

## ELABORATION AND RESEARCH OF A MODEL OF OPTIMAL PRODUCTION AND DEVELOPMENT OF INDUSTRIAL SYSTEMS TAKING INTO ACCOUNT THE USE OF THE EXTERNAL RESOURCES

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**Abstract.** The problem of optimization of investment projects related to the development of modern production systems is considered. The tasks of managing of operation and development of production systems considering external resources – the synthesis and analysis of optimal credit strategies – are posed and solved. An analysis of analogs – solutions of the variational problem of optimal development, the disadvantage of which is the difficulty of obtaining information about the state of production and the external environment, was carried out. The new solution is based on the resource approach, when external resources are taken into account in the cost of production resources. A generalized model of optimal development is used, in which the planned period of the investment project is divided into intervals. At the beginning of each interval, the optimal development strategy is adjusted taking into account the clarification of information about the future state of the active environment: actions of competitors, consumers, world markets. To determine the optimal amount and optimal distribution of credits between subsystems, the maxima of the criterion – the parameterized function of the system's efficiency – are determined at each interval. A new model has been developed based on the model of optimal development, which takes into account the use of external resources, such as loans. The method of including an external resource in the development function and the production function is considered. Examples of modeling are given.

**Keywords:** optimal aggregation, production function, development function, external resource, simulation modeling

### OPRACOWANIE I BADANIE MODELU OPTYMALNEJ PRODUKCJI I ROZWOJU SYSTEMÓW PRZEMYSŁOWYCH Z UWZGLĘDNIENIEM WYKORZYSTANIA ZASOBÓW ZEWNĘTRZNYCH

**Streszczenie.** Rozpatrywany jest problem optymalizacji przedsięwzięć inwestycyjnych związanych z rozwojem nowoczesnych systemów produkcyjnych. Postawiono i rozwiązano zadania zarządzania funkcjonowaniem i rozwojem systemów produkcyjnych z uwzględnieniem zasobów zewnętrznych – synteza i analiza optymalnych strategii kredytowych. Przeprowadzono analizę analogów – rozwiązań wariacyjnego problemu optymalnego rozwoju, którego wadą jest trudność uzyskania informacji o stanie produkcji i otoczeniu zewnętrznym. Nowe rozwiązanie oparte jest na podejściu zasobowym, kiedy to zasoby zewnętrzne są uwzględniane w kosztach zasobów produkcyjnych. Wykorzystano uogólniony model optymalnego rozwoju, w którym planowany okres realizacji projektu inwestycyjnego podzielono na przedziały. Na początku każdego interwału optymalna strategia rozwoju jest korygowana z uwzględnieniem doprecyzowania informacji o przyszłym stanie otoczenia aktywnego: działań konkurentów, konsumentów, rynków światowych. W celu określenia optymalnej ilości i optymalnej dystrybucji kredytów pomiędzy podsystemami, w każdym interwale wyznaczane są maksima kryterium – sparametryzowanej funkcji efektywności systemu. Na podstawie modelu optymalnego rozwoju opracowano nowy model, który uwzględnia wykorzystanie zasobów zewnętrznych, takich jak kredyty. Rozpatrzone metodę uwzględniania zasobu zewnętrznego w funkcji rozwoju i funkcji produkcji. Podano przykłady modelowania.

**Słowa kluczowe:** optymalna agregacja, funkcja produkcji, funkcja rozwoju, zasób zewnętrzny, modelowanie symulacyjne

### Introduction

Globalization, a steady trend of increasing the efficiency of production and services, growth of the impact of uncertainty and disturbances, taken together, have led to the parametric and structural changes in production as an object of management [18].

