the maximum number of small cell sites coalition s can activate in the area. Finally, the revenues $\widetilde{r}_s^i$ of MNO i from coalition s and the total cost $\widetilde{c}_s$ of coalition s are given as follows:

$$\widetilde{r}_s^i = \delta D\sigma_i N\rho_s(\widetilde{u}_s), \qquad \forall s \in \mathcal{S},\, \forall i \in s, \tag{11}$$

$$\widetilde{c}_s = g_s \widetilde{u}_s, \qquad \forall s \in \mathcal{S}. \tag{12}$$

IV. COMPUTATIONAL SETTING

A. *Parameter setting*

Parameters are set as shown in Table I.

We consider 3 MNOs A, B and C and a 4 km^2 area populated by 20000 users. We set $U_{\max}$ to 10000, which is an arbitrarily large number of BSs for the considered area size. Nevertheless, the number of activated BSs by any coalition does not exceed 1500 for all the considered instances. We report the key results for two values of δ: a low ($\delta = 0.75$) and a high value ($\delta = 2$). We set up 5 scenarios representing different mixtures of market shares and "spectrum shares"[6] as shown in Table II, where values of b_i, that is, of the bandwidth

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[5] Any other BS transmission will use at least a subset of the available resource blocks and therefore unavoidably interfere.

[6] The term "spectrum share" is used analogously with market share to represent, $b_i/\sum_{j\in\mathcal{O}} b_j$, that is, the weight of the spectrum of an MNO w.r.t. to the total obtained aggregating the spectrum of all MNOs.

Symbol	**Description**	**Value**
$\mathcal{O}$	Set of MNOs	$\{$A,B,C$\}$
$\mathcal{S}$	Set of coalitions	$2^{\mathcal{O}} \setminus \emptyset$
$\mathcal{S}_i$	Set of coalitions MNO $i \in \mathcal{O}$ can join	
N	Total number of users in the area	20000
$\mathcal{A}$	Area size	4 km^2
U_{max}	Max. number of BSs in the area	10000
δ	Monthly price of 1 Mbps	$\{$0.75,2$\}$€/Mbps
D	Investment lifetime	120 months [27]
η	User activity factor	0.01
$g^{\text{c,r}}_{\text{small}}$	Single-carrier small cell BS radio equipment cost	3000€ [23]
$g^{\text{c,r}}_{\text{macro}}$	Single-carrier macro cell BS radio equipment cost	20000€ [23]
ϕ	Cost coefficient per additional carrier	0.017 [24]
$g^{c,b}$	Upfront fee for backhaul	2000€ [23]
$g^{\text{o,b}}_0$	Annual leased line cost for carrier b_0=5MHz	2000€ [23]
$g^{\text{c,s}}$	Site buildout cost	2000€ [23]
ξ	O&M annual percentage	15% [25]

TABLE I: Sets, parameters and corresponding values

associated with the spectrum license of each MNO, are set to standardized bandwidths for LTE/LTE-A ($\{$1.4, 3, 5, 10, 15, 20$\}$ MHz). Scenarios S4 and S5 represent particular cases that may arise given how spectrum auctions are currently designed. We consider an extreme case in which there is only one MNO in the area that has managed to obtain a spectrum license from a recent auction: either the *smallest MNO* (S4), for instance, a new entrant which has benefited from the set-aside spectrum policy [3], or *the incumbent* (S5), which is the most likely to be the highest bidder.

	S1			**S2**			**S3**			**S4**			**S5**		
	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
σ_i	1/3	1/3	1/3	1/3	1/3	1/3	0.1	0.3	0.6	0.1	0.3	0.6	0.1	0.3	0.6
b_i	5	5	5	1.4	5	10	5	5	5	15	0	0	0	0	15

TABLE II: Scenarios

B. Simulation environment

A simulation environment was set up to derive the average user rate $\rho_s(u_s)$ for each coalition s as function of the number u_s of activated small cell BSs varying from 1 to U_{max} as in [26]. The u_s BSs and 10 sample users are uniformly distributed in a pseudo-random fashion on the considered square area. The downlink SINR of each sample user for a reference system bandwidth is obtained scaling down the interference perceived by the sample user with the load of coalition s, $l_s = 1 - (1-\eta)^{\left(\sum_{i \in s} \sigma_i N\right)/u_s}$, since users are characterized by an activity factor η. The calculated SINR is mapped to LTE spectral efficiency according to a multilevel SINR–to–spectral efficiency scheme [28]. The spectral efficiency is then multiplied by the coalition aggregated bandwidth $\widetilde{b}_s$, to obtain the nominal user rate $\rho_s^{nom}(u_s)$. The simulation is run 100 times for each value of u_s so that an average value for $\rho_s^{nom}(u_s)$ is obtained across all sample users and runs. Finally, $\rho_s(u_s)$ is derived from $\rho_s^{nom}(u_s)$ according to Equations (8).

