DYNAMIC CHARACTERIZATION OF TWO STRUCTURES REALIZED WITH DIFFERENT TECHNOLOGY: METALWORKING VS. METAL FOAM SANDWICHES

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SUMMARY: In this paper, a comparison between two different machine tool columns is presented. The two columns are realized with a different technology: classic metalworking versus metal foam sandwiches. The aim of the experimental tests is at evaluating their different mechanical performances and characteristics, with a particular focus on the damping. The comparison is carried in terms of both modal analysis and wide frequency range excitation, as described in the paper. A new method is introduced by the authors to deem which technology is the best, based on overall energy dissipated in a wide range of frequency.

KEYWORDS: metal foam, sandwiches structures, damping estimation, machine tool dynamic characterization.

1. INTRODUCTION

It is a common practice, for machine tool manufacturers, to realize structures the most rigid as possible, in order to perform extremely precise tool spindle positioning. If milling machines are considered, due to the variety of milling programs and to the high amount of cutting tool types, the machine tool will be easily forced by a broad frequency spectrum excitation, quite impossible to be forecasted during the design phase. In this case, a near-resonance working condition could easily appear and the structure stiffness is no more enough to guarantee good quality of the machining. In worst cases, instabilities might also appear, causing the so-called *“chatter”* phenomenon [1].

It is well known, for mechanical engineers, that the damping is a key parameter, which quantifies the energy dissipation in a mechanical system. It is trivial that the best design solution is resorting to extremely rigid structures, which benefit also of high damped characteristics. Nowadays, this purpose could be achieved by employing components made of metal foam sandwiches, but it is quite rare to find some experimental tests, at least on a complete structure, like a column of a milling machine, which demonstrate the effectiveness of these solutions with respect to the classical ones. Some results are described in [2] for the column of a microfactory machine, but the discussion of the results, in terms of damping, is not really detailed.

Owing to the unevenness of foamed parts, the problem of defining mechanical properties of the metallic foams has risen since their introduction, as clearly stated in handbooks, like [3]. A resonance apparatus is proposed in [4] and the difficulties, originating from random spatial density variations or anisotropy, are discussed in [5] and more detail in [6], where a procedure to overcome them is also proposed and tested. Loss factors η^c are listed for different commercial metal foams in [3]. Specific tests on mechanical behavior of metal foam sandwiches, as stand-alone components, are described in the literature; in particular, energy absorption and failure mechanism are analyzed in [7], while damping in [8].

In this paper, the authors present a comparative study about the dynamic behavior of the columns of a milling machine with table of fixed height and vertical spindle, one made according to traditional techniques, namely in metallic carpentry, and the other realized by means of panels of metal foam sandwiches, shaped and welded together. With the reference to the results presented in [2], the authors introduce a new method aimed at comparing the performances obtained between the two manufacturing solutions, in terms of dynamic behavior.

The problem is not trivial, because, as it will be illustrated in detail, despite the geometry of the column is not very different, the vibration modes and their damping are very different and, especially, several modes are excited in the range of operation of the milling machine. Therefore, a simple approach based on the comparison of the damping ratios (which are corresponding to modes at frequencies very different from each other, as it resulted also in [2]) does not give a definite answer.
To the best of authors' knowledge, it is the first time that both this kind of analysis, and the comparison method, are proposed in the literature. They allowed establishing that the column made of foam sandwich has generally superior performance in terms of vibration damping. This is a remarkable piece of information for the technology of machine tool manufacturing.

2. COLUMN DESCRIPTION AND SETUP OF EXPERIMENTAL TESTS

The two structures considered in the paper are the main columns of the milling machine, which support the vertical milling spindle. They are made by using two different technologies: the first one (hereafter *"column A"*, which is painted white in Figure 1) has a traditional "box" design and is manufactured by using classical metal carpentry, with welded steel plates of thickness up to 30 mm. The second column is a prototype (hereafter *"column B"*, bare metal in Figure 1) and is made of tiny sandwich panels filled with metal (Al based) foam, which resulted in a lighter structure. The macro-geometry is considered very similar, so that the columns are interchangeable.

Column A Column B

Figure 1 – Columns of a milling machine with table of fixed height and vertical spindle: conventional metal carpentry (left), panels of sandwich filled with metal foam (right). Note the position of the accelerometer on the column surfaces.