Main problems in the field of practice and theory of optimal management and development of production are: novelty, dynamism and significant nonlinearity; natural non-stationarity of the processes of functioning and development of the modern production system; lack of effective optimal control methods for high-dimensional problems [7, 21, 30, 31]. Works with fundamental theoretical solutions of variational problems are known [2, 3, 14, 15]. The general feature of these tasks is an object no higher than of the third order, significant restrictions on the type of functions [4, 5, 32]. To manage large-scale systems – energy systems, economic segments, market systems, „open control” („fair play”), „artificial social systems” is used [9, 19]. In these methods, the global problem of a large production system is solved by each „participant” taking into account their own goals. Such methods allowed to obtain a large number of fundamental and useful results for practice. We conducted modeling of optimal control algorithms proposed by Opoitsev [23]. With the introduction of non-smooth and non-convex „production functions” into the model, it was not possible to ensure the convergence of algorithms for finding the optimal state of equilibrium. This was the primary reason for developing a searchless method of optimal aggregation [6, 9]. A large number of studies have been conducted on the attraction and use of external resources. In [14, 33], two variants of the Hamilton function were proposed: in the form of the term „credit function” and in the form of a multiplier with a variable „cost”

in the function of production development. The development of the optimal aggregation operator for the „production, development” structure [7, 10] made it possible to modify and generalize the task of crediting the processes of optimal functioning and development of the production system.

This paper presents the solution of one of the tasks of creating a generalized mathematical model of optimal development. Section 2 of this article presents the results of research and development of the first stage, on which the development of a new model is based [11, 15, 20].

The aim of the research is to develop a new approach to optimization of the credit strategy of an enterprise in the active environment of competitors and consumers. Tasks of the research:

- formulation and solution of the optimization and development problem of structures „innovation, development, production, accumulation”, taking into account external resources,
- decomposition of the planning period into intervals to improve efficiency in the conditions of close forecasting horizons,
- development of software modules for optimization of use of external resource based on the „embedding” of loans in the functions of production and development,
- on the basis of developed modules, perform modeling of optimal development processes using an external resource.

### 1. Optimal aggregation methodology

The theoretical basis for the development is the optimal aggregation methodology [8], which: sets the rational structure for the production system; allows any loosely monotonous production functions; requires the identification of all significant resource relationships; provides a reconfiguration of the production system in case of failures in the subsystems [11, 24];

provides the adaptability of optimal control to changes in prices and technological parameters [1, 11, 23].

Working models for optimal production and development process are developed using Mathcad software, which gives us an instrument for running simulations of optimal processes of real production systems.

## 2. Analysis of the analogs of production and development of the production systems

We analyzed (with texts, graphics, and formulas) basic analogues in the field of objects and methods of modern production systems management, and assessed the existing state of this field.

Figure 1 shows the complex processes in the system „manufacturers, products of manufacturing process” of a particular production segment [8]. This is concept analysis of „globalization, a steady trend of increasing the efficiency of production and services, and the growing influence of uncertainty and disturbances, led together”. Fig. 1a shows two implementations of a random process of functioning and development of manufacturers system on the market of one product. The mathematical model of the system is assembled on the basis of classical models of microeconomics; 3D graph – a sequence of rank distributions of the producers. The dependencies of the total demand and supply on time, as well as the dynamics of the ranks of the selected manufacturer „amid” the entire system state, are located in the same phase space. Otherwise, it is „the dynamics of the element of the system of producers in the active environment of competitors and consumers”. Fig. 1b presents two implementations of random process for a multi-product system.

Besides the ranked processes, unranked processes are also presented. Simulation scenario: at the beginning of the process, manufacturers are ranked, the first half of high-rated manufacturers use classical management methods, the second half of low-rated manufacturers use innovative – risky management methods. Risky management gives a manufacturer with a low rating non-zero probability to increase significantly the rating. In the upper graph of Fig. 1b the outsider has become a leader; in the lower graph of Fig. 1b outsider remains an outsider.

We analyzed existing models of production systems considering credits.

The closest analog [6, 12, 13] researches the problem of optimal credit policy for the manufacturer to minimize total production cost. Developed model takes into account credits and reliability of produced items.

However, it does not take into account internal resource distribution between subsystems such as „development”, „production”, „innovations”.

Satisfactory models of attracting and using external resources to solve the problem of „development of production systems in an active environment” have not been found. Based on the results of developments [8, 16, 25], the structure of the management system based on simulation models of the system of producers of a certain production segment with the involvement of external resources such as lending was proposed (Fig. 2). The basis of the scheme is a predictor model and a real-time model. In the center are two examples. Above is the empirical frequency distribution of income of some enterprise of the „3 products, 4 manufacturers” system. The diagram displays the „loans” subsystem and the corresponding resource and financial ties.

