# COMPETITION AND COOPERATION BETWEEN SUPPLIERS IN MULTIMODAL NETWORK DESIGN PROBLEMS

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# 1 INTRODUCTION

The introduction of new mobility services in the transport market (e.g., carsharing, bike sharing, e-scooters, etc.) increases both the multimodal alternatives offered to travellers, and the competition among service providers for attracting customers. Sometimes cooperation between different Mobility Service Providers (MSPs) is observed, for instance when services are pooled via integrated fare systems or via the Mobility-as-a-Service paradigm, or thanks to strategic partnerships designed to offer a service with adequate resources and capture sufficient demand to be economically sustainable.

Recent works (Nair and Miller-Hooks, 2014, Nguyen et al., 2022) try to capture the interactions between classical transport services and shared services, but it remains unclear how such systems respond under various scenarios: suppliers entering or exiting the network; increasing heterogeneity of the users; changes in user preferences and external incentives/regulation (such as governmental policies or subsidies). In order to capture such phenomena, and represent the interactions between multiple MSPs and travellers of the transport network, this paper formulates an Equilibrium Problem with Equilibrium Constraints (EPEC). At the upper level, each MSP seeks to maximize profits, and their objective functions contain the specific types of costs and revenues faced by each of them. At the lower level, users are divided into classes that capture their heterogeneity in terms of daily trip chains and personal characteristics. The users' multimodal network is non-separable and therefore the equilibrium assignment is written as Variational Inequality (VI).

# 2 METHODOLOGY

In this paper the behaviours of, and relationships between MSPs and users are modelled as a supernetwork (or hypernetwork (Sheffi and Deganzo, 1978)). We do not explicitly represent the real underlying transport infrastructure; instead we divide the transport network into several unimodal networks (colored links and layers in Figure 1) each representing one type of mobility service being offered.

Travellers are divided into $K$ classes based on their personal attributes and daily trip chains. For each class, their sequence of activities is modelled as a graph (black layer in Figure 1), where nodes indicate activity locations and links indicate trips to move from one activity location to the next.

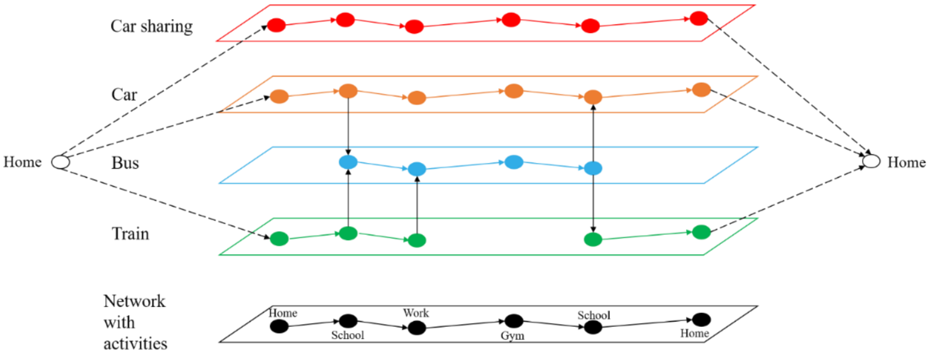

Figure 1: Supernetwork trip chain (adaptation from Carlier et al., 2003)

Each traveller must choose a path through the multimodal network in order to access their sequence of activity locations. A path (trip chain) can comprise three different types of links. Access links (black dashed lines) allow users to access a mode of transport from their origin (Home), and egress from a mode of transport to reach their final destination (Home). Mode-specific links (horizontal link) indicate trips made from one activity location to another using a specific mode of transport (designated by colour). Interchange links (vertical black link) allow users to move from one mode of transport to another.

We assume that each MSP seeks to maximize the profit arising from their mobility service, represented here as a uniquely defined layer of the supernetwork. They collect revenues based on how many travellers use the service, and accrue costs primarily depending on the size of their vehicle fleet. This is the decision variable for each MSP: the capacity or number of vehicles they provide. Moreover, at equilibrium, MSPs decide how to strategically distribute these vehicles amongst the links of their network layer. One innovative aspect of our formulation is the flexibility to represent costs and revenues for different mobility services, such as car-sharing, bike-sharing, bus, train, e-scooter and taxi, and hence calculate their profits. In the special case of only one service provider, this problem collapses into the more conventional Network Design Problem.

Each user class is assigned to the multimodal network following fixed demand traffic equilibrium, using a path-based adaptation of the multi-class and multicriteria network equilibrium model (Nagurney, 2000). Explicit enumeration of paths is used for now. The lower level equilibrium decision variables are the vector of path flows.

