Chapter

A Review on the Dynamic Response of Liquid-Storage Tanks Associated with Fluid-Structure Interaction

Ayman Mohammad Mansour and Moustafa Moufid Kassem

Abstract

Water tanks are considered one of the most important facilities in firefighting systems and municipal water supply. These critical water storage and distribution facilities should remain operable even after a severe seismic event or sustain only damages that can be readily repaired. In recent years, the seismic design of storage tanks has been aimed at fulfilling safety requirements and the environmental impact on society. This paper provides a review of research work related to seismic response of liquid-filled tanks. Major contribution from previous research works related to dynamic behavior of liquid tanks are acknowledged in this review. This paper encompasses the phenomenon of fluid-structure interaction and reviews several equivalent mechanical models for liquid storage tanks that account for this phenomenon. The application of each modeling approach and its accuracy in accounting for the fluid-structure interaction are discussed based on available literature and applicable international standards. It was shown that different equivalent modeling approaches that consider the fluid-structure interaction effects can be used to reduce the computational cost and complexity of liquid-tank systems.

Keywords: liquid-storage tanks, infrastructure, fluid-structure interaction, liquid sloshing, dynamic response, equivalent mechanical models

1. Introduction

Earthquakes result from abrupt release of energy by the slippage of two tectonic plates. The sudden release of strain generates seismic waves that are transferred to the earth's surface and result in ground motions. These dynamic vibrations create lateral movement in structures, which affects their strength and behavior. The infrastructure system is very critical and should have extra immunity against possible disasters. And natural hazards due its essential function in remaining serviceable to satisfy the water demand for drinking and firefighting purposes [1].

The dynamic vibrations of liquid-containing structures create the phenomenon of Fluid-Structure Interaction (FSI), where the momentum of the oscillating fluid generate lateral pressure on the boundaries of the structure. The study of hydrodynamic pressure on structures can be traced back to the early 1930s. The research work by Westergaard on "Water Pressure on Dams during Earthquakes" is considered the earliest study on the behavior of FSI, where the impulsive pressure on vertical dams under the effect of earthquake excitations was evaluated [2]. Subsequently, the dynamic response of liquid contained tanks and the FSI phenomenon have been subjects for extensive experimental and numerical investigations by many researchers. Such studies emerged by the efforts of Jacobsen and Ayre [3], Housner [4], Veletsos [5] and later by other researchers such as Mansour and Nazri [6], Shakib and Alemzadeh [7], Elansary and El Damatty [8], and Moslemi et al. [9]. The aim of this study is to provide a comprehensive review for the equivalent mechanical models of liquid storage tanks that account for FSI, including the added-mass, single-lumped-mass, two-lumped-mass, spring-mass, three-mass, and other models that were featured in the literature. The theoretical background, application, application, and accuracy for each model were presented based on international standards and available literature.

2. Methods

To "review" has been defined as: "To view, inspect, or examine a second time or again" [10]. Review studies are amongst the most highly sought types of articles by researchers and are the ones that provide the most substantial contribution [10]. The body of a review study can be organized in a variety of ways depending on the type and the method of the review study. The literature review refers to the generic term of review which includes published resources that give an evaluation of recent or current literature. Can cover a wide range of topics at varying levels of comprehensiveness and completeness. It's possible to include study findings. Analysis of the literature review may have different structures, namely, chronological, conceptual, thematic, etc. In a thematic approach, recurring central themes exists, in which the literature review can be divided into subsections that address different aspects of the topic. For the current study, a literature review with a thematic approach was conducted, in which the study was divided into several section; a section for each equivalent mechanical model for liquid-storage tanks associated with fluid-structure interaction.

3. Equivalent mechanical models for the liquid-tank system

Analysis of hydrodynamic pressure in structures such as liquid-storage tanks is more complicated than that of other structures. In the 1960s, Housner [4] provided a practical idealization for evaluating the hydrodynamic pressure within rectangular and cylindrical tanks that are subjected to horizontal ground motion while assuming the tank walls to be rigid. The Chilean earthquake, that took place in 1960 and damaged several large water tanks, was the main plot behind the paper by Housner [4]. FSI can be simulated using different simplified modeling approaches (added-mass approach, two lumped-mass model, spring-mass model, etc.). Livaoğlu and Doğangün [11] presented a comparison and evaluation of some of these modeling methods.

3.1 The added-mass approach

The impulsive hydrodynamic pressure is usually accounted for by introducing added masses. The added mass concept is one of the simplest methods to account for the impulsive hydrodynamic component of liquid pressure. This method has been used for decades in the design of seismic resistant structures, such as gravity dams [2] and liquid tank containers. The Added-Mass Approach (AMA) relies on few main assumptions, which are water incompressibility and the rigidity of the boundary conditions of the structure. This approach neglects the stiffness effects in the fluid and in general leads to conservative results [12]. The use of AMA is proven to be a more appropriate technique for finite element modeling than other assumptions such as those characterized using the lumped mass models [13].

