2022

Development of an App and Videos to Support the Fraction Learning Trajectory from Grades 1-7

Debbie Marie Verzosa

Ma. Louise Antonette N. De Las Peñas

Maria Alva Q. Aberin

Agnes Garciano

Jumela F. Sarmiento

See next page for additional authors

Authors
Debbie Marie Verzosa, Ma. Louise Antonette N. De Las Peñas, Maria Alva Q. Aberin, Agnes Garciano, Jumela F. Sarmiento, Juan Carlo F. Mallari, and Mark Anthony C. Tolentino

Iyer, S. et al. (Eds.) (2022). Proceedings of the 30th International Conference on Computers in Education. Asia-Pacific Society for Computers in Education

Development of an App and Videos to Support the Fraction Learning Trajectory from Grades 1-7

Debbie Marie B. VERZOSA[a*], Ma. Louise Antonette N. DE LAS PEÑAS[b], Maria Alva Q. ABERIN[b], Agnes D. GARCIANO[b], Jumela F. SARMIENTO[b], Juan Carlo F. MALLARI[b], & Mark Anthony C. TOLENTINO[b*]
[a]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*
[b]*Department of Mathematics, Ateneo de Manila University, Philippines*
* dmbverzosa@usm.edu.ph, mtolentino@ateneo.edu

Abstract: Lack of procedural fluency in fractions impedes access to advanced mathematical courses and limits opportunities for entry into STEM-related fields. This paper describes the design and pedagogical basis of the *Moving Fractions* app and supplementary fraction videos for promoting fraction learning. *Moving Fractions* utilizes game-design factors to draw students through a trajectory of fraction learning from part-whole comparisons to a more robust understanding of the measurement concept of fractions. The supplementary video immerses students in a broad range of fraction representations. The app and video are intended to form a fraction learning package for distribution in Philippine schools. Future work involves the gathering of empirical data for validating the expected benefits associated with the application of mathematics education research in the app and video design.

Keywords: mathematical app, fraction, STEM, educational video

1. Introduction

Mathematics plays a fundamental role in STEM education. Students entering STEM-related fields are those who typically perform well in high school mathematics and have positive attitudes towards mathematics (Wang, 2013). Specifically, procedural fluency in fractions was found to improve access to advanced mathematical courses (Ngo, 2018). Students who do not learn fractions properly tend to perform poorly in subsequent mathematical courses (Booth & Newton, 2012), and may demonstrate limited mathematical competencies (Schneider et al., 2018). Unfortunately, many students and even pre-service teachers do not have a robust understanding of fractions (Bansilal & Ubah, 2020). Therefore, early interventions in fraction learning are important to increase opportunities for entry into STEM-related fields.

The design of games and game-based applications (apps) in classrooms has drawn the attention of mathematics education researchers for decades (Larkin, 2013). Games have been shown to increase students' engagement and motivation (Erhel & Jamet, 2013), and increase mathematics performance in various domains such as fractions (Ninaus et al., 2016). This paper discusses the *Moving Fractions* app for PC and Android that has been designed and developed to support the learning of fractions. The app is one of the mathematical resources that has been developed under an ongoing government-funded project (De Las Peñas et al., 2021), one of the goals of which is to strengthen further the mathematical competencies in foundational concepts necessary for more advanced learning. These include place value, number sense and fraction number sense. Aside from the app, mathematical videos were also created for students to help them visualize abstract ideas. In the past two years of the pandemic, these resources have been useful in support of the blended learning modality in schools called for by the Philippine Department of Education (DepEd). The resources for instance, can be used with minimal supervision, and are ideal for distance learning. An interested reader is invited to visit https://mathplusresources.wordpress.com/ for access to the mathematical resources.

