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M. Palanikumar
Saveetha Institute of Medical and Technical Sciences

K. Arulmozhi
Bharath Institute of Higher Education and Research

Lejo J. Manavalan
Little Flower College

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TOPSIS and VIKOR Methods for Spherical Fuzzy Soft Set Aggregating Operator Framework

[1*]M. Palanikumar, [2]K. Arulmozhi and [3]Lejo J Manavalan

[1]Department of Mathematics
Saveetha School of Engineering
Saveetha Institute of Medical and Technical Sciences
Chennai-602105, India
palanimaths86@gmail.com

[2]Department of Mathematics
Bharath Institute of Higher Education and Research
Chennai-600073
Tamilnadu, India
arulmozhiems@gmail.com

[3]Department of Mathematics
Little Flower College-680103
Guruvayoor, India
lejo@littleflowercollege.edu.in

*Corresponding Author

Abstract

The Spherical Fuzzy Soft (SFS) set is a generalization of the Pythagorean fuzzy soft set and the intuitionistic fuzzy soft set. We introduce the concept of aggregating SFS decision matrices based on aggregated operations. The techniques for order of preference by similarity to ideal solution (TOPSIS) and viekriterijumsko kompromisno rangiranje (VIKOR) for the SFS approaches are the strong points of multi criteria group decision making (MCGDM), which is various extensions of fuzzy soft sets. We define a score function based on aggregating TOPSIS and VIKOR methods to the SFS-positive and SFS-negative ideal solutions. The TOPSIS and VIKOR methods provide decision-making weights. To find the optimal alternative under this condition, closeness is introduced. Also, we obtain an algorithm that deals with the MCGDM problems based on an aggregating operator. Finally, a numerical example of the MCGDM problem is given to verify the practicality of the aggregating operators.

Keywords: SFS set; MCGDM; TOPSIS; VIKOR; Aggregating Operator; SFSV-PIS; SFSV-NIS

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1. Introduction

Uncertainty can be seen everywhere in most real problems. In order to cope with the uncertainties, many uncertain theories, namely fuzzy set (FS) theory, intuitionistic fuzzy set (IFS) theory, Pythagorean fuzzy set (PFS) theory, and spherical fuzzy set theory, are put forward. A FS is a set in which every element in the universe belongs to it, but with a grade or degree of belongingness that ranges between zero and one, and such grades are known as membership values (Zadeh (1965)). Later, the notion of an IFS logic is launched by Atanassov (1986) and is classified by the condition that the sum of its membership degree and non-membership degree value is less than or equal to one. However, we may encounter a problem in decision making (DM) events where the sum of the degree of membership and non-membership of a particular attribute exceeds unity. So Yager (2014) introduced the concept of PFS and is characterized by the condition that the square sum of its degree of membership and non-membership does not exceed unity. The notion of a spherical fuzzy set is introduced by Kutlu Gundogdu and Kahraman (2018) as an extension of Pythagorean, neutrosophic, and picture fuzzy sets. Shahzaib Ashraf et al. discussed spherical fuzzy sets, which are an advanced tool of the FSs and IFSs. The concept of a spherical fuzzy set is a generalization of the PFS and it is characterized by the condition that the square sum of its degree of membership and non membership does not exceed unity.

The DM problem indicates the finding of the best optional alternatives. Hwang and Yoon (1981) had discussed using multiple criteria decision making (MCDM) methods. These two methods (TOPSIS and VIKOR) for DM problems have been studied by Adeel et al. (2019), Akram and Arshad (2019), Boran et al. (2009), Eraslan and Karaaslan (2015), Peng nd Dai (2018), Xu and Zhang (2013), and Zhang and Xu (2014). Zulqarnain et al. (2021) discussed the TOPSIS extends to interval valued intuitionistic fuzzy soft sets (shortly IVIFSS). The TOPSIS method consists of distances to positive ideal solution (PIS) and negative ideal solution (NIS), calculating a preference order that is ranked under relative closeness and finding a combination of these two distance measures. In the VIKOR method, the focal point is on ranking and selecting from a set of alternatives and computing compromise solutions for a problem with inconsistent criteria, which can help the decision makers to get a final decision (Opricovic and Tzeng (2007); Opricovic (2011)). Opricovic and Tzeng (2003) discussed the VIKOR method on fuzzy logic. Tzeng et al. (2005) discussed about the comparison of VIKOR with TOPSIS methods using public transportation problems. The purpose of this paper is to extend the concept of Pythagorean fuzzy soft sets under TOPSIS and VIKOR using MCDM methods to spherical fuzzy soft sets under TOPSIS and VIKOR using MCDM methods and derive some of their properties.

The paper is organized into five sections as follows. Section 1 is an introduction. Section 2 discusses MCGDM based on SFS sets and its properties with examples. Section 3 introduces MCGDM based on the SFS-TOPSIS aggregating operator and discusses its properties. Section 4 introduces MCGDM based on the SFS-VIKOR aggregating operator and its properties. Concluding remarks for further investigation are provided in Section 5.

2. MCGDM based on SFS sets

In this section, we introduce a concept of SFS aggregating operator and SFS aggregate matrix.

Definition 2.1.

The cardinal set of the SFS set Ψ_X is denoted by $c\Psi_X$ and is defined as $c\Psi_X = \left\{\frac{e}{\left(\vartheta_{c\delta_X}(e), \varpi_{c\zeta_X}(e), \tau_{c\varphi_X}(e)\right)} : e \in E\right\}$, where $\vartheta_{c\delta_X}$, $\varpi_{c\zeta_X}$ and $\tau_{c\varphi_X} : E \rightarrow [0,1]$, respectively, where $\vartheta_{c\delta_X}(e) = \frac{|\delta_X(e)|}{|\mathbb{U}|}, \varpi_{c\zeta_X}(e) = \frac{|\zeta_X(e)|}{|\mathbb{U}|}, \tau_{c\varphi_X}(e) = \frac{|\varphi_X(e)|}{|\mathbb{U}|}$, where $|\delta_X(e)|$, $|\zeta_X(e)|$ and $|\varphi_X(e)|$ denotes the scalar cardinalities of the SFS sets $\delta_X(e)$, $\zeta_X(e)$ and $\varphi_X(e)$, respectively, and $|\mathbb{U}|$ stand cardinality of $\mathbb{U}$. The collection of all cardinal sets of SFS sets of $\mathbb{U}$ is denoted by $cSF^{\mathbb{U}}$. If $X \subseteq E = \{e_i : i = 1, 2, ..., n\}$, then $c\Psi_X \in cSF^{\mathbb{U}}$ is defined in the matrix form by $[(p_{1j}, q_{1j}, r_{1j})]_{1\times n} = [(p_{11}, q_{11}, r_{11}), (p_{12}, q_{12}, r_{12}), ..., (p_{1n}, q_{1n}, r_{1n})]$, where $(p_{1j}, q_{1j}, r_{1j}) = \mu_{c\Psi_X}(e_j), \forall j = 1, 2, ..., n$. Hence, this matrix is called a cardinal matrix of $c\Psi_X$ of parameter E.

