

Instructional experiment on Lenz's law

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Abstract

We present a visual manifestation of Lenz's Law. The setup is composed of two coils connected by a central linear metal nucleus. Each of them has a LED plugged in parallel to its terminals. The primary is fed with a pulsating external voltage. We recorded a slow-motion video (960Hz) of the two blinking LEDs. It is possible to observe a complete out-of-phase fade-in/fade-out behavior of them, which is a direct consequence of the minus sign present in the electromagnetic induction equation. We also discuss how to introduce the apparatus in a classroom activity. The mathematical modeling and the numerical results are instructively derived along with the text. There is also a review of the technical features that must be taken into account to study the observation of high-frequency blinking LEDs.

Keywords: Electromagnetic Induction, Lenz Law, LEDs, Classroom Activity.

1. Introduction

Faraday's law of induction, together with three others, constitute the Maxwell equations, that can explain very well the physical situations involving electromagnetic phenomena. According to Faraday's observations, an electric field can be induced in a circuit if, for example, a permanent magnet is thrust into or out of the circuit. That causes the emergence of an electromotive force ε given by

$$\varepsilon = - \frac{d\phi}{dt} \quad (1)$$

in SI units, so that the induced electromotive force in the circuit is proportional to the time rate of change of magnetic flux ϕ linking the circuit [1]. The minus sign is Lenz's contribution, which states that the induced electric current is in such a direction as to oppose the flux variation through the circuit.

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Most of the devices containing an electric engine we use in everyday life are ruled by the physical principle of electromagnetic induction. Which explains why there are in the literature many simple experiments exploring this phenomenon. The Thomson experiment is one of them [2, 3], and it has been modified for different ring materials [4], and also considering differences in dimension and temperature of the ring [5]. However, when we try to explore Lenz's contribution, in the same way, we face a lack of easy-making instructional demonstrations. One example of an experiment showing Lenz's law is presented in [6]. In this experiment, it is necessary to take the data from the coupled coils system to make some plots. Although the approach is interesting, there is no visual procedure to enable the understanding of the students. In an attempt to solve this situation, we developed a educational device called EXPINEL, which is an acronym for Experimento de Indução Eletromagnética, in a free Portuguese translation.

LEDs (Light Emitting Diodes) play a central role in the EXPINEL apparatus. The fact that they can only light up when a minimum potential is applied in the correct direction is deeply explored. We can make three main experiments using the device. The first proves that the LEDs are polarized devices. The second experiment shows how the coils transform positive electric current in an alternating one. In this experiment, the electromagnetic induction can be showed using the two coils. The third experiment is the most relevant. We connect two identical LEDs with the same polarization direction in the two coils and using a slow-motion camera we observe that they blink alternately. The first and second experiments prepare the concepts necessary to understand the third one. Once the students understand the polarization of the LEDs and the electromagnetic induction due to the coils, it is possible to confirm that the alternating in the blinking is Lenz's contribution to Faraday's law.

Lenz's law is one of the most important observations in electromagnetic phenomena. However, it can remain underestimated due to the difficulty of explaining the equation that describes this law in a visual experimental form. EXPINEL can solve this issue by showing Lenz's law clearly and intuitively. Besides, many important concepts in electromagnetism can be explored using it, providing a rich and scholar knowledge building.

In this work we describe the entire EXPINEL apparatus and the Physics behind the experiments. In sections 2 and 3, we specify all the EXPINEL components as well as their possible configurations. A mathematical description of EXPINEL experiments is done in section 4. In section 5, we described some remarks in the utilization of the device, as the visualization of the LED's flickering. We conclude the discussions in section 6 and summarize some possible classroom application of the device in the Appendix.

2. Description of the Device

The entire EXPINEL device can be seen in Fig. 1 and each part of this figure is enumerated and will be described below:

1. The cover of the box in MDF (Medium Density Fiberboard) which contains the main circuit;
2. The box in MDF containing the main circuit (which will be detailed separately later). Some holes were made in the sides of the box for airflow, to help in the cooling process;
3. AC input (127V), where it was used nearly 50cm of 2.5mm diameter wire and a simple power plug;
4. A circle-shaped bent metal wire. This item is meant only to simplify the right-hand rule

demonstration in a spiral, related to electromagnetic induction.

5. Two coils inside two MDF boxes. The coils specifications are: electrical resistance 12Ω , self-inductance 35mH and maximum electrical current supported 1A ;

6. A 30cm metal bar used for intensifying the electromagnetic induction (check Section 4);

7. Four P2 output pins, linked to thin flexible 1mm diameter wire. Two 5mm high brightness LEDs connected in series with two $15\text{k}\Omega$ resistors;

8. Two MDF boxes, each one containing a P2 input and output pin connected by the 1mm diameter wire;

9. Two P2 connectors, each one composed of two input ports and one output pin;

10. Another two P2 connectors, each one made of 1mm diameter wire and two P2 output pins;

11. Two 5mm high brightness LEDs are inversely connected, both linked to a $2,5\text{k}\Omega$ resistors by the 1mm diameter wire and two P2 output pins.

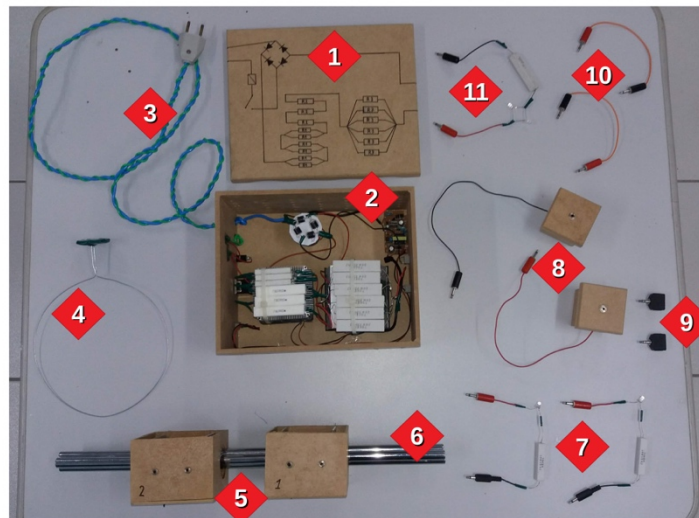


Figure 1. Outside view of the device detailed.

