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Some New Constructors for Minimal Circular Partially Balanced Neighbor Designs

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Abstract: Minimal circular neighbor designsa are economical to minimize the bias due to neighbor effects for v odd. For v even, minimal circular partially balanced neighbor designs (MCPBNDs) are used. Generators to obtain MCPBNDs-II in equal, two and three different blocks sizes are available in literature for c = 0 and 1, where c is remainder if m is divided by 4, $m = (v-2)/2$ and v is number of treatments. These designs have not been constructed for c = 2 and 3. To complete the construction of this class of neighbor designs, MCPBNDs-II are, therefore, constructed for the remaining cases. MCPNBDs-II are the neighbor designs in which $3v/2$ pairs of different treatments do not appear as neighbors.

Keywords: Neighbor effects; Direct effects; CBNDs; CSBNDs; CSGNDs.
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1 Introduction

Neighbor effect often becomes the major source of bias, which can be minimized with the use of neighbor balanced designs, see [1] and [2]. [3] used neighbor designs in virus research. A design where each pair of distinct treatments appears once as neighbors is called minimal neighbor balanced designs (NBD). [4] presented catalogue of NBDs using border plots. [5], [6], [7] and [8] are some more references for CNBDs. Minimal cicular NBDs can only be constructed for v odd, where v is the number of treatments. For v even, minimal circular partially balanced neighbor designs (MCPBNDs) are used. If each pair of distinct treatments appears as neighbors in circular design at most once, design is called MCPBND. [9] suggested that PNBDs should be used if minimal NBDs cannot be generated. [10] constructed generalized neighbor designs (GNDs) by relaxing balance property. [11], [12] and [13] constructed some classes of circular GNDs. [14] and [15] developed some infinite series to obtain the minimal circular GNDs. [16] presented list of CGNDs for blocks of sizes three.

[17] developed some generators to generate these designs in blocks of two and three different sizes using i sets of shifts for k_1 and two sets for k_2 for $c = 0$ and 1. These designs can also be constructed for $c = 2$ and 3, where c is reminder if m is divided by 4 and $m = (v-2)/2$. In this article, some generators are developed through method of cyclic shifts (Rule I) to obtain MCPBNDs-II for $c = 2$ and 3 in two different and three different block sizes. MCPBNDs-II are MCPBNDs in which $3v/2$ pairs of different treatments do not appear as neighbors while the remaining ones appear once.

2 Method of cyclic shifts

Method of cyclic shifts (Rule I) developed by [18] is explained here for the construction of MCPBNDs.

Let $S_j = [q_{j1}, q_{j2}, ..., q_{j(k-1)}]$ be i sets, where j = 1, 2,..., i, $1 \leq q_{ju} \leq v-1$, $u = 1, 2, ..., k-1$.

• If 1, 2,..., $v-1$ appears exactly once in S^* then designs will be MCBND.

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• If each of 1, 2, ..., $v-1$ appears once except $v/2$ which does not appear in S^* then designs will be MCPBND-I.
• If each of 1, 2, ..., $v-1$ appears once except $v/2$ and two others which do not appear in S* then designs will be MCPBND-II.
Where S^* contains: Each element of all S_j.
Sum of all elements (mod v) in each S_j.
Complements of all elements in (i) and (ii). In Rule I, complement of 'a'is '$v-a$'.

Rule I expresses that:
• A = [1, 2, ..., $m-2$, m] will produce MCPBNDs-II for c = 2 and 3 if sum of A is divisible by v. Otherwise, replace one or more elements with their complements to make the sum divisible by v.

Example 2.1: S_1 = [2,3,4,5] and S_2 = [6,7,10] generate MCPBND-I for v = 22, k_1 = 5, k_2 = 4.

Take v blocks to get the blocks from S_1. Consider first unit elements as 0, 1, ..., $v-1$. Add 2 (mod v) to each first unit element, to obtain second unit elements. Add 3 (mod 22) to second unit elements to obtain third unit elements, then add 4 and 5, see Table 1.

Table 1: Blocks generated from S_1.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0	1
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0	1	2	3	4
9	10	11	12	13	14	15	16	17	18	19	20	21	0	1	2	3	4	5	6	7	8
14	15	16	17	18	19	20	21	0	1	2	3	4	5	6	7	8	9	10	11	12	13

Take v more blocks for S_2 and obtain the design, see Table 2.

Table 2: Blocks generated from S_2.

23	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0	1	2	3	4	5
13	14	15	16	17	18	19	20	21	0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	0

Table 1 and Table 2 jointly present the MCPBND-II for $v = 22, k_1 = 5$ and $k_2 = 4$.

