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Analysis of Feedback Queueing Model with Differentiated Vacations under Classical Retrial Policy

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Abstract

This paper analyzes an *M/M/1* retrial queue under differentiated vacations and Bernoulli feedback policy. On receiving the service, if the customer is not satisfied, then he may join the retrial group again with some probability and demand for service or may leave the system with the complementary probability. Using the probability generating functions technique, the steady-state solutions of the system are obtained. Furthermore, we have obtained some of the important performance measures such as expected orbit length, expected length of the system, sojourn times and probability of server being in different states. Using MATLAB software, we have represented the graphical interpretation of the results obtained. Finally, the cost is optimized using the parabolic method.

Keywords: Differentiated Vacations; Bernoulli Feedback; Retrial; Parabolic Method; Probability; Performance Measures

MSC 2010 No. : 60K25, 68M20, 90B22

1. Introduction

In queueing theory, retrial vacation queues with feedback have been an intensive research topic due to their wide applications in data communication systems, production management, transportation networks, packet switching networks, video data traffic, etc. Choi et al. (1993), Martin and Gómez-Corral (1995), and Falin and Templeton (1997) did pioneering work on retrial queueing models. Sherman and Kharoufeh (2006) analyzed a single server retrial queueing system with an unreliable server. Detailed bibliographical information on retrial queues has been given by Artalejo (2010). A note on M/M/s queueing system with two reconnect and two redial orbits was given by Bouchentouf and Sakhi (2015).

The concept of customers' feedback is introduced in the retrial queueing model to make the situation more practical. For instance, the feedback queueing model under classical retrial policy is used to model the repeat request protocol in communication networks and multiple access telecommunication systems where messages are returned due to the occurrence of some error and are transmitted again. In recent years, this phenomenon has received remarkable attention from researchers due to its wide applications. In queueing theory, firstly Takacs (1963) introduced the concept of Bernoulli feedback. Choi et al. (1998) did the pioneering work on *M/M/C* retrial queueing system with geometric loss and feedback. Later on, Kumar et al. (2002) and Mokaddis et al. (2007) studied retrial queues with customer feedback and system failure. Bouchentouf et al. (2015) obtained the stability condition of a retrial queueing system with abandoned and feedback customers.

A vacation queue is a remarkable and unavoidable feature in the service facility of a queueing system to utilize the idle time of the server for different purposes. Server vacation may occur due to many reasons such as server failure, some secondary tasks given to the server, server maintenance, etc. The idea of vacation queues was first introduced by Levy and Yechiali (1975). Later on, excellent work on the vacation queueing models have been done by Doshi (1986), Takagi (1991) and Tian (2006). Pioneering work on working vacation was performed by Servi and Finn (2002). Vacation is also a special case of delay in service in queueing systems. Excellent works in the field are performed by Haghighi (2008, 2011, 2016). Tadj and Choudhury (2013) analyzed the $M^X/G/1$ queue with delayed repair and the Bernoulli vacation schedule under T-policy. Some other works on queueing model with vacations may be referred to in Ayyappan and Thamizhselvi (2016), Kadi et al. (2020), Gupta and Kumar (2021a, 2021 b), Gupta (2022), Gupta et al. (2022) and references therein.

Queue with differentiated vacations models many real-life problems such as the library, bank and supermarket, etc. The concept of differentiated vacation was first introduced by Ibe and Isijola (2014). They obtained the steady-state probabilities of the single server Markovian queueing system. Later on, the transient solution of the *M/M/1* queueing model with multiple differentiated vacation has been analyzed by Vijayashree and Janani (2018). Bouchentouf and Medjahri (2019) studied the performance analysis of differentiated multiple vacations queueing systems by incorporating the concept of balking and feedback. Sampath et al. (2020) analyzed the transient solution of the *M/M/1* queueing system with multiple differentiated vacations using the concept of waiting servers and impatient customers. Gupta and Kumar (2021c) considered state-dependent arrivals of customers in the retrial queueing system under a differentiated vacation policy. Gupta and Malik (2021) obtained system characteristics of an unreliable feedback queueing system with differentiated vacations and a waiting server.

In this paper, we have introduced the concept of differentiated vacations to classical retrial queues with Bernoulli's feedback of customers. In queueing literature, there is no work available on retrial queues with differentiated vacations under customers' feedback. The rest of the paper is organized as follows. Section 2 deals with the practical application of the model. The mathematical formulation of the quasi birth-death model is discussed in Section 3. Section 4 presents differential-difference equations for the model. The stability condition and the steady-state equations are explained in Section 5. Probability generating functions and their solutions are presented in Section 6. Section 7 describes some of the performance measures of the system. Graphical results are explained in Section 8. Finally, the conclusion and future scope are presented in Section 9.

2. Practical application of the model

Consider a denting/painting service station with one operator. The customers arriving for denting/painting of their vehicle will get immediate service if the operator is free, otherwise they will have to leave the service area and retry for their turn. When no more customers arrive for the service, the operator leaves for some secondary work. The customers arriving during the period have to wait for the service. On his return from secondary work, if he finds customers waiting for the service, he will resume the normal service, otherwise, he will leave for some short-duration tasks. When denting/painting work of a customer is completed, the satisfied customer will leave but the unsatisfied customer will re-join the service. In queueing terminology, the operator, secondary work, short duration task and unsatisfied customers correspond to the server, type I vacation, type II vacation and feedback customers respectively.