2. Pedagogical Basis: Fraction Understanding

Learning fractions offers substantial challenges for students because traditional instruction does not present fractions in a precise and coherent way (Wu, 2010). Fractions are typically presented as part-whole comparisons (like pieces of pizza), leaving students with an impoverished understanding of fractions (Lamon, 2007). To learn fractions meaningfully, students must learn what fractions are, and how correct procedures can be logically derived from the definition (Wu, 2010). According to Wu (2010), the fraction 1/*n* is defined to be the point on the number line corresponding to dividing the unit segment into *n* equal parts, and taking the first division point to the right of 0; the fraction *m*/*n* is the *m*th multiple of the fraction 1/*n*. Wu argued that a definition based on number lines is more superior than part-whole conceptions of fractions because all basic facts about fractions can be derived from the number line, but not from part-whole knowledge. Some educational systems also present alternative representations such as area and discrete models as a starting point before a number line model is introduced (Alajmi, 2012).

Instruction based on measurement concepts of fractions (i.e., thinking of *m/n* as *m* measures of 1/*n*; see Kieren, 1980) offers the strongest conceptions of fractions, allowing students to apply their knowledge to new problems and form alternative conceptions of fractions that were not taught (Lamon, 2007). The measurement construct of fractions was further delineated by Wilkins and Norton (2018). They summarized studies involving more than 300 students to propose a hierarchy of fraction schemes, beginning with part-whole to measurement concepts of fractions: (a) part-whole scheme (PWS), (b) measurement scheme for unit fractions (MSUF), (c) measurement scheme for proper fractions (MSPF), and (d) generalized measurement scheme for fractions (GMSF). The PWS and MSUF are loosely inverses of each other–partitioning a whole into *n* parts versus determining the fractional size of a unit fraction by iterating the unit fraction to produce the whole. The action associated with MSUF involves producing a whole from a proper fraction, and the action associated with GMSF involves producing a whole from any fraction (including improper fractions). The results collectively show that these schemes developed sequentially (i.e., the development of PWS preceded the development of MSUF, and so on).

These fraction schemes are aligned with the Philippine DepEd Most Essential Learning Competencies (MELCS) (DepEd, 2020), where students learn fractions in progression: from ½ and ¼ (Grade 1), followed by unit fractions and proper fractions (Grade 2), improper fractions (Grade 3), equivalent fractions and addition and subtraction of fractions (Grade 4). An app and supplementary videos based on this hierarchy are expected to develop a more robust understanding of fractions, compared to learning materials concentrating on part-whole comparisons alone (Lamon, 2007; Wilkins & Norton, 2018).

3. The *Moving Fractions* App

3.1 App Description

In *Moving Fractions,* a learner may choose to play the following levels: ½ and ¼ fractions (Figure 1(a)), unit fractions (Figure 1(b)), proper fractions (Figure 1(c)), improper fractions (Figure 1(d)) and proper, improper, and mixed fractions combined. A fraction is given on the top of the screen, such as 1/3 in Figure 1(b), while partitioned rectangles or fraction bars continuously move to the right. The learner must click on any row where the given fraction can be represented. In Figure 1(b), the fraction 1/3 can be represented by the fraction bars in the first row (with a bar divided into three) or in the third row (with a bar divided into six). If the learner clicks on any of these rows, then the fraction bar on that row would slow down its movement, and the given fraction would be shaded blue. Returning to the example in Figure 1(b), if the learner selects the first row, then the fraction bar would have an additional 1/3 shaded blue, so that the entire rectangle would be shaded blue. If this happens (i.e., a fraction bar is completely shaded), then the bar moves back to the left end of the screen. The goal is to delay the forward motion of the fraction bars and prevent them from reaching the right end of the screen by correctly selecting the fraction bars that can represent the given fraction. If a fraction bar reaches the right end of the screen, then that row would be darkened (such as the second and third rows in Figure

1(c)). Each time a row is darkened, the number of "lives" indicated by the scrollbar with a heart on the upper left of the screen decreases by one. The game ends when fraction bars in the five rows reach the right end of the screen. *Moving Fractions* is available as an Android app that can run in Android-powered smartphones or tablets or as a computer app that can be used in Windows personal computers or laptops.