In all next construction to obtain MCPBNDs-II, elements of A and B given in constructors 3.1 and 3.2 respectively, will be divided in (i) i groups of size k for $v = 2ik+4$, (ii) i groups of size k_1 and one of size k_2 for $v = 2ik_1+2k_2+4$, (iii) i groups of size k_1 and two of size k_2 for $v = 2ik_1+4k_2+4$, (iv) i groups of size k_1, one group each of size k_2 and k_3 for $v = 2ik_1+2k_2+2k_3+4$, (v) i groups of size k_1, one of size k_2 and two of size k_3 $v = 2ik_1+2k_2+4k_3+4$, such that the sum of each group should be divisible by v. Sets of shifts to produce MCPBNDs-II will be obtained by deleting any one element from each group.

3 Constructors to produce MCPBNDs-II

Here, MCPBNDs-II are constructed in two and three different block sizes for $m(mod 4) \equiv c$, where $m = (v-2)/2$ and c = 2, 3.

Construction 3.1: If c = 2 then sets of shifts obtained from A = [2, 3, ..., $m-2, m, 2m+1$] produce MCPBNDs-II.

Construction 3.2: If c = 3 then sets of shifts obtained from B = [1, 2, ..., $(m+1)/4, (m+9)/4, (m+13)/4, m-2, m, (7m+3)/4$] produce MCPBNDs-II.

4 MCPBNDs-II in two different blocks sizes

4.1 MCPBNDs-II in two different blocks sizes for c = 2

Here, MCPBNDs-II are constructed for $v = 2ik_1 + 2k_2 + 4$ and $v = 2ik_1 + 4k_2 + 4$ in two different blocks sizes, using constructor 3.1.

Generator 4.1.1. MCPBNDs-II can be constructed from i sets for k_1 and one for k_2 for $v = 2ik_1 + 2k_2 + 4$ with c = 2 and:

$k_2 = 3$:
$k_1 = 6, 10, ...,$ and i odd.
$k_1 = 5, 7, ...,$ i = 2, 6, ...

Example 4.1.1. S_1 = [3,4,5,7,9,6] and S_2 = [7,10] produce MCPBND-II for $v = 18, k_1$ = 7 and k_2 = 3.

$k_2 = 4$:
$k_1 = 1, 5, ...,$ and i = 1, 5, ...
$k_1 = 3, 7, ...,$ and i = 3, 7, ...

$k_2 = 5$:
$k_1 = 8, 12, ...,$ and i integer.
$k_1 = 6, 10, ...,$ and i even.
$k_1 = 7, 9, ...,$ and i = 4, 8, ...

Generator 4.1.2: MCPBNDs-II can be generated from i sets for k_1 and two for k_2 for $v = 2ik_1 + 4k_2 + 4$ with c = 2 and:

$k_2 = 3$:
$k_1 = 5, 9,$ ..., i = 3, 7, ...
$k_1 = 7, 11,$ ..., i = 1, 5, ...

Example 4.1.2. S_1 = [2,3,5,6,7,8], S_2 = [9,10] and S_3 = [4,12] produce MCPBND-II for $v = 30$, k_1 = 7, k_2 = 3.

$k_2 = 4$:
$k_1 = 5, 9,$ ..., i = 1, 5, ...
$k_1 = 7, 11,$ ..., i = 3, 7, ...

$k_2 = 5$:
$k_1 = 5, 9,$ ..., i = 1, 5, ...
$k_1 = 7, 11,$ ..., i = 3, 7, ...

4.2 MCPNBDs-II in two different blocks sizes for c = 3

Here, MCPBNDs-II are constructed for $v = 2ik_1 + 4k_2 + 4$ and $v = 2ik_1 + 2k_2 + 4$ in two different blocks sizes, using constructor 3.2.

Generator 4.2.1. MCPBNDs-II can be constructed from i sets of shifts for k_1 and one for k_2 for $v = 2ik_1 + 2k_2 + 4$ with c = 3 and:

$k_2 = 3$:
$k_1 = 1, 5, ...,$ and i = 3, 7, ...
$k_1 = 3, 7, ...,$ i = 1, 5, ...

$k_2 = 4$:
$k_1 = 6, 10,$ ..., i odd.
$k_1 = 5, 7,$ ..., and i = 2, 6, ...

$k_2 = 5$:
$k_1 = 1, 5, ...,$ and i = 1, 5, ...
$k_1 = 3, 7, ...,$ and i = 3, 7, ...

Example 4.2.1. S_1 = [3,4,6,7,11] and S_2 = [6,8] produce MCPBND-II for $v = 16, k_1$ = 6 and k_2 = 3.