3. Quasi birth-death model

In the present paper, we consider an *M/M/1* queueing model with feedback and differentiated vacations under classical retrial policy. The detailed description of the proposed model is as follows:

1. The customers arrive in the system according to Poisson distribution with parameter λ. If the server is in a normal working state, the arriving customer immediately receives service, otherwise, they are obliged to join the free pool (orbit) to wait for their turn.
2. Customers waiting in the orbit retry for their request with classical retrial rate β and inter-retrial times are exponentially distributed.
3. The service time in a normal busy period is exponentially distributed with parameter μ.

4. As the system gets empty the server leaves for a type I vacation without waiting for customers. On completion of type I vacation, if the server finds no customers in the system he leaves for type II vacation of shorter duration, otherwise, the server returns to normal free-state to serve the customers waiting in the orbit. On the other hand, on completion of vacation of type II, if the server finds no customers, it remains in type II vacation. The period for both vacations follows exponential distributions with parameters v_1 and v_2 respectively with $v_2 > v_1$.
5. On completion of service, there are chances that the customer leaves the system with probability $\bar{f}$ (= 1-f) or rejoins the orbit for another service with probability f. The schematic diagram of the model is shown below.

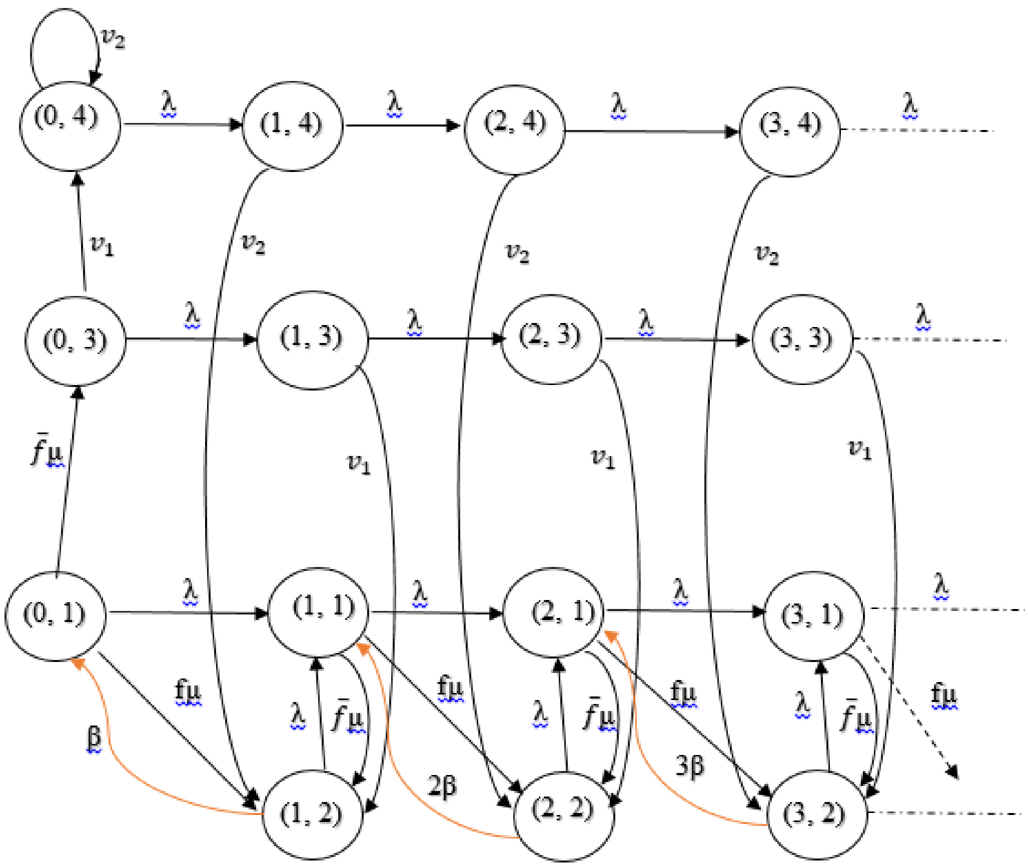

Figure 1. State transition diagram of the model.

4. Differential - difference equations

Denoting the number of customers in the orbit by *N(t)* at a given time *t* and the state of the server at a given time t by *H(t)*. The possible values of the server states are:

$$H(t) = \begin{cases} 1, \text{the server is busy during normal service} \\ 2, \text{the server is free during normal service} \\ 3, \text{the server is in type I vacation state} \\ 4, \text{the server is in type II vacation state} \end{cases}$$

Then, $\{N(t), H(t)\}$ is a continuous Markov process with state-space

$$S = \{(n, i), n \geq 0, 1 \leq i \leq 4\} - \{(0, 2)\}.$$

On considering $p_{n\,i}(t)$ as the probability of n customers in orbit in i^th^ state at time t, we have obtained the system of Kolmogorov differential-difference equations as follows:

$$\frac{d}{dt}p_{0\,1}(t) = \beta p_{1\,2}(t) - \big(\lambda + f\mu + \mu(1-f)\big)p_{0\,1}(t), \tag{1}$$

$$\frac{d}{dt}p_{n\,1}(t) = \beta(n+1)p_{n+1\,2}(t) + \lambda p_{n-1\,1}(t) + \lambda p_{n\,2}(t) - \big(\lambda + f\mu + \mu(1-f)\big)p_{n\,1}(t), \qquad n \geq 1, \tag{2}$$

$$\frac{d}{dt}p_{n\,2}(t) = v_1 p_{n\,3}(t) + v_2 p_{n\,4}(t) + f\mu p_{n-1\,1}(t) + \mu(1-f)p_{n\,1}(t) - (\lambda + n\beta)p_{n\,2}(t) \qquad n \geq 1, \tag{3}$$