3.2 The single-lumped-mass model

The representation of a single-lumped-mass model for Elevated Water Tanks (EWTs) can be seen in **Figure 1**. This concept was introduced in the 1950s [14] and has two main assumptions. First, for a completely full liquid tank, the water sloshing behavior will not have any vertical movement thus allowing the system to behave as a system with a Single Degree-Of-Freedom (SDOF). Second, the supporting structure acts as a cantilever and is considered to have uniform rigidity along its elevation.

According to ACI-371R [15], this model should be used if the weight of water equals or exceeds 80% of the overall weight of the system. The lumped mass consists of the own-weight of the tank, two-thirds (at maximum) of the own-weight of the supporting structure, and the weight of the contained water.

Previous studies have shown that the use of single lumped-mass model representation of EWTs yields similar results relative to experimental testing and other mechanical models [13]. In addition, the convective mass may have negligible influence on the natural characteristics of EWTs depending on the geometrical shape of the tank.

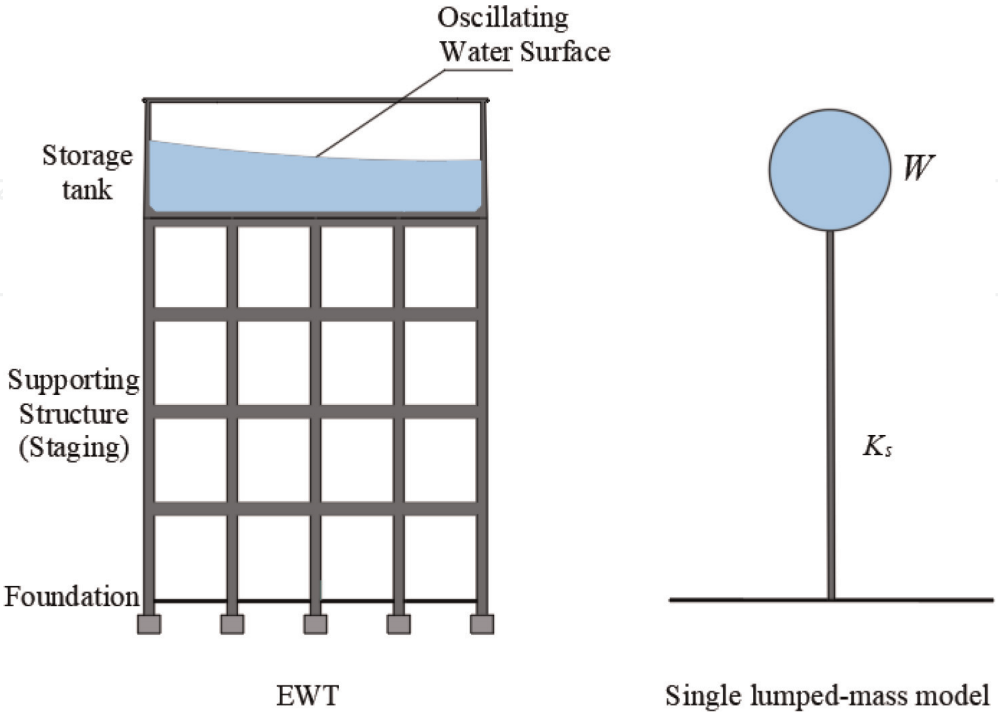

Figure 1.
An EWT and its single lumped-mass representation.

3.3 The two-lumped-mass model

The addition of a convective lumped mass relies on the assumption that was presented by Housner [16] on the relative motion between the storage tank, contained liquid, and the ground. According to his concept, analysis of EWTs can be performed by considering three conditions. First, if the water tank is empty then the sloshing water effect is absent, or if the tank is completely filled with water, then the sloshing effect in the tank is negligible. In this case the EWT will behave as a system with a SDOF, or a one-mass structure. However, the sloshing effect is not neglected where the tank is partially filled. Thereby, this gives the EWT an additional degree-of-freedom, making it a two-mass structure. Consequently, the dynamic analysis of equivalent models must include at least a two-mass system. More lumped convective masses may also be added for ground supported water tanks.

In the simplified analysis procedure for fixed-base EWTs ([17], ACI-371R), a two lumped masses model usually used to represent the fluid-tank system (**Figure 2**). Housner [4] assumed that the two masses to be uncoupled and the seismic forces on the support were evaluated by assuming two separate SDOF systems. The upper mass represents the convective mass of water, which characterizes the motion of the free-liquid-surface. The lower mass represents the impulsive mass of the fluid and the mass of the structure, which is derived by the own-weight of the storage tank plus a portion of the supporting structure's own-weight [11]. Lu et al. [13] showed in his study that the equivalent two-mass model can predict the natural characteristics of water sloshing effect with reasonable accuracy similar to that derived by the much more advanced fluid Finite-Element (FE) technique.