Column A has a mass of 958 kg, while that of column B is 805 kg, namely about 16% less.
A preliminary analysis was performed using FEM, but the results were not deemed reliable due to several uncertainties in the modelling. They were related to the constraints between the columns and the slideways and to the effects of the welding on the borders of the plates, especially in the case of metal foam sandwiches, in which the heating causes an unavoidable modification of the cell distribution and the adhesion of the metallic foam to the plates. Therefore, an experimental campaign was setup.

2.1. Test setup

The experimental tests have been performed by following the same procedure for both the columns and aimed at evaluating the main dynamic differences between them, in order to assess the advantages of installing columns of type B in mass production machines.
The columns were fixed on a very massive suspended foundation by means of four special machined supports, with realized a very stiff connection. The stiffness of the connection was checked by installing one accelerometer on each support and another one close to it on the column side and by comparing the results of the measured impulse response function during bump tests. Once the stiff connection has been verified, both columns have been

instrumented with 16 piezoelectric accelerometers (see Figure 1), 4 accelerometers for each lateral surface. Despite the columns were structurally different, the accelerometers have been installed in homologous positions, since the external geometry is quite similar.
The columns were excited by means of an impact hammer and several excitation points were considered (up to 7 positions) in order to avoid nodes of vibrating surface modes. In addition, in order to evaluate possible non-linearities of column responses (especially for column B), several impulsive tests with different input force amplitudes were carried on. Tests were performed resorting to a pendulum with the impulsive hammer installed. No particular differences appeared in the impulse frequency response functions (FRFs), considering four different amplitudes of force. Thus, both columns shown linear behavior. This fact is a first remarkable result of the test campaign.

2.1.1. *Sensors and data acquisition*

The system response to the input force has been measured by means of the following devices:

- 16 PCB Piezotronics 333B30 piezoelectric accelerometers (±50g pk) conditioned with a PCB Piezotronics 483C50 conditioning unit;
- 5 National Instruments NI9239 boards (4 channels each, 24 Bit, ±10 V, da 50 kS/s, antialiasing filtered) installed on a National Instrument cDAQ 9172 (8 slots).

The impulsive excitation was provided by using a PCB Piezotronics 086D05 dynamometric hammer, equipped with metal tip. The accelerometer positions on the column surfaces are shown in Figure 2, with a red dot (●), for the $n_{ch} = 16$ accelerometers. The hammer impact points m_i are also shown in Figure 2, with a blue cross (×).

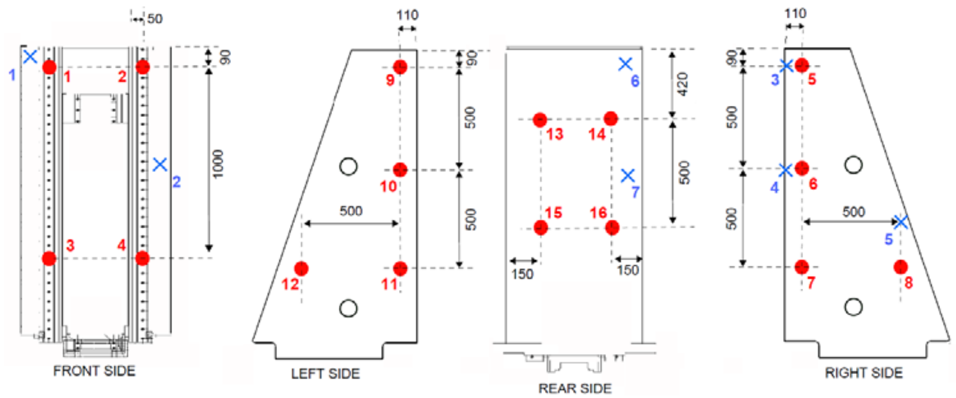

Figure 2 – Position of the accelerometers (●) and of the hammer impact points (×) on the columns.

Each time history has been acquired for 5 s at the sampling frequency of 12500 Hz, then windowed around the decay. In order to reduce the uncorrelated measurement noise, each test was repeated $n_p = 15$ times. The impulse FRFs have been calculated according to the H_1 estimator:

$$H_1(f) = \frac{G_{xy}(f)}{G_{xx}(f)} \tag{1}$$

where G_{xy} and G_{xx} represent the cross and auto spectra respectively, referred to the input signal $X(f)$ and of the outputs $Y(f)$ spectra. The cross and auto spectra are defined as:

$$G_{xy}(f) = \frac{1}{n_p}\sum_{i=1}^{n_p} X_i^*(f)Y_i(f)\ ;\ \ G_{xx}(f) = \frac{1}{n_p}\sum_{i=1}^{n_p} X_i^*(f)X_i(f) \tag{2}$$

where the star (*) indicates the complex conjugate. Figure 3 shows the magnitude of the calculated FRFs in accelerometer positions #2 and #9, for both kinds of columns.
In addition, the coherence functions were calculated as:

$$\gamma_{xy}^2 = \frac{H_1(f)}{H_2(f)} = \frac{G_{xy}(f)G_{yx}(f)}{G_{xx}(f)G_{yy}(f)} \tag{3}$$