$$\frac{d}{dt}p_{0\,3}(t) = \mu(1-f)p_{0\,1}(t) - (\lambda + v_1)p_{0\,3}(t) \tag{4}$$

$$\frac{d}{dt}p_{n\,3}(t) = \lambda p_{n-1\,3}(t) - (\lambda + v_1)p_{n\,3}(t)\,, \qquad n \geq 1\,, \tag{5}$$

$$\frac{d}{dt}p_{0\,4}(t) = v_1 p_{0\,3}\,(t) - \lambda p_{0\,4}\,(t), \tag{6}$$

$$\frac{d}{dt}p_{n\,4}(t) = \lambda p_{n-1\,4}(t) - (\lambda + v_2)p_{n\,4}(t), \qquad n \geq 1. \tag{7}$$

5. Stability condition and steady-state equations

The stability condition of the model is $\lambda < \bar{f}\mu$.

Under the stability condition, on taking limit $t \to \infty$, the differential-difference equations reduce to the following set of steady-state equations:

$$\big(\lambda + f\mu + \mu(1-f)\big)p_{0\,1} = \beta p_{1\,2}, \tag{8}$$

$$\big(\lambda + f\mu + \mu(1-f)\big)p_{n\,1} = \beta(n+1)p_{n+1\,2} + \lambda p_{n-1\,1} + \lambda p_{n\,2}, \qquad n \geq 1, \tag{9}$$

$$(\lambda + n\beta)p_{n\,2} = v_1 p_{n\,3} + v_2 p_{n\,4} + f\mu p_{n-1\,1} + \mu(1-f)p_{n\,1}, \qquad n \geq 1, \tag{10}$$

$$(\lambda + v_1)p_{0\,3} = \mu(1-f)p_{0\,1}, \tag{11}$$

$$(\lambda + v_1)p_{n\,3} = \lambda p_{n-1\,3}, \qquad n \geq 1, \tag{12}$$

$$\lambda p_{0\,4} = v_1 p_{0\,3}, \tag{13}$$

$$(\lambda + v_2) p_{n\,4} = \lambda p_{n-1\,4}, \qquad n \geq 1. \tag{14}$$

6. Probability generating functions and solutions

Define the probability generating functions as

$$P_i(z) = \sum_{n=0}^{\infty} p_{n\,i} z^n, \qquad i = 1, 3, 4, \tag{15}$$

$$P_2(z) = \sum_{n=1}^{\infty} p_{n\,2} z^n. \tag{16}$$

Multiplying equations (11) and (12) with appropriate powers of z, summing over n and making use of Equation (15), we have

$$(\lambda + v_1 - \lambda z) P_3(z) = \mu(1-f) p_{0\,1}$$
$$= (\lambda + v_1) p_{0\,3},$$

Which implies

$$P_3(z) = \frac{\lambda + v_1}{\lambda + v_1 - \lambda z} p_{0\,3}. \tag{17}$$

On similar steps Equations (12), (13) and (14) yield

$$(\lambda + v_2 - \lambda z) P_4(z) = v_1 p_{0\,3} + v_2 p_{0\,4}$$
$$= \frac{v_1(\lambda + v_2)}{\lambda} p_{0\,3},$$

$$P_4(z) = \frac{v_1(\lambda + v_2)}{\lambda(\lambda + v_2 - \lambda z)} p_{0\,3}. \tag{18}$$

Multiplying Equations (8) and (9) with suitable powers of z, summing over n and using P.G.Fs, we arrive at

$$(\lambda + \mu - \lambda z) P_1(z) = \beta P_2'(z) + \lambda P_2(z). \tag{19}$$

Similarly, Equations (10) and (16) give

$$\lambda P_2(z) + \beta z P_2'(z) = \big(\mu + f\mu(z-1)\big) P_1(z) + v_1 P_3(z) + v_2 P_4(z) - A p_{0\,3}, \tag{20}$$

where

$$A = \lambda + v_1 + \frac{v_1(\lambda + v_2)}{\lambda}. \tag{21}$$

From Equations (19) and (20) we get

$$P_2'(z) + \frac{\lambda(\lambda + f\mu)}{\beta(f\mu - \mu + \lambda z)} P_2(z) = \frac{(\lambda + \mu - \lambda z)(v_1 P_3(z) + v_2 P_4(z) - Ap_{0\,3})}{\beta(f\mu - \mu + \lambda z)(1 - z)} . \tag{22}$$

Using $\text{I.F} = (|f\mu - \mu + \lambda z|)^{\frac{\lambda + f\mu}{\beta}}$, (23)

The solution of the differential equation (16) is as follows,

$$P_2(z) = |f\mu - \mu + \lambda z|^{\frac{-(\lambda + f\mu)}{\beta}} \int_0^z I.F \frac{(\lambda + \mu - \lambda z)(v_1 P_3(z) + v_2 P_4(z) - Ap_{0\,3})}{\beta(f\mu - \mu + \lambda z)(1 - z)}\, dz . \tag{24}$$

Using Equation (19) in Equation (20), we get

$$\big((\lambda + \mu - \lambda z)z - \mu - f\mu(z - 1)\big) P_1(z) = \lambda(z - 1)P_2(z) + v_1 P_3(z) + v_2 P_4(z) - Ap_{0\,3},$$

$$P_1(z) = \frac{\lambda(z - 1)P_2(z) + v_1 P_3(z) + v_2 P_4(z) - Ap_{0\,3}}{(\lambda + \mu - \lambda z)z - \mu - f\mu(z - 1)}$$

$$= \frac{\lambda P_2(z)}{\mu - \lambda z - f\mu} + \frac{v_1 P_3(z) + v_2 P_4(z) - Ap_{0\,3}}{(1 - z)(\lambda z - \mu + f\mu)} . \tag{25}$$

Thus, all the probability generating functions are implicitly expressed in terms of $p_{0\,3}$ only which can be obtained by using total probability law.

