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STOCHASTIC TRAFFIC DEMAND PROFILE: INTERDAY VARIATION FOR GIVEN TIME AND DAY OF WEEK

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ABSTRACT

Traffic demand prediction is one of the major elements of traffic planning and modelling. Traffic surveys routinely estimate the profile of traffic demand on a certain road section, showing the expected evolution of the demand over a day or week. However, the actual demand fluctuates around this value on day-to-day basis and thus can exceed otherwise sufficient capacity and consequently cause congestion due to the capacity drop. This type of traffic demand variability has not yet been properly studied although it can play significant role in traffic modelling and engineering. The relevance of this variability is further increasing with the growing popularity of stochastic traffic models. This paper presents results of a statistical analysis of the demand variability in five-minute aggregation intervals. Normal, lognormal and gamma distributions all show reasonably well fit to the data for individual intervals and often do not differ on statistically significant level. Based on the count of the best fits, the lognormal distribution seems to be most suitable, while the gamma distribution is the most universal and with generally acceptable fit. There appears to be a pattern where certain distributions have better fit in different times of the day and week. The regularity and magnitude of demand probably both play a role in this, as well as the aggregation interval. Two simple models for modelling the variability are proposed for practical applications when there is not enough data to perform similar analysis.

KEYWORDS

Random traffic demand, Probability distribution, Traffic model, Goodness-of-fit

INTRODUCTION

The paper deals with the variability of traffic demand around its time-varying mean value given by the traffic demand profile. Traffic demand profiles are commonly estimated via a traffic survey at specific location for workday or weekend. They show the estimated average progression of traffic demand over the whole day based on the collected data, which are usually extrapolated and transformed using coefficients of daily or seasonal variation specified in TP 189 [1] in the Czech Republic. This data can further be used to calculate annual average daily traffic (AADT). However, there is usually no information about the possible variation of the traffic volumes from day to day at any time. This variability can play important role if the intensity is close to capacity levels. Once the capacity, which is also a stochastic variable [2–4], is exceeded, the capacity drops to queue discharge flow level [5–7], making it more difficult for the congestion to resolve even after the demand drops again.

Most of the existing research on traffic demand focused on OD (origin-destination) matrices. Those model the traffic relations among points of interest on the traffic network on variable scale and are then used (mostly) in the four-stage traffic models as one of the inputs for modelling traffic flow [8]. Historically, mostly deterministic OD matrices, which, at best, reflected the difference between the morning and afternoon peak hours, were used. In the past two decades, many new more dynamic models were developed, which utilize the development of IT and big data [9]. While researches were dealing with the randomness of morning peak even earlier, practitioners preferred simple deterministic models which were sufficient for the applications at the time, according to Alfa [10].

If only local road section is modelled, the traffic demand or intensity cannot be modelled via the four-stage model and must be defined otherwise. This is often resolved by simply using a uniform distribution or at best with an exponential distribution with constant parameter based on an expected hourly volume in the peak hour. However, for stochastic models the traffic demand should be also modelled in more detailed statistical manner, especially if methods such as Monte Carlo simulation are used. Such model then allows, for example, to estimate the probability of TF breakdown at given time of day/week or development of a queue, or to accurately model behaviour of a traffic actuated system such as traffic lights, variable message signs and others. Using a fixed TF intensity would contradict the purpose of such model.

The problem of variability of traffic flow (TF) intensity on given day of week and time of day across following weeks, which could be called stochastic traffic demand profile, have not yet been significantly dealt with in the literature. While researchers and practitioners were and are certainly both aware of the random fluctuations on day-to-day and week-to-week basis, little attention is paid to it. If such variability is concerned, it is usually in the form of a random multiplicative coefficient with normal distribution and zero mean, e.g. Brilon [11] used random coefficient coming from N~(1, 0.1).

Better understanding of the phenomenon and the variability patterns could lead to better traffic modelling and prediction or to optimization of different traffic control algorithms, as well as more accurate traffic-engineering evaluations and designs.