C. Equilibria calculation

We calculate the equilibria of the GNEP approximating it with a discrete strategy space. For each MNO i and for each coalition $s \in \mathcal{S}_i$ we discretize the cost fraction α_s^i with a step of 0.01 and thus the strategy set for each player i consists of the Cartesian product between the finite set of possible cost fractions $\{0, 0.01, 0.02, \dots 1\}$ and the set $\mathcal{S}_i$. Strategy profiles that lead to infeasible outcomes (e.g., the cost fractions of a coalition do not sum to 1 or coalitions selected by MNOs do not coincide) are discarded. Among the equilibria of the discretized GNEP *Pareto dominated* ones are identified: if E_1 and E_2 are two equilibria and p_1^i and p_2^i the corresponding payoff of each player i, then E_1 is Pareto dominated by E_2 if $p_2^i \geq p_1^i$ for any i and E_2 is strictly preferred over E_1 by at least one player.

V. ANALYSIS OF NUMERICAL RESULTS

For 3 MNOs, there are 5 possible coalitional structures: *all MNOs build separate networks* (A,B,C), *2 MNOs build a shared network whereas the third one builds its own* (AB,C)/(AC,B)/(BC,A) and *all MNOs collaborate and build a single/shared network* (ABC). Tables III–XII illustrate the obtained equilibria for each considered scenario (Table II) for the two values of δ: for each coalitional structure representing an equilibrium (row), we report the range of stable percentage of costs for each member (column).

For instance, in scenario S1 (Table III) (AB,C) is an equilibrium if A pays at least 39% of the cost of coalition (AB) since B is willing to pay at most 61% of the cost and vice versa since MNOs have equal market shares. C instead builds its own network and pays all of its costs (represented by a cost fraction 100%). The color code is the following: dark grey cells represent cases where no network of small cells is built because it is either not profitable, that is, MNOs obtain a non-negative return on investment (e.g., MNO A has only 1.4 MHz of bandwidth and δ is low (Table V)) or it is not feasible since no MNO in the coalition has a spectrum license (e.g., in scenario S4 (Tables IX, X), B and C have no spectrum license and therefore cannot invest neither together nor by themselves regardless of δ). Equilibria that are not Pareto dominated are represented by white cells whereas Pareto dominated ones are highlighted in grey. However, for the same coalitional structure there can be both dominated and non-dominated equilibria depending on how the coalition cost is split among its members (see, e.g., Table IX: for $\delta = 0.75$, if the cost of (AC) is split between A and C as 0%:100%, (AC,B) is not Pareto dominated, but it is if split, for instance, as 15%:85%).

A. Equilibria and scenarios

For certain ranges of cost fractions, all the coalitional structures are equilibria for all the considered scenarios. — (A,B,C) is always an equilibrium of the game since, given that 2 MNOs decide to invest by themselves, the third one has no choice but to do the same. Instead, since each MNO has always the choice of investing by itself given any strategy of the other MNOs, the fact that any coalition of 2 (AB, AC, BC) or the grand coalition (ABC) are equilibria for certain ranges of cost fractions means that any MNO is better off collaborating with at least another MNO, due either to the combined gain of spectrum pooling and cost sharing (scenarios S1–S3) or purely to benefit of sharing the network cost (scenarios S4 and S5, for which only one MNO has a spectrum license).