The alternating electric current can be quite useful to study the LEDs' polarization. It would just be necessary to obtain from the original external power source a low rectified electric current or at least one that was just "positive" or just "negative", not alternating. Then, for decreasing the electric current we have used a resistors composition combined with two heat-sink coolers to low the temperature of the resistors. This part of the device can be used to easily explain the resistor's operation and how they can decrease the electric current at the same time they get hotter in consequence of the Joule's effect. Proceeding, the decreased electric current gets in the diode bridge rectifier, which is an arrangement made of four diodes that transform the alternating current into an only positive Pulsed Direct Current (PDC). So, having the electric current in this correct way, there are different schemes for connecting the coils and the LEDs.

Now we have the main circuit in evidence in Fig. 2, which is inside the MDF box

indicated by number two in the Fig. 1. In the same way as the previous image, it will be described each number in the Fig. 2:

1. AC input (127V);
2. Fuse holder and fuse - the fuse specifications will be according to the maximum circuit current. In this case, its value should vary around 1A;
3. Power switch button (type "On-off");
4. Pilot light (connected to the 12V power source);
5. Resistors ($\sim 20W$): 3 ones with 15Ω , 6 ones with 22Ω , and 6 ones with 330Ω (the connection configuration will be described later);
6. Two heat-sink coolers;
7. Diode bridge rectifier;
8. A 12V power source for the coolers to work;
9. Two P2 input ports.

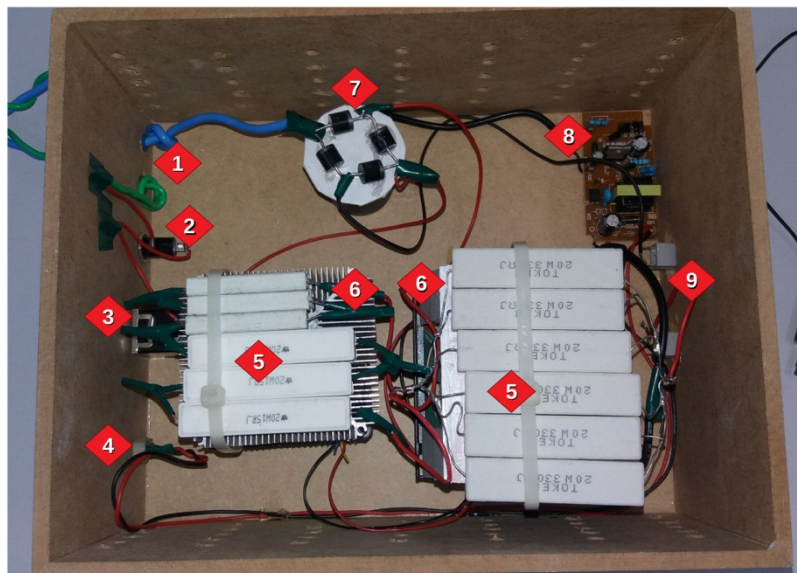


Figure 2. Inside view of the device detailed.

It is worth noticing that all the components used can be replaced as long as the dimension of the circuits is maintained. By dimensioning the circuit we mean to use the appropriate materials according to the voltage and electric current specifications. For example, for the high values of electric current from the power source, it is mandatory to use thicker wires and high-power electrical resistors. But the coils can be replaced with no loss in the intended effect of the experiment. Besides, the metal bar (item 6) can be neglected or exchanged. However, it improves the magnetic field effect, so it is advisable to use it.

It is necessary to emphasize that this setup is just one of all possible that could have been done to decrease the electric current intensity. In our case, it has been made with the resistors that were available in the laboratory and these can be replaced once the equivalent resistance is around 133Ω . That way, the decreased electric current would be less than 1A. It

is also advisable to use high-power resistors. Section IV explains the functioning of the coils and how the electromagnetic induction effect occurs. Another way for decreasing the current could be the insertion of a transformer, instead of high-power resistors. In that case, we would lose a noticeable Joule effect. Although some energy is lost through this effect, they were put in EXPINEL to be used as a pedagogical tool. The two big coolers working on these resistors show to the students that the thermal effect could become relevant enough that the electric energy would all be dissipated in form of heat, if they were not there.

The Figures 3, 4 and 5 show other different perspectives of our device.

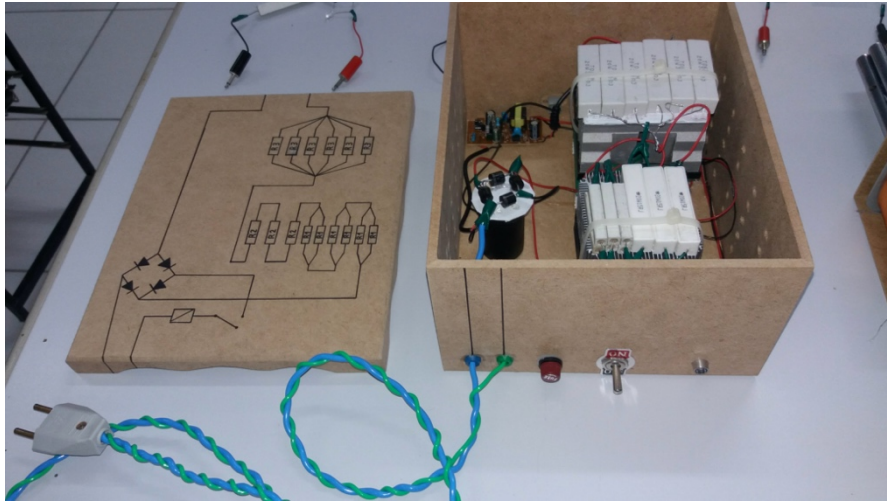


Figure 3. Front side view of the device. The box cover can be used to draw the circuit inside the box.

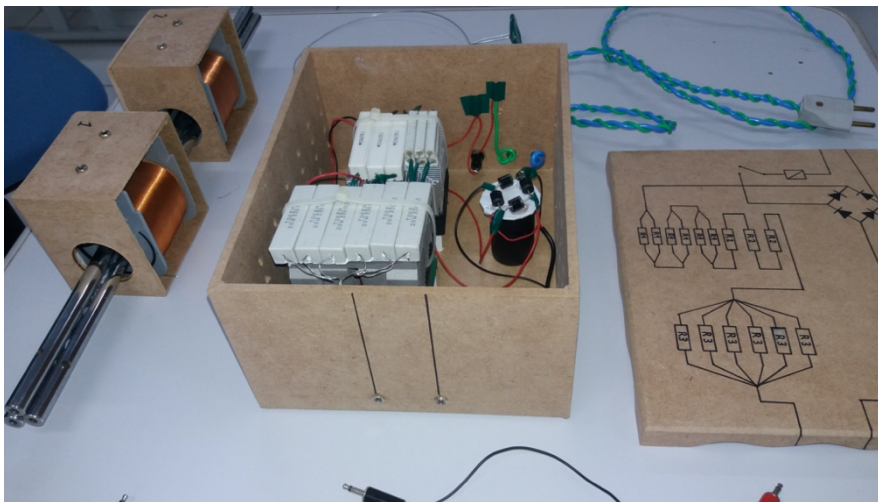


Figure 4. Back side view of the device.