METHODS

The traffic flow intensity (demand) data used for this study come from overhead ASIM by Xtrail tri-tech (microwave, infrared, ultrasonic) traffic detectors on motorway D5 at km 32.9, direction Prague, from June to December 2015 (29 weeks in total). Traffic congestions do not regularly occur in this location or on nearby on- or off-ramps. Therefore, the measured intensity can be considered as equal to the traffic demand with reasonable reliability. The original data is aggregated in 5-minute intervals and differentiates 6 vehicle categories. Firstly, the different categories were merged and transformed to passenger cars by using passenger car equivalent (PCE) PCE = 2 for trucks and busses to take their larger effect on the TF into consideration. Hence, pc/h was used as a unit of TF intensity. All the TF records were grouped into groups based on day-of-week, hour, and starting minute, together reflecting the whole week-long TF demand profile segmented into 5-minute intervals. Therefore, 2016 groups (7 days × 24 hours × 12 5-minute intervals) were created, each with up to 29 measurements. The outliers with Z-score > 3 were discarded from each group.

Each of the groups was fitted with five different random distributions using R script: normal (norm), gamma, Weibull (Weib), inverse Weibull (invWeib) and log-normal (lognorm). Those five were chosen based on theoretical assumptions and histograms of few randomly chosen groups as possible candidates for reasonable fit. The parameters of each of the fitted distribution for each of the group, along with the Akaike information criterium (AIC) which describes the goodness-of-fit of the model with the underlying data, were obtained as a result.

The choice of the best-fitting distribution was run in several phases as it turned out that the results are far from unambiguous. The best distribution was to be chosen based on the number of groups for which it had the best fit (lowest AIC). Since the first results led to no clear conclusions, each of the groups was expanded with data from the neighbouring groups, assuming they are highly correlated in terms of the TF intensity. That allowed to triple the sample size in each group (up to 87 measurements) at the cost of negligible bias, leading to more accurate fitting of the distributions. All the presented results are based on the data with extended groups. To further clarify the conclusions, groups from the night hours and weekends were omitted from the evaluation for two reasons. First, they can be reasonably assumed to have different statistical distribution thanks to different traffic patterns and second, they play little role in traffic engineering application. Eventually, the counts of best fits were also evaluated for each day and even hour, separately, and for each of the subset of groups, the average AIC was also calculated for each of the distributions. That allows to compare to what extent is the best distribution better than the others. Small AIC difference means both (or more) distributions are almost equally good at fitting the data.

RESULTS AND DISCUSSION

Tab. 1 presents the aggregated results after expanding the groups with the neighbouring intervals for both whole week and for only workdays from 6 AM to 6 PM.

The overall results of the performed analyses are provided in Tab. 2. The left side of each table a-e (one for each workday) shows the count of best fits of each model within each time interval, the right side shows the average AIC of each model within the corresponding interval.

Tab. 1: Summary of the counts of best fit among the five probability distributions with and without the night and weekend intervals.

Distribution	norm	gamma	Weib	invWeib	lognorm
No. of best fits (Mon-Sun)	475	615	215	60	651
No. of best fits (Mon-Fri 6-18 h)	221	150	63	33	253

The relative likelihood of each model is given by $\exp((AIC_{min} - AIC_i)/2)$ where AIC_{min} is the AIC of the seemingly best-fitting model and AIC_i of the *i*-th model. It gives the probability that the *i*-th model minimizes the loss of information better than the model with lowest AIC and can be used to choose the best model(s) based on statistical significance. Unlike likelihood-ratio test, the compared models do not need to be nested, but the fitted data set obviously must be the same, i.e. the comparison can only be made within each time interval, e.g. compare the models on Friday 17-18 (Tab. 2 e). If the common level of significance 0.05 is used, the difference needed for the model with higher AIC to be statistically significantly worse than the one with the lowest AIC is circa 6. Therefore, if the difference in AIC is less than 6, we can say the two given models are not significantly different. In the case of Friday 17-18 this means that while gamma distribution seems to have the best fit, both normal and lognormal distributions are not statistically significantly worse in representing the empirical data. The same conclusions can be made for most of the presented intervals even though the best model differs. In few cases the Weibull distribution has the best fit but that seems more like a coincidence within the overall results.

Tab. 2 (a-e): Aggregated results of goodness-of-fit on hour-to-hour and day-to-day basis for each workday between 6 AM and 6 PM. Tables a-e show results for individual workdays. The results are colour-coded for each section (and separately for each line on the right side) for better clarity.

