Applications and Applied Mathematics: An International Journal (AAM)

Volume 17 | Issue 2 | Article 7

12-2022

(R1978) Heated Laminar Vertical Jet of Psudoplastic Fluids-Against Gravity

Manisha Patel
Sarvajanik College of Engineering and Technology

M. G. Timol
Veer Narmad South Gujarat University

Available at
http://pvamu.edu/aam
Appl. Appl. Math.
ISSN: 1932-9466

Applications and Applied Mathematics:
An International Journal
(AAM)

Vol. 17, Issue 2 (December 2022), pp. 408 – 420

Heated Laminar Vertical Jet of Psudoplastic Fluids-Against Gravity

[1*]Manisha Patel and [2]M. G. Timol

[1*]Department of Mathematics
Sarvajanik College of Engineering and Technology
Surat-395001, Gujarat, INDIA
manishapramitpatel@gmail.com

[2] Department of Mathematics
Veer Narmad South Gujarat University
Magdalla Road, Surat-395007, Gujarat, India
mgtimol@gmail.com

*Corresponding author

Received: March 24, 2022; Accepted: August 24, 2022

Abstract

A heated laminar jet of Pseudo-plastic fluid flowing vertically upwards from a long narrow slit into a region of the same fluid which is at a rest and at a uniform temperature is considered. The governing non-linear Partial differential equations (PDEs) for the defined flow problem are transformed into non-linear ordinary differential equations using the effective similarity technique-one parameter deductive group theory method. The obtained non-linear coupled Ordinary differential equations are solved and the results are presented by graphs. The effect of the Prandtl number and Grashof number on the velocity and temperature of the jet flow is discussed. Also, a detailed discussion of some interesting applications of the vertical jet flow of pseudoplastic fluids in different fields of engineering and geophysics are provided.

Keywords: Laminar; Vertical jet; Prandtl number; Grashof number; Pseudo-plastic fluid; Non-linear PDE; Lava; Bore holes

MSC 2020 No.: 20D06, 76A05, 76D10, 76D25, 86A70

Nomenclature

u, v- Components of velocity in the x and y directions
ρ - Mass density

θ - Temperature
Cp - Specific heat at a constant pressure
K - Thermal conductivity
υ - Kinematic viscosity
n - Flow behavior index
g - Gravitational acceleration
β - Coefficient of thermal/volume expansion
Q - Heat flux across any cross section perpendicular to the jet axis
L- Fundamental length
T_o - Temperature at the wall
T_∞ - Temperature of the medium at infinity
Gr - Grashof number
Pr - Prandtl number
ψ - Stream function
G - Group
$M_1(a)$, $M_2(a)$ - Real constants
a - Group parameter
η - Similarity variable
f_1, f_2 - Similarity functions
$\alpha_1,...,\alpha_4,\beta_1,...,\beta_4$ - Real constants

1. Introduction

The theory of jet is an important and highly developed branch of hydromechanics. The first problem of jet for ideal fluid was solved by Helmholtz (1868). Kirchhoff (1869) substantially developed and generalized Helmholtz's method. Potential flow under gravity with free surfaces has been somewhat neglected in classical hydromechanics. Advanced technology required more and more understanding of the problems of jet flows. There are numerous situations in aerodynamics, engineering processes, meteorology where jet flows occur in a natural way.

Schlichting (1934, 1968) was the first to expand the boundary layer theory to the theory of jet flows. The numerical solution of the governing ordinary differential equation of the steady two-dimensional free jet flow was determined by him. An analytic solution to this problem was discussed by Bickely (1937). Stehr et al. (2000) discussed the resulting effects because of the interaction of the induced flow and the jet. They have applied both numerical and analytical methods for the solution of the governing equations. The model and computation of an inviscid liquid jet emerging from a rotating drum were analysed by Decent et al. (2002). A slender liquid jet immersed in a liquid zone due to a circulating orifice was investigated by Wallwork et al. (2002). The author developed the model for the slender non-linear inviscid jet with the assumption of the stationary centerline of the jet and in the presence of surface tension. Patel et al. (2014)

investigated the axisymmetrical and two-dimensional jet flow for an incompressible Pseudoplastic fluid. The jet flow of a laminar compressible electrically conducting fluid issuing from circular orifices in the presence of the radial magnetic field is analyzed by Patel et al. (2015).

The laminar jet flow due to an Incompressible Newtonian fluid coming out from a circular hole or a narrow slit was investigated by Patel et.al. (2016). They have discussed both the cases, Free jet and dissipative jet. They considered the variable physical properties: thermal conductivity and viscosity vary with temperature. Magan et al. (2016) analyzed the free jet of power-law fluid for which the Reynold number is considered in terms of the viscosity of the power-law fluid. The free jet is modelled by making the boundary layer approximation perpendicular to the axis of symmetry. Danial (2017) developed the governing equations along with the boundary conditions for the flow of the nonlinear rotating slender jet.