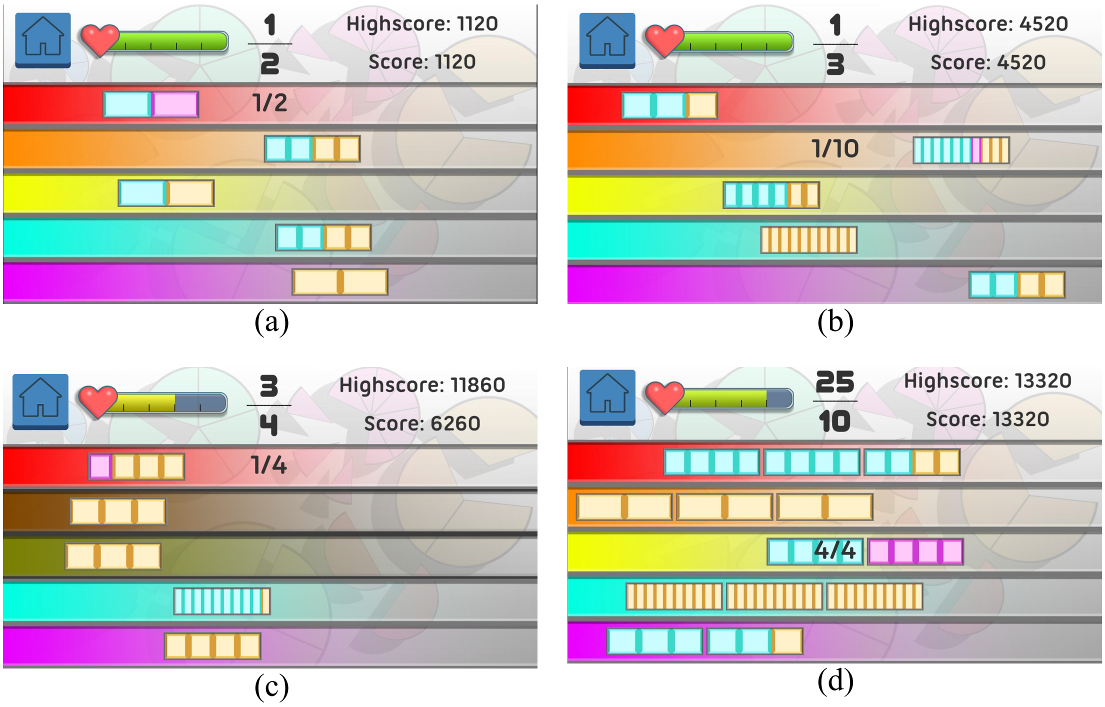

Figure 1. Screenshots showing four of the five levels in *Moving Fractions*.

In playing the different levels in *Moving Fractions*, learners understand fractions by developing the four schemes proposed by Wilkins and Norton (2018). When learners play the 1/2 and 1/4 level (Figure 1(a)) and the unit fractions level (Figure 1(b)), they see the relative size of a unit fraction to a whole and realize that *n* iterations of 1/*n* produce 1 whole. For example, 10 iterations of 1/10 produce 1 whole, thus developing the MSUF (Figure 2(a)). In playing the proper fractions level (Figure 1(c)), learners see the relative size of a proper fraction *m/n* to a whole and to the unit fraction 1/*n*, thus developing PWS and MSPF (Figures 2(b) and 2(c)). Learners operating with MSPF realize that a whole can be reproduced by partitioning 3/4 into three equal parts and iterating one of those 1/4 parts four times to produce a whole (Figure 2(c)). Similarly, when playing the improper fractions level (Figure 1(d)) learners see the relative size of an improper fraction *m/n* to a whole and to the unit fraction 1/*n,* hence developing GMSF.

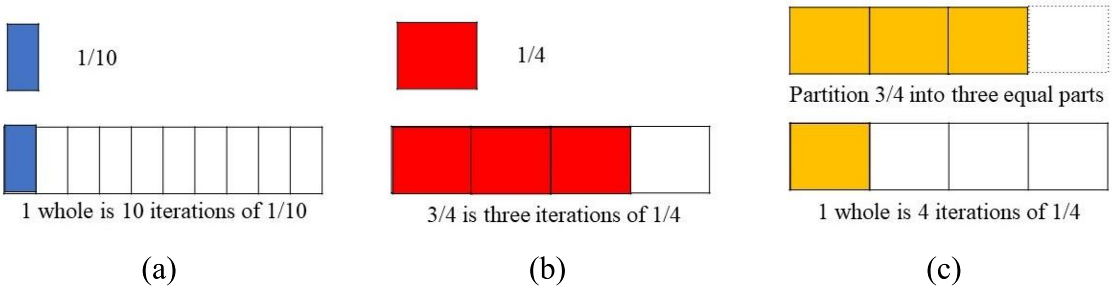

Figure 2. Three of the four fraction schemes are (a) MSUF, (b) PWS and (c) MSPF.