Taking limit $z \to 1$ in Equation (17), (18) and (24), we obtain

$$P_2(1) = |f\mu - \mu + \lambda|^{\frac{-(\lambda + f\mu)}{\beta}} \int_0^1 I.F \frac{(\lambda + \mu - \lambda z)(v_1 P_3(z) + v_2 P_4(z) - Ap_{0\,3})}{\beta(f\mu - \mu + \lambda z)(1 - z)}\, dz , \tag{26}$$

$$P_3(1) = \frac{\lambda + v_1}{v_1} p_{0\,3} , \tag{27}$$

$$P_4(1) = \frac{v_1(\lambda + v_2)}{\lambda v_2} p_{0\,3} . \tag{28}$$

Differentiating Equations (17) and (18) successively and on taking limit $z \to 1$, we have

$$P_3'(1) = \frac{\lambda(\lambda + v_1)}{{v_1}^2} p_{0\,3} , \tag{29}$$

$$P_3''(1) = \frac{2\lambda^2(\lambda + v_1)}{{v_1}^3} p_{0\,3} , \tag{30}$$

$$P_4'(1) = \frac{v_1(\lambda + v_2)}{{v_2}^2} p_{0\,3} , \tag{31}$$

$$P_4''(1) = \frac{2 v_1 \lambda(\lambda + v_2)}{{v_2}^3} p_{0\,3} . \tag{32}$$

Taking limit $z \to 1$ in Equation (25) and using L'Hospital's rule we obtain

$$P_1(1) = \frac{\lambda P_2(1)}{\mu - \lambda - f\mu} + \frac{v_1 P_3'(1) + v_2 P_4'(1)}{\mu - \lambda - f\mu} \ . \tag{33}$$

After the rearrangement of terms in Equation (19) and taking limit $z \to 1$, we get

$$P_2'(1) = \frac{\mu P_1(1) - \lambda P_2(1)}{\beta} \ . \tag{34}$$

Differentiating Equation (25) and using L' Hospital's rule twice, we obtain

$$P_1'(1) = \frac{v_1 P_3''(1) + v_2 P_4''(1) + 2\lambda P_2'(1)}{2(\mu - \lambda - f\mu)} + \frac{\lambda^2 P_2(1) + \lambda(v_1 P_3'(1) + v_2 P_4'(1))}{(\mu - \lambda - f\mu)^2} \ . \tag{35}$$

Using total probability law

$$\sum_{n=0}^{\infty} p_{n\,1} + \sum_{n=1}^{\infty} p_{n\,2} + \sum_{n=0}^{\infty} p_{n\,3} + \sum_{n=0}^{\infty} p_{n\,4} = 1 \ , \tag{36}$$

we have,

$$p_{0\,3} = \frac{\mu - \lambda - f\mu}{\mu(1-f)} \left[\frac{\{\lambda v_2(\lambda + v_1) + {v_1}^2(\lambda + v_2)\}}{\lambda v_1 v_2} + |\mu - \lambda - f\mu|^{\frac{-(\lambda + f\mu)}{\beta}} \int_0^1 \frac{(\lambda + \mu - \lambda z)}{\beta(1-z)(f\mu - \mu + \lambda z)} \left(\frac{v_1(\lambda + v_1)}{\lambda + v_1 - \lambda z} \ z + \frac{v_1 v_2(\lambda + v_2)}{\lambda(\lambda + v_2 - \lambda z)} - A \right) |f\mu - \mu + \lambda z|^{\frac{\lambda + f\mu}{\beta}} dz \right]^{-1} . \tag{37}$$

7. Some important system performance measures

In the present section, we present some of the important system performance measures of the system as follows:

$\mathrm{E}[L_0]$ = The expected number of customers in the orbit

$$= P_1'(1) + P_2'(1) + P_3'(1) + P_4'(1) \ , \tag{38}$$

$\mathrm{E}[L_s]$ = The expected number of customers in the system

$$= \mathrm{E}[L_0] + P_1(1) \ , \tag{39}$$

$E[W_0]$ = Expected sojourn time of customers in orbit

$$= \frac{\mathrm{E}[L_0]}{\lambda} \ , \qquad (Little's\ formula), \tag{40}$$

$E[W_s]$ = Expected sojourn time of customers in the system

$$= \frac{\mathrm{E}[L_s]}{\lambda}, \qquad (Little's\ formula), \tag{41}$$

Pr_{V1} = Probability of server in type I vacation

$$= P_3(1)$$

$$= \sum_{n=0}^{\infty} p_{n\,3}, \tag{42}$$

Pr_{V2} = Probability of server in type II vacation

$$= P_4(1)$$

$$= \sum_{n=0}^{\infty} p_{n\,4}, \tag{43}$$

Pr_V = Probability of server on vacations

$$= Pr_{V1} + Pr_{V2}, \tag{44}$$

Pr_N = Probability of server in working (active) state

$$= P_1(1) + P_2(1)$$

$$= \sum_{n=0}^{\infty} p_{n\,1} + \sum_{n=1}^{\infty} p_{n\,2}. \tag{45}$$

8. Graphical illustrations

In this section, we illustrate the behavior of some of the performance measures of the system with variations in various parameters. We have also optimized the expected cost per unit time concerning service rate.