Definition 2.2.

Let $\Psi_X \in SF^{\mathbb{U}}$ and $c\Psi_X \in cSF^{\mathbb{U}}$. The SFS set aggregation operator $SFS_{agg} : cSF^{\mathbb{U}} \times SF^{\mathbb{U}} \rightarrow SFS(\mathbb{U}, E)$ is defined as $SFS_{agg}(c\Psi_X, \Psi_X) = \left\{\frac{u}{\mu_{\Psi^*_X}(u)} : u \in \mathbb{U}\right\} = \left\{\frac{u}{\left(\vartheta_{\delta^*_X}(u), \varpi_{\zeta^*_X}(u), \tau_{\varphi^*_X}(u)\right)} : u \in \mathbb{U}\right\}$, which is an called aggregate spherical fuzzy set of Ψ_X. The positive membership function $\vartheta_{\delta^*_X}(u) : \mathbb{U} \rightarrow [0,1]$ by $\vartheta_{\delta^*_X}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\vartheta_{c\delta_X}(e), \vartheta_{\delta_X}(e)\right)(u)$, neutral membership function $\varpi_{\zeta^*_X}(u) : \mathbb{U} \rightarrow [0,1]$ by $\varpi_{\zeta^*_X}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\varpi_{c\zeta_X}(e), \varpi_{\zeta_X}(e)\right)(u)$, and negative membership function $\tau_{\varphi^*_X}(u) : \mathbb{U} \rightarrow [0,1]$ by $\tau_{\varphi^*_X}(u) = \frac{1}{|E|}\sum_{e\in E}\left(\tau_{c\varphi_X}(e), \tau_{\varphi_X}(e)\right)(u)$. The set $SFS_{agg}(c\Psi_X, \Psi_X)$ is expressed in matrix form as $[(p_{i1}, q_{i1}, r_{i1})]_{m\times 1} = (p_{11}, q_{11}, r_{11}), (p_{21}, q_{21}, r_{21}), \ldots (p_{m1}, q_{m1}, r_{m1})$, where $[(p_{i1}, q_{i1}, r_{i1})] = \mu_{\Psi^*_X}(u_i)$ and i various from 1 to m. The above matrix said to be SFS aggregate matrix of $SFS_{agg}(c\Psi_X, \Psi_X)$ over $\mathbb{U}$.

The MCGDM is based on SFS sets by the following algorithms.

Algorithm I

Step 1: Form the SFS set Ψ_X over $\mathbb{U}$.

Step 2: Calculate the cardinalities and cardinal set $c\Psi_X$ of Ψ_X.

Step 3: Compute aggregate SFS set Ψ^*_X of Ψ_X is $M_{\Psi^*_X} = \frac{M_{\Psi_X} \times M^{\mathbb{T}}_{c\Psi_X}}{|E|}$, where $M_{\Psi_X}, M_{c\Psi_X}, M^*_{\Psi_X}$ be the matrices of $\Psi_X, c\Psi_X, \Psi^*_X$, respectively.

Step 4: Find the score function $S(u) = \vartheta^2_u - \varpi^2_u - \tau^2_u, \forall\, u \in \mathbb{U}, -1 \leq S(u) \leq 1$.

Step 5: Find the best alternative by $\max_i S(u_i)$.

Example 2.1.

An automobile company produces ten different types of car $\mathbb{U} = \{C_1, C_2, ..., C_{10}\}$ and five parameters, namely $E = \{e_1, e_2, ..., e_5\}$, consisting of fuel economy, acceleration, speed, ride comfort, and power steering, respectively. In our problem, the customer has to establish which car to purchase. Each car is evaluated, which is a subset of parameters. That is, $X = \{e_1, e_2, e_3, e_5\} \subseteq E$ and we appeal to Algorithm I.

Step 1: SFS set Ψ_X of $\mathbb{U}$ is formed, which is defined as

$$\Psi_X = \Bigg\{ \left(e_1, \left\{\frac{C_1}{(0.35,0.2,0.7)}, \frac{C_4}{(0.4,0.6,0.1)}, \frac{C_7}{(0.3,0.7,0.15)}, \frac{C_9}{(0.5,0.55,0.1)}, \frac{C_{10}}{(0.15,0.25,0.2)}\right\}\right),$$
$$\left(e_2, \left\{\frac{C_2}{(0.6,0.3,0.15)}, \frac{C_3}{(0.4,0.4,0.3)}, \frac{C_5}{(0.3,0.6,0.1)}, \frac{C_8}{(0.25,0.55,0.35)}, \frac{C_{10}}{(0.7,0.1,0.15)}\right\}\right),$$
$$\left(e_3, \left\{\frac{C_3}{(0.2,0.25,0.55)}, \frac{C_4}{(0.1,0.35,0.4)}, \frac{C_6}{(0.7,0.1,0.5)}, \frac{C_8}{(0.5,0.25,0.3)}, \frac{C_9}{(0.3,0.7,0.15)}\right\}\right),$$
$$\left(e_5, \left\{\frac{C_1}{(0.6,0.5,0.2)}, \frac{C_2}{(0.75,0.15,0.3)}, \frac{C_4}{(0.7,0.2,0.25)}, \frac{C_6}{(0.2,0.5,0.15)}, \frac{C_7}{(0.45,0.3,0.25)}, \frac{C_{10}}{(0.25,0.2,0.5)}\right\}\right) \Bigg\}.$$

Step 2: The cardinal set $c\Psi_X = \left\{\frac{e_1}{(0.17,0.23,0.13)}, \frac{e_2}{(0.23,0.2,0.11)}, \frac{e_3}{(0.18,0.17,0.19)}, \frac{e_5}{(0.3,0.19,0.17)}\right\}$.