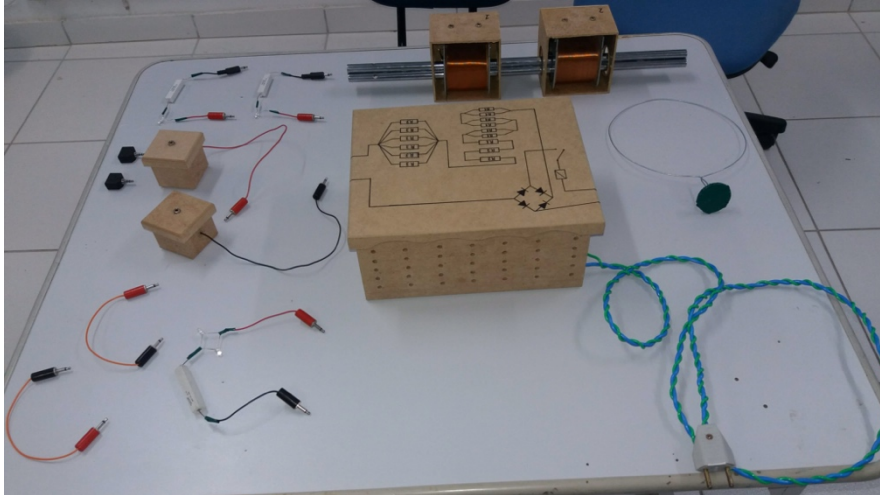


Figure 5. Back side view of the device.

3. Operation and Functioning

The EXPINEL has been projected to be an easy using device in the classroom, where the students could interact with and observe each experiment step. In this section, the EXPINEL operation will be detailed as well as its possible configurations.

A. The main circuit

The main circuit can be divided in two parts: the rectifier source and the high power resistors. Looking to the Fig. 6 from left to right, after the fuse protection step, the next component is the set of diodes, considering the switch on. This is the simplest rectifier circuit since we haven't used a resistor between the two central diodes. We were interested just in building a positive direct current source. Moreover, it should pulse in time (characterizing a positive PDC), since we meant to work with blinking LEDs (polarized components) as the final test charge at the experiment.¹ We shall consider this procedure the source part of the product. In fact, it is responsible to turn the negative values of the alternate external source into positive ones: it is analogous to apply the modulus mathematical operator to the oscillating input of the circuit.

¹ We could have used just one diode to archive this criterion, but in this case, during half of the time of the complete circuit period, there would be no current. This would bring some difficulties to interpret the LED behavior we are interested in.

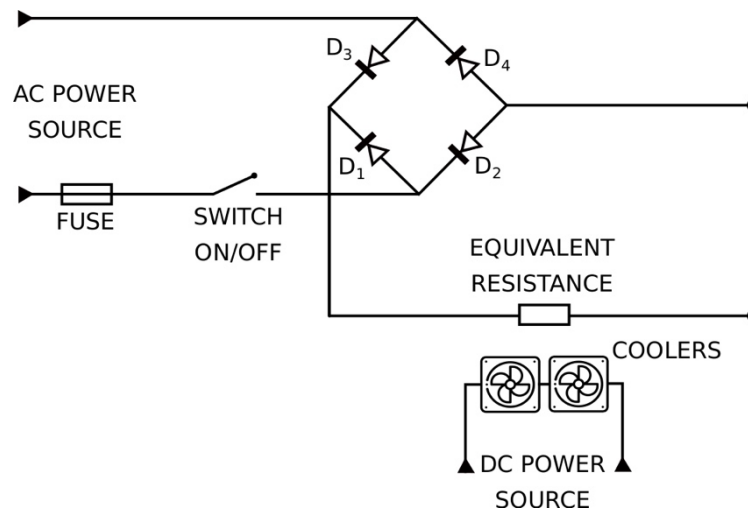


Figure 6. The main circuit diagram with the components showed in Fig. 2. The DC power is a 12V external power supply to the coolers and the pilot light.

This set of resistors needs to cool down because of the heating caused by Joule's Effect. Then we used two heat-sinks attached to two coolers that are powered by a 12V source. We point out that this resistors' composition was set to provide a high enough current in order to make the induction effect sensible along the length of the metal we used. On the other hand, this arrangement should be low enough to not overheat the primary coil. For that, we used 6 resistors R1 in parallel in pairs and each pair in series, 3 resistors R2 in series, and 6 resistors R3 in parallel; see Fig. 7.

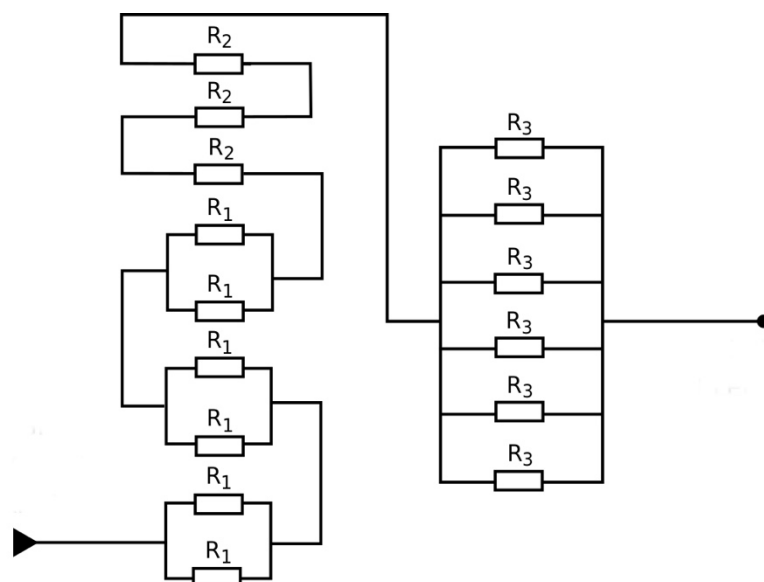


Figure 7. The resistor association. The resistors can be replaced since the equivalent resistance is around 130Ω .

B. The experiment part I:

Once the electric current is only positive in the outputs of the main circuit, the first part of the experiment can be performed. Using the stations of input (the little boxes, item 8 in Fig. 1), we plug the two 5mm inversely connected LEDs (item 11 in Fig. 1). It is expected to be observed that only one LED lights up in this configuration (outlined in Fig. 8), showing that in fact, the electric current is flowing only in one direction since the LEDs are connected with inverse polarization. The video available in [7] shows the effect above discussed between 1:05s and 1:18s.