One aspect that increases the complexity of the problem is the interdependency between flows on parallel links of the supernetwork that represent the same real transport link of the underlying infrastructure network. Consequently, the link costs are non-separable. The users' equilibrium is therefore formulated as a VI, and the algorithm used to calculate it is the Extragradient method (or modified projection method) (Nagurney, 1999).

# 3 EXAMPLE AND DISCUSSION

In this section, we show the application of the described methodology solving the users' equilibrium at the lower level for one class of travellers. The upper level equilibrium for different MSPs defined in our formulation it is illustrated below, but not explicitly solved it in this paper.

We first illustrate a simple scenario in which a user class with a Home-Work-Home tour has two modes available: private car and a bus service (Figure 2 left). The costs perceived by users are flow dependent. More specifically, the private car's travel time will increase with congestion, while the

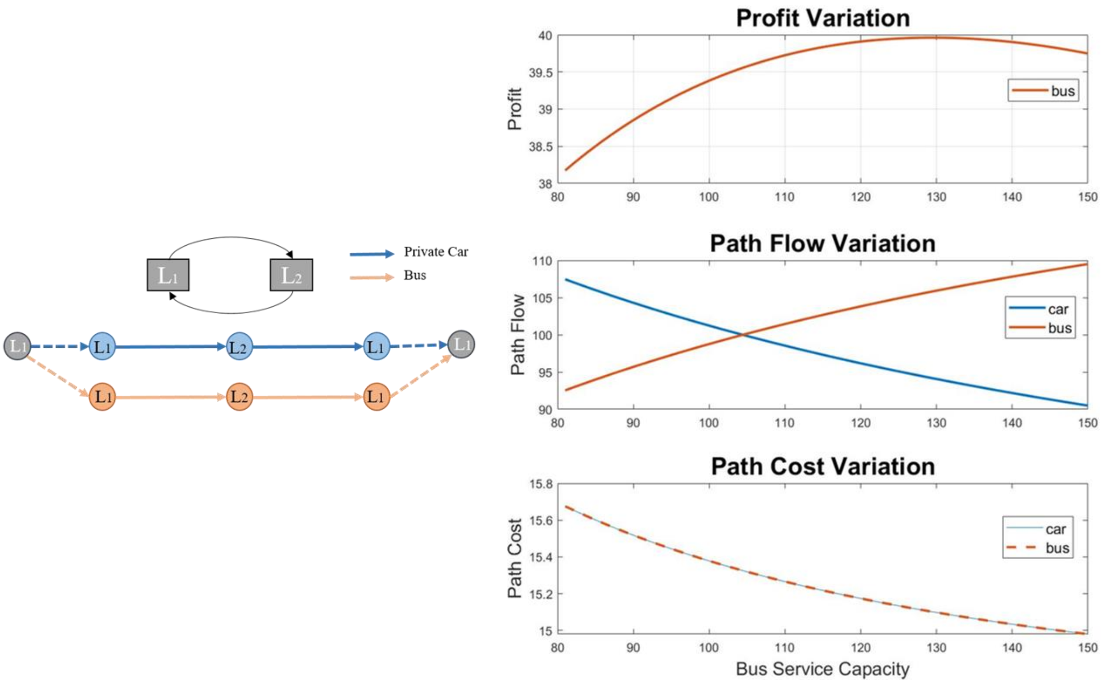

Figure 2: Example with two modes of transport and one MSP

attractiveness of the bus service will decrease based on the waiting time at the bus stop. Here the bus company is the only MSP in the network, and they seek to maximize their profit by varying the capacity of their service (“Profit Variation” in Figure 2).

The equilibrium between a bus provider and a single user class is shown on the right side of Figure 2. The abscissa of all plots shows increasing the capacity (frequency) of the bus. As expected, this results in users swapping from private car to bus (“Path Flow Variation”). The lower right graph of Figure 2 (“Path Cost Variation”) shows the change in path costs, and that they coincide. Convergence of the equilibrium algorithm was assured using a standard path cost gap function.