(a)

Monday	norm	gamma	Weib	invWeib	lognorm	norm	gamma	Weib	invWeib	lognorm
Sum 6-18	44	31	20	7	42	719.60	721.45	726.61	751.44	724.34
6-7	3	0	9	0	0	740.81	757.11	737.89	825.59	768.93
7-8	5	0	7	0	0	773.28	791.70	771.53	865.11	805.19
8-9	9	0	3	0	0	748.45	757.38	751.50	815.24	765.09
9-10	7	3	1	0	1	699.82	701.47	705.07	734.60	703.85
10-11	2	8	0	0	2	727.38	726.05	734.89	748.09	726.92
11-12	2	4	0	0	6	707.81	705.61	717.13	723.74	705.74
12-13	9	3	0	0	0	683.93	684.58	689.25	711.56	686.15
13-14	2	5	0	0	5	688.50	686.11	698.76	700.38	686.00
14-15	0	2	0	2	8	693.79	689.75	705.84	693.27	688.56
15-16	0	1	0	3	8	721.37	715.86	734.69	718.83	714.15
16-17	4	3	0	0	5	720.51	718.44	730.05	739.04	718.90
17-18	1	2	0	2	7	729.55	723.37	742.64	741.86	722.62

(b)

Tuesday	norm	gamma	Weib	invWeib	lognorm	norm	gamma	Weib	invWeib	lognorm
Sum 6-18	41	21	1	14	67	744.71	742.48	753.32	762.15	743.20
6-7	10	2	0	0	0	749.33	752.29	752.30	792.58	756.48
7-8	9	2	1	0	0	765.42	770.41	769.68	819.42	775.91
8-9	10	2	0	0	0	775.36	778.07	780.29	820.87	782.57
9-10	7	2	0	0	3	754.09	754.95	760.26	792.84	758.19
10-11	3	4	0	0	5	732.26	730.23	739.10	753.25	731.09
11-12	0	1	0	3	8	736.49	730.32	748.13	736.09	728.66
12-13	1	1	0	0	10	737.04	732.28	748.25	742.70	731.26
13-14	1	0	0	1	10	724.38	720.02	736.90	730.29	719.04
14-15	0	1	0	0	11	765.15	759.22	775.85	763.83	757.60
15-16	0	0	0	5	7	756.32	749.19	769.30	748.55	746.81
16-17	0	4	0	5	3	719.37	714.83	729.31	717.35	713.48
17-18	0	2	0	0	10	721.36	717.90	730.51	728.07	717.28

(c)

Wednesday	norm	gamma	Weib	invWeib	lognorm	norm	gamma	Weib	invWeib	lognorm
Sum 6-18	52	31	7	1	53	730.69	731.07	737.47	757.84	733.21
6-7	9	3	0	0	0	742.48	745.36	745.28	784.92	749.69
7-8	8	0	4	0	0	805.89	811.91	806.48	855.89	818.03
8-9	10	1	1	0	0	790.46	802.47	793.31	873.02	814.16
9-10	4	4	0	0	4	737.96	736.66	746.07	761.61	737.89
10-11	0	4	0	1	7	728.70	725.25	738.47	737.43	724.67
11-12	3	2	1	0	6	724.60	722.98	732.82	742.58	723.31
12-13	4	2	0	0	6	735.87	734.47	744.15	757.02	735.36
13-14	4	4	0	0	4	736.64	735.37	745.00	760.13	736.21
14-15	7	2	0	0	3	707.79	707.94	714.03	735.26	709.45
15-16	0	4	1	0	7	701.25	698.91	708.90	714.82	699.04
16-17	1	3	0	0	8	678.32	675.55	688.33	685.12	675.08
17-18	2	2	0	0	8	678.38	675.91	686.82	686.28	675.60

(d)