B. Cost repartition and δ

Cost repartition is influenced by the value of δ — For instance, in scenarios S4 and S5, when MNOs gain little for each unit of service (low δ), the maximum the MNO with a spectrum license would pay to be in a coalition does not exceed its share of users in the coalition (Tables IX, XI): in scenario S4 (Table IX), MNO A, which is the only one with a spectrum license, is willing to pay at most 10% of the (ABC) cost, 27% of the (AB) cost and 15% of the (AC) cost which approximately represent its respective user share in these coalitions. Instead, as B and C own no spectrum license, they are willing to pay from 8 to 24% more than their respective user shares in any coalition in which A is a member. When MNOs gain more from their investment (high δ), the MNO which brings the spectrum to the coalition can leverage it to the point of not paying any network cost at all. For instance, in scenario S5, $\delta = 2$ (Table XII), in equilibrium (ABC) with cost split as, e.g., 25%:75%:0% among A, B and C, respectively, C pays no cost for the deployed network while A and B share the cost proportionally to their market shares. Similarly for scenario S4 (Table X).

C. Cost repartition, market share and spectrum share

Given a coalitional structure, the equilibrium coalition cost division among its members reflects their user and spectrum shares — For our baseline scenario S1 (Tables III, IV) the symmetry in both the market and spectrum shares is reflected in equal ranges of cost fractions for all members of a coalition. In scenario S2 (Tables V, VI), in which MNOs have equal market shares but different spectrum shares, the larger the amount of spectrum an MNO brings to a coalition the smaller the minimum cost fraction it has to pay to be part of it, which is amplified by the increase of δ. For instance, for $\delta = 0.75$, MNO C contributes with 2/3 of the aggregated spectrum of coalition (BC), and has to pay at least 33% of its costs whereas B at least 42%, whereas for $\delta = 2$, their cost fractions are reduced to 3% and 26%, respectively. Scenario S3 (Tables VII, VIII) illustrates the impact of the market share in the coalition cost division when MNOs have equal spectrum shares. We can see how the maximum cost fraction they are willing to pay to be in any coalition reflects approximately their user shares (see e.g., equilibrium (ABC) in Tables VII and VIII, where although the maximum cost fraction increases with δ for each MNO, the relative difference tends to follow the user shares; e.g., B is willing to pay approximately 3 times more w.r.t. A to be in (ABC) for both values of δ).

In scenarios S1–S3, as the price per unit of service increases, so does the fraction of cost MNOs are willing to pay to be in any coalition. For instance, in scenario S3, B would pay up to 40% of the costs of coalition (BC) for $\delta = 0.75$ but up to 56% for $\delta = 2$, while C goes from 80% to 100%, which means on the other hand that the minimum cost fraction they should pay becomes smaller as δ increases. Similarly for all other coalitions (Tables VII, VIII). In scenarios S4 and S5 instead, it is only the MNOs with no spectrum license that significantly increase the maximum cost fraction they are willing to pay to be in any coalition in which the MNO with a spectrum license is part of. For instance, in scenario S4, MNO A (the only one owning a spectrum license) pays up to 10% of the cost of (ABC) for $\delta = 0.75$ but at most 9% for $\delta = 2$, while the other 2 MNOs would pay more with the increase of δ (Tables IX, X). In scenario S5 instead, in which the only MNO with a spectrum license, C, is also the incumbent, it would pay only 3% more to be in (BC), 2% more to be in (AC) and only 1% more to be in (ABC) as δ grows from 0.75 to 2 (Tables XI, XII).

D. Dominated and non-dominated equilibria

All grand coalition (ABC) equilibria are not Pareto dominated by any other equilibrium for all considered instances — In scenarios S1–S3 and S5, for each equilibrium of a coalitional structure different from the grand coalition (ABC) there is an (ABC) equilibrium that Pareto dominates it, which shows the gain of spectrum pooling. Instead, in scenario S4 (Tables IX, X), there are also other non-dominated coalitional structure equilibria, e.g., for $\delta = 0.75$, (AC,B) with the cost of AC split as 0%:100%. While B clearly prefers (ABC) over (AC,B) since it cannot build a network by itself without a spectrum license, interestingly A prefers (AC) to (ABC), even if it were to pay no fraction of the (ABC) cost, as (AC) is a less congested coalition and all of its cost is paid by the incumbent MNO (C). Instead, C prefers coalition (ABC) to (AC) when it pays up to 70% of the (ABC) cost, otherwise it prefers to pay for all the cost of (AC).