The learning progression from PWS to GMSF was used in the development of other apps, such as in the CandyFactory app (Aslan, Norton, & Balci, 2012). However, *Moving Fractions* has distinct game mechanics that differentiate it from the CandyFactory app. In the CandyFactory app, children learn partitioning and iterating within the context of a company that needs to fill orders of candy that meet certain requirements in terms of its size. *Moving Fractions* demonstrates the fraction schemes more indirectly and through a faster-paced game-like approach. The next section discusses the game-design features of *Moving Fractions* in more detail.

3.2 Game-design Factors

Aligned with the objectives of the ongoing project (De Las Peñas et al., 2021), the authors have been developing mathematical resources that are based on sound pedagogical research and that can prove beneficial even for students who are in distance learning modes. One way this has been achieved is by developing mathematical apps such as *Moving Fractions*. By integrating game-based elements in *Moving Fractions*, it is aimed for learners to be more engaged and interested with studying fractions that may lead to their development of analytical thinking, strategizing, and problem-solving that are important skills in STEM education (Vahidy, 2019). Furthermore, it is envisioned that *Moving Fractions* can be a mathematical resource that is "not only aligned with the curriculum but [is] also creative, fun and provide[s] opportunities to learn" (Miller, 2018, p. 9).

The development of the *Moving Fractions* app as an educational game is also motivated by previous studies where classroom teaching activities have been transformed into game-based versions. Hu and Shang (2018), for example, designed gamified teaching activities using mobile applications. More recently, Verzosa et al. (2021) developed a game-based version (i.e., an app called *Catch the Carrot*) of traditional number estimation activities for grade school students.

Moving Fractions has been developed in consideration of the 11 interrelated game-design factors in the Game-based Learning (GBL) Design Model of Shi & Shih (2015). These game-design factors are game goal, game mechanism, game fantasy, game value, interaction, freedom, narrative, sensation, challenges, sociality, and mystery.

As previously mentioned, the *game goal* of *Moving Fractions* is, at each turn, to identify the occurrence of a given fraction in five rows each containing one or more partitioned rectangles. The game goal facilitates the learning of fractions through the fraction schemes of Wilkins and Norton (2018). Figure 3 following the layout used by Shi & Shih for the GBL Design Model (Shi & Shih, 2015), further presents the game-design factors as they relate to *Moving Fractions*.