Thursday	norm	gamma	Weib	invWeib	lognorm	norm	gamma	Weib	invWeib	lognorm
Sum 6-18	62	22	32	4	24	722.44	727.49	727.18	770.19	732.70
6-7	7	0	4	0	1	708.10	721.98	708.81	793.07	733.25
7-8	8	0	4	0	0	761.52	778.82	762.31	857.83	792.92
8-9	9	0	3	0	0	779.23	791.27	780.41	852.74	802.06
9-10	8	1	3	0	0	719.05	723.98	721.29	768.27	728.67
10-11	9	1	1	0	1	719.74	722.81	722.44	759.43	726.09
11-12	3	4	1	0	4	687.39	686.11	696.99	708.47	686.60
12-13	2	1	1	1	7	687.67	685.47	698.71	702.90	685.57
13-14	4	1	1	2	4	707.17	706.28	715.49	731.30	707.46
14-15	3	5	1	0	3	720.22	718.89	728.24	745.26	719.77
15-16	3	1	3	1	4	722.99	724.74	729.86	755.82	727.47
16-17	2	8	2	0	0	739.98	740.34	746.11	777.76	743.37
17-18	4	0	8	0	0	716.27	729.15	715.46	789.39	739.13

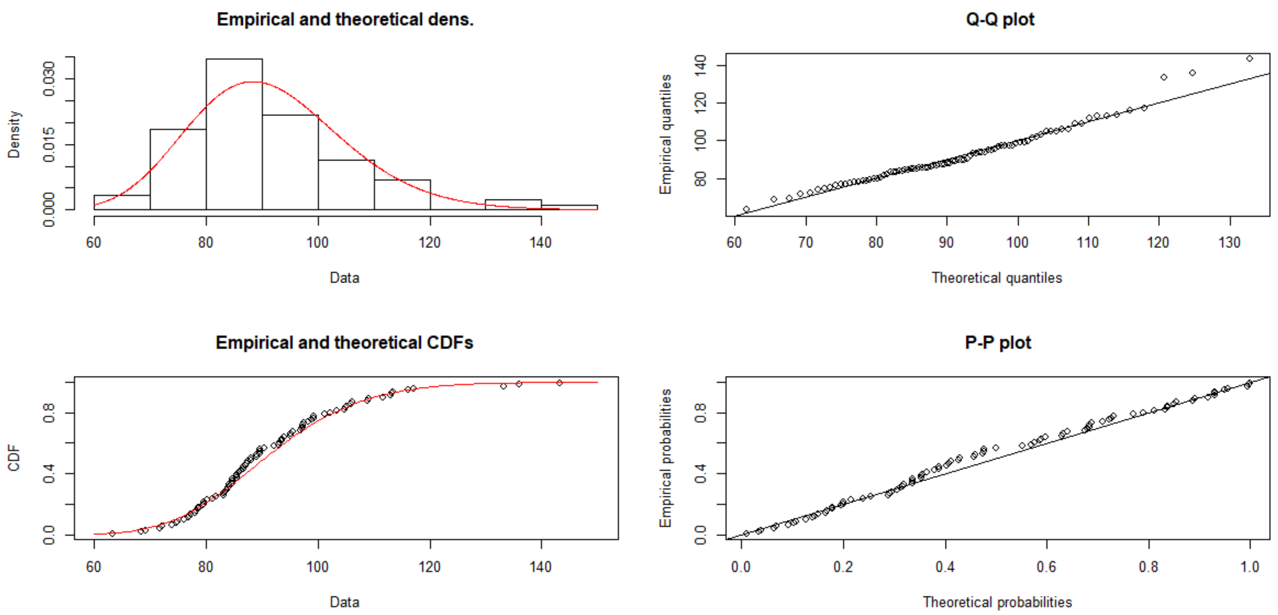
(e)

Friday	norm	gamma	Weib	invWeib	lognorm	norm	gamma	Weib	invWeib	lognorm
Sum 6-18	22	45	3	7	67	698.42	695.84	706.51	713.31	696.02
6-7	0	3	0	1	8	660.36	656.43	669.03	662.54	655.48
7-8	0	2	0	2	8	698.30	693.08	708.46	701.10	691.82
8-9	0	2	0	0	10	688.56	684.59	698.44	695.04	683.83
9-10	0	1	0	1	10	667.42	663.51	677.42	671.89	662.60
10-11	2	2	1	3	4	674.39	671.92	682.53	685.48	671.90
11-12	4	4	2	0	2	674.73	674.96	679.80	697.84	676.29
12-13	0	5	0	0	7	684.42	681.80	692.80	696.52	681.67
13-14	3	7	0	0	2	718.79	717.54	727.35	746.73	718.72
14-15	2	7	0	0	3	731.76	730.12	739.45	756.21	731.31
15-16	3	4	0	0	5	737.00	734.17	745.81	759.40	734.70
16-17	3	5	0	0	4	749.14	747.19	755.48	772.37	748.33
17-18	5	3	0	0	4	696.20	694.78	701.51	714.58	695.60