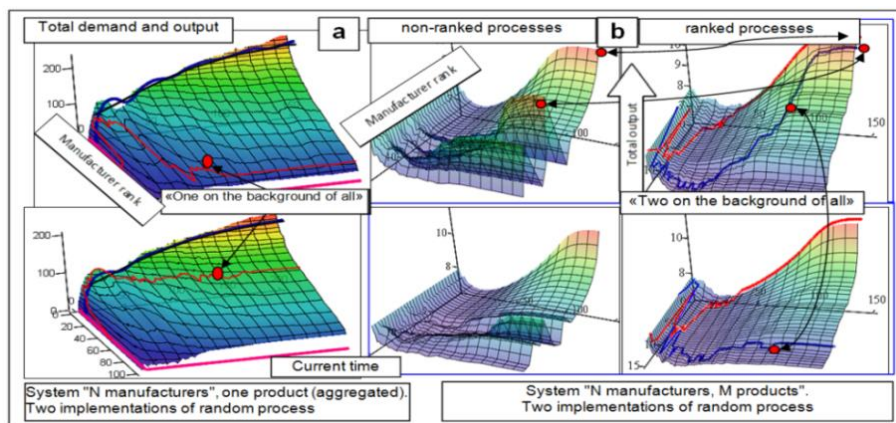


Fig. 1. Modeling of manufactures system dynamics

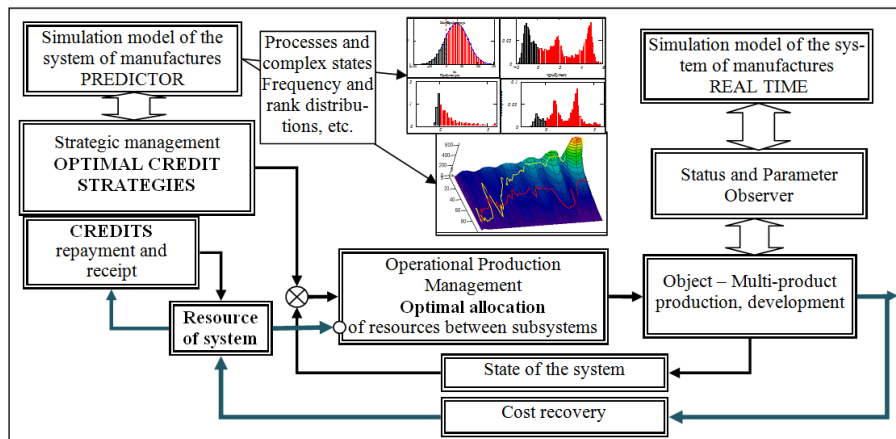


Fig. 2. Diagram of ASUP based on a simulation model of the class „one against the background of all”