ACI 350.3 (ACI-350.32006) permits the idealization of EWTs as uncoupled single-lumped masses in order to estimate the natural characteristics of the convective and impulsive components of EWTs. Mansour and Nazri [6] evaluated the FSI effect on

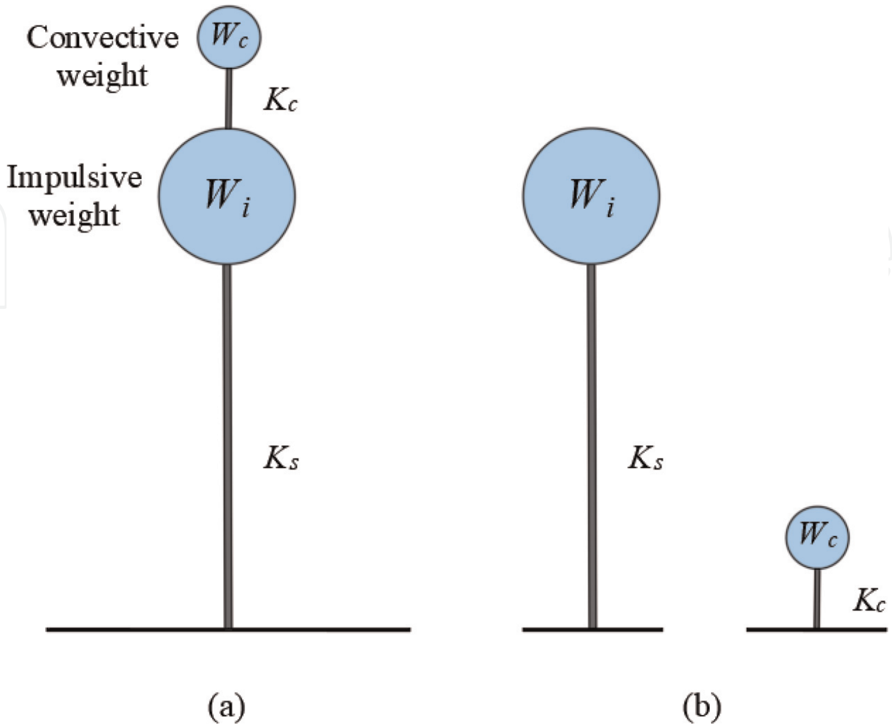

Figure 2.
(a) The two mass model for EWT proposed by Housner [4] and (b) the equivalent uncoupled system.

the dynamic response of EWTs. The periods of vibration corresponding to the impulsive and convective components of the EWTs were predicted using the equivalent two-mass model to validate the developed Three-Dimensional (3D) models. By considering the supporting structure of EWTs as a vertical cantilever, the periods of vibration the significant modes (i.e., T_i and T_c, which are m_i and m_s related, respectively) of EWT systems can be obtained using the equations presented in **Table 1**.

3.4 The spring-mass model

The FE model for a liquid-tank system can be represented by the spring-mass concept, which was originally proposed by Housner [4]. In the spring-mass model, also referred to as the equivalent mechanical model, the liquid is replaced by two lumped masses: the impulsive and convective masses. The impulsive mass is rigidly connected to the tank walls and the convective mass is connected using elastic springs. **Figure 3** illustrates the spring-mass model representation for EWTs based on the principle proposed by Housner [4]. This modeling technique has been used by researchers as a simplified approach for the assessment of the seismic vulnerability of liquid tanks as opposed to much more complex and computationally intensive approaches, such as the continuum liquid-medium models [4, 6, 18].

The parameters of the mechanical spring-mass model are calculated based on the aspect ratio of the liquid-filled tank [19]. According to American Concrete Institute (ACI) (ACI-350.32006), the parameters of this model models can be evaluated using the equations presented in **Table 2**.

The spring-mass model representation of liquid-tank system based on Housner's analogy is considered adequate for modeling of EWTs and is a widely used concept in many international guidelines for seismic design of tanks and buildings such as Eurocode, ACI, and NZS ([20, 21], ACI-350.32006, [15]). These design guidlines have tweaked Housner's method with a few changes due to the findings of the subsequent studies on the seismic design of liquid-containing tanks [22, 23]. The accuracy and efficiency of the two-mass representation of the EWTs was proved by Shepherd [24], who compared the theoretical results of a prestressed RC EWT to the experimental testing results. A detailed study conducted by Dutta, Dutta et al. [25] on RC EWTs integrating soil-structure-fluid interaction shows relatively small differences in in total structural response represented using Westergaard's AMA and the lumped-mass mechanical analogy.