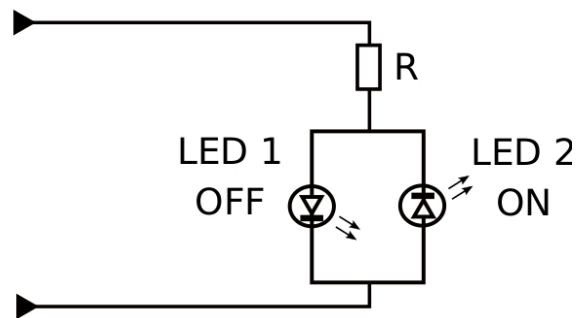


Figure 8. The configuration of the experiment part I. The connected LEDs are the item 11 in Fig. 1.

C. The experiment part II:

In the next demonstration, the wires from the only positive PDC are connected to one of the coils, and the two 5 mm inversely connected LEDs (item 11 on Fig. 1) are connected to the other coil. Besides, the two coils are close to each other and the metal bar (item 6 on Fig. 1) is placed inside both the coils, this configuration is outlined in Fig. 9. Now, it is expected to observe that both LEDs light up due to the electromagnetic induction effect “produced” by the coils. It is important to notice that the current getting out from the main circuit needs to be only positive to show that the coils produce the alternating current in fact. It is more instructive than simply powering the device in an alternating current source. In this step, it can be interesting to use a smartphone camera in the slow-motion function. Although the LEDs seem to be on, they are blinking very quickly in a frequency superior to that the human eye can capture.

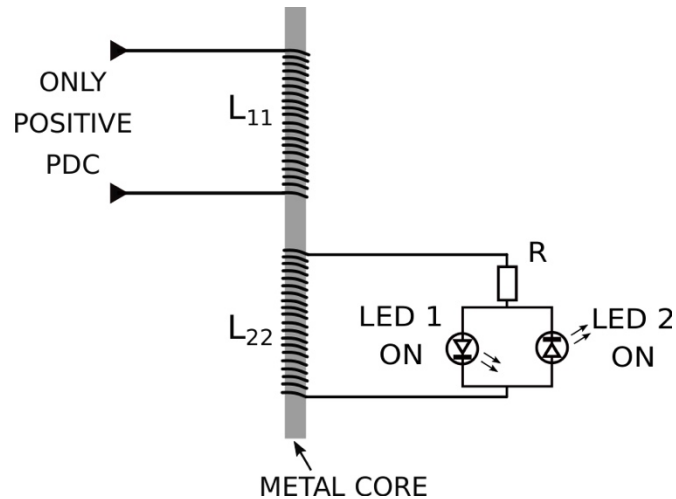


Figure 9. The configuration of the experiment part II. The same item is used in part I, but here it is connected to the coil L_{22} . Note that it is not connected to the main circuit. The LEDs turn on due to electromagnetic induction.

D. The experiment part III:

At this part of the experiment, outlined in Fig. 10, the procedure is identical to the previous one. But now we connect the two single LEDs (item 7 on Fig. 1) in both coils. Using again the slow-motion function to film the LEDs, it will be observed that both LEDs blink alternately.

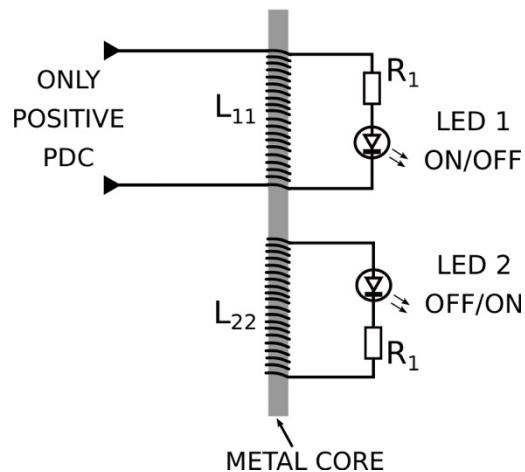


Figure 10. The configuration of the experiment part III. The metal core improves the electromagnetic induction effect.

This point is especially important in the experiment because it captures the essence of Lenz's law. It is worth noticing that the coils must have the same orientation, which means both have the identical winding direction, as in Fig. 11(a). This setup is shown in Fig. 11(b), with the LEDs having the same polarization, which leads to an alternating blinking.

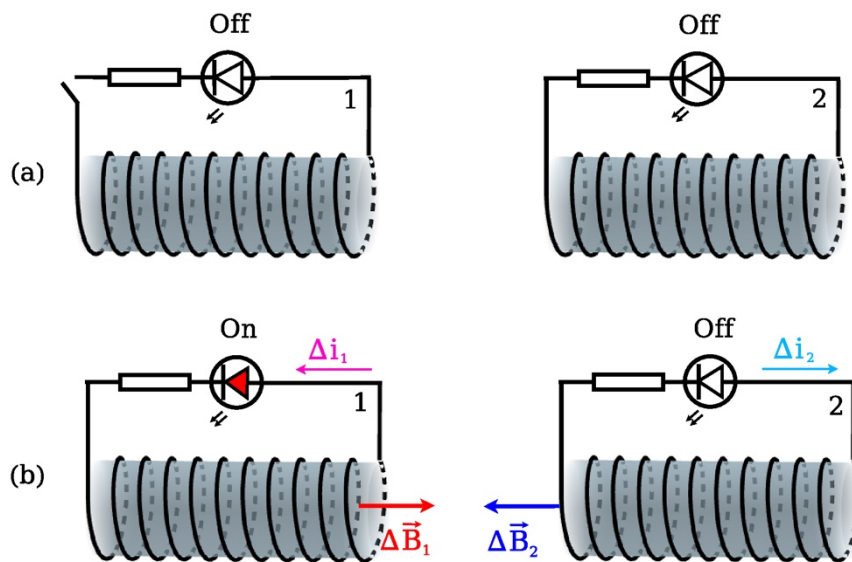


Figure 11. Schematic description of the coils in Fig. 10 with both having the same wrapping wire orientation. (a) Open circuit. (b) LEDs with identical polarization direction.

However, other configurations are feasible changing the LEDs' polarization or the winding direction on the coils, as in Fig. 12(a). Some of these configurations can make the LEDs blink simultaneously, holding yet Lenz's law. Therefore, forecasting the LEDs' state depending on the setup can be an activity to be proposed for the students, and it is detailed in the appendix.