An empirical functional relation known as Power-law is widely used to characterize fluids of this type. Other empirical relations which have been used to describe Pseudoplastic behaviour are Prandtl, Eyring, Powell-Eyring, Williamson, etc. (Wilkinson (1960)). The derivation of the governing equations of a Pseudoplastic fluid was first provided by Schowalter (1960). Acrivos et al. (1960) obtained the numerical analysis of the boundary layer equations of both Pseudoplastic fluids (Shear-thinning) and Dilatant fluids (shear-thickening). A brief instruction and classification on various fluid models of non-Newtonian fluid are given in detaile by Patel et al. (2010, 2013).

The technique used in the present investigation is deductive group transformation, which leads to a similarity representation of the problem. In 1967 and 1968 Moran et al. (1967, 1968a, 1968b, 1968c) presented a general systematic formalism for similarity analysis, where a given system of partial differential equations was reduced to a system of ordinary differential equations. Details of the theory are found in Moran's above-mentioned papers. The two- parameter group-theoretic transformation method has been applied by Patil et al. (2012) and Darji et al. (2014). Recently, the deductive group of transformation has been successfully applied to various flow problems by many researchers (like Abd-el-Malek et al. (2002), Partemar et al. (2011), Adnan et al. (2011), Darji et al. (2013), Jain et al. (2015) and Patel et al. (2017)).

Much less work has been carried out for the heated vertical jet of non-Newtonian fluid in the past. Probably Kalathia (1975) was the first to derive the governing equations for the heated vertical jet flow for power-law fluid. In the present work, we expanded the work of Kalathia (1975) by obtaining the similarity solutions and then numerical solutions along with the graphical presentation.

Nowadays the Study of Pseudoplastic fluid (shear-thinning fluids) is essential due to the important applications of the fluids fall in this category. Drilling fluids, polymer solutions, Blood, catsup and nail polish are some of examples of Pseudoplastic behaviour. While drilling the boreholes into the surface of the earth or drilling the oil or natural gases, the purpose of a Drilling fluid (mud) is to clean the bottom of the hole, lubricate the bit maintain the walls of the hole and transport the cuttings to surface. This type of behaviour is characteristic of suspensions of asymmetric particles or solutions of a high polymer such as cellulose derivatives. For these types of fluids, the apparent viscosity continues to decrease with increasing rate of shear until no further alignment along the streamlines is possible and the flow curve becomes linear.

Much less work had been carried out in the literature for a vertical jet of Pseudoplastic fluids. In this paper we study a heated laminar jet of Pseudoplastic fluid flowing vertically upwards from a long narrow slit into a region of the same fluid which is at a rest and at a uniform temperature. The flow through the slit is produced by a constant pressure difference. It is assumed that the flow is steady, laminar (i.e., having a small Reynolds number) and incompressible. The existence of temperature difference between a jet and its surrounding will give rise to buoyant forces. As soon as the jet immerse it will be under the effect of these forces. In a steady-state, the free convection velocity, variation in density, etc., will depend on these buoyant forces.

For simplicity, the constant properties are postulated except for a small change in density due to temperature differences. Such simplified treatments do not appear unreasonable as long as the temperature differences involved is not too large. The coupled equation of motion is solved within the limitations of the physical aspects of the problem.

2. Basic Equations

Consider a heated laminar jet of Pseudoplastic fluid flowing vertically upwards from a long narrow slit into a region of the same fluid which is at a rest and at a uniform temperature. The geometry of the flow is shown in Figure 1. Following Kalathia (1975), who had derived the governing equations for the heated vertical jet flow for power-law fluid, the governing equations for the heated vertical jet flow for Pseudoplastic fluid (after neglecting viscous dissipation) can be written as

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{1}$$

$$\overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = \upsilon \frac{\partial}{\partial \overline{y}}\left(\left| \frac{\partial \overline{u}}{\partial \overline{y}} \right|^{n-1} \frac{\partial \overline{u}}{\partial \overline{y}} \right) + g\beta\overline{\theta}, \quad 0 < n < 1, \tag{2}$$

$$\rho C_p \left(\overline{u}\frac{\partial \overline{\theta}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{\theta}}{\partial \overline{y}} \right) = K \frac{\partial^2 \overline{\theta}}{\partial \overline{y}^2}. \tag{3}$$