Generator 4.2.2: MCPBNDs-II can be generated from i sets for k_1 and two for k_2 for $v = 2ik_1 + 4k_2 + 4$ with c = 3 and:

$k_2 = 3$:
$k_1 = 4, 8$, ..., i integer.
$k_1 = 6, 10$, ..., i even.
$k_1 = 5, 7$, ..., i = 4, 8, ...

Example 4.2.2. S_1 = [1,8,11], S_2 = [6,7,9], S_3 = [4,13] and S_4 = [2,3] produce MCPBND-II for $v = 32, k_1$ = 4 and k_2 = 3.

$k_2 = 4$:
$k_1 = 6, 10$, ..., i odd.
$k_1 = 5, 7$, ..., i = 2, 6, ...

$k_2 = 5$:
$k_1 = 8, 12$, ..., i integer.
$k_1 = 6, 10$, ..., i even.
$k_1 = 7, 9$, ..., i = 4, 8, ...

5 MCPBNDs-II in three different blocks sizes

5.1 MCPBNDs-II in three different blocks sizes for c = 2

Here, MCPBNDs-II are constructed for $v = 2ik_1 + 2k_2 + 4k_3 + 4$ and $v = 2ik_1 + 2k_2 + 4k_3 + 4$ in three different blocks sizes, using constructor 3.1.

Generator 5.1.1: MCPBNDs-II can be constructed from i sets for k_1, one set for k_2 and two for k_3 for $v = 2ik_1 + 2k_2 + 2k_3 + 4$ with c = 2 and:

$k_3 = 3$:
$k_1 = 8, 12, ..., k_2 = k1 - 1$, and i integer.
$k_1 = 6, 10, ..., k_2 = k_1 - 1$, and i even.
$k_1 = 5, 7, ..., k_2 = k_1 - 1$, and i = 2, 6, ...
$k_1 = 8, 12, ..., k_2 = k_1 - 2$, and i integer.
$k_1 = 6, 10, ..., k_2 = k_1 - 2$, and i odd.
$k_1 = 7, 9, ..., k_2 = k1 - 2$, and i = 3, 7, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 3$, and i = 4, 8, ...
$k - 1 = 7, 11, ..., k_2 = k_1 - 3$, and i = 2, 6, ...

Example 5.1.1. $S_1 = [5, 9, 10, 11, 13], S_2 = [3, 6, 7, 8]$ and $S_3 = [11, 14]$ produce MCPBND-II for $v = 26, k_1 = 6, k_2 = 5$ and $k_3 = 3$.

$k_3 = 4$:
$k_1 = 1, 5, ..., k_2 = k_1 - 1$, and i = 1, 5, ...
$k_1 = 3, 7, ..., k_2 = k_1 - 1$, and i = 3, 7, ...
$k_1 = 9, 13, ..., k_2 = k_1 - 2$, and i = 2, 6, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 2$, and i = 4, 8, ...
$k_1 = 8, 12, ..., k_2 = k_1 - 3$, i integer.
$k_1 = 10, 14, ..., k_2 = k_1 - 3$, and i odd.
$k_1 = 9, 11, ..., k_2 = k_1 - 3$, and i = 3, 7, ...

$k_3 = 5$:
$k_1 = 1,5,...,k_2 = k_1 - 1$, and i = 4, 8, ...
$k_1 = 3,7,...,k_2 = k_1 - 1$, and i = 2, 6, ...
$k_1 = 10,14,...,k_2 = k_1 - 2$, i even.
$k_1 = 9,11,...,k_2 = k1 - 2$, and i = 1, 5, ...

Generator 5.1.2: MCPBNDs-II can be constructed from i sets for $_1$, one set for k_2 and two for k_3 for $v = 2ik_1 + 2k_2 + 4k_3 + 4$ with c = 2 and:

$k_3 = 3$:
$k_1 = 8,12,...,k_2 = k_1 - 1$, i integer.
$k_1 = 6,10,...,k_2 = k_1 - 1$, i odd.
$k_1 = 5,7,...,k_2 = k_1 - 1$, i = 3, 7, ...
$k_1 = 9,13,...,k_2 = k_1 - 2$, i = 4, 8, ...
$k_1 = 7,11,...,k_2 = k_1 - 2$, i = 2, 6, ...

Example 5.1.2. $S_1 = [3,7,8,10,11], S_2 = [2,5,6,12], S_3 = [9,14]$ and $S_4 = [4,16]$ produce MCPBND-II for $v = 38, k_1 = 6, k_2 = 5$ and $k_3 = 3$.