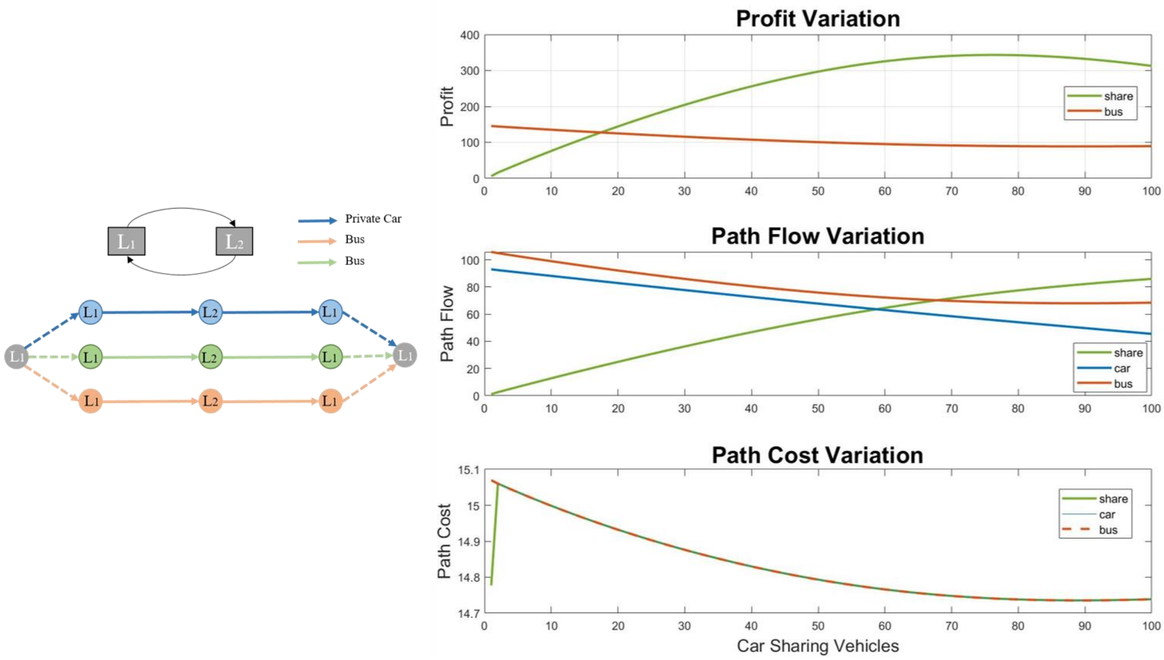

Figure 3: Example with Three modes of transport and two MSPs

Building on this base scenario, we introduce a car sharing service. Bus service capacity is fixed corresponding to maximum profit from above, the number of vehicles of the shared service is increased (abscissa of all plots in Fig 3). The system equilibrium changes drastically (Figure 3). While the bus service has a dedicated lane, the same roads are used by private car and car sharing who therefore both contribute to the congestion experienced. Initially the car sharing service is unused, due to the absence of vehicles in the network. As the number of car sharing vehicles increases, users are less attracted from private car and bus; the bus MSP’s profit naturally decreases.

From this example, it is clear that the bus provider needs to respond (in terms of their service frequency) to the car sharing MSP’s fleet size, to mitigate against loss of profits. In Figure 4 we show the variation in profits for car share MSP and bus MSP at different combinations of their decision variables, illustrating the upper level MSP equilibrium problem.

Despite this simple illustrative example, the methodology accommodates multiple MSPs at the upper level adopting different strategies. Typically they compete in pursuit of profit, and to become market leader. In other contexts that we are currently investigating, cooperation may occur with the aim of MSPs collectively increasing their share of demand and hence revenues.

The lower level equilibrium representing users’ choices depends on the MSPs’ behaviour: the modes of transport available, type of mobility subscriptions, prices and fleet sizes. At the same time, the user equilibrium influences MSPs’ decisions at the upper level via revenues accrued. A poorly used mobility service will push the MSP to change strategy, or it will disappear from the market because it is unprofitable. Additionally, we are now applying our model to study how a new MSP entering the market affects users' equilibrium at the lower level, and competition at the upper level where MSPs will change their strategies in order to protect their profits.

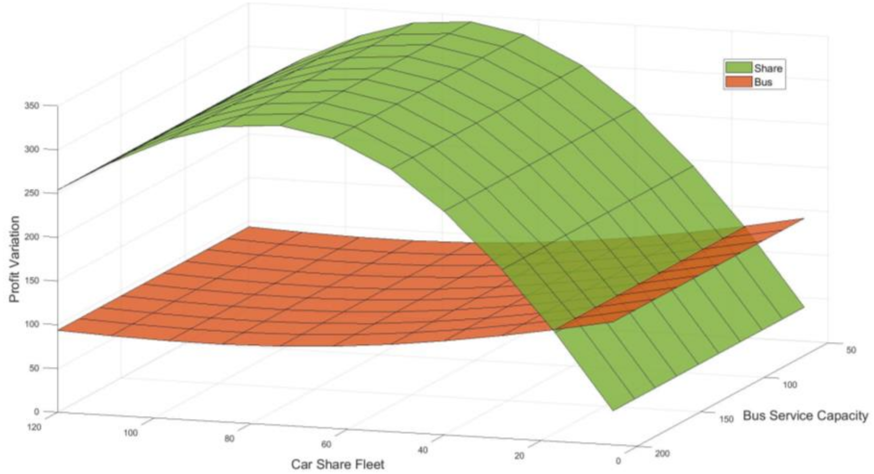

Figure 4: Profit Variation with capacity

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