The necessary boundary conditions are

$$\overline{v} = \frac{\partial \overline{u}}{\partial \overline{y}} = \frac{\partial \overline{\theta}}{\partial \overline{y}} = 0 \quad at \quad \overline{y} = 0, \tag{4}$$

$$\overline{u} = \overline{\theta} = 0 \quad at \quad \overline{y} \to \infty. \tag{5}$$

It may be noted that the usual condition of constancy of flux of momentum will not be satisfied in the case of buoyant jets. However, in accordance with the assumptions made, we shall have (whether the jet is buoyant or not)

$$Q = \rho C_p \int_0^\infty \bar{u}\,\bar{\theta}\; d\bar{y} = C_1 = \text{Constant.} \tag{6}$$

Introducing the non-dimensional variable as follows:

$$x = \frac{\bar{x}}{L}, y = \frac{\bar{y}}{L}\left(\frac{L^2}{\upsilon}\right)^{\frac{n-1}{n+1}}, u = \frac{L\bar{u}}{\upsilon}, v = \frac{L\bar{v}}{\upsilon}\left(\frac{L^2}{\upsilon}\right)^{\frac{n-1}{n+1}}, \theta = \frac{\bar{\theta}}{\left(T_0 - T_\infty\right)}. \tag{7}$$

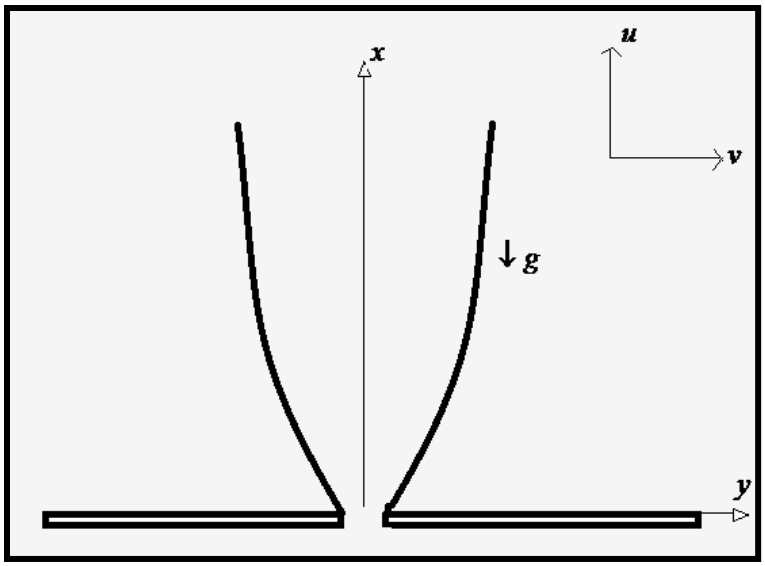

Figure 1. Co-ordinate system for two-dimensional vertical jet

The governing equations (1)-(6) in dimensionless form are given by Equations (8) through (13):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\upsilon^{n-1}}{\alpha^n}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) + Gr\;\theta, \tag{9}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}. \tag{10}$$

The boundary conditions are

$$v = \frac{\partial u}{\partial y} = \frac{\partial \theta}{\partial y} = 0 \quad at \quad y = 0, \tag{11}$$

$$u = \theta = 0 \qquad at \quad y \to \infty. \tag{12}$$

The momentum flux will be

$$Q = \rho C_p \int_0^\infty u\,\theta\, dy = \frac{C_1}{\upsilon(T_0 - T_\infty)} = \text{Constant.} \tag{13}$$

Introduce the stream function ψ as usual and hence Equations (8) through (13) come to the form

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = n\left(\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\frac{\partial^3\psi}{\partial y^3} + Gr\,\theta, \tag{14}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}. \tag{15}$$

The boundary conditions are

$$\frac{\partial\psi}{\partial x} = \frac{\partial^2\psi}{\partial y^2} = \frac{\partial\theta}{\partial y} = 0 \quad at \quad y = 0, \tag{16}$$

$$\frac{\partial\psi}{\partial y} = \theta = 0 \qquad at \quad y \to \infty. \tag{17}$$

The momentum flux will be

$$Q = \int_0^\infty \frac{\partial\psi}{\partial y}\theta\, dy \quad = 1. \tag{18}$$

Now, to reduce the above equations (14) through (18) with two independent variables in the equations with one independent variable, we apply the one-parameter deductive group theory technique (Moran (1967), (1968a), (1968b), (1968c)).