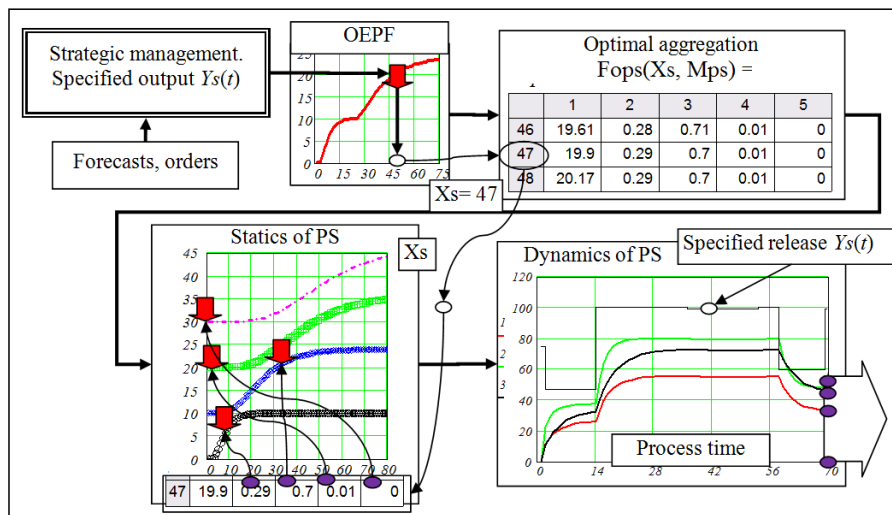


Fig. 3. Operational production management based on optimal aggregation

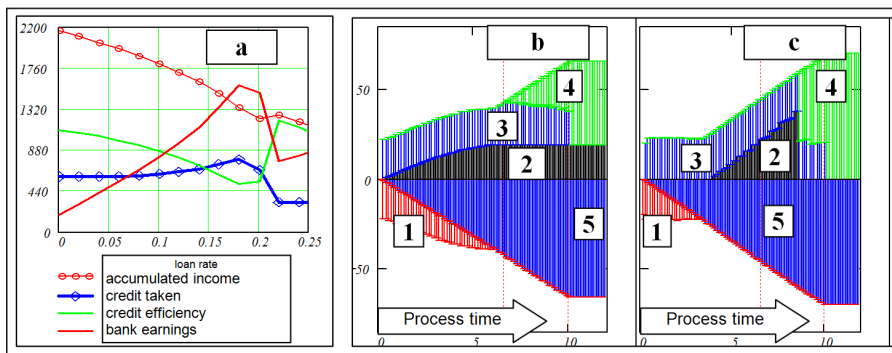


Fig. 4. Credit Policy Analysis Examples

The characteristic feature of the control system in Fig. 2 – availability of a system of specialized models of the system of manufacturers and a production system: real-time simulation models and a predictor model; state and parameter observer. These models create information support for the subsystems of strategic and operational management. The diagram highlighted the subsystem „credits”, which is element of the external resource in this article.

Fig. 3 gives a visual representation of production management based on optimal aggregation methods. The „dynamics” block contains transients in subsystems for a given input effect of the production system. The „strategic management” block determines a specified output, for which the module „optimal aggregation” calculates the minimum of the total costs  $X_s$ , and their distribution over the elements of the system. The „static” block represents the definition of subsystem outputs for a specified resource distribution. The block „dynamics” represents the rate of the output in the subsystems.

Analysis and selection of optimal use of external resources. We analyzed analogs of the model of the optimal development of the enterprise, taking into account lending [12, 17]. The area of these model’s adequacy is „passive” external environment, independent of the actions of producers and consumers. The novelty of this research is the development of adequate enterprise management, taking into account the active environment of competitors and consumers.

Fig. 4 presents an example of a comparative analysis of credit strategies. Fig. 4a – dependences on the credit rate of the following indicators: „accumulated income of the manufacturer”, „took loans”, „effectiveness of loans”, „bank income”. Let us analyze the dependence of the indicator „took loans” from the rate: from zero to 6% – a slight drop, the more expensive the loans, the less we take. The interval is from 6 to 18% – the higher the rate, the more we take loans, the reason – it is necessary

to overcome the „inefficiency threshold” of production. The interval from 18 to 25%: expensive loans are not covered – less credits are taken. Let us compare the dependencies of „bank income” and „credit efficiency” (for production): the complete opposite. Relevant for the manufacturer indicator: „accumulated income” monotonously decreases with the increase of the loans rate.

Figs. 4b and 4c show two alternative strategies of optimal development (description: 1 – credits strategy; 2 – credit return strategy; 3 – development spending; 4 – accumulation rate – criterion increment; 5 – production rate). Processes have different credit recovery strategies. In Fig. 4c it is: „all profits (3) go to the return of debt on credits”. In practice, the strategy of „deferred payments” is used. These credit recovery strategies are part of a set of models and programs [12, 21, 26]. Regarding the use of working models, presented in Fig. 3, it should be noted that they are adequate to the reality only for an „autonomous” enterprise with an unlimited market, independent on the actions of competitors and the „whims” of the consumers.