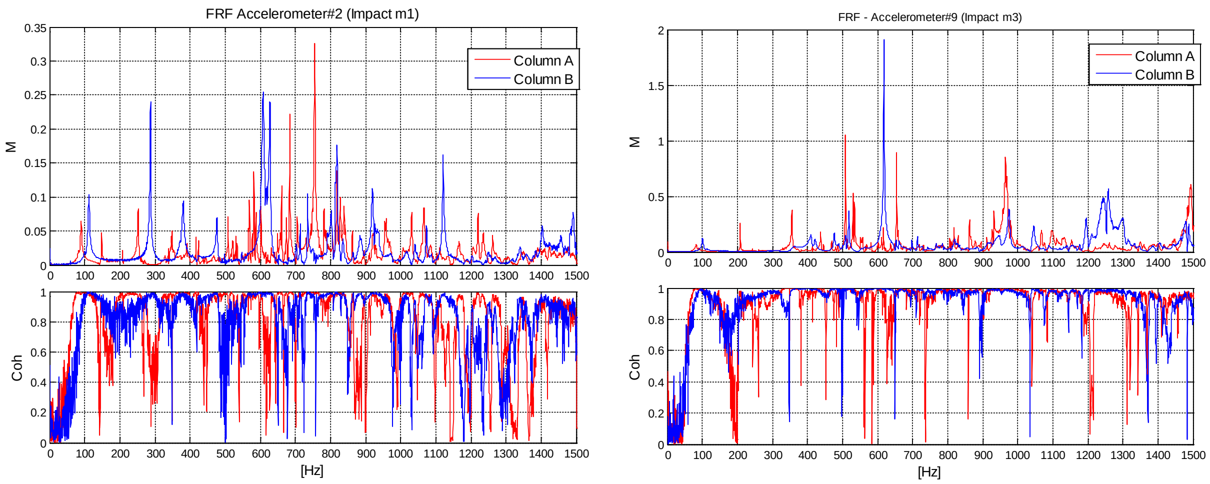

Figure 3 – Magnitude of FRF and coherence functions for the two columns: accelerometer #2 - impact in m_1 (left) and accelerometer #9 - impact in m_3 (right).

where G_{yy} is immediately obtained from eq. (2) by replacing $X^*(f)$ with $Y^*(f)$. Coherence values over 0.9 are commonly considered acceptable. Figure 3 shows the calculated coherences for sensor positions #2 and #9.

2.2. Signal processing

The modal identification of the mechanical parameters of the columns has been made by using the experimental FRFs of eq. (1). The procedure has been divided in two steps.

1) the first one used the LSCE (*Least Squares Complex Exponentials*, see [9]) algorithm. In this way, for each frequency range selected, each pole stability was evaluated at the increasing of the order of the system.
2) In the second step, a numerical optimization of the analytical FRF was carried out with respect to the experimental one. An analytical FRF is essentially defined as a limited sum of single degree of freedom systems of second order, with upper and lower residual terms too:

$$\alpha_{jk}(\omega) = \frac{1}{\omega^2 M^r_{jk}} + \sum_{r=n_1}^{n_2} \frac{{}_r A_{jk}}{\omega_r^2 - \omega^2 + 2i\xi\omega\omega_r} + \frac{1}{K^r_{jk}} \tag{4}$$

Actually, the modal identification relies on a multi-channel and multi-modal procedure, which allows performing a simultaneous analysis of all the measurement channels for a quite wide frequency range.
Finally, modal frequencies, modal masses and modal damping values are obtained. In order to compare the different identified modes between column A and B, *MAC* (*Modal Assurance Criterion*, see [10]) indexes were calculated as:

$$MAC(\phi_{\varepsilon_A}, \phi_{\varsigma_B}) = \frac{\left| \sum_{i=1}^{n_{ch}} (\phi_{\varepsilon_A})_i (\phi_{\varsigma_B})_i \right|^2}{\sum_{i=1}^{n_{ch}} (\phi_{\varepsilon_A})_i (\phi_{\varepsilon_A})_i \sum_{i=1}^{n_{ch}} (\phi_{\varsigma_B})_i (\phi_{\varsigma_B})_i} \tag{5}$$

where ϕ_{ε_A} represents the $\varepsilon - th$ modal shape of column A previously identified, compared with the $\varsigma - th$ modal shape ϕ_{ς_B} of column B. Also *AutoMAC* indexes were calculated, in order to compare the modes of the same column, by simply replacing index A with B, or vice versa, in eq. (5).