In the below graphs, we have fixed λ=1.3, μ=5, β=2, $v_1 = 0.7, v_2 = 1,\ f = 0.3$ unless they are varied in the graphs.

(a) Sensitivity analysis

From Figure 2, it is obvious that expected orbit length $\mathrm{E}[L_0]$, expected system length $\mathrm{E}[L_s]$ and probability of server in normal active state Pr_N all increase whereas the probability of the server in vacations Pr_V decreases with increasing feedback probability$f$; this is because the chances of feedback of customers increase with increase in f thereby increasing the expected orbit length, system length and probability of server being in normal state.

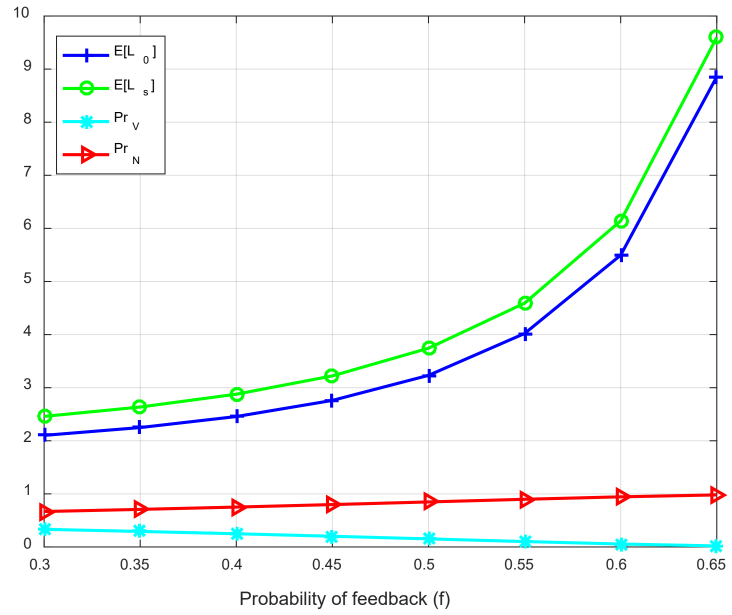

Figure 2. Effect of feedback probability on system performance measures

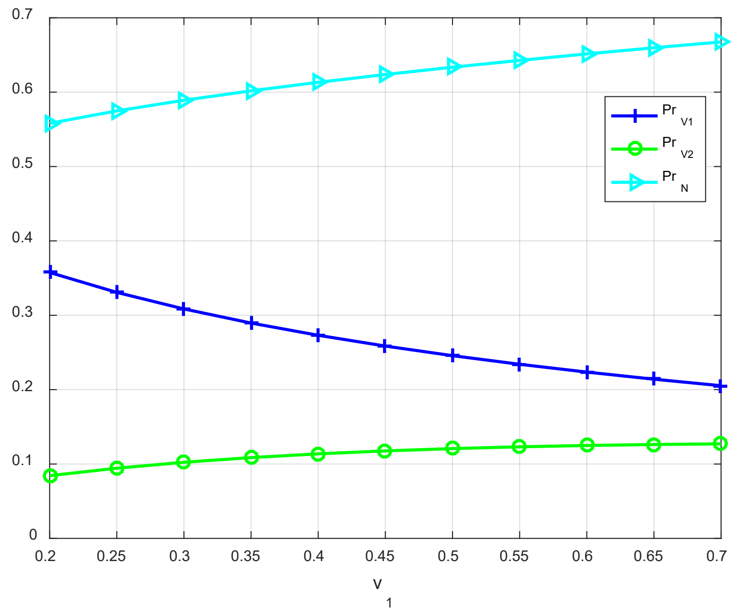

Figure 3. Effect of change in v_1 on probabilities of various states

Figure 3 illustrates the effect of type I vacation rate v_1 on Pr_{V1}, Pr_{V2}, Pr_N. We observe that with an increase in v_1, probabilities Pr_{V2} and Pr_N both increase but Pr_{V1} decreases. This is because as v_1 increases, the period of vacation of type I decreases, thereby increasing the probability of server being in type II vacation, normal state probability and decreasing that of being in type I vacation.

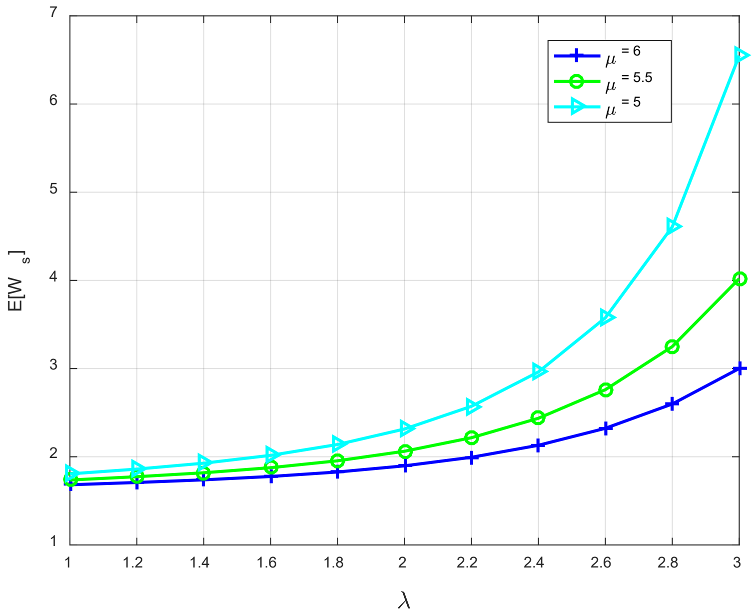

Figure 4. Effect of arrival rate (λ) on expected sojourn time of customers in the system

Figure 4 represents that the expected waiting time of customers in system $E[W_s]$ increases with an increase in λ; this is because inter-arrival times decrease as λ increases. This increase in expected waiting time becomes more obvious with decrease in μ because the service time increases, and hence, the expected waiting time in the system increases.