Step 3: The aggregate SFS set Ψ_X^* of Ψ_X is $M_{\Psi_X^*} = \frac{M_{\Psi_X} \times M_{c\Psi_X}^{\mathbb{T}}}{|E|}$.

Thus, $M_{\Psi_X^*} = \left\{\frac{C_1}{(0.0473, 0.0277, 0.0241)}, \frac{C_2}{(0.07125, 0.01725, 0.01305)}, \frac{C_3}{(0.0252, 0.02385, 0.0272)}, \frac{C_4}{(0.0585, 0.04655, 0.02595)}, \frac{C_5}{(0.0135, 0.0234, 0.0021)}, \frac{C_6}{(0.037, 0.0218, 0.02395)}, \frac{C_7}{(0.03675, 0.0433, 0.012)}, \frac{C_8}{(0.02925, 0.0297, 0.01875)}, \frac{C_9}{(0.0278, 0.0484, 0.0082)}, \frac{C_{10}}{(0.05135, 0.0228, 0.02465)}\right\}$.

Step 4: Find the score function $S(C_i)$ as below.

$$S(C_i) = \left\{\frac{S(C_1)}{0.00089}, \frac{S(C_2)}{0.00461}, \frac{S(C_3)}{-0.00067}, \frac{S(C_4)}{0.00058}, \frac{S(C_5)}{-0.00037}, \frac{S(C_6)}{0.00032}, \frac{S(C_7)}{-0.00067}, \frac{S(C_8)}{-0.00038}, \frac{S(C_9)}{-0.00164}, \frac{S(C_{10})}{0.00151}\right\}.$$

Step 5: Here $\max_i S(C_i) = 0.00461$.

Algorithm II

Step 1: Form SFS matrix.

Step 2: Case I: Calculate the choice matrix for the positive, neutral, and negative membership of the SFS matrix when weights are equal.

Case II: Find the choice matrix for the positive, neutral, and negative-membership of the SFS matrix when weights are unequal.

Step 3: Find the score function $S(u) = \vartheta_u^2 - \varpi_u^2 - \tau_u^2, \forall\, u \in \mathbb{U}$ and $-1 \leq S(u) \leq 1$.

Step 4: Find the best alternative by $\max_i S(u_i)$.

Case I: Let $X = (\vartheta_{ij}, \varpi_{ij}, \tau_{ij}) \in SFSM_{m\times n}$. Then the choice matrix of SFS X is

$\mathcal{C}(X) = \left[\left(\frac{\sum_{j=1}^{n}(\vartheta_{ij})^2}{n}, \frac{\sum_{j=1}^{n}(\varpi_{ij})^2}{n}, \frac{\sum_{j=1}^{n}(\tau_{ij})^2}{n}\right)\right]_{m\times 1}$ for every i. By Example 2.1,

$$\mathcal{C}(X) = \left\{ \begin{bmatrix} 0.0965 \\ 0.1845 \\ 0.04 \\ 0.132 \\ 0.018 \\ 0.106 \\ 0.0585 \\ 0.0625 \\ 0.068 \\ 0.115 \end{bmatrix}, \begin{bmatrix} 0.058 \\ 0.0225 \\ 0.0445 \\ 0.1045 \\ 0.072 \\ 0.052 \\ 0.116 \\ 0.073 \\ 0.1585 \\ 0.0225 \end{bmatrix}, \begin{bmatrix} 0.106 \\ 0.0225 \\ 0.0785 \\ 0.0465 \\ 0.002 \\ 0.0545 \\ 0.017 \\ 0.0425 \\ 0.0065 \\ 0.0625 \end{bmatrix} \right\}.$$

$$S(C_i) = \left\{ \frac{S(C_1)}{-0.00529}, \frac{S(C_2)}{0.03303}, \frac{S(C_3)}{-0.00654}, \frac{S(C_4)}{0.00434}, \frac{S(C_5)}{-0.00486}, \frac{S(C_6)}{0.00556}, \frac{S(C_7)}{-0.01032}, \frac{S(C_8)}{-0.00323}, \frac{S(C_9)}{-0.02054}, \frac{S(C_{10})}{0.00881} \right\}.$$

Case II: Weighted choice matrix of SFS X (for every i, where $w_j > 0$ are weights) is

$\mathcal{C}_w(X) = \left[\left(\frac{\sum_{j=1}^{n} w_j(\vartheta_{ij})^2}{\sum w_j}, \frac{\sum_{j=1}^{n} w_j(\varpi_{ij})^2}{\sum w_j}, \frac{\sum_{j=1}^{n} w_j(\tau_{ij})^2}{\sum w_j}\right)\right]_{m\times 1}$.

Weights $(w_j) = \{0.27, 0.15, 0.18, 0.21, 0.19\}$ and by Example 2.1,

$$\mathcal{C}_w(X) = \left\{ \begin{bmatrix} 0.101475 \\ 0.160875 \\ 0.0312 \\ 0.1381 \\ 0.0135 \\ 0.0958 \\ 0.062775 \\ 0.054375 \\ 0.0837 \\ 0.09145 \end{bmatrix}, \begin{bmatrix} 0.0583 \\ 0.017775 \\ 0.03525 \\ 0.12685 \\ 0.054 \\ 0.0493 \\ 0.1494 \\ 0.056625 \\ 0.169875 \\ 0.025975 \end{bmatrix}, \begin{bmatrix} 0.1399 \\ 0.020475 \\ 0.06795 \\ 0.043375 \\ 0.0015 \\ 0.049275 \\ 0.01795 \\ 0.034575 \\ 0.00675 \\ 0.061675 \end{bmatrix} \right\}.$$

$$S(C_i) = \left\{ \frac{S(C_1)}{-0.01267}, \frac{S(C_2)}{0.02515}, \frac{S(C_3)}{-0.00489}, \frac{S(C_4)}{0.0011}, \frac{S(C_5)}{-0.00274}, \frac{S(C_6)}{0.00432}, \frac{S(C_7)}{-0.0187}, \frac{S(C_8)}{-0.00145}, \frac{S(C_9)}{-0.0219}, \frac{S(C_{10})}{0.00388} \right\}.$$