The given examples of the solution of variational development problems, taking into account the use of external resources [9, 22, 27] – is a small part of the research and development in the field of optimal development of the systems. The possibilities of obtaining new scientific and practical results are not exhausted within the framework of the developed models of optimal development and new information technologies of designing new models for new tasks. This should be an operational optimal decision on the allocation of loans: which subsystem, in what time interval, in what volume and taking into account the state of the system and the external environment. Further the development of credit strategies of manufacturers in the active environment of competitors and consumers is presented. This development is the approximation to the requirements.

### 3. Statement and solution of the problem of optimal aggregation taking into account loans

Development of a model for attracting external resources based on aggregation „production, development”.

We used the methodology of optimal aggregation, the main point of which is the possibility of equivalent replacement of a rational production system by an optimal equivalent element with a production function:  $Y_s = Fops(X_s, Mps)$ , where  $X_s$ ,  $Y_s$  – total costs and output of the system;  $Fops$  – optimal equivalent production function (OEPF);  $Mps = augment(vpp, vpr, vpc, vpk)$  – parameters matrix, OEPF, composed of appropriate vectors. Here:  $vpp$ ,  $vpr$  – vectors of parameters of production and development functions,  $vpc$  – vector of price parameters and  $vpkr$  – vector of credit parameters.

The input data for the optimal aggregation problem are the production functions of the elements of the system, which are given in a discrete form, in the form of a table of the relationship between the spent resource and produced value. The result is the optimal equivalent production function, which is also provided in a discrete form, and shows the optimal distribution of resources between the elements of the system at each step of the process. The paper uses a „resource” approach, in which inputs and outputs are measured in abstract units.

We choose the following alternatives to return credits: from the incomes of production and the strategies of credits return, which are presented in Fig. 4.

We modified the production and development functions taking into account credits as a specific „borrowed resource” and displayed it in the parameters of operators of optimal aggregation of parallel structures and structures „production, development”. No close analogues of this approach were found, therefore at the first stage of development we use structural and logical methods of applied system analysis [7, 8, 16].

Fig. 5 presents the analysis of the credit impact of subsystems „production” and „development”. We assume that the return of loans is related to the costs of production at the level of the production system on the whole. In this paper, we consider the tasks at the resource level, and the prices are included as parameters. Models of the markets where prices are formed are described in work of Borovska [9, 11]. On the left graph of Fig. 5 three functions of production are presented: excluding and taking into account the cost of debts repayment. The situation when loans supplement own resources is considered. We see that the output at constant total costs may differ significantly. On the right graph of Fig. 5 four functions of production are presented: the initial, after the cost of a loan for development and two functions, taking into account the cost of paying debts. We see that the output at constant production costs may differ.

The model in Fig. 5 allows to identify the specific features of production processes development. „Short” loans should

be taken in case when maximum return in the form of products, production facilities is expected. It is advisable to return credits with a deferred payment. In a multi-product system, it is possible and desirable to form the chains of „production, development” processes in different phases of implementation [27, 28, 29].

General conclusions regarding the analysis of credits: credits should be spent on improving the efficiency of the production system on the levels of „production”, „development”, and „innovation”.

### 4. Mathematical model of optimal development taking into account the external resources. Embedding credits in production and development functions

Based on the materials, proposed above, we solve the problem of optimal aggregation for the structure of „production, development”, taking into account external resources. Optimal aggregation of a binary structure is the solution of one-dimensional problem of non-linear programming for a pair of elements of the „costs, output” class. The solution is found for the entire interval of determining the variable „costs”, and the result of the aggregation is the „optimal equivalent production function” (OEPF) of a given binary structure. Optimal aggregation operations could be made associative. This enables you to create the „algebra of production functions”, similar to the algebra of transfer functions in Automatic control theory [16].

Binary operators and operands of optimal aggregation algebra. The binary operator can be represented by a mathematical model, and then by the appropriate computer program or immediately – a „working model”. The development of the binary operator of optimal aggregation is the main intellectual component of optimal aggregation methods. The result of the development is a multidimensional nonlinear programming problem, which is decomposed into a system of one-dimensional problems, solved by the searchless method. The form of solving the problem of optimal aggregation is: a function of the total resource, as well as of the parameters. Parameters could be numbers, vectors, matrices, and subroutines.