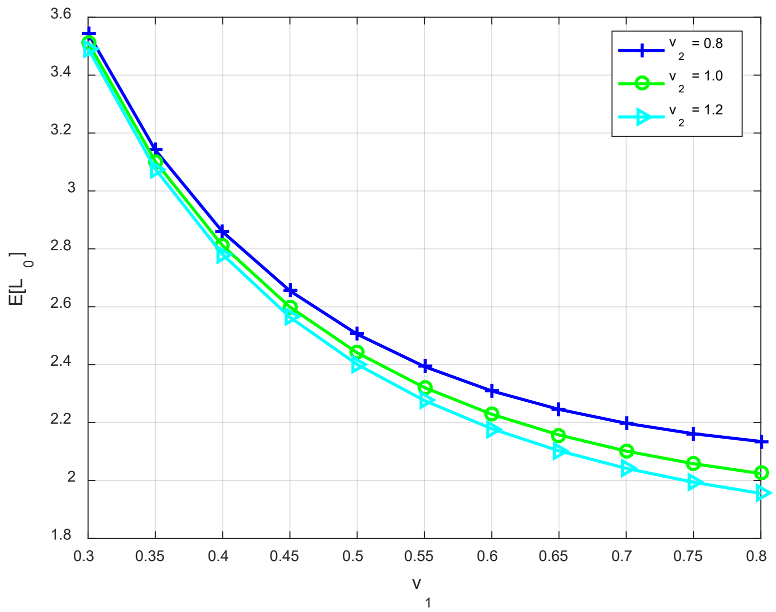

Figure 5. Effect of v_1 on expected orbit length for different values of v_2

Figure 5 reveals that as v_1 increases, the expected orbit length E$[L_0]$ decreases; because the period of vacation of type I becomes shorter hence the normal service starts earlier causing the reduction in number of customers in orbit. This decrease becomes more and more prominent with an increase in v_2; the reason is that the period of vacation of type II goes on decreasing.

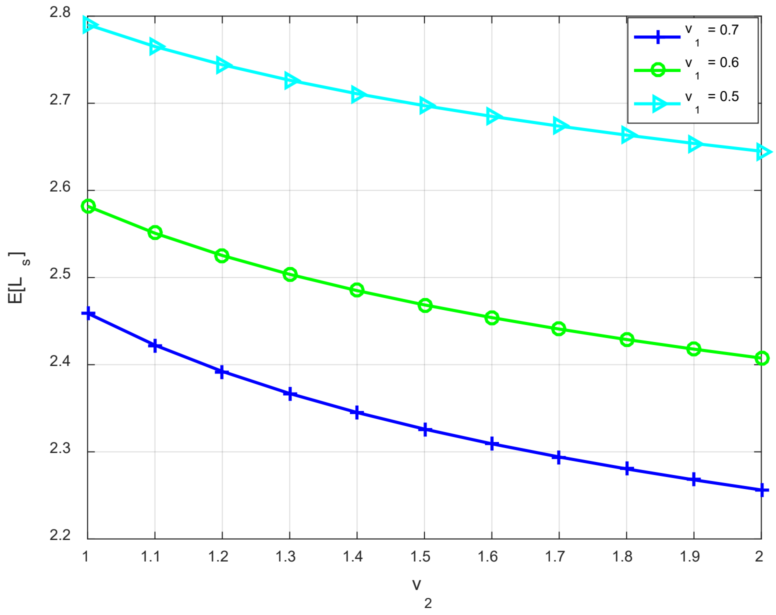

Figure 6. Effect of v_2 on expected system length for different values of v_1

Figure 6 illustrates that as v_2 increases, the expected system length $\mathrm{E}[L_s]$ decreases and it decreases further with the increase in v_1 as expected. Since as v_2 increases, the vacation of type II becomes of shorter duration thereby reducing the mean system length.