3. RESULTS

As expected, the two columns show quite different dynamic behavior, due to the different manufacturing method, and only few modes can be directly compared.

The identified modes are reported for both columns in Table 1. As a rule of thumb, the modes of the lighter column B have higher natural frequencies.

Table 1a – Column A: natural frequencies and damping ratios.

Mode	*Frequencyf [Hz]*	*Damping ratio ξ [%]*
1	84.0	0.4089
2	89.5	0.3104
3	148.5	0.0779
4	208.5	0.0484
5	253.0	0.1005
6	343.5	0.1301
7	346.5	0.2061
8	355.5	0.1005
9	343.5	0.0810
10	424.5	0.0319
11	469.0	0.0938
12	508.5	0.0288
13	530.5	0.0363
14	569.0	0.0299

Table 1b – Column B: natural frequencies and damping ratios.

Mode	*Frequency f [Hz]*	*Damping ratio ξ [%]*
1	101.0	0.3410
2	112.0	0.2714
3	287.5	0.0678
4	381.0	0.1642
5	410.5	0.1484
6	476.0	0.0784
7	518.5	0.0385
8	608.0	0.0949
9	619.0	0.0296
10	698.0	0.0201
11	735.0	0.0265
12	756.5	0.0279
13	802.5	0.0466
14	934.0	0.0594
15	974.0	0.1283
16	1045.5	0.0590

The *AutoMAC* indexes are not displayed for the sake of brevity. While the diagonal terms are obviously equal to one, the off-diagonal terms are much smaller than one, thus indicating that enough sensors have been used to determine the modes and that they are well distinguished. The *MAC* values between the modes of columns A and B are shown in Figure 4.

By considering only the mode pairs which show high *MAC* values (above 0.7 in Figure 4), which are the only ones for which any comparison might have sense, a first possible result is reported in Table 2. For the considered mode pairs, frequencies exhibit a sensible increase, up to 25%, in case of the structure made of metal foam sandwiches. However, the product $2\pi f \cdot \xi$, which is the actual system damping for the given frequency *f*, does not show any clear trend. In authors' opinion, the analysis focused on the damping, which can consider to four modes only and neglects several others, is not adequate for a manufacturer who wants a reliable criterion for choosing the suitable technology for the machine column.

3.1 Authors' proposed method

For these reasons, the authors are now introducing a criterion, which should be able to perform a wide frequency range analysis and a reliable comparison. By supposing that the quality of the milling depends only on the vibrations in terms of displacements (which is rather likely), the idea is to compute the RMS of the displacements, supposing an unitary exciting force for the whole spectra (i.e. $F(f) = F = 1[\mathrm{N}]$) of excitation. According to Parseval's theorem, it follows that:

$$RMS_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N} x_i(t)^2} = \sqrt{\sum_{i=1}^{N}\left|\frac{\overline{X}_i(f)}{N}\right|^2} \quad \text{where } \overline{X}(f) = \mathrm{FFT}\langle x(t)\rangle \tag{6}$$

where $\overline{X}_i(f)$ is the *i-th* element of $X_i(f)$ which has been computed according to the fast Fourier transform (FFT) algorithm. Then, considering the same spectrum defined only for positive frequencies, it follows that:

$$RMS_x = \sqrt{\sum_{i=1}^{N}\left|\frac{\overline{X}_i(f)}{N}\right|^2} = \sqrt{2\sum_{i=1}^{n=N/2}\left|\frac{\widehat{X}_i(f)}{2}\right|^2} \quad \text{where } \widehat{X}_i(f) = \Im'\langle x(t)\rangle \tag{7}$$

where $\widehat{X}_i(f)$ is the the *i-th* elements of $\widehat{X}(f)$ which is the one-sided Fourier transform $\Im'\langle\cdot\rangle$ of the original signal $x(t)$. By supposing an exciting force $F(f)$ equal to 1 N for each frequency, introduced in a mechanical system described by the experimental FRF $G(f)$ expressed in term of receptance, then $\widehat{X}(f)=G(f)\cdot F(f)$. Considering the positive frequencies, the root mean square RMS_{disp} of the displacement can be calculated as:

$$RMS_{disp} = \sqrt{2\sum_{i=1}^{n=N/2}\left|\frac{G_i(f)\cdot 1\mathrm{N}}{2}\right|^2} = \sqrt{2\sum_{i=1}^{n=N/2}\left|\frac{G_i(f)}{2}\right|^2} \qquad (8)$$

Table 2 – Comparison between modes with high MAC.