The equivalent spring-mass model has been used by many researchers to simulate the dynamic behavior of EWTs using a simplified approach. Mansour et al. [1] investigated the non-linear seismic vulnerability of a set of EWTs with structural variables

Equations for the period of vibration		
The impulsive component	$T_i = 2\pi\sqrt{\frac{m_i+m_s}{K_s}}$	(1)
The convective component	$T_c = \frac{2\pi}{\sqrt{3.68\tanh\left(3.68\frac{H_L}{D}\right)}}\sqrt{\frac{D}{g}}$	(2)

**Where:* K_s *is the horizontal translation stiffness of the EWT's supporting structure,* m_s *is the lumped structural mass, whivelech includes mass of water tank and two-thirds of staging mass,* m_i *is the impulsive mass,* g *is the acceleration due to gravity, equal to 9.81 m/s*^{2}*, and* HL *and* D *correspond to the tank's geometry, i.e., the height and the diameter.*

Table 1.
The vibration period of the significant modes of the EWT system.

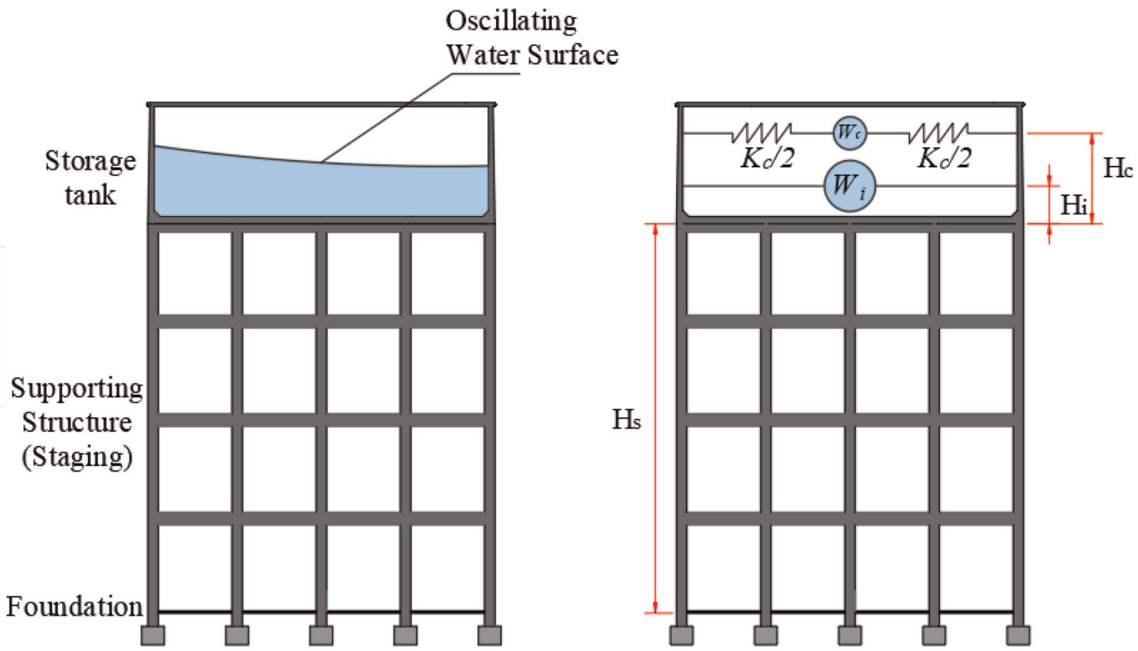

Figure 3.
The spring mass model representation of an EWT.