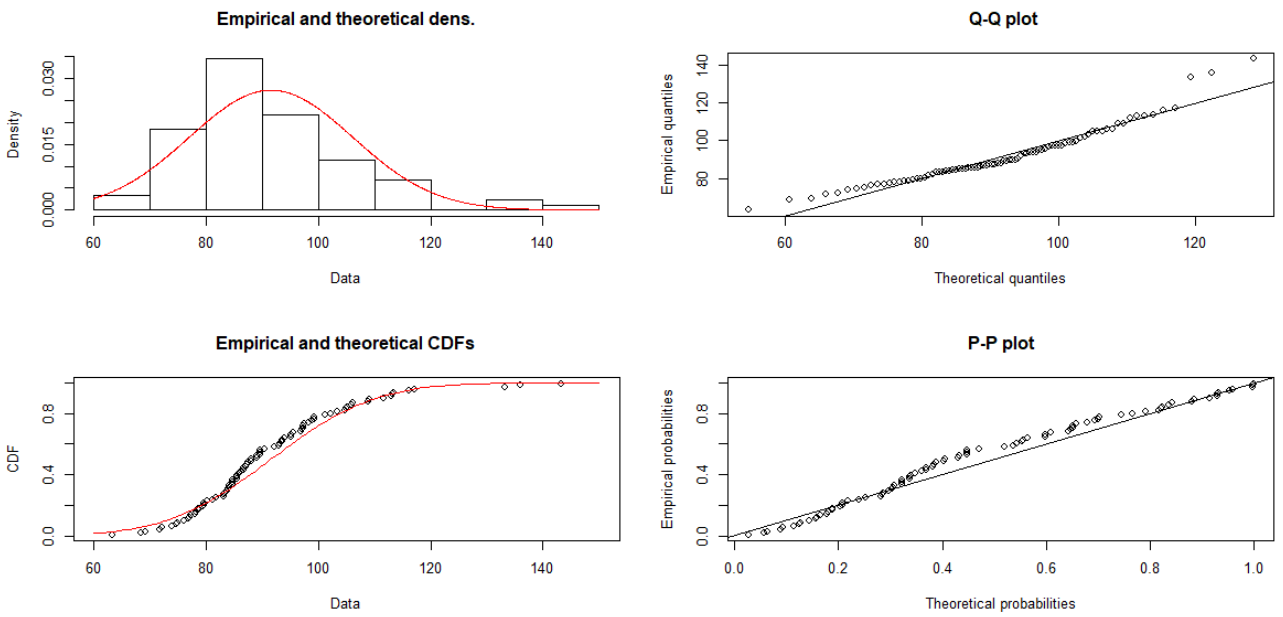
The counts of best fits for each model are also highly skewed towards the three distributions while Weibull and especially inverse Weibull distributions lag far behind. In most cases, even their AIC is much higher than that of the three other distributions, while in the cases when they do have the best fit, the three distributions are in most cases not significantly worse. Obviously, if either Weibull or inverse Weibull distribution has good fit, the other has the worst fit by far.

Fig. 1 finally shows an illustrative comparison of goodness-of-fit of three different distributions to one particular group of TF data. The three distributions illustrate good, acceptable, and unsuitable fitting model, using different graphic tools of goodness-of-fit measure. Note that the AIC difference between log-normal and normal distribution is more than 10 but, visually, even the normal distribution has relatively good fit, despite being statistically significantly worse. That shows that often more than one distribution can model the empirical data reasonably well, which is important.