Algorithm III

Step 1: Find the spherical fuzzy weighted averaging numbers (SFWANs) under aggregated operation, $C_i = \left(\sum_{j=1}^{n} w_j \vartheta_{ij}, \sum_{j=1}^{n} w_j \varpi_{ij}, \sum_{j=1}^{n} w_j \tau_{ij}\right)$.

Step 2: Obtain the score function $S(u) = \vartheta_u^2 - \varpi_u^2 - \tau_u^2, \forall\, u \in \mathbb{U}$ and $-1 \leq S(u) \leq 1$.

Step 3: Find the best alternative by $\max_i S(u_i)$.

Weights $(w_j) = \{0.27, 0.15, 0.18, 0.21, 0.19\}$ and by Example 2.1,

$$C_i = \left\{ \begin{bmatrix} 0.2085 \\ 0.2325 \\ 0.096 \\ 0.259 \\ 0.045 \\ 0.164 \\ 0.1665 \\ 0.1275 \\ 0.189 \\ 0.193 \end{bmatrix}, \begin{bmatrix} 0.149 \\ 0.0735 \\ 0.105 \\ 0.263 \\ 0.09 \\ 0.113 \\ 0.246 \\ 0.1275 \\ 0.2745 \\ 0.1205 \end{bmatrix}, \begin{bmatrix} 0.227 \\ 0.0795 \\ 0.144 \\ 0.1465 \\ 0.015 \\ 0.1185 \\ 0.088 \\ 0.1065 \\ 0.054 \\ 0.1715 \end{bmatrix} \right\}.$$

$$S(C_i) = \left\{ \frac{S(C_1)}{-0.03026}, \frac{S(C_2)}{0.04233}, \frac{S(C_3)}{-0.02255}, \frac{S(C_4)}{-0.02355}, \frac{S(C_5)}{-0.0063}, \frac{S(C_6)}{0.000085}, \frac{S(C_7)}{-0.04054}, \frac{S(C_8)}{-0.01134}, \frac{S(C_9)}{-0.04255}, \frac{S(C_{10})}{-0.00668} \right\}.$$

Hence, the car C_2 for the customer to be purchased.

3. MCGDM based on SFS-TOPSIS aggregating operator

Algorithm IV (SFS-TOPSIS)

Step 1: Assume that there are finite decision makers, $\mathcal{D} = \{\mathcal{D}_i : i \in \mathbb{N}\}$, a finite collection of alternatives, $\mathcal{C} = \{\ddot{x}_i : i \in \mathbb{N}\}$, and a finite family of parameters, $\mathcal{D} = \{e_i : i \in \mathbb{N}\}$.

Step 2: Form a linguistic term with an obtained weighted parameter matrix (weight ω_{ij} means $\mathcal{D}_i$ to $\ddot{x}_j$ by considering linguistic variables),

$$\mathcal{P} = [\omega_{ij}]_{n\times m} = \begin{bmatrix} \omega_{11} & \omega_{12} & \ldots & \omega_{1m} \\ \omega_{21} & \omega_{22} & \ldots & \omega_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{i1} & \omega_{i2} & \ldots & \omega_{im} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{n1} & \omega_{n2} & \ldots & \omega_{nm} \end{bmatrix}.$$

Step 3: Obtain a weighted normalized decision matrix ($\widehat{\eta}_{ij} = \frac{\omega_{ij}}{\sqrt{\sum_{i=1}^{n}\omega_{ij}^2}}$ and $\mathcal{W} = (m_1, m_2, ..., m_m)$, where $m_i = \frac{\omega_i}{\sqrt{\sum_{l=1}^{n}\omega_{li}}}$ and $\omega_j = \frac{\sum_{i=1}^{n}\widehat{\eta}_{ij}}{n}$),

$$\widehat{\mathcal{N}} = [\widehat{\eta}_{ij}]_{n\times m} = \begin{bmatrix} \widehat{\eta}_{11} & \widehat{\eta}_{12} & \dots & \widehat{\eta}_{1m} \\ \widehat{\eta}_{21} & \widehat{\eta}_{22} & \dots & \widehat{\eta}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{\eta}_{i1} & \widehat{\eta}_{i2} & \dots & \widehat{\eta}_{im} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{\eta}_{n1} & \widehat{\eta}_{n2} & \dots & \widehat{\eta}_{nm} \end{bmatrix}.$$

Step 4: Create an SFS decision matrix,

$$\mathcal{D}_i = [x^i_{jk}]_{l\times m} = \begin{bmatrix} x^i_{11} & x^i_{12} & \dots & x^i_{1m} \\ x^i_{21} & x^i_{22} & \dots & x^i_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x^i_{j1} & x^i_{j2} & \dots & x^i_{jm} \\ \vdots & \vdots & \ddots & \vdots \\ x^i_{l1} & x^i_{l2} & \dots & x^i_{lm} \end{bmatrix}.$$

Here, x^i_{jk} means that SFS element for i^{th} decision maker $\mathcal{D}_i$ for each i. Find the aggregating matrix by $\mathcal{X} = \frac{\mathcal{D}_1+\mathcal{D}_2+...+\mathcal{D}_n}{n} = [\dot{x}_{jk}]_{l\times m}$.