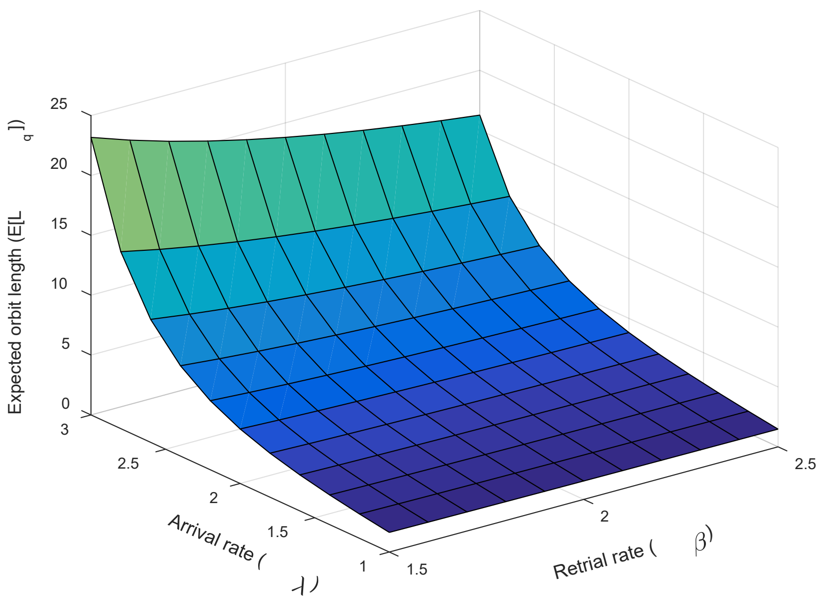

Figure 7. $E[L_q]$ versus λ and β

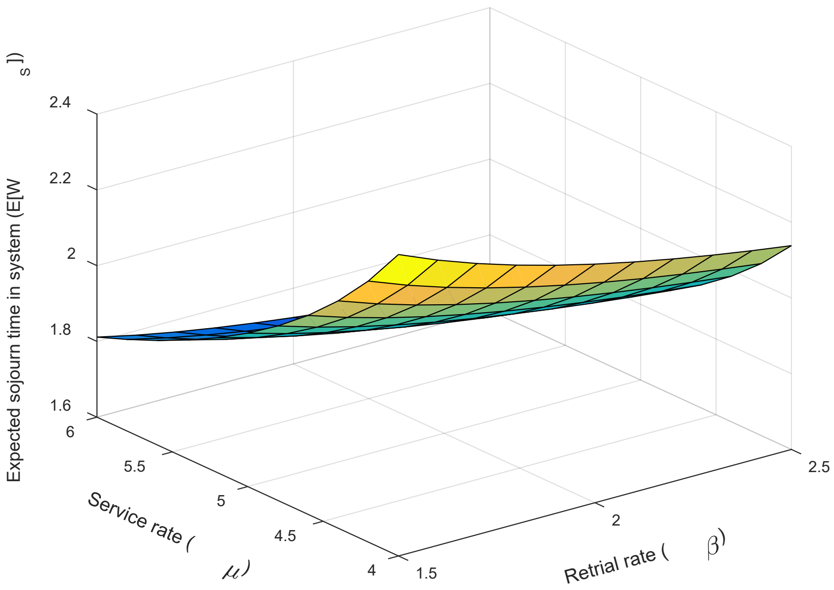

Figure 8. $E[W_S]$ versus μ and β

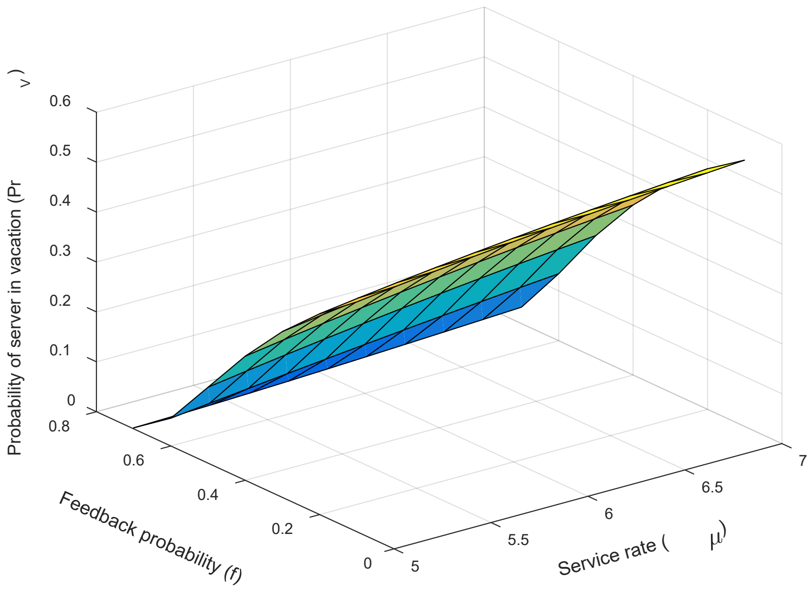

Figure 9. Pr_V versus f and μ

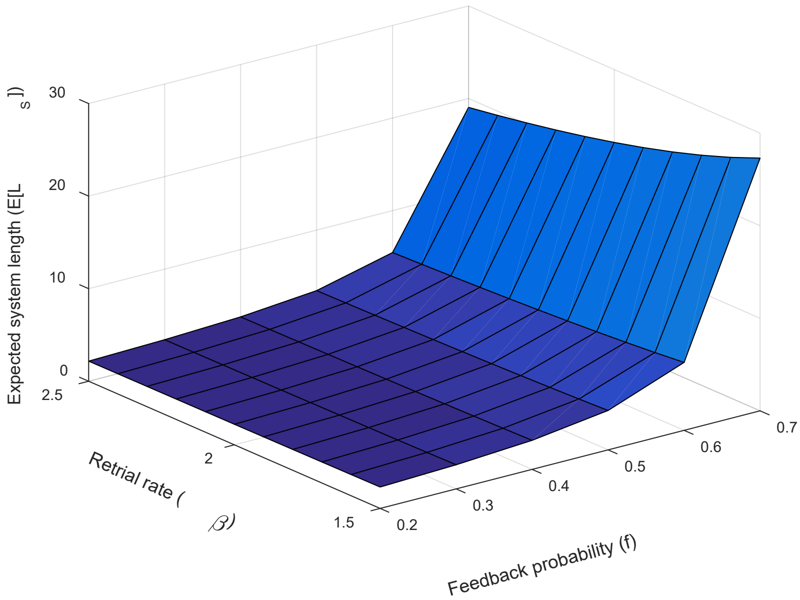

Figure 10. $E[L_S]$ versus β and f

Figures 7 and 8 represent variation in expected orbit length and expected sojourn times in the system in three dimensions. Figure 9 shows variation in the probability of the server on vacations with feedback probability and service rate in three dimensions. Figure 10 shows the effect of retrial rate β and feedback probability f on expected system length in three dimensions.