Mathematical model of structure with parametric resource connection „production, development”. Substantial background: modern high-tech, innovative productions are integrated with the subsystems of development and maintenance of production units, subsystems of management, control and testing. Organizational integration makes management and financial integration desirable and possible. These transformations generate a new class of optimization problems and corresponding mathematical models. We considered the basic model of the integrated system „production, development” [11, 35]. In this paper, we modify this model to account the use of credits.

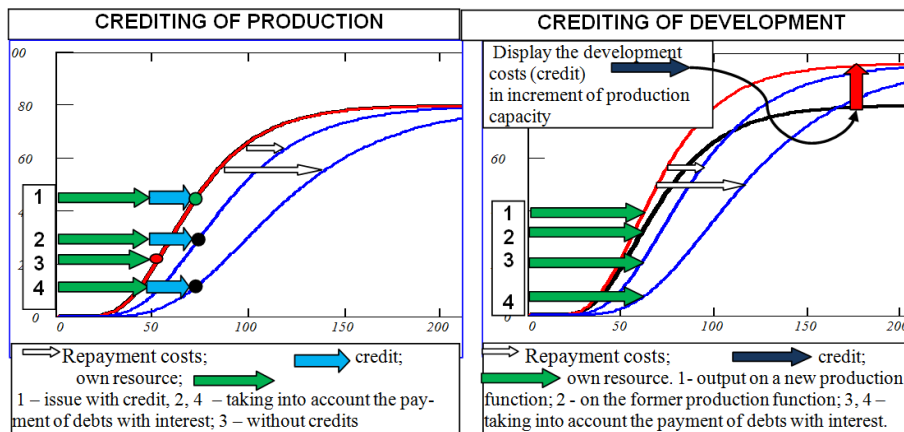


Fig. 5. Resource schemes of credits for the production and development

We write down the functions of production (PF) and development (DF):

$$fp(xp) = fpp(xp, vPp(xr, vPp0)); fr(xr) = fc(xr, vPr) \quad (1)$$

where  $xp, xr$  – production and development resources,  $vPp0$  – the initial value of the vector of parameters of the production function (PF),  $vPr$  – vector of development function parameters (DF).

We consider in details the parametric coupling function of the production function (PF) with the development function (DF):

$$vPp(xr, vPp0) \Rightarrow vPp = VP2(\alpha, \Delta xs, vPp0, vPr) \quad (2)$$

where  $VP2()$  – a function that characterizes the transformation of the costs of the „development” subsystem into a change in the parameters of the production function (PF) – increase of the efficiency and production capacity, decrease of service costs,  $\Delta xs \cdot \alpha$ ,  $0 \leq \alpha \leq 1$  – quantum of system resource and the proportion of its distribution between development and production. The initial state of the system is set:  $xp0$  – production rate;  $vPp0$  – parameter vector of the production function (PF);  $vPr$  – vector of development function parameters (DF).

We write the equation of state of the system „production, development” after using a quantum of resource:

$$xp = xp0 + \alpha \cdot \Delta xs; xr = (1 - \alpha) \cdot \Delta xs \quad (3)$$

Authors display development costs in the changes in production capacity  $yr = fr(xr, vPr)$ , taking into account (3)

and obtain  $yr = fr((1 - \alpha) \cdot \Delta xs, vPr)$ . We use three-parameter models of the production function (PF) and development functions (DF), we assemble into the user vector function: the dependencies

of the parameters of the production function model (PF) for specific segments of production and technologies:

$$vPp = VP2(\alpha, \Delta xs, vPp0, vPr) \quad (4)$$

After using the resource quantum for the development and production, taking into account (3) and (4), rate of production will have following form:

$$yp = fp(xp, vPp) = fp((xp0 + \alpha \cdot \Delta xs), (vPp0 + \delta vPp)) \quad (5)$$

Taking into consideration (4), (5), we form the function of the user – „new rate of release”:

$$yp(\Delta xs, \alpha) = fp((xp0 + \alpha \cdot \Delta xs), VP2(\alpha, \Delta xs, vPp0, vPr)) \quad (6)$$

and user function „output increment”:

$$\delta yp(\Delta xs, \alpha) = yp(\Delta xs, \alpha) - yp0 = yp(\Delta xs, \alpha) - fp(xp0, vPp0) \quad (7)$$

Expression (7) is the optimization criterion in the basic problem of optimal aggregation of the integrated system „production, development”. Modification of the binary operator „production, development” to account he credits. Fig. 6 presents an example of the development of a transition operator between development process intervals.