Step 5: Find decision of the weighted SFS matrix (where $\ddot{x}_{jk} = m_k \times \dot{x}_{jk}$),

$$\mathcal{Y} = [\ddot{x}_{jk}]_{l\times m} = \begin{bmatrix} \ddot{x}_{11} & \ddot{x}_{12} & \dots & \ddot{x}_{1m} \\ \ddot{x}_{21} & \ddot{x}_{22} & \dots & \ddot{x}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{x}_{j1} & \ddot{x}_{j2} & \dots & \ddot{x}_{jm} \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{x}_{l1} & \ddot{x}_{l2} & \dots & \ddot{x}_{lm} \end{bmatrix}.$$

Step 6: Find the values of SFSV-PIS and SFSV-NIS. Now,

SFSV-PIS = $[\ddot{x}^+_1, \ddot{x}^+_2, ..., \ddot{x}^+_l] = \{(\vee_k\ddot{x}_{jk}, \wedge_k\ddot{x}_{jk}, \wedge_k\ddot{x}_{jk}) : k = 1, 2, ..., m\}$ and,

SFSV-PIS = $[\ddot{x}^-_1, \ddot{x}^-_2, ..., \ddot{x}^-_l] = \{(\wedge_k\ddot{x}_{jk}, \vee_k\ddot{x}_{jk}, \vee_k\ddot{x}_{jk}) : k = 1, 2, ..., m\}$.

Step 7: Find the SFS-Euclidean distance from SFSV-PIS and SFSV-NIS. Now,

$(d^+_j)^2 = \sum_{k=1}^{m}\left\{(\vartheta^+_{jk} - \vartheta^+_j)^2 + (\varpi^+_{jk} - \varpi^+_j)^2 + (\tau^+_{jk} - \tau^+_j)^2\right\}$, and

$(d^-_j)^2 = \sum_{k=1}^{m}\left\{(\vartheta^-_{jk} - \vartheta^-_j)^2 + (\varpi^-_{jk} - \varpi^-_j)^2 + (\tau^-_{jk} - \tau^-_j)^2\right\}$, where $j = 1, 2, ..., n$.

Step 8: Find the closeness of an ideal solution by $C^*(\ddot{x}_j) = \frac{d_j^-}{d_j^+ + d_j^-} \in [0, 1]$.

Step 9: Find the rank of alternatives using closeness coefficients in the order of decreasing or increasing.

Step 10: The conclusion of the best alternative.

Example 3.1.

A company plans to invest some cash in the stock exchange by purchasing some shares of the best five companies. They decide to invest a percentage of their cash in the amounts of 30, 25, 20, 15, and 10 to reduce the factor. Find the top five ranked companies.

Step 1: A finite set of decision makers, $\mathcal{D} = \{\mathcal{D}_i : i = 1, 2, 3, 4, 5\}$, the collection of companies/alternatives, $\mathcal{C} = \{\ddot{x}_i : i = 1, 2, ..., 10\}$. and a finite family of parameters, $D = \{e_i :$ i = 1 to 5 $\}$, $e_1 =$ Momentum, $e_2 =$ Value, $e_3 =$ Growth, $e_4 =$ Volatility, $e_5 =$ Quality.

Step 2: Obtain a weighted parameter matrix under the linguistic variables.

Table 1. Linguistic variables.

Linguistic variables	Fuzzy weights
Very Good Crucial(VGC)	0.95
Good Crucial (GC)	0.80
Average Crucial (AC)	0.65
Poor Crucial (PC)	0.50
Very Poor Crucial (VPC)	0.35

Construct a weighted parameter matrix

$$\mathcal{P} = [w_{ij}]_{5\times5} = \begin{bmatrix} GC & VGC & PC & VPC & AC \\ AC & GC & VPC & PC & VGC \\ PC & AC & VGC & VGC & VPC \\ VGC & PC & AC & GC & PC \\ AC & VPC & VGC & VGC & PC \end{bmatrix} = \begin{bmatrix} 0.80 & 0.95 & 0.50 & 0.35 & 0.65 \\ 0.65 & 0.80 & 0.35 & 0.50 & 0.95 \\ 0.50 & 0.65 & 0.95 & 0.95 & 0.35 \\ 0.95 & 0.50 & 0.65 & 0.80 & 0.50 \\ 0.65 & 0.35 & 0.95 & 0.95 & 0.50 \end{bmatrix}.$$

Here ω_{ij} means the weight of the $\mathcal{D}_i$ to the $\ddot{x}_j$.

Step 3: The normalized weighted decision matrix

$$\widehat{\mathcal{N}} = [\widehat{\eta}_{ij}]_{5\times5} = \begin{bmatrix} 0.4926 & 0.6214 & 0.3101 & 0.2085 & 0.4658 \\ 0.4002 & 0.5233 & 0.2171 & 0.2979 & 0.6807 \\ 0.3079 & 0.4251 & 0.5892 & 0.5660 & 0.2508 \\ 0.5850 & 0.3270 & 0.4031 & 0.4766 & 0.3583 \\ 0.4002 & 0.2289 & 0.5892 & 0.5660 & 0.3583 \end{bmatrix}.$$

Weighted vector values are $\mathcal{W} = (0.1231, 0.1308, 0.124, 0.1192, 0.1433)$.

Step 4: The aggregated decision matrix $\mathcal{X} = \frac{\mathcal{D}_1+\mathcal{D}_2+\mathcal{D}_3+\mathcal{D}_4+\mathcal{D}_5}{5} =$

$$\begin{bmatrix} (0.61,0.48,0.33) & (0.72,0.45,0.26) & (0.18,0.46,0.65) & (0.56,0.65,0.39) & (0.78,0.17,0.41) \\ (0.18,0.32,0.84) & (0.31,0.19,0.74) & (0.74,0.12,0.28) & (0.3,0.8,0.21) & (0.14,0.62,0.42) \\ (0.35,0.45,0.52) & (0.42,0.37,0.42) & (0.42,0.35,0.45) & (0.41,0.5,0.52) & (0.22,0.26,0.35) \\ (0.31,0.67,0.22) & (0.64,0.21,0.12) & (0.12,0.47,0.61) & (0.31,0.82,0.21) & (0.52,0.19,0.47) \\ (0.78,0.31,0.11) & (0.3,0.8,0.21) & (0.21,0.11,0.68) & (0.41,0.14,0.56) & (0.14,0.82,0.31) \\ (0.84,0.23,0.29) & (0.2,0.81,0.39) & (0.39,0.13,0.73) & (0.33,0.49,0.28) & (0.19,0.68,0.23) \\ (0.3,0.8,0.21) & (0.5,0.39,0.46) & (0.56,0.59,0.42) & (0.59,0.4,0.55) & (0.42,0.56,0.29) \\ (0.71,0.37,0.48) & (0.37,0.7,0.42) & (0.58,0.27,0.43) & (0.27,0.8,0.35) & (0.23,0.5,0.28) \\ (0.53,0.61,0.19) & (0.61,0.58,0.18) & (0.3,0.8,0.21) & (0.41,0.29,0.64) & (0.18,0.77,0.31) \\ (0.23,0.82,0.29) & (0.83,0.1,0.29) & (0.39,0.62,0.45) & (0.8,0.16,0.43) & (0.26,0.39,0.62) \end{bmatrix}$$