(b) Cost analysis

As service rate μ increases, the mean operating cost is observed to first decrease and then increase as illustrated in Figure 11. In this section, we construct a cost function to find the service rate μ for which the expected cost is optimized. For this purpose, we define some cost elements as follows:

C_L = cost per customer present in the orbit per unit time,

C_μ = cost per unit time for service in a normal busy period,

C_{v_1}= fixed cost per unit time during the vacation of type I,

C_{v_2}= fixed cost per unit time during the vacation of type II.

We construct an operating cost function per unit time as

$$F(\mu) = C_L \, E[L_0] + C_\mu \mu + C_{v_1} v_1 + C_{v_2} v_2 \, .$$

Taking cost elements as C_L = 24,C_μ =30, C_{v_1}=14, and C_{v_2}=12; from Figure 8, we observe that there is an optimal value of service rate. To obtain the optimal service rate, we use the parabolic method. The point at which F(x) is optimum in three-point pattern $\{x_1, x_2, x_3\}$ is given by

$$x_L = \frac{0.5(F(x_1)(x_2{}^2 - x_3{}^2) + F(x_2)(x_3{}^2 - x_1{}^2) + F(x_3)(x_1{}^2 - x_2{}^2))}{F(x_1)(x_2 - x_3) + F(x_2)(x_3 - x_1) + F(x_3)(x_1 - x_2)}.$$

The new value obtained replaces one of the three points to improve the current three-point pattern. The process is repeatedly applied until an optimum value is obtained up to the desired degree of accuracy.

Taking 10^{-4} as the stopping tolerance and closely observing Figure 11, we choose initial points. Table 1 shows that optimum value $F(\mu)$=193.899635 corresponding to μ=3.344503 with the permissible error of 10^{-4}, which is verified by Figure 11.

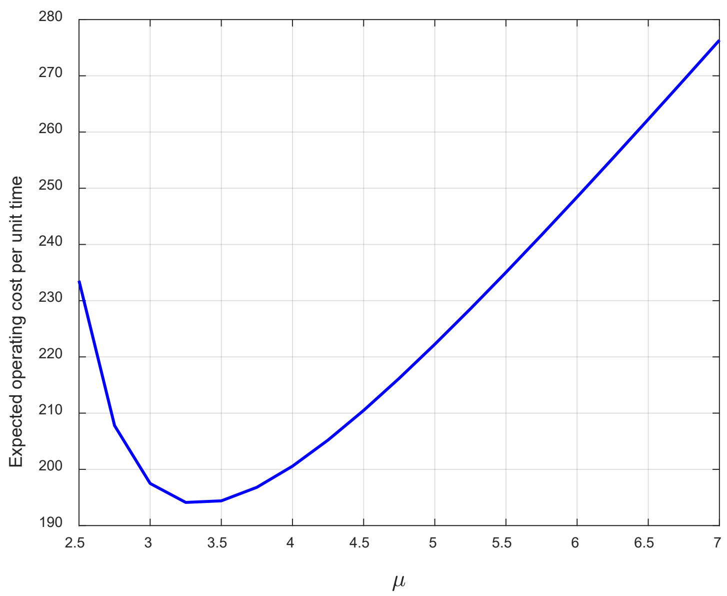

Figure 11. Effect of service rate (μ) on Expected operating cost per unit time

Table 1. Parabolic method for optimization of Expected cost function per unit time

x_1	x_2	x_3	$F(x_1)$	$F(x_2)$	$F(x_3)$	x_L
3.00	3.50	3.75	197.486512	194.396990	196.808353	3.396428
3.00	3.396428	3.50	197.486512	193.959136	194.396990	3.367692
3.00	3.367692	3.396428	197.486512	193.911768	193.959136	3.353324
3.00	3.353324	3.367692	197.486512	193.901419	193.911768	3.348323
3.00	3.348323	3.353324	197.486512	193.899974	193.901419	3.346002
3.00	3.346002	3.348323	197.486512	193.899688	193.899974	3.345116
3.00	3.345116	3.346002	197.486512	193.899643	193.899688	3.344725
3.00	3.344725	3.345116	197.486512	193.899635	193.899643	3.344570
3.00	3.344570	3.344725	197.486512	193.899634	193.899635	3.344503

9. Conclusion and future scope

In the present paper, we have analyzed the Markovian feedback queueing model with differentiated vacation under classical retrial policy using the quasi birth-death process and probability generating function technique. We have derived closed-form expressions for various system performance measures. The variation in expected orbit length, system length, expected sojourn times and various states of the server are studied numerically with various system parameters like arrival rate, service rate, rates of two types of vacations etc. and are graphically represented via MATLAB software. As expected, the mean orbit and system lengths are found to decrease with type I and type II vacation rates and increase with increase in arrival rate. The three-dimensional behaviour of expected system length, mean sojourn time spent by customers in the system, expected orbit length and probability of server in vacations have been observed graphically relative to different parameters. Finally, the optimal service rate for the model is obtained using the parabolic method. It will be useful further to extend the model with general service time or general retrial times in multi-server environment.

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