(a)


Empirical and theoretical dens.
Density
Data
Q-Q plot
Empirical quantiles
Theoretical quantiles
Empirical and theoretical CDFs
CDF
Data
P-P plot
Empirical probabilities
Theoretical probabilities

(b)


Empirical and theoretical dens.
Density
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Q-Q plot
Empirical quantiles
Theoretical quantiles
Empirical and theoretical CDFs
CDF
Data
P-P plot
Empirical probabilities
Theoretical probabilities

(c)

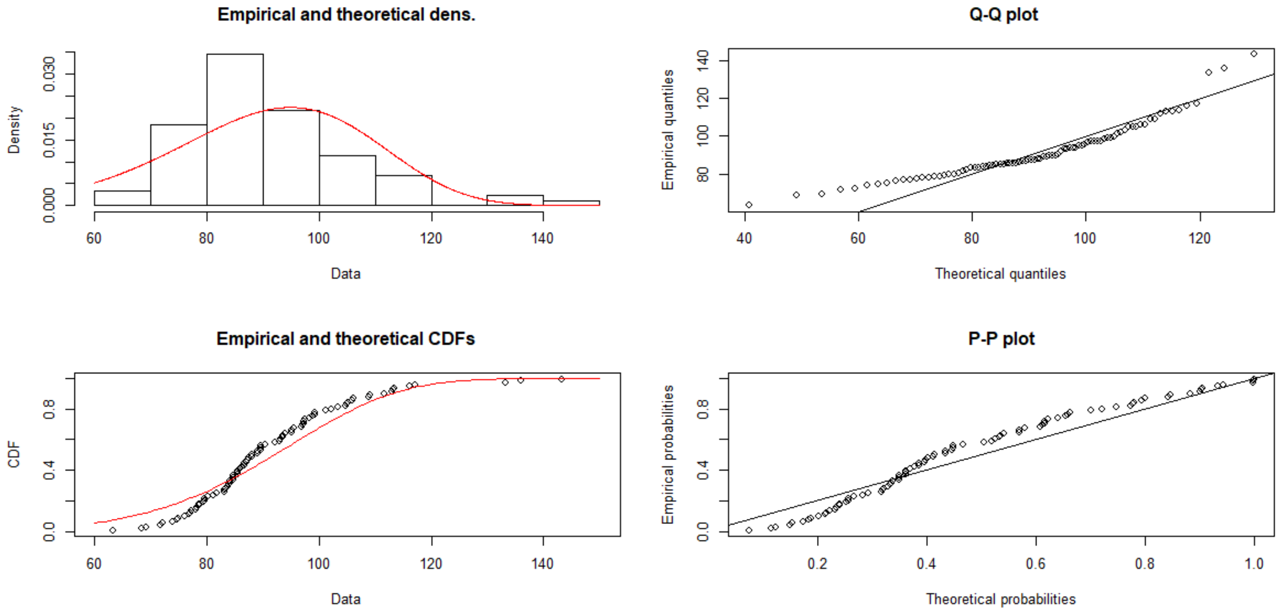

Fig. 1 – Illustrative comparison of three probability distribution functions and Q-Q and P-P plots for the Monday 15:20-15:35 group: (a) log-normal distribution (AIC 706.8), (b) normal distribution (AIC 717.3), (c) Weibull (AIC 735.2).

Based on the results in Tab. 2, there seems to be a trend where normal distribution performs best during the morning peak but after that, gamma or lognormal distribution take over as best fitting, on average. The exception seems to be Friday where gamma, and partially lognormal distribution have the best fit over the whole day. It is possible that this happens due to different patterns in the traffic flow during the morning peak which tends to happen more regularly for various reasons, most obviously due to schools starting at 8 am so parents driving their children always commute at the same time. Further, the capacity of road network may have played a role during certain times of day (especially the morning peak) and the effect would differ on different locations.

Theoretical arguments can be made for all the three distributions – central limit theorem might point to the normal distribution, while right-skewed, lower-bound data suggest lognormal or gamma distributions. However, as gamma distribution is the most flexible (it is in fact a whole family of distributions), it may be the most suitable candidate for modelling this variable. This is supported by the results, where it was almost always among the two best-fitting distributions.