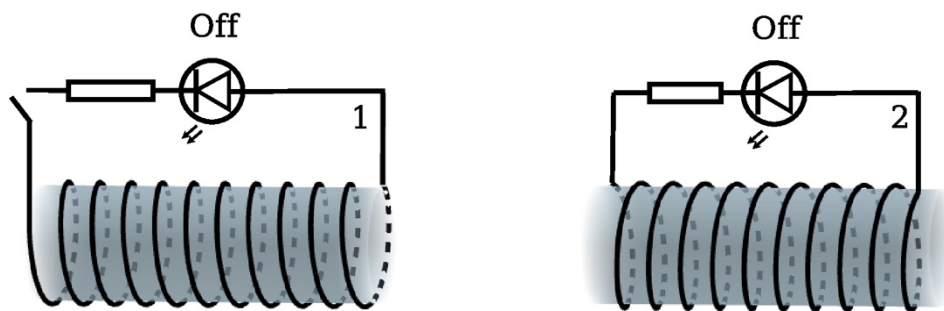


Figure 12. Schematic description of the coils in Fig. 10 with both having inverted wrapping wire orientation.

4. Mathematical Modeling and Estimations

The idea of this section is to make a review on the proper functioning of a transformer and apply it to describe basic features of our device. Our mathematical model is intended to explain in a pedestrian way the basic behavior of the circuits involved. We point out, though, that this is a first approximation to the actual EXPINEL description. As explained in Section III,

we have used a bridge rectifier which provides a pulsating DC source. To avoid analytical obstacle imposed by modulus functions and the corresponding derivatives, we proceed the further calculations with smooth trigonometric functions. Moreover, we only expect to find superior bounds to the corresponding currents and tensions of interest. This restriction is related to the LEDs functioning - they are truly non-ohmic devices, with varying resistance. Our description coincides to the case when we are, at least, providing the minimum tension required to light it up. In this case, the LED resistance tends to zero, allowing the current to pass. This scenario yields estimated values to be compared with measured ones.

We assume that the circuits are fed by an electromotive power source given by

$$\varepsilon(t) = \varepsilon_0 \cos(\omega t) \quad (2)$$

ε_0 gives the maximum value the electromotive force can achieve and ω is the angular frequency. The source is connected to a primary coil with self-inductance given by L_{11} . The resistance of the first circuit is resumed in one resistor R_0 . The secondary coil, with no electrical contact to the former, is characterized by a self-inductance L_{22} and resistance R . As in our experiments, the coils can move and the closer they are, the greater is the mutual inductance. We name it L_{12} (L_{21}) for the net effect the first (second) feels due to the second (first). The Fig. 13 sketches the basic configuration we would like to describe. We emphasize that $\varepsilon(t)$ is the PDC and R_0 stands for the equivalent resistance in Fig. 6. We set the ammeters represented by the arrows that indicate operationally how to plug them into the circuit.

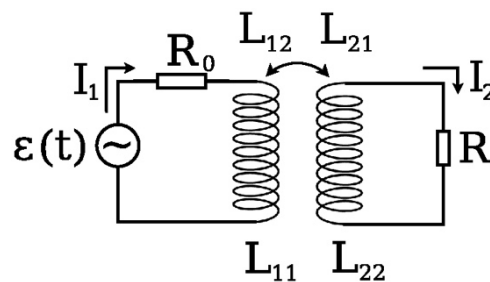


Figure 13. A transformer basic structure, describing the experiment part II in Figures 6 and 9.

Applying the expression (1) to both parts of our setup gives,

$$-\varepsilon(t) + R_0 I_1(t) = -L_{11} \frac{dI_1}{dt} - L_{12} \frac{dI_2}{dt} \quad (3)$$

$$R I_2(t) = -L_{21} \frac{dI_1}{dt} - L_{22} \frac{dI_2}{dt} \quad (4)$$

We are interested in the stationary solutions. Since we have an external source with a fixed frequency, it is reasonable to expect that, for $t \rightarrow \infty$, all currents oscillate with this very frequency, in analogy to mechanical forced oscillations. In this case, all the time dependence can be resumed in a complex exponential: $e^{i\omega t}$, as long as we remember to project the solution on the real space once all the desired calculations have been performed. Using this trick of complexifying the variables, any time derivative may be substituted by a multiplication according to

$$\frac{d}{dt}(e^{i\omega t}) = i\omega e^{i\omega t} \Rightarrow \frac{d}{dt} \equiv i\omega. \quad (5)$$

The “complexified” functions will be hatted. With this notation, together with (5), the initial system of coupled differential equations (3) - (4) is transformed in a linear system in the \hat{I}_1 and \hat{I}_2 unknowns,

$$(R_0 + i\omega L_{11})\hat{I}_1 + i\omega L_{12}\hat{I}_2 = \hat{\varepsilon}, \quad (6)$$

$$i\omega L_{21}\hat{I}_1 + (R_0 + i\omega L_{22})\hat{I}_2 = 0. \quad (7)$$

whose solution is given by

$$\hat{I}_1(t) = \frac{\varepsilon(t)(R_0 + i\omega L_{22})}{R_0 R_0 + i\omega(R_0 L_{11} + R_0 L_{22}) - \omega^2(L_{11} L_{22} - L_{12} L_{21})}, \quad (8)$$

$$\hat{I}_2(t) = - \frac{\varepsilon(t)(i\omega L_{21})}{R_0 R_0 + i\omega(R_0 L_{11} + R_0 L_{22}) - \omega^2(L_{11} L_{22} - L_{12} L_{21})}. \quad (9)$$

Before we move on, let us comment the approximation concerning the inductance that will be adopted henceforth. First of all, once the coils are identical, the two self-inductances are equal, that is, $L_{11} = L_{22}$. Also, one may show that $L_{12} = L_{21}$ [8]. It is possible to infer that the auto and self-inductances are approximately the same according to the following argument. We have that [9]

$$L_{12}^2 = L_{21}^2 = kL_{11}L_{22}, \quad 0 \leq k < 1 \quad (10)$$

in which k is the inductive coupling coefficient. Then, the term $\omega^2(L_{11}L_{22} - L_{12}L_{21})$ in eqs. (8) and (9) can be rewritten as $\omega^2 L_{11}^2(1 - k)$. The electric power in Brazil oscillates with frequency $f = 60\text{Hz}$. In our case, once the source was rectified, it doubles the frequency, as explained in Section 3, that implies $\omega = 2\pi 120\text{s}^{-1}$. Moreover, the auto-inductance is $L_{11} = 35\text{mH}$, which is the nominal value provided by the manufacturer. Considering these values, we have that $\omega^2 L_{11}^2(1 - k) \approx 10^4(1 - k)$. However, once the precision of the multimeter is 10^{-2} this entire term can be neglected if we consider k as $k = 0.999999$. This consideration of the inductive coupling coefficient being not equal but really close to one means that the two coils are in a strong coupling regime, commonly stated for transformers dimensioning. This allows us to use the approximation $L_{12} = L_{21} \approx L_{11} = L_{22}$. It is important noticing that this approximation will provide maximum values of physical quantities (Max), to be compared with experimental ones (Meas). Then, from now on, $\hat{I}_1(t)$ and $\hat{I}_2(t)$ are said to be maximum values of electric currents. Therefore, the eqs. (8) and (9) become