$= [\dot{x}_{jk}]_{10\times 5}$.

Step 5: Find the weighted SFS decision matrix $\mathcal{Y} = m_k \times \dot{x}_{jk} =$

$$\begin{bmatrix} (0.0751,0.0591,0.0406) & (0.0942,0.0589,0.034) & (0.0223,0.0571,0.0806) \\ (0.0222,0.0394,0.1034) & (0.0406,0.0249,0.0968) & (0.0918,0.0149,0.0347) \\ (0.0431,0.0554,0.064) & (0.0549,0.0484,0.0549) & (0.0521,0.0434,0.0558) \\ (0.0382,0.0825,0.0271) & (0.0837,0.0275,0.0157) & (0.0149,0.0583,0.0757) \\ (0.0961,0.0382,0.0135) & (0.0392,0.1047,0.0275) & (0.026,0.0136,0.0843) \\ (0.1034,0.0283,0.0357) & (0.0262,0.106,0.051) & (0.0484,0.0161,0.0905) \\ (0.0369,0.0985,0.0259) & (0.0654,0.051,0.0602) & (0.0695,0.0732,0.0521) \\ (0.0874,0.0456,0.0591) & (0.0484,0.0916,0.0549) & (0.0719,0.0335,0.0533) \\ (0.0653,0.0751,0.0234) & (0.0798,0.0759,0.0235) & (0.0372,0.0992,0.026) \\ (0.0283,0.101,0.0357) & (0.1086,0.0131,0.0379) & (0.0484,0.0769,0.0558) \\ (0.0667,0.0774,0.0465) & (0.1118,0.0244,0.0588) & \\ (0.0357,0.0953,0.025) & (0.0201,0.0889,0.0602) & \\ (0.0489,0.0596,0.062) & (0.0315,0.0373,0.0502) & \\ (0.0369,0.0977,0.025) & (0.0745,0.0272,0.0674) & \\ (0.0489,0.0167,0.0667) & (0.0201,0.1175,0.0444) & \\ (0.0393,0.0584,0.0334) & (0.0272,0.0975,0.033) & \\ (0.0703,0.0477,0.0655) & (0.0602,0.0803,0.0416) & \\ (0.0322,0.0953,0.0417) & (0.033,0.0717,0.0401) & \\ (0.0489,0.0346,0.0763) & (0.0258,0.1104,0.0444) & \\ (0.0953,0.0191,0.0512) & (0.0373,0.0559,0.0889) & \end{bmatrix} = [\ddot{x}_{jk}]_{10\times 5}.$$

Step 6: SFSV-PIS and SFSV-NIS can be written as

$\ddot{x}_1^+ = (0.1118, 0.0244, 0.034), \ddot{x}_2^+ = (0.0918, 0.0149, 0.025), \ddot{x}_3^+ = (0.0549, 0.0373, 0.0502),$

$\ddot{x}_4^+ = (0.0837, 0.0272, 0.0157), \ddot{x}_5^+ = (0.0961, 0.0136, 0.0135), \ddot{x}_6^+ = (0.1034, 0.0161, 0.033),$

$\ddot{x}_7^+ = (0.0703, 0.0477, 0.0259), \ddot{x}_8^+ = (0.0874, 0.0335, 0.0401), \ddot{x}_9^+ = (0.0798, 0.0346, 0.0234),$

$\ddot{x}_{10}^+ = (0.1086, 0.0131, 0.0357)$, and $\ddot{x}_1^- = (0.0223, 0.0774, 0.0806),$

$\ddot{x}_2^- = (0.0201, 0.0953, 0.1034), \ddot{x}_3^- = (0.0315, 0.0596, 0.064), \ddot{x}_4^- = (0.0149, 0.0977, 0.0757),$

$\ddot{x}_5^- = (0.0201, 0.1175, 0.0843), \ddot{x}_6^- = (0.0262, 0.106, 0.0905), \ddot{x}_7^- = (0.0369, 0.0985, 0.0655),$

$\ddot{x}_8^- = (0.0322, 0.0953, 0.0591), \ddot{x}_9^- = (0.0258, 0.1104, 0.0763), \ddot{x}_{10}^- = (0.0283, 0.101, 0.0889).$

Step 7: For each alternative, the SFS Euclidean distances from SFSV-PIS and SFSV-NIS,

$d_1^+ = 0.1448$, $d_2^+ = 0.2026$, $d_3^+ = 0.0459$, $d_4^+ = 0.1567$, $d_5^+ = 0.2117$, $d_6^+ = 0.198$,

$d_7^+ = 0.0961$, $d_8^+ = 0.1317$, $d_9^+ = 0.1497$, $d_{10}^+ = 0.1802$, and

$d_1^- = 0.1651$, $d_2^- = 0.1822$, $d_3^- = 0.0515$, $d_4^- = 0.172$, $d_5^- = 0.2111$, $d_6^- = 0.185$,

$d_7^- = 0.1082$, $d_8^- = 0.1117$, $d_9^- = 0.1501$, $d_{10}^- = 0.1907$.

Step 8: We calculate the closeness coefficients from SFSV-PIS and SFSV-NIS for each alternative,

$C_1^* = 0.5328$, $C_2^* = 0.4736$, $C_3^* = 0.5286$, $C_4^* = 0.5233$, $C_5^* = 0.4994$, $C_6^* = 0.4831$,

$C_7^* = 0.5295$, $C_8^* = 0.459$, $C_9^* = 0.5007$, $C_{10}^* = 0.5142$.