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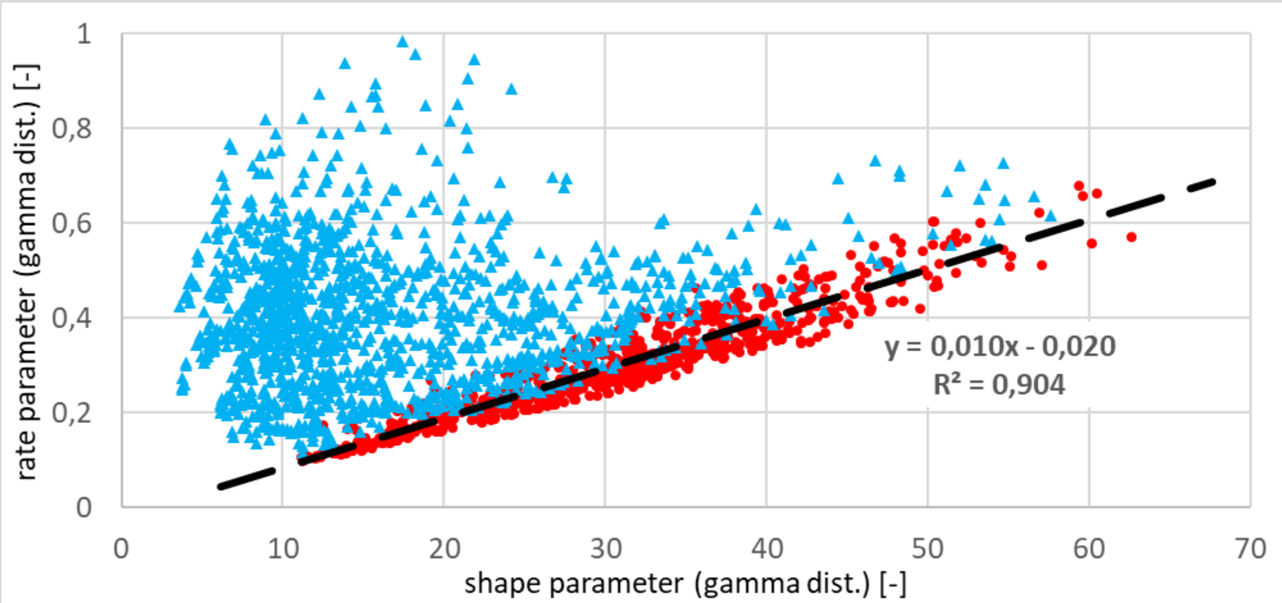

Fig. 2 – Relationship of the shape and rate parameters of gamma distribution. Red circles represent intervals on workdays from 8-18 hours, for completeness, weekends and night intervals are shown as blue triangles.

While understanding what distribution is suitable for modelling this type of data is important, it is only the starting point for any practical application. If there is a large set of historical traffic demand data, it can be used to fit a theoretical distribution to it and later the distribution can be used to generate traffic in a model. However, there is often not enough data (never in the case of manual traffic surveys) to do that. Thus, a generally acceptable model that could be used when only the estimated mean values are known is necessary for many practical applications. As the gamma distribution was regarded as most universal, it is a natural candidate for such a model. Moreover, as Figure 2 shows, there is even very strong corelation (R^2 = 0,904) between the two parameters of the distribution if only intervals with strong and regular traffic (workdays 8-18) are considered. Unfortunately, there is no meaningful way to connect the mean value of expected traffic demand to the shape or scale parameter of the gamma distribution, rendering the gamma distribution unusable if it is not possible to fit it to a large set of data. On the contrary, while it is possible to directly transform the mean expected traffic intensity to the mean of a lognormal distribution, there is much weaker correlation between the parameters of lognormal distribution. In fact, if only the intervals for workdays 8-18 are plotted, they form a circular shape with virtually no correlation.

As was shown earlier, the normal distribution is also a good candidate for modelling the variability and it has a strong benefit in that one of its parameters, the mean, is always known, or rather estimated with varying degree of certainty. Moreover, as Figure 3 shows, there is also fairly strong correlation (R^2=0,84) between the mean and standard deviation. Therefore, only knowing the mean expected TF intensity in any 5-minute interval of a day is sufficient to model its random fluctuations relatively accurately on day-to-day (or week-to-week, depending on the way the mean is calculated) basis.