The unpacking of the user function  $Foppt(MP1, MP2)$  – OEPF test system is performed. The details and logic of the transition between the matrices of parameters of the integrated system „production, development” are presented. In the upper part – the initial data; in the middle part – the dependencies between the elements of the parameter matrices; in the lower part – the transition between the states of the matrix of parameters of the optimally aggregated production system, which has a parametric resource connection between the subsystems „production” and „development”, for two points in time. In the right part of Fig. 6 the results of calculations for the three steps of „working out” „quanta” of the resource are shown.

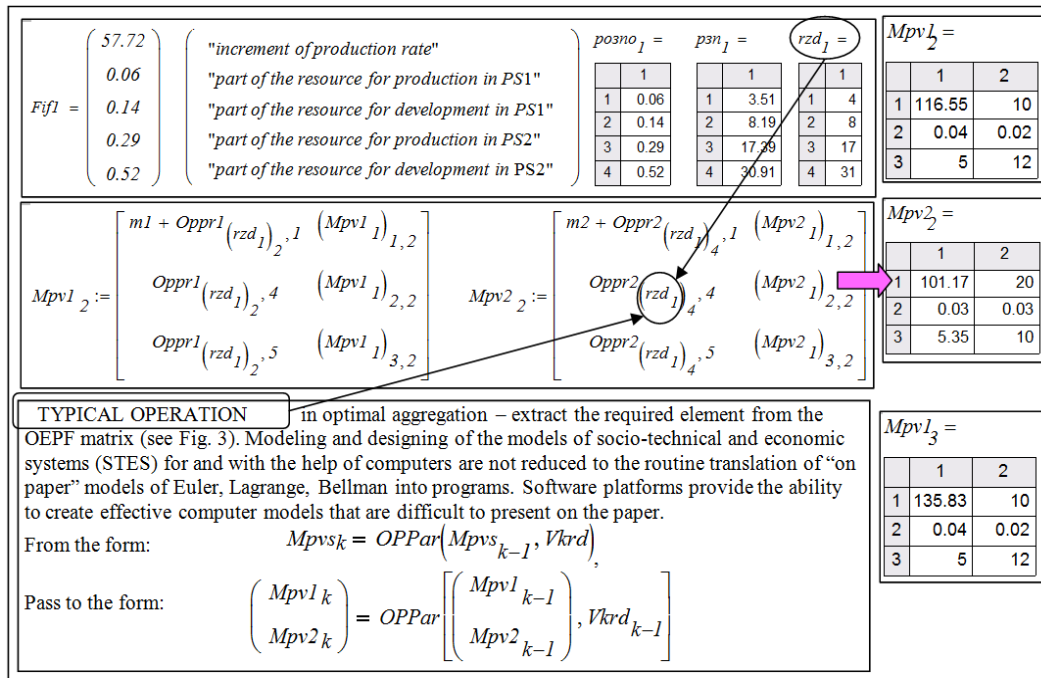


Fig. 6. Obtaining of the operator of the transition between the development process intervals

### 5. Testing of modules of optimal development and analysis of the structure of the complex models

All elements of the developed complex models have been tested. On a set of models, testing was performed to reproduce the available empirical data, to match the results obtained on alternative models. Fig. 7 shows an example of testing a key component – the operator of the transition between intervals.

Fig. 7a – testing of the „response” of the system to the dynamics of the magnitude of the resource quanta: with increasing and decreasing. Results are represented by graphs, numbers, vectors  $Ra, Rb$ . Fig. 7b – construction of optimal equivalent efficiency functions for three development intervals, and the results (outputs and resource distributions) of resource quantum development with and without credits are derived. Also areas of maximum efficiency loans are highlighted.