$$\hat{I}_1(t) = \frac{\varepsilon(t)(R+i\omega L_{22})}{R_0 R+i\omega(RL_{11}+R_0 L_{22})} \quad (11)$$

and

$$\hat{I}_2(t) = -\frac{\varepsilon(t)(i\omega L_{21})}{R_0 R+i\omega(RL_{11}+R_0 L_{22})}. \quad (12)$$

There is a list of interesting features that can be explored through these solutions. Let us enumerate them.

1. The mutual inductance L_{21} depends on the geometrical configuration one sets the coils together. In our case, clearly it depends on the axial gap d between our moving coils transformer. L_{21} increases when $d \rightarrow 0$ and, conversely, decreases as $d \rightarrow \infty$. It explains why the LED in experiment part III shines brighter when the coils are close to each other.

2. In the stationary regime, (9) explicitly shows that \hat{I}_2 oscillates with the same frequency ω of the source and its real value is given by

$$I_2(t) = \frac{\omega L_{21} \varepsilon_0}{\sqrt{(R_0 R)^2 + \omega^2 (RL_{11} + R_0 L_{22})^2}} \sin(\omega t - \phi); \phi = \arctg\left(\frac{\omega(RL_{11} + R_0 L_{22})}{R_0 R}\right). \quad (13)$$

Let us estimate, at least in a first approximation, the value obtained above. We use the corresponding root mean square (RMS), since the solution (13) oscillates in time. The nominal values are given by $R_0=133\Omega$, $R=1.53 \text{ k}\Omega$, $\varepsilon_0=\sqrt{2}(127 - 1.4) \text{ V} = 177.62\text{V}$ and $\omega = 754\text{s}^{-1}$. The difference in ε_0 is due to the tension stolen by the diode bridge. We set $L_{11} = L_{22} \approx L_{21}=35\text{mH}$, the nominal value provided by the manufacturer. That way

$$I_{2(Max)}^{RMS} = 15.91 \text{ mA}. \quad (14)$$

This result shows that the apparatus was well dimensioned to prevent the LED damage. We also point out that this result was obtained in first approximation, in accordance with the simplest circuit modeling that has been adopted here. The actual measured value of I_2 is less than the one in (14) for several reasons so exposed. It is given by

$$I_{2(Meas)}^{RMS} = 6.25 \text{ mA}. \quad (15)$$

and was measured by an ammeter Hikari HM-2080. Moreover, the metal core also diminishes the value in (14). Although it increases the inductances one by one, the combination in (13) is decreased, which can be seen directly by inspection.

3. In the previous comment, we have calculated the current that feed the LED. Let us now obtain the maximum tension δV_{Max} available at the secondary coil. Operationally, one plugs a voltmeter in the place of the resistance R . Mathematically, it corresponds to take the limit

$$RI_2^{RMS} = \frac{\omega L_{21} \varepsilon_0}{\sqrt{2(R_0^2 + \omega^2 L_{11}^2)}} = 24.41 \text{ V} \quad (16)$$

since an idealized voltmeter is supposed to have infinity resistance. For the same reasons

used for the previous comment, the experimental value should be lesser than the theoretical one. In fact, the measurement gives

$$\delta V_{Meas} = 12.25 V. \tag{17}$$

Once again, we used the voltmeter Hikari HM-2080, confirming our prediction.

4. This last comment is more related to a general feature of transformers than to our device. It is commonly stated that the iron core in transformers is used to channel the oscillating magnetic field responsible to the induction process. There is another important (and underestimated) application to it. The situation where $R \rightarrow \infty$ represents that the secondary coil is not in use. In this case,

$$\hat{I}_1 \rightarrow \frac{\varepsilon}{R_0 + i\omega L_{11}} \neq 0. \tag{18}$$

In order to avoid energy loss, one is interested in diminishing $|\hat{I}_1|$ in (18). It is not smart to raise R_0 to do so, as we would lose more energy due to Joule effect, for example. Instead, we could adjust L_{11} . In vacuum, $L_{11} \sim \mu_0$. When the spatial region permeated by the magnetic field (whose flux provides L_{11}) is fulfilled by a ferromagnetic material, L_{11} increases once the permeability also increases accordingly. Thus, it is possible to reduce the spurious current $|\hat{I}_1|$, as desired, without touching in the real resistance R_0 . Now we turn our attention to the experiment part III. The watchful reader will note that the primary coil is connected to the resistor R_1 (and the LED), see Fig. 1. This mounting is equivalent a voltage divider as shown

in Fig. 14, where we denoted $\Delta V(t) = \frac{R_1}{R_0 + R_1} \varepsilon(t)$ and $R_P = R_0 R_1 / (R_0 + R_1)$.

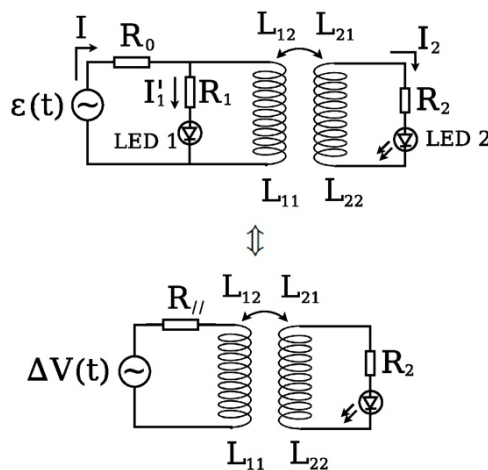


Figure 14. Voltage divider in experiment part III.