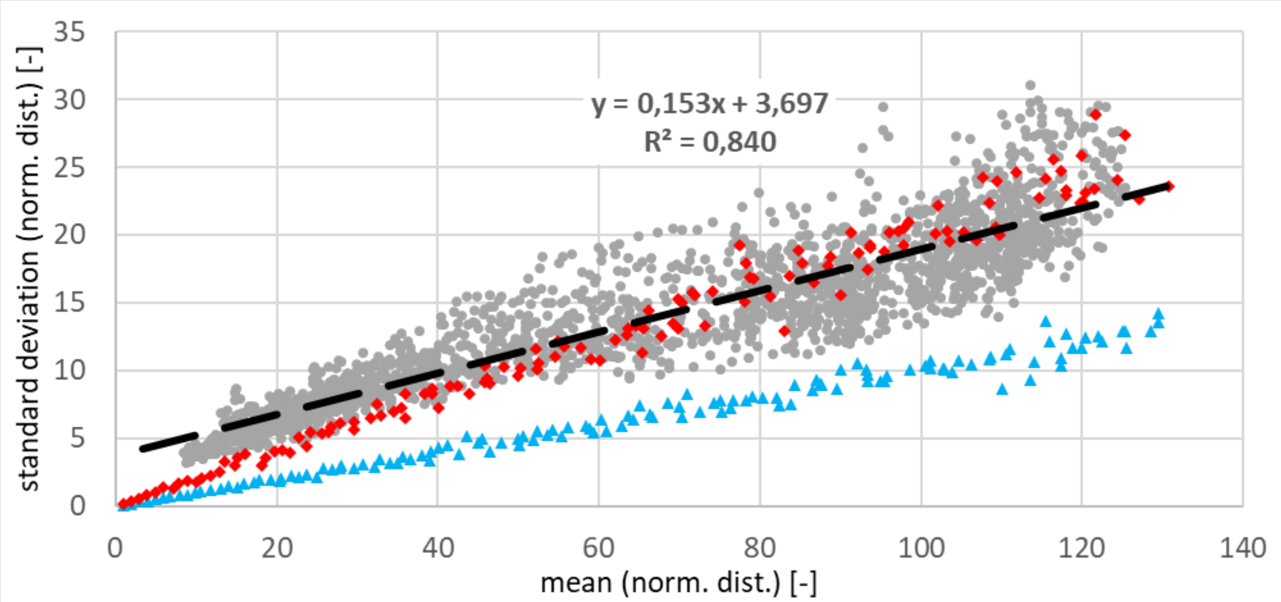

Fig. 3 – Relationship of the mean and standard deviation in grey circles (all 2016 datapoints), interpolated with a black line. For comparison, datapoints modelled based on the Brilon's approach [11] are shown: the original N~(1, 0.1) – blue triangles; modified N~(1, 0.2) red diamonds.

The standard deviation (SD) can be estimated as $3.7 + 0.153 \times \mu$, where μ is the mean expected TF intensity. Figure 3 also compares it to the model implemented by Brilon [11]. He originally used multiplicative coefficient from N~(1, 0.1), which is shown to greatly underestimate the actual variability of the traffic flow intensities, at least for 5-minute intervals. However, if the standard deviation of the random coefficient is increased to 0.2, the result is much closer to the data. The average SD seems to be very similar, but it is underestimated for low intensities and slightly overestimated for high intensities. Overall, the Brilon's modified approach with SD increased to 0.2 is very robust and simple and seems good enough to use if no better data is available. However, for situations similar to that for which this study was based upon (5-minute aggregation interval, motorway), the proposed model N~(μ, 3.7+0.153×μ) should provide more accurate results.

The whole concept discussed in this study is only concerned with the demand variability and assumes that the mean expected demand for given time/day is known with reasonable reliability and won't significantly change within the modelled timeframe. Any significant changes in the mean, such as long-term increase of traffic, new capacity restrictions, or possible detours, should be reflected by other means as they would be normally.

CONCLUSIONS

Based on the presented results, normal, lognormal, and gamma distribution all seem like good adepts for describing the inter-day variability of traffic flow at given time of day with quite similar counts of best fits among them with the other two usually not being statistically significantly worse. On the other hand, both Weibull and inverse Weibull distributions can be dismissed as potential candidates given the presented results. Two different approaches for modelling random day-to-day or week-to-week fluctuations in traffic demand are recommended for situations when only the mean expected values are known.

Based on the case study, the TF distribution seems quite variable during the week and perhaps no single theoretical distribution is perfect for describing it. Even larger data sample, different aggregation intervals, or involving some sort of time series in the modelling might help to bring more clarity in future studies. Further, similar experiments should be performed on various locations and types of roads to see if the patterns would change with respect to the surrounding road network and its capacity.

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