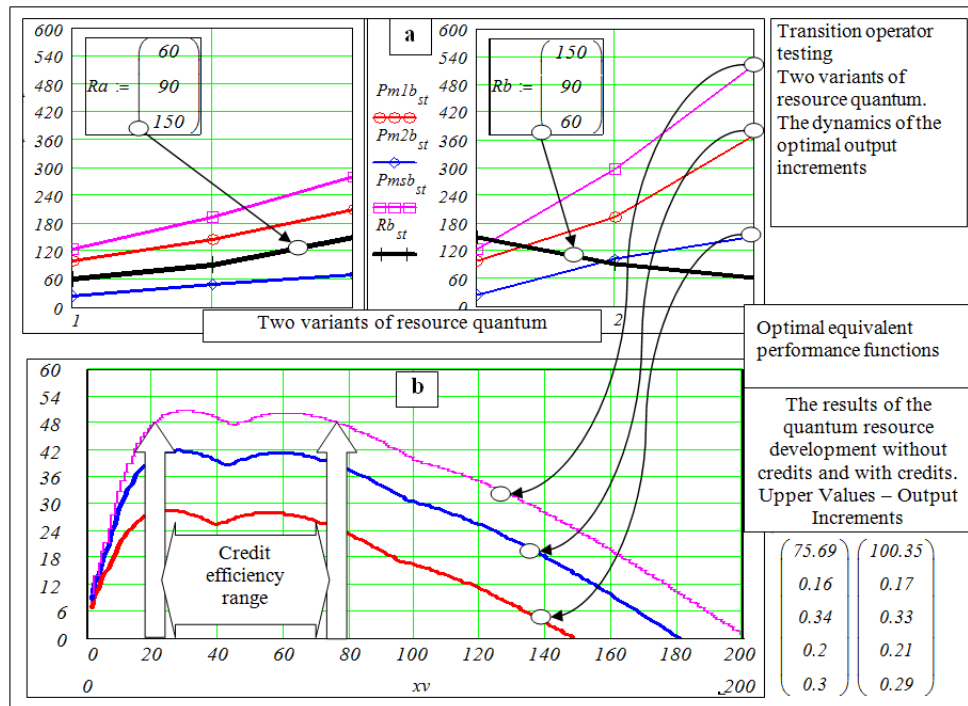


Fig. 7. Testing of the transition operator between the intervals of the development process – the optimal development of a quantum of resource. Example

### 6. Conclusions

Analysis of analogues and prototypes showed the constant relevance of optimal development problems for production systems of all the levels. Fundamental mathematical models for the simplified problems and objects of nohigher than the third order are obtained. The solutions obtained for variational problems were not generalized. There were no adequate and effective mathematical development models using external resources.

The goal of generalizing the problem of optimal development of production systems based on the optimal aggregation methodology is set so that the changes in the control system during the transition to the new object – with new technologies, new production products, are reduced to parametric tuning.

The search and analysis of direct analogues was performed. New tasks have been set to achieve the goal. The development base is the classical formulation of a variation development problem and its solution by the maximum principle method and the optimal aggregation methodology. Scientific novelty:

- new task of optimal aggregation of structures with parametric connections „innovation, development, production, accumulation” has been solved. The result of aggregation is the optimal distribution of the system resource between development subsystems and production subsystems;
- decomposition of the planning period into intervals to improve efficiency in the conditions of close forecasting horizons is performed. At the beginning of each interval, the optimal development strategy is calculated for the remainder of the planned period;
- software optimization of use of external resource based on the „embedding” of loans in the functions of production and development is proposed and implemented.

The software system has been developed, modeling of optimal development processes using an external resource has been performed.

Practical significance. Research and development allow for any moment of the process the optimum volume of credits depending on the current pace of production and demand forecasts. Optimal rapid allocation of own resources between production reduces the need for loans. Prospects. The next stage for this development is the creation of mathematical models and software module „virtual reality” statistics based on Predictor models. This will take into account the risks in the optimal management of the production and development. The results are used for further research to develop optimal adaptive control system of modern enterprise.

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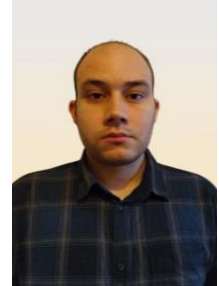
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