The solution for both $I_1(t)$ and $I_2(t)$ can be promptly imported from our previous solution, see (11) and (12).

$$\hat{I}_1(t) = \frac{\Delta V(t)(R_2 + i\omega L_{22})}{R R_2 + i\omega(R_2 L_{11} + R L_{22})}, \tag{19}$$

$$\hat{I}_2(t) = - \frac{\Delta V(t)(i\omega L_{21})}{R_p R_2 + i\omega(R_2 L_{11} + R_p L_{22})}. \quad (20)$$

We are interested though in the current \hat{I}'_1 that lights the LED plugged to the primary coil. The Kirchhoff's law applied to the closed contour connecting the source, R_0 and R_1 , together with the constraint $\hat{I} = \hat{I}_1 + \hat{I}'_1$ provides

$$\hat{I}'_1(t) = \frac{\varepsilon(t) - R_0 \hat{I}_1}{R_0 + R_1} = \frac{i\omega R_2 L_{11} \Delta V(t)}{R_1 [R_p R_2 + i\omega(R_2 L_{11} + R_p L_{22})]}. \quad (21)$$

The evolution of currents in time is governed by the differential equations provided by the induction and Lenz laws. In this case, a comparison among the solutions is mere consequence of these basic principles. A close look to the solutions (20) and (21) between the currents $\hat{I}_2(t)$ and $\hat{I}'_1(t)$ shows that they have a phase factor given by π ,

$$\hat{I}'_1(t) = \frac{R_2 L_{11}}{R_1 L_{21}} e^{i\pi} \hat{I}_2(t). \quad (22)$$

This 'delay' is responsible for the fading in/out described in the Section 5.

5. Observing the Flickering Effect

The observation of the Flickering LED is not a simple task. Here we are dealing with the analysis of the perception that a student with no previous introduction about the measuring light intensity methods has of the phenomena.

If one looks directly at a high-frequency blinking LED, the light seems to be smooth, with no off phases. It is a similar condition we have with the old incandescent bulb lamps, that works at 60Hz. Everyone agrees that the lamp is simple on while it is emitting light. On the other hand, some interesting answers can be reported on an experiment in which a light source with controlled varying emitting frequency is used. Some parameters on the complete feedback of the observer's perception of the phenomena are independent of the person and some don't. The perception of the intensity of the light, its frequency, and even its color (from the same fixed source) can be different, for example, from person to person.

The measurement of the human sensation in this kind of experiment is very well described in the literature. For instance, there is a dependence in the perception of the emitted colors of a flickering white LED with its frequency, which is shown in [11]. Moreover, if the oscillation of the frequency is controlled, it is possible to find other correlations. In [12], the spatial dependence of wave is varied and the results for the perception are different even if the same global frequency is used. It is also influenced by the wavelength of light. This dependence was carefully treated in [13].

More recently, we find works that deal with these influences in some applied technologies. There is a very complete analysis of the absorption frame rate of the human eye with a large set of CCD commercial cameras in [14]. Many parameters are analyzed and the perception of an oscillation light source is described also for the electronic devices. In [15], there is a rich discussion about the visual persistence effect with the cinema projections rooms as a background.

In general, the focus of these works is the measuring of the critical flicker frequency (CFF), which is a quantity which shows the maximum frame rate that the human eye can resolve for a given set of parameters of the source light and the environment. The definition is not fixed and can vary a bit in different areas of application. For a flickering red LED experiment, in which its frequency is gradually increased step-by-step for the same amount, the CFF would be the last value in which the observer noted something different looking to the light. This is not a very exact measurement, since it may depend on the state of the person at the moment of the check. There are also some physiological parameters of a group of individuals that can also influence the results. Nevertheless, is possible to take a lot of these parameters in control and find many coherent answers for this approach. Some equations describe this phenomenon as the Weber-Fechner law, which relates the perception of the light-emitting source with the human perception of it. There is an old interesting paper that shows its theoretical development and the historical motivations, [16]. This law seems to agree with all the experimental data, even dealing with inputs that are, in principle, completely independent.

In our specific case, we are trying to use the simplest method to show the students the lag between the turn-on of the LED on the first coil and the turn-off of the LED on the second. We ask them to look directly at the LEDs and their answer came with no surprise: it is obvious that both of them are completely turned-on emitting continuous light. It is expected since it is working at high frequency. Then, we use a cellphone camera in slow motion ($\sim 120\text{Hz}$) and it is possible to see that the LEDs are blinking. But, the interesting results came when the super-slow motion cellphone camera ($\sim 960\text{Hz}$) is switched on and it is possible to see the fade-in of the first LED while the second one is fading out. This effect can be seen at [7]. It occurs simultaneously and it is a direct consequence of the Lenz law, see Section IV. At this point, on the classroom, the explanation starts to come from the students to their pairs. The teacher at this moment can just watch their discussions.

If one wants to repeat this movie, it is necessary to have some patience in this work. The difficulty is on the cellphone camera perception of the phenomena. As it was discussed above, electronic devices in this kind of experiment can show different feedback to the same light stimulus. Besides, the cellphone is processing in real-time the image as it is doing many other tasks. The lag between the arrival of light at the camera lens and its projection on the screen varies with the latency of the data processing. And the speed of processing this data is clearly not continuous for a cellphone. This varying lag is a source of discrepancies on the perception of the phenomena, but the expected fade in/fade out of the LEDs occur more often than the chaotic sequence that is shown during some periods.

Anyway, the search for the correct moment to perform the video analysis and also the correct configuration of the cameras associate with the more efficient way to film the experiment can be a goal to the teacher try to achieve with their students. Lenz law is now on a controlled situation in which the student has the correct tools to search for it.

6. Conclusion

In this work, we present a visual effect derived directly from Lenz's law. The phenomenon consists of two LEDs blinking on a completely out of phase regime generated by the voltage induction between two coils. We explore the fact that it could not be explained without the minus sign of the electromagnetic induction law.

Due to the advanced level of math modeling for high school students, we leave it to the

discretion of the teacher. From the beginning, we tried to elaborate a demonstration that could be performed on a sequence of steps. At the end of the process, the students would be able to conclude that the unique explanation to the observations is to suppose that the induced magnetic field of the second coil has the opposite direction to the one on the primary.

First, we constructed an educational electric power source to provide a PDC output voltage. Then, we guarantee that the input voltage on our test charges should make the LED light only when it is plugged in the polarization direction. Students can see that LEDs only allow the current to pass in a one-way, and our power source can make it blink with high frequency if its circuit is correctly plugged.

Before we move on, questions of the students about the components inside the MDF box will naturally arise. It is the best moment to talk about the Joule effect on the set of resistors and the cooling system used to cool it down. The configuration of the parallel and series connections on this circuit can also be explained. The dimensioning of electric circuits is another issue to discuss. The current passing through the electric circuit must be such that the resistors will work at a temperature in order to prevent damage. It must be clear to the students that this is one of the possible arrangements of these connections that would allow an approximately one-ampere electric current modulus to pass through the ceramic resistors. It is also possible to insert a puzzling activity challenging the students to imagine other possible setups to the resistors.