	A	**B**	**C**
(**A**),(**B**),(**C**)	100%	100%	100%
(**AB**),(**C**)	39-61%	39-61%	100%
(**AC**),(**B**)	39-61%	100%	39-61%
(**BC**),(**A**)	100%	39-61%	39-61%
(**ABC**)	18-41%	18-41%	18-41%

TABLE III: S1, $\delta = 0.75$

	A	**B**	**C**
(**A**),(**B**),(**C**)	100%	100%	100%
(**AB**),(**C**)	10-90%	10-90%	100%
(**AC**),(**B**)	10-90%	100%	10-90%
(**BC**),(**A**)	100%	10-90%	10-90%
(**ABC**)	0-69%	0-69%	0-69%

TABLE IV: S1, $\delta = 2$

	A	**B**	**C**
(**A**),(**B**),(**C**)		100%	100%
(**AB**),(**C**)	46-55%	45-54%	100%
(**AC**),(**B**)	48-64%	100%	36-52%
(**BC**),(**A**)		42-67%	33-58%
(**ABC**)	20-42%	20-42%	16-38%

TABLE V: S2, $\delta = 0.75$

	A	**B**	**C**
(**A**),(**B**),(**C**)	100%	100%	100%
(**AB**),(**C**)	36-100%	0-64%	100%
(**AC**),(**B**)	42-100%	100%	0-58%
(**BC**),(**A**)	100%	26-97%	3-74%
(**ABC**)	0-82%	0-71%	0-55%

TABLE VI: S2, $\delta = 2$

	A	**B**	**C**
(**A**),(**B**),(**C**)	100%	100%	100%
(**AB**),(**C**)	11-30%	70-89%	100%
(**AC**),(**B**)	0-18%	82-100%	100%
(**BC**),(**A**)	100%	20-40%	60-80%
(**ABC**)	0-12%	14-37%	51-74%

TABLE VII: S3, $\delta = 0.75$

	A	**B**	**C**
(**A**),(**B**),(**C**)	100%	100%	100%
(**AB**),(**C**)	0-41%	59-100%	100%
(**AC**),(**B**)	0-24%	100%	76-100%
(**BC**),(**A**)	100%	0-56%	44-100%
(**ABC**)	0-20%	0-62%	18-100%

TABLE VIII: S3, $\delta = 2$

	A	**B**	**C**
(**A**),(**B**),(**C**)	100%		
(**AB**),(**C**)	1-27%	73-99%	
(**AC**),(**B**)	1-15%		85-99%
	0%		100%
(**BC**),(**A**)	100%		
(**ABC**)	0-10%	13-38%	52-77%

TABLE IX: S4, $\delta = 0.75$

	A	**B**	**C**
(**A**),(**B**),(**C**)	100%		
(**AB**),(**C**)	5-24%	76-95%	
	0-4%	96-100%	
(**AC**),(**B**)	5-14%		86-95%
	0-4%		96-100%
(**BC**),(**A**)	100%		
(**ABC**)	0-9%	0-76%	15-100%

TABLE X: S4, $\delta = 2$

	A	**B**	**C**
(**A**),(**B**),(**C**)			100%
(**AB**),(**C**)			100%
(**AC**),(**B**)	16-18%		82-84%
(**BC**),(**A**)		36-41%	59-64%
(**ABC**)	4-12%	30-38%	50-58%

TABLE XI: S5, $\delta = 0.75$

	A	**B**	**C**
(**A**),(**B**),(**C**)			100%
(**AB**),(**C**)			100%
(**AC**),(**B**)	14-29%		71-86%
(**BC**),(**A**)		33-80%	20-67%
(**ABC**)	0-25%	16-76%	0-59%

TABLE XII: S5, $\delta = 2$

VI. Conclusions

In this work, we model the strategic problem of coalition formation and coalition cost division in a RAN and spectrum sharing scenario as a GNEP. We consider multiple MNOs with fixed market shares and allocated spectrum that are planning to upgrade their radio access of a dense urban area by deploying small cell BSs. Each coalition of MNOs is assumed to exploit all the aggregated spectrum resources of its members and deploy a number of BSs that maximizes its global profit. The strategy of each MNO consists of selecting a coalition and a fraction of the coalition cost to pay which maximizes its individual return on investment. A numerical analysis of several scenarios representing different network and economic configurations shows how for certain ranges of cost fractions coalitions of at least two MNOs are always equilibria and that the grand coalition equilibria are never Pareto dominated. This shows how spectrum pooling and cost sharing incentivize shared network configurations, while the stable cost divisions reflect the MNOs' relative user and spectrum shares.

The proposed GNEP can be a starting point for more complex scenarios where MNOs are able to differentiate service offered to their users and/or adopt different pricing models.

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