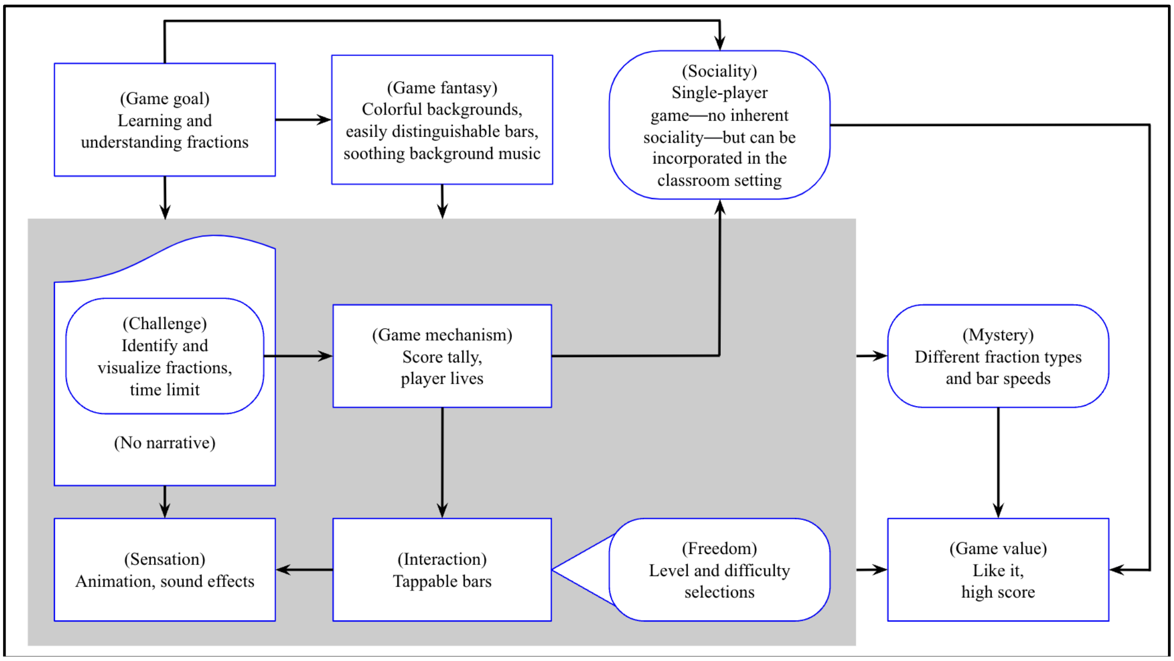

Figure 3. The GBL Design Model for *Moving Fractions*.

The *game mechanism* involves an arcade-type format in which learners must score as much as they can with a limited number of lives. Learners must keep track of the horizontally moving rectangles and quickly tap (*interaction*) correct rows—this increases the game's excitement and replayability.

Moreover, to allow learners to adjust to their current ability levels, the game provides the learners the *freedom* to select the level and difficulty they want to play.

As *Moving Fractions* is intended to be a straightforward replayable game with simple objectives, it has been decided not to integrate *narrative* and *sociability* elements into the game. Instead, the focus is to build the *game fantasy* through colorful backgrounds and soothing music. The animations and sound effects also enhance *sensation* within the game, making it more immersive and exciting for learners. It must be noted, however, that while *sociability* is not built inherently into the game, it may be incorporated when the game is played by multiple learners.

The *game value* depends largely on whether the learner likes the game and is motivated to achieve high scores. This is enhanced by the game's *challenges* that correspond to the increasing difficulty as one progresses through topics on fractions and applying this knowledge to play the game. The availability of multiple levels also adds to the game's *mystery* as learners can become curious about the new elements that can appear in the next levels.

4. Supplementary Fraction Videos

A supplementary tool developed to help promote a deep conceptual understanding of fractions is a series of videos demonstrating various ways that fractions may be represented. These videos are different from existing videos that are available online. First, these are short yet contain three models or representations: the area model, set model and number line model. The use of multiple representations provides a powerful means by which learners can visualize the idea of fractions. Second, the videos also present some questions at the end, to engage the viewers in the thinking process. Third, the text is minimized to lessen the viewers' cognitive demand.

The first video introduces the learner to the concept of ½ and ¼. A representation of these fractions using the area model shows objects (such as a rectangle, circle, triangle, heart, fruit) breaking into two or four equal parts (Figure 4(a)). This helps a learner grasp the idea of these fractions in terms of a part-whole scheme (Wilkins & Norton, 2018). The set model representations of ½ and ¼ shows a set of discrete objects being partitioned into two groups with the same number of objects is shown (Figure 4(b)). Here the learner relates ½ to the notion of segregating one set of objects into two smaller groups with an equal number of elements. Following the definition given above (Wu, 2010), a third representation of the unit fractions is through a number line model. For instance, the portion of the number line from 0 to 1 is shown as being split by a midpoint mark and the fraction ½ is visualized by the animation of a jump from 0 to the midpoint mark (Figure 4(c)). A similar number line model is used to depict ¼.

(a) (b) (c)

Figure 4. Representations of ½ using (a) area model; (b) set model; and (c) number line model.