	Frequency [Hz]			*$2\pi f\cdot\xi$*		
	Column A	*Column B*	$\Delta_{f_{BA}}$ *[%]*	*Column A*	*Column B*	$\Delta_{2\pi f\xi_{BA}}$ *[%]*
ϕ_{1_A},ϕ_{1_B}	84.0	101.0	20.2	216	216	0
ϕ_{2_A},ϕ_{2_B}	89.5	112.0	25.1	175	191	9
ϕ_{5_A},ϕ_{3_B}	253.0	287.5	13.7	160	122	-24
ϕ_{12_A},ϕ_{9_B}	508.5	619.0	21.7	92	115	25

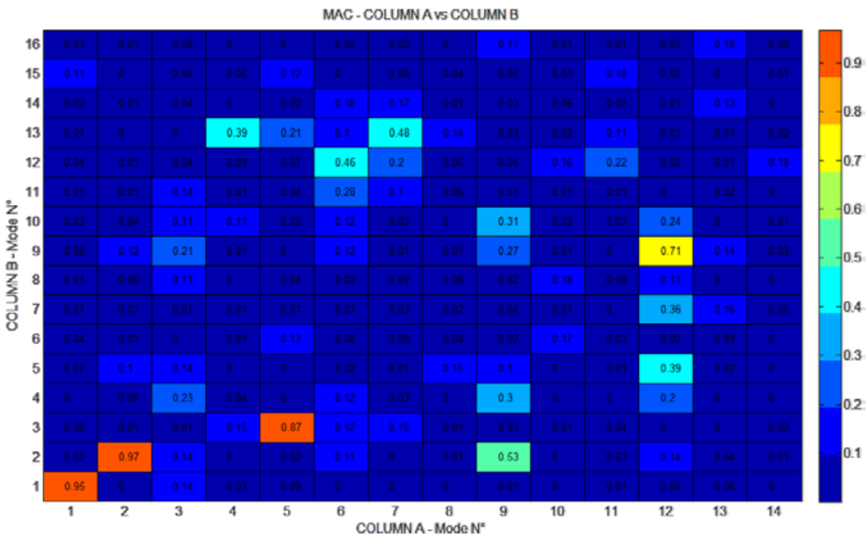

Figure 4 – *MAC* values for column A and B modes.

Then, a wide band analysis is carried out, by supposing four different tool-working ranges (corresponding to spindle rotational speeds) and two different milling conditions: roughing and finishing (see Table 3). In addition, different number of cutters are considered mounted on the tool (from 2 up to 8). The FRFs in correspondence of the n_{ch} points of the columns has been calculated as described, after being integrated in the frequency domain, in order to obtain the receptance values; naturally, the RMS value has been computed inside each frequency band defined as $\Delta f_k = f_{h_k} - f_{l_k}$. The number of cutters obviously influences the final limits band definition (each band limits of Table 3 are multiplied for the number of cutters).
The results are reported in Table 4. For each band and for each number of cutters, the RMS values of the modally estimated displacements are reported in terms of percentage difference between column B and A.
For easy interpretation, color-coding is used in Table 4. Positive values are highlighted in yellow, meanwhile negative ones (which mean vibration reduction for column B) in green.
It is rather easy to see that the lower levels of RMS vibrations are obtained by column B in the majority of the frequency bands, for both roughing and finishing. Therefore, the use of the column made of panels of sandwich metal foam in mass production machines will guarantee, in general, better quality of milled surfaces.