Step 9: Order C_i^* is $\ddot{x}_1 \geq \ddot{x}_7 \geq \ddot{x}_3 \geq \ddot{x}_4 \geq \ddot{x}_{10} \geq \ddot{x}_9 \geq \ddot{x}_5 \geq \ddot{x}_6 \geq \ddot{x}_2 \geq \ddot{x}_8$.

Step 10: It concludes that the firm should have $\ddot{x}_1$ investment 30%, $\ddot{x}_7$ investment 25%, $\ddot{x}_3$ investment 20%, $\ddot{x}_4$ investment 15% and $\ddot{x}_{10}$ investment 10%.

4. MCGDM based on SFS-VIKOR aggregating operator

Algorithm V (SFS-VIKOR)

Step 1: Assume that there are finite decision makers, $\mathcal{D} = \{\mathcal{D}_i : i \in \mathbb{N}\}$, a finite collection of alternatives, $\mathcal{C} = \{\ddot{x}_i : i \in \mathbb{N}\}$, and a finite family of parameters, $\mathcal{D} = \{e_i : i \in \mathbb{N}\}$.

Step 2: Form a linguistic term with the obtained weighted parameter matrix,

$$\mathcal{P} = [\omega_{ij}]_{n\times m} = \begin{bmatrix} \omega_{11} & \omega_{12} & \ldots & \omega_{1m} \\ \omega_{21} & \omega_{22} & \ldots & \omega_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{i1} & \omega_{i2} & \ldots & \omega_{im} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{n1} & \omega_{n2} & \ldots & \omega_{nm} \end{bmatrix}.$$

Step 3: Obtain a weighted normalized decision matrix,

$$\widehat{\mathcal{N}} = [\widehat{\eta}_{ij}]_{n\times m} = \begin{bmatrix} \widehat{\eta}_{11} & \widehat{\eta}_{12} & \dots & \widehat{\eta}_{1m} \\ \widehat{\eta}_{21} & \widehat{\eta}_{22} & \dots & \widehat{\eta}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{\eta}_{i1} & \widehat{\eta}_{i2} & \dots & \widehat{\eta}_{im} \\ \vdots & \vdots & \ddots & \vdots \\ \widehat{\eta}_{n1} & \widehat{\eta}_{n2} & \dots & \widehat{\eta}_{nm} \end{bmatrix}.$$

Step 4: Create an SFS decision matrix,

$$\mathcal{D}_i = [x^i_{jk}]_{l\times m} = \begin{bmatrix} x^i_{11} & x^i_{12} & \dots & x^i_{1m} \\ x^i_{21} & x^i_{22} & \dots & x^i_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x^i_{j1} & x^i_{j2} & \dots & x^i_{jm} \\ \vdots & \vdots & \ddots & \vdots \\ x^i_{l1} & x^i_{l2} & \dots & x^i_{lm} \end{bmatrix}.$$

In this case, x^i_{jk} denotes the SFS element for i^{th} decision maker $\mathcal{D}_i$ for each i. Find the aggregating matrix by using $\mathcal{X} = \frac{\mathcal{D}_1+\mathcal{D}_2+\ldots+\mathcal{D}_n}{n} = [\dot{x}_{jk}]_{l\times m}$.

Step 5: Find the decision of the weighted SFS matrix (where $\ddot{x}_{jk} = m_k \times \dot{x}_{jk}$),

$$\mathcal{Y} = [\ddot{x}_{jk}]_{l\times m} = \begin{bmatrix} \ddot{x}_{11} & \ddot{x}_{12} & \dots & \ddot{x}_{1m} \\ \ddot{x}_{21} & \ddot{x}_{22} & \dots & \ddot{x}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{x}_{j1} & \ddot{x}_{j2} & \dots & \ddot{x}_{jm} \\ \vdots & \vdots & \ddots & \vdots \\ \ddot{x}_{l1} & \ddot{x}_{l2} & \dots & \ddot{x}_{lm} \end{bmatrix}.$$

Step 6: Calculate the values of SFSV-PIS and SFSV-NIS. However, as of now,

SFSV-PIS = $[\ddot{x}^+_1, \ddot{x}^+_2, ..., \ddot{x}^+_l] = \{(\vee_k \ddot{x}_{jk}, \wedge_k \ddot{x}_{jk}, \wedge_k \ddot{x}_{jk}) : j = 1, 2, ..., l\}$ and,

SFSV-PIS = $[\ddot{x}^-_1, \ddot{x}^-_2, ..., \ddot{x}^-_l] = \{(\wedge_k \ddot{x}_{jk}, \vee_k \ddot{x}_{jk}, \vee_k \ddot{x}_{jk}) : j = 1, 2, ..., l\}$.

Step 7: Find the values of utility $\mathcal{S}_i$, individual regret $\mathcal{R}_i$ and compromise $\mathcal{Q}_i$, where $\mathcal{S}_i = \sum_{j=1}^m m_j \left(\frac{d(\ddot{x}_{ij},\ddot{x}^+_j)}{d(\ddot{x}^+_j,\ddot{x}^-_j)}\right)$, $\mathcal{R}_i = \max_{j=1}^m m_j \left(\frac{d(\ddot{x}_{ij},\ddot{x}^+_j)}{d(\ddot{x}^+_j,\ddot{x}^-_j)}\right)$ and $\mathcal{Q}_i = \kappa\left(\frac{\mathcal{S}_i-\mathcal{S}^-}{\mathcal{S}^+-\mathcal{S}^-}\right) + (1-\kappa)\left(\frac{\mathcal{R}_i-\mathcal{R}^-}{\mathcal{R}^+-\mathcal{R}^-}\right)$, where $\mathcal{S}^+ = \max_i \mathcal{S}_i$, $\mathcal{S}^- = \min_i \mathcal{S}_i$, $\mathcal{R}^+ = \max_i \mathcal{R}_i$ and $\mathcal{R}^- = \min_i \mathcal{R}_i$. The real number κ is called the coefficient of a decision mechanism. The role of κ is that if majority compromise solution when $\kappa > 0.5$; and consensus compromise solution when $\kappa = 0.5$; and veto compromise solution when $\kappa < 0.5$. Let m_j said to be j^{th} parameter of weight.