These multiple representations of fractions are reinforced in succeeding videos that depict proper and improper fractions, equivalent fractions, addition, and subtraction of fractions (Figure 5). The concepts demonstrated in the videos build on each other. A video on *Equivalent Fractions* shows the equivalence of the (improper) fractions 5/3 and 10/6 using the area model (Figure 5(d)) and the number line model (Figure 5(e)). The repetitive use of various representations to illustrate other fractions which are equivalent helps the learner grasp the notion of equivalent fractions in various ways.

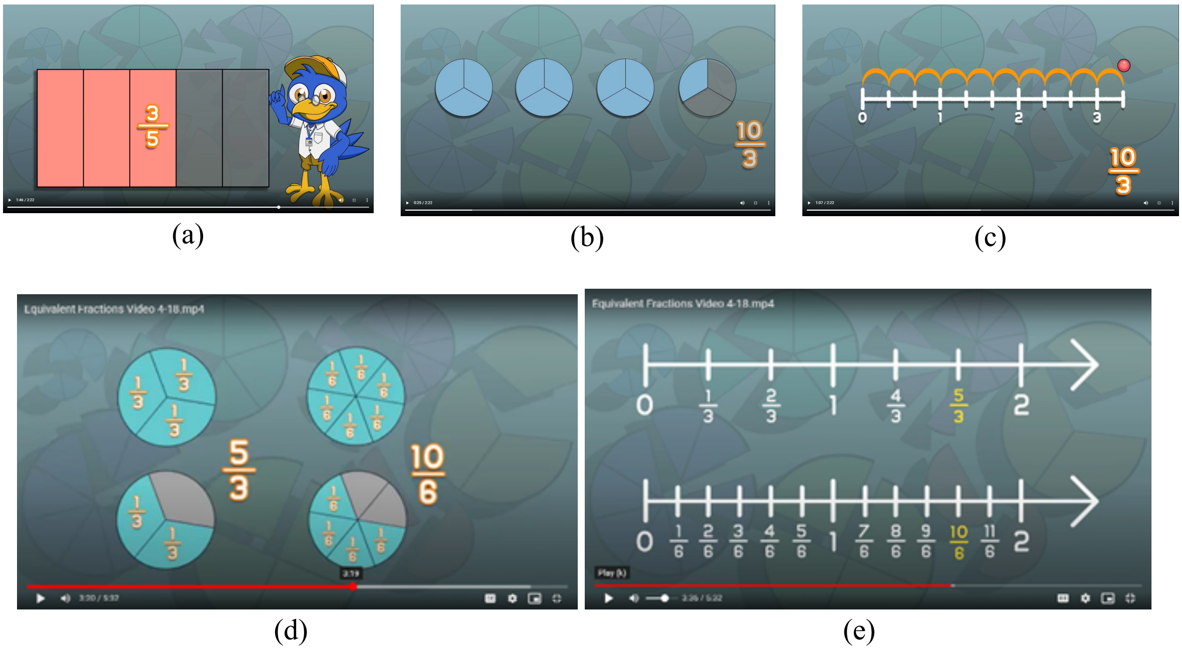

Figure 5. (a) Proper fraction ⅗ (area model), (b) improper fraction 10/3 (area model), (c) improper fraction 10/3 (number line model), (d) equivalent fractions 5/3 and 10/3 (area model), and (e) equivalent fractions 5/3 and 10/3 (number line model).

Another video on *Addition and Subtraction* of similar fractions animates the idea of putting together or removing several copies of a unit. For instance, the operation 5/2 + 7/2 is depicted in Figure 6(a) which shows five and six copies of ½ of a papaya, respectively. The sum is shown by putting together the unit pieces to make 12/2 or 6 papayas (Figure 6(b)). The notion of subtraction is similarly made more accessible by animating the removal of unit pieces from a whole (Figure 6(c)).