Table 3 – Milling band definition

		Spindle rotational speed range [rpm]	
		Lower limit (f_l)	***Higher limit (f_h)***
Roughing milling condition	Band #1	800 (13.3 Hz)	1100 (18.3 Hz)
	Band #2	1100 (18.3 Hz)	1400 (23.3 Hz)
	Band #3	1400 (23.3 Hz)	1700 (28.3 Hz)
	Band #4	1700 (28.3 Hz)	2000 (33.3 Hz)
Finishing milling condition	Band #1	2000 (33.3 Hz)	4500 (75.5 Hz)
	Band #2	4500 (75.5 Hz)	7000 (116.7 Hz)
	Band #3	7000 (116.7 Hz)	9500 (158.3 Hz)
	Band #4	9500 (158.3 Hz)	12000 (2000 Hz)

4. CONCLUSIONS

An accurate analysis of the performance in milling operations, using the machine column made of panels of metal foam sandwiches compared to the traditional manufacturing in carpentry (currently used in mass production), is presented in this paper. The comparison is made by means of a modal characterization of the two columns and the identification of an analytical model. A first interesting result is the linearity of the response of the column made of metal foam sandwiches, which not obvious a priori.

Since the two columns are comparable only for the geometric dimensions (they are interchangeable on the machine), but differ in terms of construction, a comparison based simply on modal damping does not have much meaning. The modal analysis has shown that few modes are directly comparable, but they have very different frequencies, so that the comparison of the damping is not significant and does not provide any clear conclusion.

The method proposed by the authors is based, instead, on the calculation of the vibration response of the models of the two columns in the frequency range of machine operation and on the calculation of the RMS value of the overall vibration, deemed as the influential parameter on the machined part finishing. By means of this method, it was possible to demonstrate that the column made of metal foam sandwiches has generally less vibration under the same wide-frequency excitation. Thus, it can ensure best finishing, once installed in the machine tool considered.

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COLUMN A vs COLUMN B - ROUGH MILLING (800-2000 rpm)

RMS percentage difference of B wirh respect to A [%]

	2 cutters				3 cutters				4 cutters				5 cutters				6 cutters				7 cutters				8 cutters			
Band	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
Ch. #1	29,03	27,32	11,86	-3,61	22,91	-6,27	-75,48	-69,20	-10,98	-82,44	168,56	679,42	-80,02	75,01	336,91	-49,90	-45,10	835,22	-45,17	-10,55	146,47	-12,50	-6,68	-63,66	688,28	-51,33	-60,00	-66,64
Ch. #2	218,54	193,81	85,74	13,81	133,54	15,04	-75,60	-78,18	12,34	-85,49	125,79	668,48	-82,11	55,33	362,85	-31,98	-54,89	799,83	-25,20	36,02	118,31	15,75	38,49	-57,30	663,68	-33,99	-50,40	-78,44