Following Patel (2017), who had defined the group for the solution of boundary layer flow of Prandtl fluid past a flat surface, we defined the below group G for our flow problem as

$$G: \begin{cases} \tilde{x} = h^x(a)\; x + k^x(a), \\ \tilde{y} = h^y(a)\, y + k^y(a), \\ \tilde{\psi} = h^\psi(a)\psi + k^\psi(a), \\ \tilde{\theta} = h^\theta(a)\theta + k^\theta(a). \end{cases} \tag{19}$$

Equation (14) is said to be transformed invariantly for some function $M_1(a)$, whenever,

$$\frac{\partial\tilde{\psi}}{\partial\tilde{y}}\frac{\partial^2\tilde{\psi}}{\partial\tilde{x}\partial\tilde{y}} - \frac{\partial\tilde{\psi}}{\partial\tilde{x}}\frac{\partial^2\tilde{\psi}}{\partial\tilde{y}^2} - n\left(\frac{\partial^2\tilde{\psi}}{\partial\tilde{y}^2}\right)^{n-1}\frac{\partial^3\tilde{\psi}}{\partial\tilde{y}^3} - Gr\tilde{\theta}$$

$$= M_1(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - n\left(\frac{\partial^2\psi}{\partial y^2}\right)^{n-1}\frac{\partial^3\psi}{\partial y^3} - Gr\theta\right]. \tag{20}$$

Similarly we can say for the equation (15) to be absolutely invariant for some function $M_2(a)$, whenever,

$$\frac{\partial\tilde{\psi}}{\partial\tilde{y}}\frac{\partial\tilde{\theta}}{\partial\tilde{x}} - \frac{\partial\tilde{\psi}}{\partial\tilde{x}}\frac{\partial\tilde{\theta}}{\partial\tilde{y}} - \frac{1}{\Pr}\frac{\partial^2\tilde{\theta}}{\partial\tilde{y}^2} = M_2(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} - \frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}\right]. \tag{21}$$

Therefore, from the above two equations (20) and (21), the following relations (22) and (23) are obtained, respectively,

$$\frac{h^{2\psi}}{h^x h^{2y}} = \frac{h^{2\psi}}{h^x h^{2y}} = \left(\frac{h^{\psi}}{h^{2y}}\right)^{n-1}\frac{h^{\psi}}{h^{3y}} = h^{\theta} = M_1(a), \tag{22}$$

$$\frac{h^{\psi}h^{\theta}}{h^x h^y} = \frac{h^{\theta}}{h^{2y}} = M_2(a). \tag{23}$$

Also, from the invariance of the auxiliary conditions,

$$k^y = k^{\theta} = 0. \tag{24}$$

From the equations (22), (23) and (24),

$$h^x = h^{3y},\ h^{\psi} = h^{2y},\ h^{\theta} = h^{-y},\ k^{\theta} = k^y = 0. \tag{25}$$

Using (25) in (19), the group G is

$$G:\begin{cases}\tilde{x} = h^{3y}x + k^x,\\ \tilde{y} = h^y y,\\ \tilde{\psi} = h^{2y}\psi + k^{\psi},\\ \tilde{\theta} = h^{-y}\theta,\end{cases} \tag{26}$$

$$\sum_{i=1}^{4}(\alpha_i S_i + \beta_i)\frac{\partial g}{\partial S_i} = 0; \quad S_i = x, y, \psi, \theta, \tag{27}$$

$$\Rightarrow (\alpha_1 x + \beta_1)\frac{\partial g}{\partial x} + (\alpha_2 y + \beta_2)\frac{\partial g}{\partial y} + (\alpha_3\psi + \beta_3)\frac{\partial g}{\partial\psi} + (\alpha_4\theta + \beta_4)\frac{\partial g}{\partial\theta} = 0, \tag{28}$$

Here,

$$\alpha_i = \frac{\partial h^{S_i}}{\partial a} \ \& \ \beta_i = \frac{\partial k^{S_i}}{\partial a}; \quad i = 1, 2, 3, 4, \tag{29}$$

$$\begin{aligned}
&\alpha_1 = \frac{\partial h^x}{\partial a} = \frac{\partial h^{3y}}{\partial a} = 3h^{2y}\frac{\partial h^y}{\partial a} = 3\frac{\partial h^y}{\partial a} = 3\alpha_2 \quad ,(\because h^y \text{ is identity at } a_0),\\
&\alpha_2 = \frac{\partial h^y}{\partial a},\\
&\alpha_3 = \frac{\partial h^\psi}{\partial a} = \frac{\partial h^{2y}}{\partial a} = 2h^y\frac{\partial h^y}{\partial a} = 2\frac{\partial h^y}{\partial a} = 2\alpha_2 \quad ,(\because h^y \text{ is identity at } a_0),\\
&\alpha_4 = \frac{\partial h^\theta}{\partial a} = \frac{\partial h^{-y}}{\partial a} = -h^{-2y}\frac{\partial h^y}{\partial a} = -\frac{\partial h^y}{\partial a} = -\alpha_2 \, ,(\because h^y \text{ is identity at } a_0),
\end{aligned} \tag{30}$$

$$\beta_1 = \frac{\partial k^x}{\partial a}, \ \beta_2 = \frac{\partial k^y}{\partial a} = 0, \ \beta_3 = \frac{\partial k^\psi}{\partial a}, \ \beta_4 = \frac{\partial k^\theta}{\partial a} = 0. \tag{31}$$