Equations for the parameters of the spring-model		
The impulsive and convective weights (W_i and W_c, respectively) can be obtained using Eqs. (3) and (4) where, D is the inside diameter of a circular tank, and H_L is the maximum water level.	$\frac{W_i}{W_l} = \frac{\tanh\left[0.866\left(\frac{D}{H_L}\right)\right]}{0.866\left(\frac{D}{H_L}\right)}$	(3)
	$\frac{W_c}{W_l} = 0.23\left(\frac{D}{H_L}\right)\tanh\left[3.68\left(\frac{H_L}{D}\right)\right]$	(4)
The heights of the impulsive and convective masses from the bottom of the tank wall can be determined from Eqs. (5)–(7) where, H_i is the height of the center of gravity of the impulsive mass measured from above the base of the tank wall, and H_c is the height of the center of gravity of the convective mass measured from above the base of the tank wall.	$\frac{H_i}{H_L} = 0.5 - 0.09375\left(\frac{D}{H_L}\right)$ for $\frac{D}{H_L} < 1.333$	(5)
	$\frac{H_i}{H_L} = 0.375$ for $\frac{D}{H_L} > 1.333$	(6)
	$\frac{H_c}{H_L} = 1 - \frac{\cosh\left[3.68\left(\frac{H_L}{D}\right)\right] - 1}{3.68\left(\frac{H_L}{D}\right)\sinh\left[3.68\left(\frac{H_L}{D}\right)\right]}$ for all tanks	(7)
The stiffness of convective mode can be obtained by Eqs. (8)–(10) where, λ is the circular frequency coefficient, g is the gravitational acceleration taken as 9.81 m/s^2, ω_c is the circular frequency of oscillation of the first sloshing mode (convective mode), and K_c is the spring stiffness of convective mode.	$\lambda = \sqrt{3.68g\tanh\left[3.68\left(\frac{H_L}{D}\right)\right]}$	(8)
	$\omega_c = \frac{\lambda}{\sqrt{D}}$	(9)
	$K_c = \frac{W_c}{g}{\omega_c}^2$	(10)
Alternatively, the stiffness of the convective mode can be obtained using the combined Eq. (11)	$K_c = 3.68\frac{W_c}{D}\tanh\left(3.68\frac{H_L}{D}\right)$	(11)

Table 2.
The equations and calculation procedure of the spring-mass model parameters.

(i.e., variable staging patterns and contained liquid fill levels) using the performance-based earthquake engineering methodology. The study utilized a developed seismic evaluation tool – the collapse margin indicator – to investigate the dynamic behavior of frame supported EWTs and considered the FSI effect by adding lumped masses that are connected to the tanks' walls either rigidly or elastically through oscillators as shown in **Figure 4**.

3.5 The three-mass model

Earlier studies considered rigid tank walls when evaluating the hydrodynamic pressure induced by ground motion records [4, 18, 26]. However, following a series of powerful earthquakes in Japan and the United States that caused severe damage to liquid storage tanks, it was realized that modeling tanks using the rigid-tank concept is insufficient since real tanks experienced significant deformation when subjected earthquake loads. Subsequently, multiple studies were conducted, and it was established that accounting for the tank flexibility and the interaction between the contained fluid and the vibration of the walls can significantly affect the hydrodynamic pressure and consequently the impulsive component of the structural response [27, 28]. Over time, assumptions concerning tank properties have been refined progressively to take account of the tank deformability and flexibility of the container and soil interaction effects [27].

Haroun and Housner [27] proposed the three-mass model representation for cylindrical tanks subjected to seismic loading as illustrated in **Figure 5**. The three masses in this equivalent mechanical model correspond to the impulsive mass, the convective mass, and the mass representing the tank wall's flexibility. In the following studies, Haroun and Ellaithy [23] imple1mented the three-mass model to evaluate the dynamic response of EWT and to assess the influence of the tank walls' flexibility on the dynamic behavior of EWTs. The effect of higher modes of convective masses on the pressure exerted on the vessel may be not significant, even when the fundamental frequency of the structure is close to the natural frequency of convective mode. A later study by Jaiswal et al. [29] show negligible differences in the parameters of the equivalent spring-mass mode obtained from rigid and flexible tank wall.

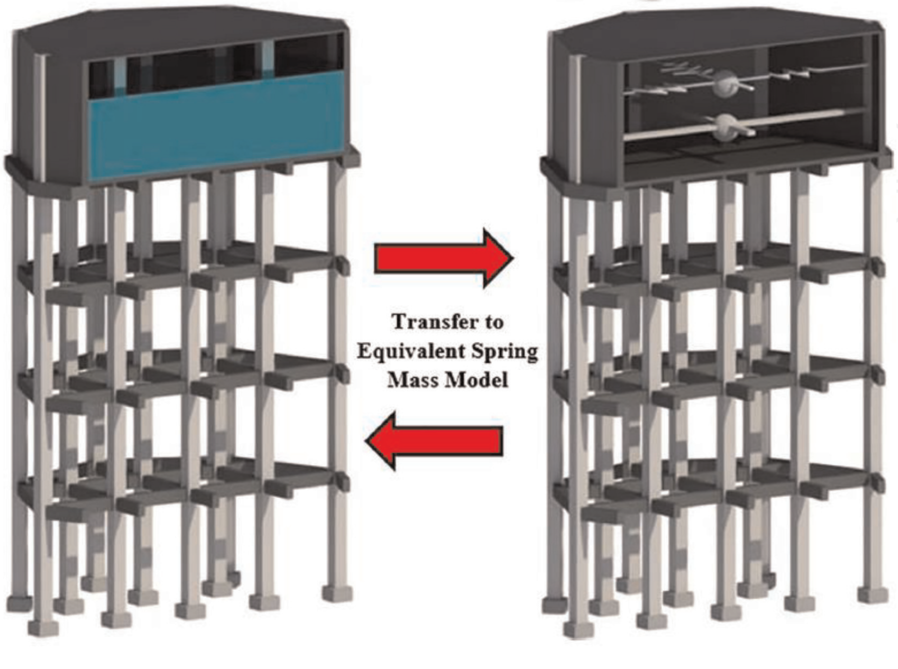

Figure 4.
The spring-mass model for an EWT [1].