$k_3 = 4$:
$k_1 = 6,10,...,k_2 = k_1 - 1$, i even.
$k_1 = 7,9,...,k_2 = k_1 - 1$, i = 1, 5, ...
$k_1 = 9,13,...,k_2 = k_1 - 2$, i = 2, 6, ...
$k_1 = 7,11,...,k_2 = k_1 - 2$, i = 4, 8, ...

$k_3 = 5$:
$k_1 = 8,12,...,k_2 = k_1 - 1$, i integer.
$k_1 = 10,14,...,k_2 = k_1 - 1$, i odd.
$k_1 = 7,9,...,k_2 = k_1 - 1$, i = 3, 7, ...
$k_1 = 9,13,...,k_2 = k_1 - 2$, i = 4, 8, ...
$k_1 = 11,15,...,k_2 = k_1 - 2$, i = 2, 6, ...

5.2 MCPBNDs-II in three different blocks sizes for c = 3

Here, MCPBNDs-II are constructed for $v = 2ik_1 + 2k_2 + 4k_3 + 4$ and $v = 2ik_1 + 2k_2 + 4k_3 + 4$ in three different blocks sizes, using constructor 3.2.

Generator 5.2.1: MCPBNDs-II can be constructed from i sets for k_1, one set for k_2 and two for k_3 for $v = 2ik_1 + 2k_2 + 2k_3 + 4$ with c = 3 and:

$k_3 = 3$:
$k_1 = 5,9,...,k_2 = k_1 - 1$, and i = 3, 7, ...
$k_1 = 7,11,...,k_2 = k_1 - 1$, and i = 1, 5, ...
$k_1 = 9,13,...,k_2 = k_1 - 2$, and i = 4, 8, ...
$k_1 = 7,11,...,k_2 = k_1 - 2$, and i = 2, 6, ...
$k_1 = 10,14,...,k_2 = k_1 - 2$, and i even.
$k_1 = 10,14,...,k_2 = k_1 - 3$, and i even.
$k_1 = 7,9,...,k_2 = k_1 - 3$, and i = 1, 5, ...

$k_3 = 4$:
$k_1 = 7,9,...,k_2 = k_1 - 1$, and i = 2, 6, ...
$k_1 = 8,12,...,k_2 = k_1 - 2$, and i integer.
$k_1 = 10,14,...,k_2 = k_1 - 2$, and i odd.
$k_1 = 7,9,...,k_2 = k_1 - 2$, and i = 2, 6, ...
$k_1 = 9,13,...,k_2 = k_1 - 3$, and i = 4, 8, ...
$k_1 = 11,15,...,k_2 = k_1 - 3$, and i = 2, 6, ...

$k_3 = 5$:
$k_1 = 10, 14, ..., k_2 = k_1 - 1$, and i even.
$k_1 = 7, 9, ..., k_2 = k_1 - 1$, and i = 2, 6, ...
$k_1 = 1, k_2 = k_1 - 2$, and i = 2, 6, ...
$k_1 = 11, 15, ..., k_2 = k_1 - 2$, and i = 4, 8, ...

Generator 5.2.2: MCPBNDs-II can be generated from i sets for k_1, one set for k_2 and two for k_3 obtained from for $v = 2ik_1 + 2k_2 + 4k_3 + 4$ with c = 3 and:

$k_3 = 3$:
$k_1 = 5, 9, ..., k_2 = k_1 - 1$, i = 4, 8, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 1$, i = 2, 6, ...
$k_1 = 6, 10, ..., k_2 = k_1 - 2$, i even.
$k_1 = 7, 9, ..., k_2 = k_1 - 2$, i = 1, 5, ...

Example 5.2.2. $S_1 = [8, 9, 10, 11, 13, 14], S_2 = [2, 3, 7, 12], S_3 = [4, 17], S_4 = [1, 5]$ produce MCPBND-II for $v = 40, k_1 = 7, k_2 = 5$ and $k_3 = 3$.

$k_3 = 4$:
$k_1 = 9, 13, ..., k_2 = k_1 - 1$, i = 2, 6, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 1$, i = 4, 8, ...
$k_1 = 8, 12, ..., k_2 = k_1 - 2$, i integer.
$k_1 = 10, 14, ..., k_2 = k_1 - 2$, i odd.
$k_1 = 7, 11, ..., k_2 = k_1 - 2$, i = 3, 7, ...

$k_3 = 5$:
$k_1 = 9, 13, ..., k_2 = k_1 - 1$, i = 4, 8, ...
$k_1 = 7, 11, ..., k_2 = k_1 - 1$, i = 2, 6, ...
$k_1 = 10, 14, ..., k_2 = k_1 - 2$, i even.
$k_1 = 9, 11, ..., k_2 = k_1 - 2$, i = 1, 5, ...

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