Step 8: Obtain the rank of choices and derive a compromise solution. Arrange $\mathcal{S}_i$, $\mathcal{R}_i$ and $\mathcal{Q}_i$ in increasing order. The alternative $\ddot{x}_\alpha$ will be declared a compromise solution if it ranks first (has the lowest value) in $\mathcal{Q}_i$ and both of the following conditions are met at the same time. $C1$ admissible: If $\ddot{x}_\alpha$ and $\ddot{x}_\beta$ denotes top alternatives in $\mathcal{Q}_i$, then $\mathcal{Q}(\ddot{x}_\beta) - \mathcal{Q}(\ddot{x}_\alpha) \geq \frac{1}{n-1}$, n is the counting of parameters. $C2$ admissible: $\ddot{x}_\alpha$ it greatest ranked by any of the case ($\mathcal{S}_i$ and $\mathcal{R}_i$) and ($\mathcal{S}_i$ or $\mathcal{R}_i$). If $C1$ and $C2$ are not satisfied simultaneously, then there exist multiple compromise solutions. (i) If $C1$ is true, then both alternatives $\ddot{x}_\alpha$ and $\ddot{x}_\beta$ are said to be compromise solutions: (ii) If $C1$ is false, then the alternatives $\ddot{x}_\alpha$, $\ddot{x}_\beta$,..., $\ddot{x}_\zeta$ are said to be compromise solutions, where $\ddot{x}_\zeta$ is founded by using $\mathcal{Q}(\ddot{x}_\zeta) - \mathcal{Q}(\ddot{x}_\alpha) \geq \frac{1}{n-1}$.

Example 4.1.

We can apply Example 3.1 to Algorithm V. We conclude that the first five steps are all the same process. Hence, we solve Example 3.1 using the VIKOR method by entering **Step 6**.

Step 6: SFSV-PIS and SFSV-NIS are listed as follows,

$\ddot{x}_1^+ = (0.1034, 0.0283, 0.0135), \ddot{x}_2^+ = (0.1086, 0.0131, 0.0157), \ddot{x}_3^+ = (0.0918, 0.0136, 0.026),$

$\ddot{x}_4^+ = (0.0953, 0.0167, 0.025), \ddot{x}_5^+ = (0.1118, 0.0244, 0.033)$ and

$\ddot{x}_1^- = (0.0222, 0.101, 0.1034), \ddot{x}_2^- = (0.0262, 0.106, 0.0968), \ddot{x}_3^- = (0.0149, 0.0992, 0.0905),$

$\ddot{x}_4^- = (0.0322, 0.0977, 0.0763), \ddot{x}_5^- = (0.0201, 0.1175, 0.0889).$

Step 7: Taking $\kappa = 0.5$, for each alternative $\ddot{x}_i$, we discovered utility $\mathcal{S}_i$, individual regret $\mathcal{R}_i$, and compromise $\mathcal{Q}_i$.

Table 2. Compromise values.

$\ddot{x}$	$\mathcal{S}_i$	$\mathcal{R}_i$	$\mathcal{Q}_i$
$\ddot{x}_1$	0.2805	0.0926	0.0907
$\ddot{x}_2$	0.427	0.1163	0.8354
$\ddot{x}_3$	0.3531	0.0837	0.2478
$\ddot{x}_4$	0.3508	0.1037	0.4452
$\ddot{x}_5$	0.3923	0.1323	0.8816
$\ddot{x}_6$	0.392	0.1139	0.6909
$\ddot{x}_7$	0.3498	0.085	0.2489
$\ddot{x}_8$	0.3744	0.1061	0.5509
$\ddot{x}_9$	0.4074	0.1231	0.8388
$\ddot{x}_{10}$	0.3166	0.0991	0.2816

Step 8: The rank of alternatives for $\mathcal{Q}_i$: $\ddot{x}_1 \leq \ddot{x}_3 \leq \ddot{x}_7 \leq \ddot{x}_{10} \leq \ddot{x}_4 \leq \ddot{x}_8 \leq \ddot{x}_6 \leq \ddot{x}_2 \leq \ddot{x}_9 \leq \ddot{x}_5$. Now, $\mathcal{Q}(\ddot{x}_3) - \mathcal{Q}(\ddot{x}_1) = 0.1571 \ngeq \frac{1}{4}$. Thus, C1 is false. Furthermore $\mathcal{Q}(\ddot{x}_4) - \mathcal{Q}(\ddot{x}_1) = 0.3544 \geq \frac{1}{4}$. As a result, we decide that $\ddot{x}_1, \ddot{x}_3, \ddot{x}_7, \ddot{x}_{10}, \ddot{x}_4$ are multiple compromise solutions. Hence, the

company should have investment 30% on $\ddot{x}_1$, 25% on $\ddot{x}_3$, 20% on $\ddot{x}_7$, 15% on $\ddot{x}_{10}\%$, and 10% on $\ddot{x}_4$.

5. Conclusion:

In the present attention, Algorithm I, Algorithm II and Algorithm III are followed by MCGDM based on SFS. Also, Algorithm IV and Algorithm V are followed by SFS linguistic TOPSIS and VIKOR approaches based on aggregation operators. Again, we interact with the SFS aggregation operator and score function values based on some technique. The TOPSIS and VIKOR methods assume a scalar component for each criterion, and these two methods are different from the normalization approach. In TOPSIS, they utilize the vector normalization approach, and VIKOR utilizes the linear normalization approach. The major difference between the two methods is in the aggregation function. We can find the ranking of values using an aggregating function. The best ranked alternative by VIKOR is closest to the ideal solution. However, the best ranked alternative by TOPSIS is the one using the ranking index, but it's not the ideal solution. Hence, the advantage of VIKOR gives a compromise solution. In my future research, we may apply the TOPSIS and VIKOR approaches to cubic fuzzy soft sets.

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