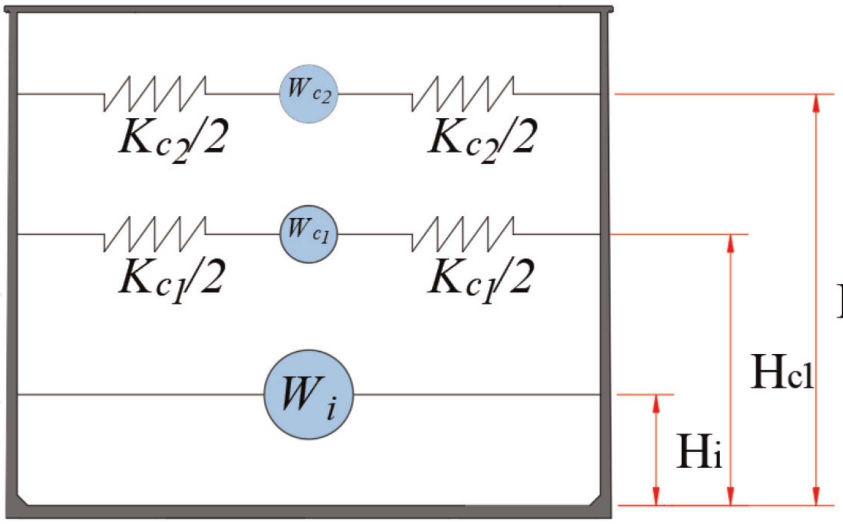

Figure 5.
Equivalent 2D system for liquid-filled storage tank with flexible tank wall behavior.

Some studies have identified the hydrodynamic pressure developing inside deformable cylindrical tanks. Haroun and Housner [27] analyzed the response of flexible liquid-containing tanks using modal superposition. The tank's walls were modeled as shell elements using the finite element method and the fluid domain was considered using a mathematical boundary solution technique. Previous studies showed that the flexibility of tank causes it to experience rocking wall and base translation, which result in longer impulsive periods and increased effective damping. However, due to convective mode having long period of oscillation, the convective mass can be computed without considering the tank wall and supporting soil flexibilities [30]. Using two-dimensional space FE modeling, Ghaemmaghami and Kianoush [31] examined the seismic behavior of two different tank configurations, tall and shallow, while taking the effects of FSI and wall flexibility into account. The results show that incorporating the fluid damping properties and the wall flexibility can drastically affect the dynamic response of the liquid tanks.

3.6 Other equivalent models

While simplified models, such as those developed by Housner [4] and Haroun and Housner [27], generate a dynamic response similar to that of a continuum liquid 3D-tank-model [32], it may not, however, take into account certain aspects that affect the accuracy of the analysis results. In a recent study, Papadrakakis and Fragiadakis [33] investigated the seismic performance of unanchored liquid-storage tanks having variable tank diameters and liquid-filling heights using two nonlinear FE computational methods; coupled Eulerian-Lagrangian and spring-mass analogy. Results show that the traditional equivalent masses-springs analogy does not consider the effect of uplifting history for ground unanchored liquid tanks and its influence on the tank's dynamic behavior.

Studies by Sweedan and El Damatty [34] and El Damatty et al. [35] verified the application of the previously established analytical and numerical models on combined conical tanks by experimentally identifying their dynamic characteristics. In order to further improve the seismic study of EWTs, Sweedan [36] suggested a mechanical model to duplicate forces produced in combined EWTs experiencing