Now, the characteristic equation from Equation (28) is:

$$\frac{dx}{(\alpha_1 x + \beta_1)} = \frac{dy}{(\alpha_2 y + \beta_2)} = \frac{d\psi}{(\alpha_3 \psi + \beta_3)} = \frac{d\theta}{(\alpha_4 \theta + \beta_4)}. \tag{32}$$

Solving the first two relations of Equation (32) for η, we have

$$\eta = y(\alpha_1 x + \beta_1)^{-\frac{1}{3}}. \tag{33}$$

Solving the first and third relations of Equation (32) for $f_1(\eta)$, we have

$$\psi = (\alpha_1 x + \beta_1)^{\frac{2}{3}} f_1(\eta) - \frac{\beta_3}{2\alpha_1}. \tag{34}$$

Solving the first and last relations of Equation (32) for $f_2(\eta)$, we have

$$\theta = (\alpha_1 x + \beta_1)^{-\frac{1}{3}} f_2(\eta). \tag{35}$$

Using the equations (33), (34) and (35) and its derivatives in Equations (14) through (18), the below similarity equations (36) through (40) are obtained:

$$n\left(\frac{d^2 f_1}{d\eta^2}\right)^{n-1}\frac{d^3 f_1}{d\eta^3} + \frac{2}{3}\alpha_1 f_1(\eta)\frac{d^2 f_1}{d\eta^2} - \frac{1}{3}\alpha_1\left(\frac{df_1}{d\eta}\right)^2 + Gr\ f_2(\eta) = 0, \tag{36}$$

$$\frac{1}{\Pr}\frac{d^2 f_2}{d\eta^2}+\frac{1}{3}\alpha_1 f_2(\eta)\,\frac{df_1}{d\eta}+\frac{2}{3}\alpha_1 f_1(\eta)\frac{df_2}{d\eta}=0, \tag{37}$$

$$f_1(\eta)=0,\ \frac{d^2 f_1}{d\eta^2}=0,\ \frac{df_2}{d\eta}=0 \qquad at \quad \eta=0, \tag{38}$$

$$\frac{df_1}{d\eta}=0,\ f_2(\eta)=0 \qquad at \quad \eta\to\infty. \tag{39}$$

$$Q=\int_0^\infty f_2(\eta)\frac{df_1}{d\eta}\,d\eta\ =1. \tag{40}$$

3. Results and Discussions

The obtained similarity equations with auxiliary conditions (Equations (36) through (40)) are solved using the bvp4c- MATLAB ODE solver. The variation in velocity and temperature are presented graphically. The analysis of the effect of the buoyant force and the gravitational forces for the temperature difference and velocity variation is very important for this type of problem. Figure 2, Figure 3 and Figure 4 present the graphs for the velocity profile for variation in the values of Prandtl number (*Pr*), Grashof number (*Gr*) and fluid index ($n < 1$), respectively. The velocity of jet flow remains constant for some interval of eta and then decreases rapidly with the increase in Prandtl number as shown by the graph in Figure 2. In the other words, fluid is more viscous with a high Prandtl number, so velocity remains constant for a small interval and then decreases speedily. *Gr* and *n* are fixed for this case. For the graph in Figure 3, *Pr* and *n* are fixed, the velocity decreased more quickly with the increase in *Gr*. Particularly, the gravitational force effect is clearly shown in Figure 3, because at a high value of *Gr*, the fluid velocity is constant for a very small interval then it decreased quickly. Figure 4 represents the velocity decreased rapidly when the value of *n* decreased.

The variation in the temperature profile is shown in figures 5 and 6. Figure 5 represents the temperature profile for different values of Grashof number *Gr*. The wall temperature increase when the value of *Gr* or any buoyant-related parameter increase. Because of this, the fluid particle bonding becomes weak and the internal friction decrease. And therefore the gravity effects become stronger. From Figure 5, the temperature increased rapidly with increases in the value of *Gr*, reaches its peak value and then decrease briskly. Similarly, the temperature increased quickly to its peak and then decreased uniformly with the increase in the value of *Pr*, which is given in Figure 6.