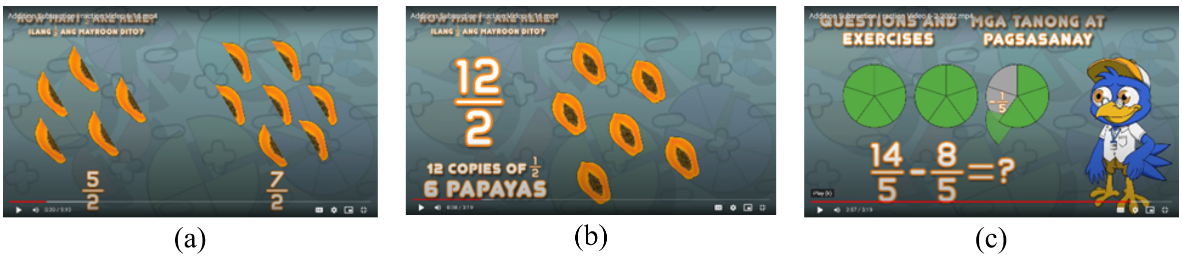

Figure 6. Illustrations of (a) 5/2 + 7/2, (b) sum of 5/2+7/2, and (c) 14/5 – 8/5.

5. Integration and Use of *Moving Fractions*

This section describes how Moving Fractions can be utilized as a pedagogical tool based on the RAT (Replacement, Amplification, Transformation) framework (Hughes, Thomas, & Scharber, 2006). The choice of replacement, amplification, transformation or combination of any of this may depend on the classroom context, nature of students and curriculum content and goals.

Based on this framework, the Moving Fractions app can serve as a replacement for the traditional method of teaching fraction understanding without changing content and learning goals in relation to fractions remain the same. For example, the fraction strips or circular regions which are often used by teachers to illustrate and represent fractions are now replaced by the moving fraction bars in the app. The task of representing fractions (unit, proper, improper) remains the same but the static method of presenting them is now changed to the colorful and dynamic representations of fractions. Such representations can be used by the teacher to discuss relationships between different types of fractions.

Moreover, integration of the app can go beyond the replacement category. The framework suggests that technology can amplify learning. This happens when concepts are presented more clearly and efficiently. The app provides a more precise and accurate representation of fractions than can be done without technology. For example, the fraction ⅕ is difficult to illustrate by hand. But with the app, this fraction is presented accurately. The design of the app can also help students understand different representations of fractions (e.g. ½ is 2/4 or ⅓ is 2/6). Teachers can maximize this feature of the app so that misconceptions about fractions can be addressed.

Finally, technology can involve transformation of instructional methods, learning process, and/or the content. The app may spur a transformation in teachers' instructional practices from being teacher-centric or teacher-controlled to student-centered. By playing with the app, the learning process is transformed as the feedback mechanism heightens student engagement and facilitates opportunities for discovery. The app can be used as a tool for collaborative activity where students can communicate their thinking and experiences in playing the app. Such activity also helps students to work with others and create a positive learning environment. Learning need not come solely from the teacher's input but from the app itself. Further, the game-like environment of the app allows students to create efficient strategies in order to prolong their life in the game.

6. Conclusion and Future Direction

Competency in fractions is identified as an essential skill for entry into STEM-related fields. The *Moving Fractions* app and the supplementary fraction videos were designed to develop learners' knowledge of fractions. The app utilizes game-design principles to concretize the learning trajectory of fraction learning, from part-whole comparisons to a more robust understanding of the measurement concept of fractions. Meanwhile, the fraction videos immerse learners in a range of fraction representations that can extend their initial part-whole understanding of the fraction concepts. The next step is to package the app and videos and gather empirical data on how the fraction learning package can promote fraction understanding.

Acknowledgements

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the mathematical resources for Grades 1 to 11 Mathematics. Acknowledgement also goes to Dr. Ma. Mercedes T. Rodrigo and the Ateneo Laboratory for the Learning Sciences (ALLS) for providing technical assistance. We thank our app developers Victor Antonio M. Ortega, Mr. Nigel Benedict Cargo, Mr. Jose Teodoro Lacson, and Mr. Amiel Damian F. Justiniani for the apps and video art assets.

References

Alajmi, A. H. (2012). How do elementary school textbooks address fractions? A review of mathematics textbooks in the USA, Japan, and Kuwait. *Educational Studies in Mathematics, 79,* 239-261.

Aslan, S., Norton, A., & Balci, O. (2012). CandyFactory educational game for iPad (Version 2.0). Blacksburg: Virginia Tech. https://apps.apple.com/us/app/candyfactory-educational-game-for-ipad/id533213891

Bansilal, S. & Ubah, I.J.A. (2020). The use of cross multiplication and other mal–rules in fraction operations by pre-service teachers. *The Journal of Mathematical Behavior, 58,* 100781.