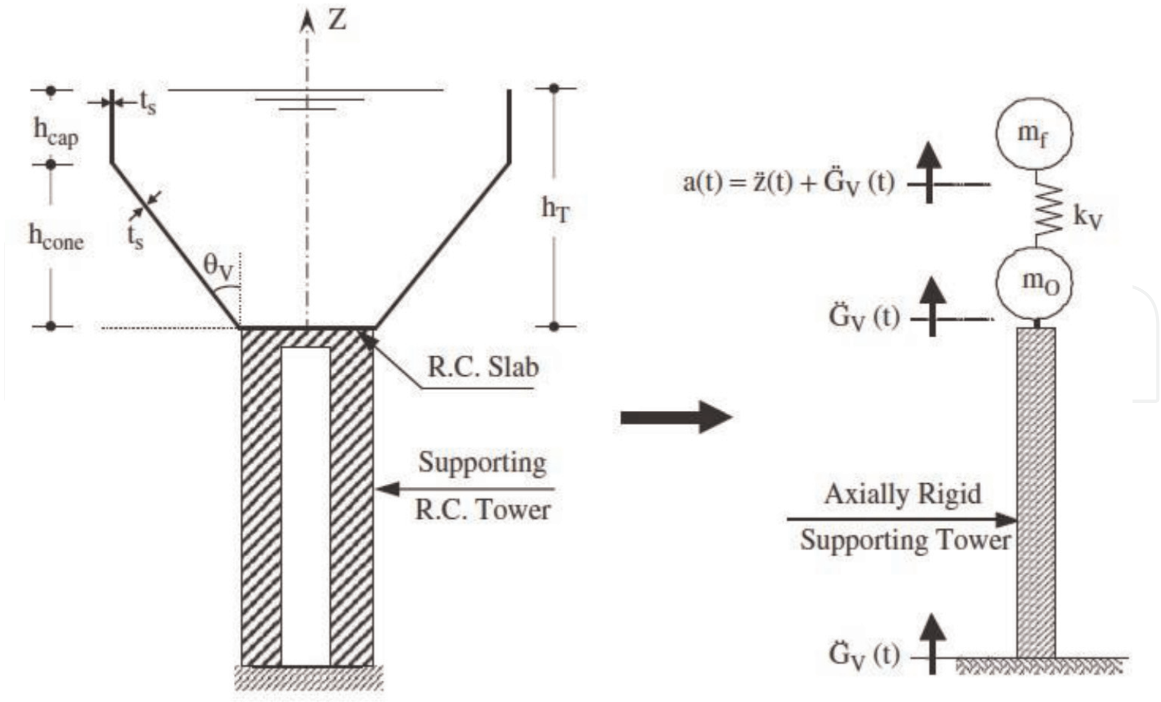

Figure 6.
Equivalent model for vertically excited combined tanks proposed by Sweedan [36].

vertical ground excitation. A schematic of the equivalent idealization is shown in **Figure 6**.

Vathi and Karamanos [37] studied the base uplifting behavior of ground cylindrical liquid tanks subjected to strong horizontal seismic excitations. A simplified liquid-tank model was developed using the spring-mass model improved by an appropriate rotational spring at its base to take into consideration the tank's rotation, or rocking, by the impulsive motion due to uplifting (**Figure 7**). The results from this study mark a significant influence of tank base uplifting on the dynamic response of unanchored tanks.

Algreane et al. [38] introduced an alternative impulsive masses configuration to the dynamic behavior of reinforced concrete EWTs. The proposed model suggests

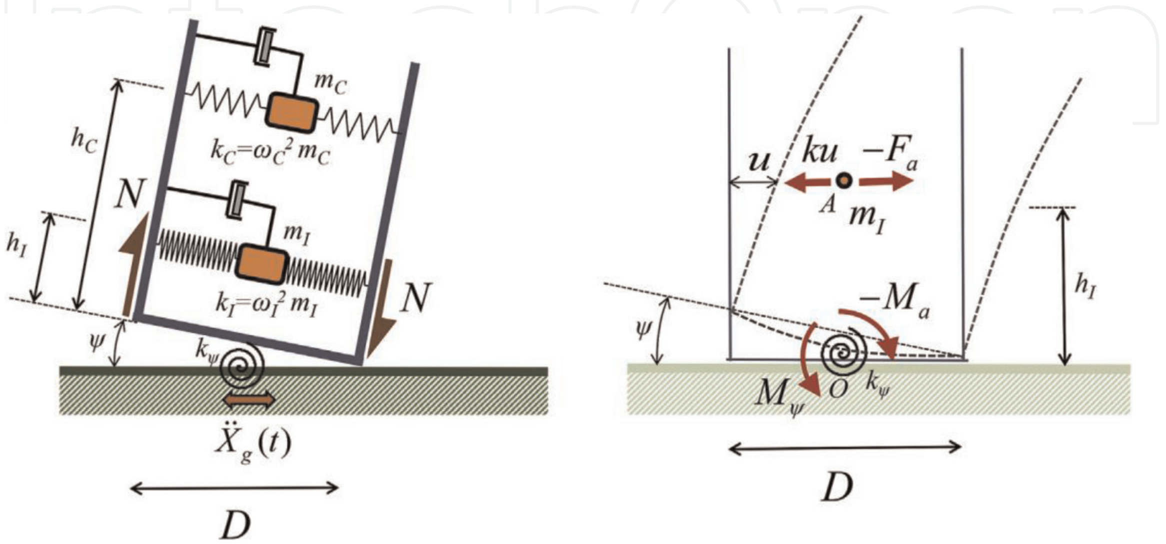

Figure 7.
A simplified model for an unanchored liquid storage tank accounting for base uplifting [37].