4. Graphical Presentation

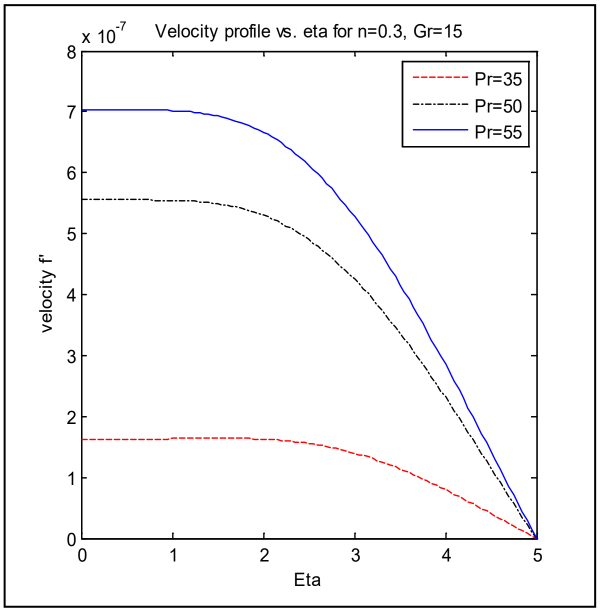

Figure 2. Velocity Profile for different values of Pr

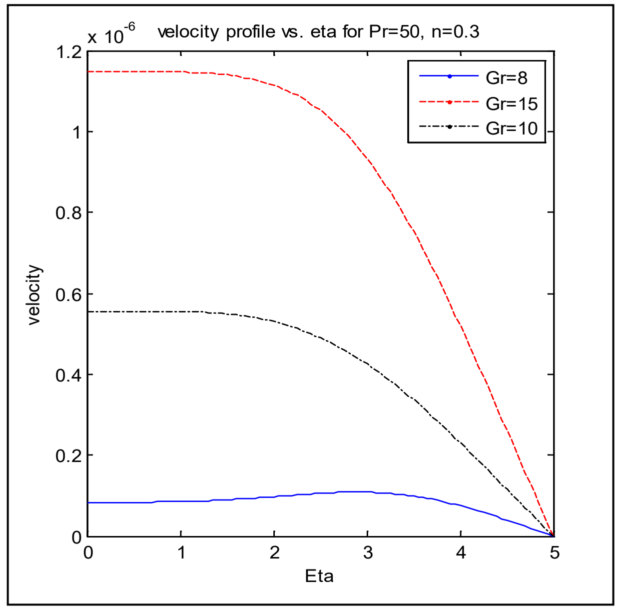

Figure 3. Velocity profile for different values of Gr

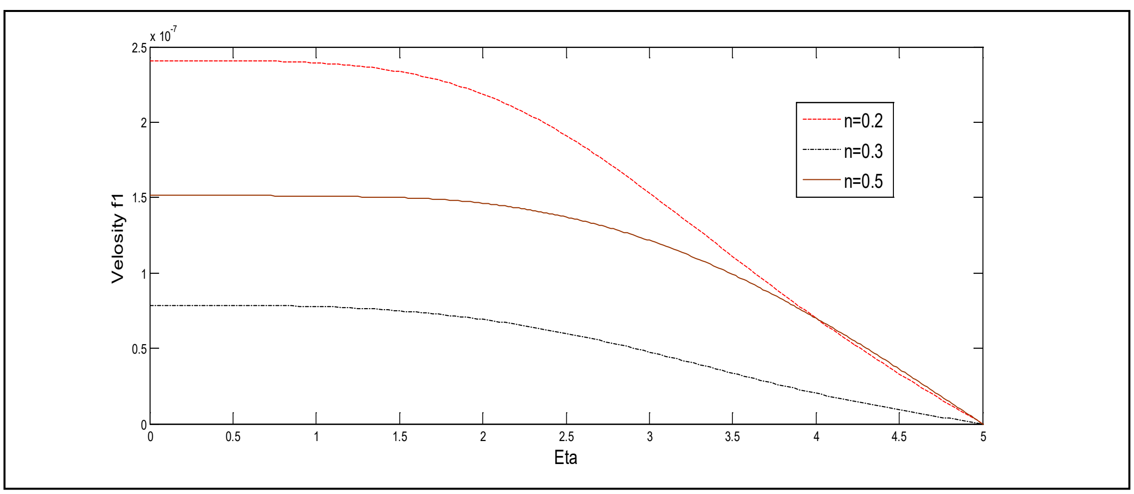

Figure 4. Velocity profile for different values of *n* for *Pr*=50 and *Gr*=10

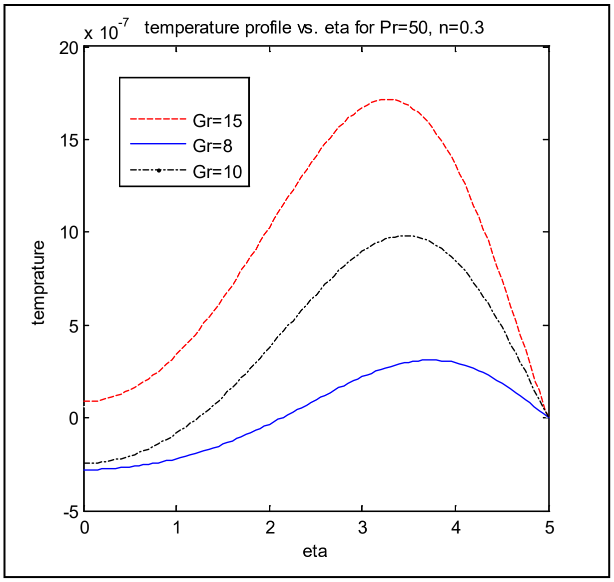

Figure 5. Temperature Profile for different values of *Gr*

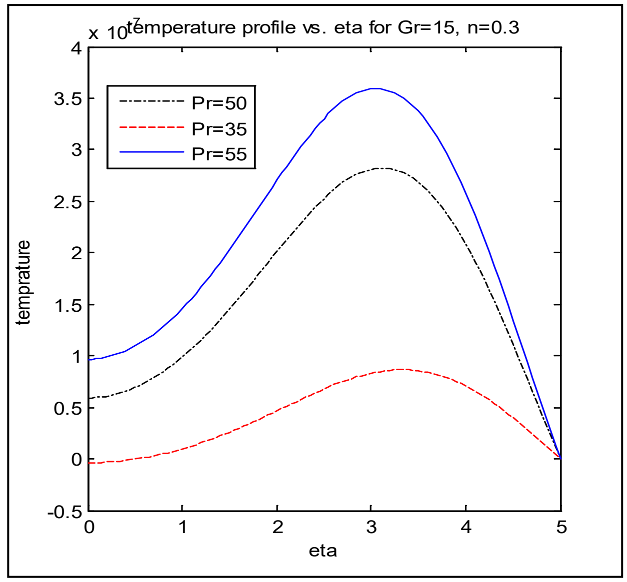

Figure 6. Temperature profile for different values of *Pr*

5. Conclusion

The changes in velocity and temperature profiles are slower for the present jet compared with the Newtonian fluid jet. This is quite consistent with the nature of Psudoplasctic fluids. It is observed that when fluid crosses the slit (i.e., at orifice), there is a sudden decrease in cross sectional area, and consequently there is a considerable increase in velocity. The problem discussed in the present paper is probably applicable to drilling boreholes, natural gas or oil into the surface of the soil using drilling fluids. Also, it may be applicable to the problems related to the volcanic eruption. In a Hawaiian eruption, lava (magma) is immersed from a vent as a fire fountain or lava jet (a huge natural vertical jet of non-Newtonian fluid). This is also an example of the vertical jet of Pseudoplastic fluid.

REFERENCES