The anti-parallel set of LEDs are connected to the power source, and we recapture their polarization characteristics. After knowing the basic working features of the product, it is time to use the two coils. A cell phone camera with super slow-motion function (at least 960Hz of video capture capability) must be used to make a movie of the two LEDs plugged in the same positive chosen direction.² During a considerable time interval, it can be possible to see the fade-in behavior of the first LED while the second one is fading out.

That can be the main point of the class if the students have no doubts about the previous steps. It will turn into a simple experimental confirmation of the theory. It must be clear that the hypothesis of the LEDs blinking in phase would lead to a plus incorrect sign on Lenz's law. Therefore, most importantly, the observable out-of-phase flickering behavior is a direct consequence of the minus sign.

Is it possible to provoke the class to imagine a more stable procedure to film the LED? They can also be challenged to improve the power source circuit to have a better output signal (using some filters, for example). Moreover, if it is possible to construct a didactic teaching sequence adapted to the reality of the school by which the students could build their version of the EXPINEL, the engagement should increase exponentially.

It is relevant to notice that the idea of out-of-phase anti-parallel LEDs blinking can be archived with many other devices. Low-cost multiprocessors (Arduino, PIC, etc.), simple digital circuits, or even simply a very high-power resistor and a piece of enameled wire could display the same effect. However, our device not only shows Lenz's law but processes the information step-by-step. Thus, the students can understand the LEDs' functioning, the role of the coils in the electromagnetic induction, and finally, the opposite direction of the magnetic field. This knowledge-building could promote a more solid learning process than only show the

² This direction can be taken, for example, as the direction of the magnetic field on the bar at the moment that the switch button is turned on.

phenomenon and expect the students to accept it.

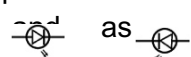

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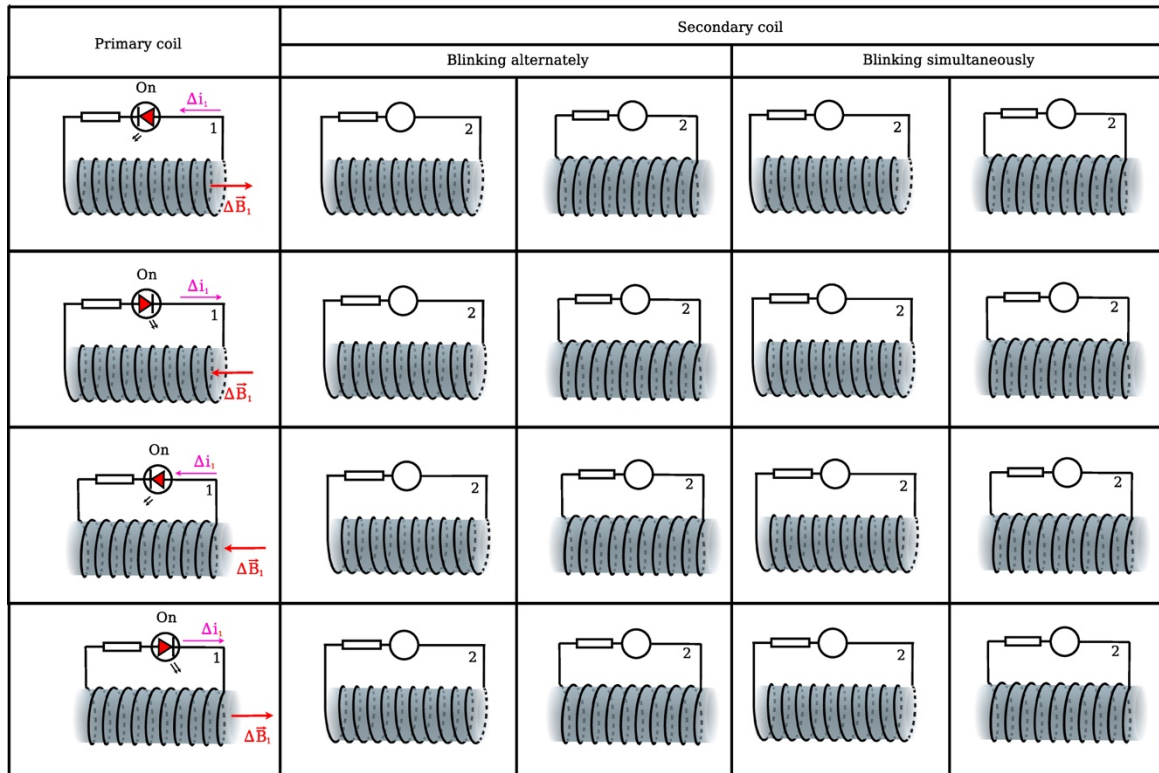
Appendix – Classroom Activity Proposition

This appendix is devoted to providing one possible implementation of our device in the classroom. The application is based on the so-called POE methodology, which stands for Predict - Observe - Explain [17]. In the first stage, the prediction, the students are asked about possible outcomes and the corresponding explanations about an experiment to be shown. The answers are supposed to be written by the apprentices. This step is intended to make the students conscious of their understanding of the science subjects involved [18]. In the second stage, the observation, the teacher exposes the experiment once hypothesized by the class. Finally, comes the explanation step. Not only does the teacher elucidate the physical phenomena involved, but also compares it to the prior answers provided by the students. This procedure induces a clash between the previous explanation and the observations, called cognitive conflict. Solving this friction seems to encourage curiosity and interest in the audience. It is also expected that it may make the previous concepts more flexible, enhancing learning by an investigative approach [19, 20].

Let us now apply the POE for the Expinel. The zeroth step consists of presenting the apparatus to the class. Here the teacher will only perform the experiments one and two proposed in section III, then it is expected to the students to understand concepts such as the polarization of the LED and electromagnetic induction produced by the coils. However, in the third experiment, the teacher is only allowed to introduce the idea without giving too many details. Turning now to the predicting stage, the students are invited to complete the table I representing all the possible configurations for the two coils with the LEDs  as  well as the change of current Δi_2 and the magnetic field $\Delta \vec{B}_2$ induced in the secondary coils.

The explanations are supposed to be written by the students, to be compared with the experimental results. Following up to the observation, the Expinel is set on operation and, finally, the results are compared - explanation stage -with the previous answers, as prescribed by the POE methodology.

Table 1. Table for the students to complete with the LEDs, the change of current and the magnetic field induced in the secondary coils in the predict stage of POE methodology. After the explanation stage the students can analyze their answers in the table.



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