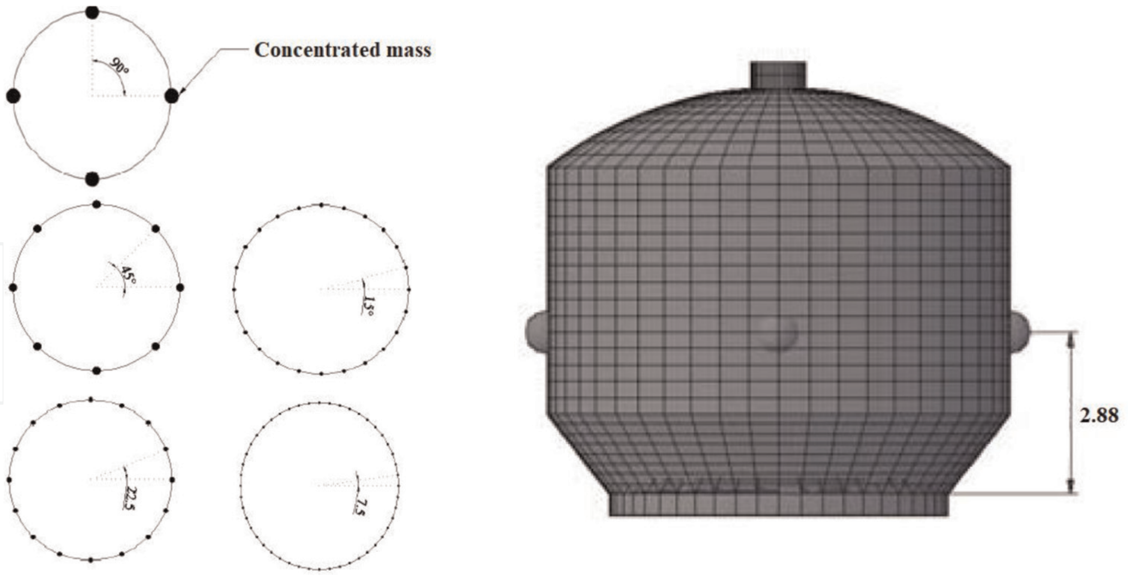

Figure 8.
Alternative masses distribution proposed by Algreane et al. [38].

the distribution of the impulsive mass by different alternative configurations in an effort to simplify Westergaard's AMA technique and reduce computational time. The impulsive mass is divided into 4, 8, 16, 24 and 48 masses, and distributed into wall panels of the tank at the center of gravity of an empty container as shown in **Figure 8**.

4. Conclusion

The functionality of liquid-storage tanks should be ensured during and after natural disasters, e.g., earthquakes, such that under intense ground motions the structural collapse is prevented. Therefore, it is crucial to quantify the safety margin against the structural collapse state for water tanks, which are considered as lifeline structures. Furthermore, the literature shows that the dynamic behavior of these structures is governed by many factors including the tank shape, fluid properties, structural flexibility. Soil characteristics, and the type of supporting structure.

Researchers tend to prefer simple and straightforward modeling techniques for the FE analysis of liquid-tank systems. Different simplified modeling approaches that consider the FSI effects can be used to reduce the complexity and computational cost of liquid-tank systems. The dynamic behavior of liquid-tank systems that is obtained using these modeling techniques can have high accuracy that can match that obtained from continuum liquid-tank systems. A simplified single-mass model could be used if the weight of water equals or exceeds 80% of the overall weight of the liquid-tank system. In the two-mass model representation of liquid-tank system, the hydrodynamic pressure developing within the liquid resulting from the dynamic motion of the liquid tank can be divided into two parts. The liquid mass in the top zone of the tank, called the convective mass, characterizes the motion of the free-liquid-surface. The liquid in the bottom zone of the tank, called the impulsive mass, represents the remaining mass of the fluid and the mass of the structure. A FE model can be represented by a spring-mass model based on Housner's analogy in which the liquid is replaced by two lumped masses: the impulsive and convective masses. The impulsive mass is connected to the tank walls using rigid links and the convective mass is connected through elastic springs. This modeling approach is effective in reducing the reduce the computational cost and complexity of liquid-tank systems while resulting in moments and forces that are comparable to that obtained from continuum liquid-medium models subjected to the same ground motion records.

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Author details

Ayman Mohammad Mansour and Moustafa Moufid Kassem*
School of Civil Engineering, Universiti Sains Malaysia, Penang, Malaysia

*Address all correspondence to: moustafa.mk@usm.my

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