Abd-el-Malek, M. B., Badran, N. A.and Hassan, H. S. (2002). Solution of the Rayleigh problem for a power law non-Newtonian conducting fluid via group method, Int. J. Eng. Sci., Vol. 40, pp. 1599–1609.

Acrivos, A., Shah, M. J. and Peterson, E. E. (1960). Momentum and heat transfer in laminar boundary-layer flows of non-Newtonian fluids past external bodies, AIChEJ., Vol. 6, pp. 312-317.

Adnan, K.A., Hasmani, A.H. and Timol, M. G. (2011). A new family of similarity solutions of three dimensional MHD boundary layer flows of non-Newtonian fluids using new systematic group theoretic approach, Appl. Math. Sci., Vol. 5, pp. 1325–1336.

Bickley, W.G. (1937). The plane jet, Philos. Mag., Vol. 23, pp. 727–731.

Daniel, N.R. (2017). Modeling and computation of nonlinear rotating polymeric jets during Force spinning process, International Journal of Non-Linear Mechanics, Vol. 92, pp. 1–7.

Darji, R.M. and Timol, M.G. (2013). Group symmetry and similarity solutions for MHD mixed-convection flow of power-law fluid over a non-linear stretching surface, Int. J. Math. Sci. Comput., Vol. 3, pp. 32–36.

Darji, R.M. and Timol, M.G. (2014). Similarity analysis for unsteady free convective boundary layer flow of power law fluids via group theory, JAMA,Vol. 3, No. 1, pp. 9-19.

Decent, S.P., King, A.C. and Wallwork, I. M. (2002). Free jets spun from a prilling tower, J. Eng. Math., Vol. 42, pp. 265–282.