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*, 247–253.

De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., Tolentino, M. A. C., Verzosa, D. M. B. (2021). *Annual report (Department of Science and Technology-Philippine Council for Industry,*

Energy and Emerging Technology Research and Development): Mathematical resources for distance learning utilizing community LTE networks and television frequencies.
Department of Education. (2020). *K to 12 most essential learning competencies with corresponding CG codes*. Retrieved July 12, 2020, from https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf.
Erhel, S. & Jamet, E. (2013). Digital game-based learning: impact of instructions and feedback on motivation and learning effectiveness. *Computers in Education, 67*, 156–167.
Hu, R., & Shang, J. (2018). Application of gamification to blended learning in elementary math instructional design. In: S. Cheung, L. Kwok, K. Kubota, LK. Lee, & J. Tokito (Eds.), *Blended Learning. Enhancing Learning Success. ICBL 2018. Lecture Notes in Computer Science* (Vol. 10949, pp. 93-104). Springer, Cham.
Hughes, J. E., Thomas, R., & Scharber, C. (2006). Assessing technology integration: The RAT – Replacement, Amplification, and Transformation – Framework. In C. M. Crawford, R. Carlsen, K. McFerrin, J. Price, R. Weber, & D. A. Willis (Eds.), *Proceedings of SITE 2006--Society for Information Technology & Teacher Education International Conference* (pp. 1616-1620). Orlando, Florida, USA: Association for the Advancement of Computing in Education (AACE).
Kieren, T. (1980). The rational number construct–Its elements and mechanisms. In T. Kieren (Ed.), *Recent research on number learning* (pp. 125–149). Columbus: OH: ERIC/SMEAC.
Lamon, S. J. (2007). Rational numbers and proportional reasoning: towards a theoretical framework for research. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629–667). Reston: NCTM.
Larkin, L. (2013). Mathematics education: Is there an app for that? In V. Steinle, L. Ball, & C. Bardina (Eds.), *Mathematics education: Yesterday, today and tomorrow.* Melbourne: MERGA.
Miller, T. (2018). Developing numeracy skills using interactive technology in a play-based environment. *International Journal of STEM Education, 5*, 1-11.
Ninaus, M., Kiili, K., McMullen, J., & Moeller, K. (2016). A game-based approach to examining students' conceptual knowledge of fractions. In R. Bottino & J. Jeuring (Eds.) *GALA 2016* (pp. 37-49). Utrecht: Springer.
Ngo, F. (2018). Fractions in college: How basic math remediation impacts community college students. *Research in Higher Education*, *60*(4), 485-520.
Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: a meta-analysis. *Child Development, 89*(5), 1467-1484.
Shi, Y.-R., & Shih, J.-L. (2015). Game factors and game-based learning design model. *International Journal of Computer Games Technology, 2015*, 1-11.
Vahidy, J. (2019). Enhancing STEM learning through technology. *Technology and the Curriculum: Summer 2019.* Lethbridge, AB, Canada: Power Learning Solutions.
Verzosa, D. M. B., De Las Peñas, M. L. A. N., Aberin, M. A. Q., Garciano, A. D., Sarmiento, J. F., & Tolentino, M. A., C. (2021). Development of a gamified number line app for teaching estimation and number sense in grades 1 to 7. In M. M. T. Rodrigo, S. Iyer, & A. Mitrovic (Eds.), *Proceedings of the 29th International Conference on Computers in Education* (pp. 648-656). Asia-Pacific Society for Computers in Education.
Wang, Z. (2013). Why students choose STEM majors: Motivation, high school learning and postsecondary context of support. *American Educational Research Journal*, *50*(5), 1081-1121.
Wilkins, J. L. M., & Norton, A. (2018). Learning progression toward a measurement concept of fractions. *International Journal of STEM Education, 5,* 27.
Wu, H., (2010). *Understanding numbers in elementary school mathematics.* Providence, RI: American Mathematical Society.