Ch. #3	34,75	50,01	1,42	-16,48	30,03	-20,55	-78,42	-74,46	-24,64	-85,54	126,08	520,32	-82,75	54,91	255,46	-44,80	-51,59	631,87	-39,89	-9,80	114,12	-11,78	-10,14	-46,96	536,71	-44,76	-37,49	-84,85
Ch. #4	21,64	5,65	15,66	-22,85	11,33	-15,68	-75,12	-74,46	-12,84	-84,81	140,98	633,40	-80,96	59,08	304,70	-47,75	-50,98	785,70	-42,52	-2,39	123,33	-12,75	1,38	-34,47	657,31	-48,48	-30,28	-66,09
Ch. #5	92,84	135,64	72,81	9,11	105,01	6,56	-78,31	-25,13	0,15	-75,47	386,71	288,71	-75,10	217,42	169,68	28,22	0,33	367,84	36,13	58,34	302,48	61,09	66,85	-93,22	388,89	37,04	-91,33	-53,67
Ch. #6	138,63	84,84	49,86	-10,18	60,39	-7,29	-79,93	-35,74	-10,19	-77,94	282,32	239,07	-77,41	154,59	130,71	-32,20	-11,86	286,47	-26,10	-4,70	219,84	4,24	-3,91	-91,13	291,20	-25,11	-90,16	-87,67
Ch. #7	14,68	36,43	33,62	13,21	44,50	13,52	-77,47	-36,51	4,74	-77,07	328,83	289,04	-74,33	173,46	152,48	-2,99	-12,08	341,69	1,20	-5,30	254,84	30,36	-2,01	-76,93	345,73	-3,15	-74,98	-81,95
Ch. #8	60,16	107,01	121,75	34,12	117,60	34,06	-75,51	-34,47	28,58	-75,63	295,92	134,39	-72,72	169,42	63,62	-19,82	-16,04	172,11	-14,75	7,16	227,23	2,80	6,28	5,16	193,62	-14,45	20,04	-60,56
Ch. #9	-8,10	30,64	-24,08	-37,85	-2,26	-36,73	-81,97	-29,03	-40,00	-78,23	356,84	278,10	-79,10	203,62	157,31	9,35	-1,48	346,27	15,17	36,44	282,72	40,87	28,12	-85,63	365,71	7,41	-83,19	-36,83
Ch. #10	-18,63	6,54	-26,50	-23,62	-9,71	-24,92	-80,74	-34,90	-28,94	-78,15	300,74	189,72	-77,86	173,80	97,16	-19,68	-10,30	246,07	-15,85	-42,57	238,83	10,01	-37,24	-88,87	265,75	-27,73	-87,15	-90,00
Ch. #11	26,83	26,62	-8,64	-36,68	11,03	-33,94	-80,70	-33,72	-32,29	-77,49	306,15	169,28	-77,09	171,18	84,38	-16,82	-12,10	213,90	-13,89	-14,51	240,05	3,62	-18,82	-81,32	242,77	-22,92	-78,40	-88,28
Ch. #12	103,87	69,14	18,30	5,38	44,34	-2,06	-79,64	-32,99	-8,34	-77,40	300,67	165,59	-76,72	163,75	90,63	4,03	-7,68	213,33	6,62	-3,11	229,41	30,74	-4,40	12,04	249,66	-6,55	13,25	-58,18
Ch. #13	13,36	3,46	-9,52	-28,11	1,14	-31,77	-84,90	-83,34	-35,05	-90,30	49,02	172,89	-88,86	4,02	17,44	-93,33	-67,14	213,19	-92,67	-21,45	40,83	-88,57	-29,95	-93,95	145,12	-93,11	-93,04	-82,88
Ch. #14	-5,32	2,83	-36,61	-48,42	-18,13	-52,70	-85,98	-82,08	-54,64	-90,20	49,21	172,81	-89,43	4,22	19,73	-92,13	-65,04	216,64	-91,32	-26,08	40,27	-86,89	-26,26	-95,95	155,68	-91,91	-95,47	-87,27
Ch. #15	-61,24	-71,32	-71,70	-64,49	-69,78	-65,85	-83,99	-79,56	-64,34	-89,26	87,80	174,29	-87,66	24,98	17,96	-92,14	-61,12	224,49	-91,43	-38,37	71,59	-87,06	-41,44	-96,18	120,49	-91,33	-95,77	-83,63
Ch. #16	25,74	58,54	44,63	-13,79	51,85	-10,61	-75,70	-82,36	-8,62	-87,71	46,60	170,70	-84,28	4,51	23,85	-91,47	-65,35	211,33	-90,69	-43,63	40,58	-85,95	-41,04	-94,01	128,93	-90,92	-93,23	-84,95
CFR TOT	34,01	-61,19	-69,15	-77,24	-65,11	-77,36	-94,46	-88,56	-78,17	-95,13	-23,34	8,01	-94,74	-49,79	-43,76	-93,91	-81,29	26,79	-93,34	-76,34	-33,71	-90,21	-76,30	-97,19	7,33	-93,71	-96,77	-94,97