Helmholtz, H.L.F. (1868). Über discontinuirliche Flüssigkeitsbewegungen, Monatsbericht Akad, Wiss. Berlin S, pp. 215–228.

Jain, N. and Timol, M.G. (2015). Similarity solutions of quasi three dimensional power law fluids using the method of satisfaction of asymptotic boundary conditions, Alexandria Engineering Journal, Vol. 54, pp. 725–732.

Kalathia, N. L. (1975). Laminar jets and flow over a wedge: Some problems, A Doctoral Thesis, IIT Kanpur.

Kirchhoff, G. (1869). Zur Theorie freier Flüssigkeitsstrahlen, Z. Reine Angew. Math., Vol. 70, pp. 289-298.

Magan, A.B., Mason, D.P. and Mahomed, F.M. (2016). Analytic solution in parametric form for the two-dimensional free jet of power-law fluid, International Journal of Non-Linear Mechanics, Vol. 85, pp. 94–108.

Moran, M. J. and Gaggioli, R. A. (1967). Similarity analysis if compressible boundary layer flows via group theory, Technical Summary Report No. 838, Mathematical Research Center, U.S. Army, Madison, Wisconsin.

Moran, M. J. and Gaggioli, R. A. (1968a). Similarity analysis via group theory, AIAA J., Vol. 6, pp. 2014-2016.

Moran, M. J. and Gaggioli, R. A. (1968b). A new symmetric formalism for similarity analysis, with application to boundary layer flows, Technical Summary Report No. 918, Mathematical Research Center, U. S. Army, Madison, Wisconsin.

Moran, M. J. and Gaggioli, R. A. (1968c). Reduction of the number of variables in systems of partial differential equations with auxiliary conditions, SIAM J. Appl. Math., Vol. 16, pp. 202-215.

Parmar, H. and Timol, M.G. (2011). Deductive group technique for MHD coupled heat and mass transfer natural convection flow of non-Newtonian power law fluid over a vertical cone through porous medium, Int. J. Appl. Math. Mech., Vol. 7, No. 2, pp. 35–50.

Patel, M. and Timol, M. G. (2010). The general stress–strain relationship for some different visco-inelastic non-Newtonian fluids, International Journal of Applied Mechanics and Mathematics (IJAMM), Vol. 6, No. 12, pp. 79-93.

Patel, M. and Timol, M. G. (2014). On the axisymmetrical and two-dimensional jet flow of an incompressible pseudo-plastic fluids, Int. J. of Appl. Math and Mech., Vol. 10, No. 8, pp. 45-60.

Patel, M. and Timol, M. G. (2015). Numerical solution of two-dimensional laminar circular jet in a variable magnetic field, International journal of mathematics and scientific computing, Vol. 5, No. 1, pp. 14-18.

Patel, M. and Timol, M. G. (2016). Jet with variable fluid properties: Free jet and dissipative jet, International Journal of Non-Linear Mechanics, Vol. 85, pp. 54–61.

Patel, M., Surati, H., Chanda, M. S. and Timol, M. G. (2013). Models of various non-Newtonian fluids, International E-Journal of Education and Mathematics (IEJEM), Vo. 2, No. 4, pp. 21-36.

Patel, M., Patel, J. and Timol, M. G. (2017). On the solution of boundary layer flow of Prandtl fluid past a flat surface, Journal of Advanced Mathematics and Applications, Vol. 6, pp. 1-6.

Patil, V. S. and Timol, M.G. (2012). Three dimensional unsteady incompressible magneto-hydrodynamic boundary layer equations of non-Newtonian power law fluids, JAMA, Vol. 1, No. 2, pp. 264-268.

Schlichting , H. (1933). Laminare strahlausbreitung, Z. Angew. Math. Mech., Vol. 13, pp. 260–263.

Schlichting , H. (1968). *Boundary-layer theory*, McGraw-Hill, NewYork.

Schowalter, W. R. (1960). The application of boundary-layer theory to power-law pseudoplastic fluids: Similar solutions, AIChEJ., Vol. 6, pp. 24-28.

Stehr, H. and Schneider, W. (2000). Jet flows in non-Newtonian fluids, Z. Angew. Math. Phys., Vol. 51, pp. 922-941.

Wallwork, I. M., Decent, S.P., King, A.C. and Schulkes, R.M.S.M. (2002). The trajectory and stability of a spiraling liquid jet: Part I. inviscid theory, J. Fluid Mech., Vol. 459, pp. 43–65.

Wilkinson, W. L. (1960). *Non-Newtonian Fluids: Fluid Mechanics, Mixing and Heat Transfer*, Pergamon Press, London.