COLUMN A vs COLUMN B - FINISHING MILLING (2000-12000 rpm)

RMS percentage difference of B wirh respect to A [%]

	2 cutters				3 cutters				4 cutters				5 cutters				6 cutters				7 cutters				8 cutters			
Band	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Ch. #1	-9,08	-57,97	151,16	-32,06	289,14	108,55	-10,40	-4,26	58,85	-17,56	53,33	-35,34	57,75	33,92	35,28	1,48	63,29	34,58	-13,80	38,16	72,76	24,72	7,07	40,03	115,58	30,38	38,54	14,72
Ch. #2	-10,75	-47,38	72,90	6,65	253,56	63,22	27,86	-43,18	32,39	25,94	-1,73	-38,34	37,39	-4,95	26,38	18,35	42,82	-7,01	2,28	20,50	44,29	-5,45	18,28	47,98	90,96	34,31	30,72	43,48
Ch. #3	-15,23	-46,66	40,11	-13,89	237,24	26,17	66,24	14,18	23,58	45,33	14,01	-21,67	19,59	32,06	-12,47	-29,41	37,25	8,18	9,51	-25,33	57,61	-55,06	-23,61	39,28	53,14	-14,35	-12,77	22,77
Ch. #4	-12,57	-45,12	112,04	-14,53	282,00	52,28	52,06	4,16	57,23	34,80	6,61	-8,44	39,65	25,29	12,93	-15,46	68,72	3,43	3,00	-34,22	79,64	-40,41	-27,50	40,80	39,50	1,89	-11,46	40,69
Ch. #5	2,58	-77,63	3,83	-69,89	266,25	-33,09	-38,00	-39,95	-58,15	-39,21	27,52	-52,71	-74,44	-16,43	87,72	-42,95	-54,76	19,30	-50,00	-10,54	-34,63	20,14	-33,20	108,23	-40,28	59,07	5,51	23,85
Ch. #6	-8,53	-83,48	-11,76	-89,20	155,19	-22,07	-77,69	-73,65	-39,48	-74,39	-72,91	-75,80	-60,58	-69,65	-62,16	-65,81	-59,22	-72,87	-66,73	-67,66	-55,85	-79,90	-66,03	-74,28	-58,13	-61,43	-69,23	-75,76
Ch. #7	-6,78	-59,76	50,79	-62,60	266,90	43,16	-37,83	-13,18	27,75	-31,51	22,79	-21,34	-5,42	0,16	79,19	-10,85	0,08	17,77	1,93	43,03	11,06	-3,81	7,23	24,67	23,82	63,08	53,60	23,84
Ch. #8	-12,00	0,59	84,58	-34,08	313,86	29,98	19,13	-60,23	68,29	0,65	-56,37	89,46	1,10	-33,65	15,85	7,53	24,37	-48,95	34,08	43,67	8,26	-41,07	27,11	11,15	-19,59	14,10	43,78	7,17
Ch. #9	2,05	-69,12	-27,27	-60,32	259,71	-44,11	-27,18	-36,88	-52,81	-29,26	20,43	-50,56	-72,13	-11,11	87,41	-52,21	-48,63	13,28	-49,64	-9,01	-27,56	17,25	-42,19	222,89	-34,42	59,25	19,76	4,96
Ch. #10	-7,47	-76,80	-18,49	-87,02	195,31	-27,18	-80,88	-79,99	-37,09	-76,89	-80,33	-72,34	-63,50	-72,76	-67,48	-57,09	-62,11	-78,71	-61,46	-71,39	-62,02	-81,05	-59,49	-76,44	-63,60	-60,71	-72,67	-58,46
Ch. #11	-10,43	-65,91	29,93	-58,60	239,84	18,01	-37,18	-19,57	12,07	-34,17	7,69	-34,19	-11,66	-1,40	48,90	-10,85	-6,87	2,14	11,91	8,86	0,83	-15,94	3,37	27,02	19,09	37,14	28,25	16,14
Ch. #12	-5,73	1,73	115,27	-24,14	368,08	55,86	51,85	-34,19	92,71	25,98	-30,81	82,85	37,47	-8,99	33,40	46,02	61,08	-27,05	95,44	37,45	46,20	-25,03	48,10	6,49	15,36	44,03	36,53	-8,21
Ch. #13	-46,10	-92,95	26,21	-89,86	-28,82	-34,73	-88,91	-78,48	-76,28	-87,21	-69,86	-80,43	-63,09	-86,72	-60,03	-67,39	-70,79	-75,87	-68,06	-66,31	-70,55	-67,03	-69,35	-40,69	-71,12	-61,51	-53,99	10,22
Ch. #14	-43,02	-94,04	-10,17	-91,07	-38,80	-37,76	-87,79	-78,16	-78,59	-86,20	-69,42	-81,26	-71,30	-85,18	-63,19	-62,28	-74,71	-75,58	-65,47	-74,39	-67,81	-68,88	-70,06	-38,69	-68,11	-61,23	-59,82	-31,19
Ch. #15	-38,42	-94,09	-35,85	-65,38	-42,65	-59,94	-59,67	-34,56	-77,98	-64,27	-14,35	-15,46	-75,68	-45,12	19,56	-24,50	-65,38	-24,29	-9,15	26,02	-49,42	-5,11	-17,67	28,10	-49,56	14,09	28,00	56,42
Ch. #16	-41,04	-91,62	-26,06	-75,29	-26,33	-53,55	-66,97	-19,56	-67,88	-68,30	-28,83	-29,88	-70,07	-45,96	-15,16	-14,45	-60,25	-42,26	-1,57	14,33	-52,59	-28,78	-12,32	35,06	-53,04	-17,43	33,52	-1,80
CFR TOT	-17,29	-95,60	-67,93	-92,02	-52,40	-76,23	-89,23	-85,43	-85,77	-89,12	-81,24	-81,96	-86,26	-86,00	-72,53	-82,07	-85,52	-82,92	-78,89	-77,27	-83,23	-81,79	-81,15	-67,68	-83,64	-74,29	-75,00	-73,60

Table 4 – Comparison of the RMS values of the modally estimated displacements.