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Procedia CIRP 36 (2015) 6 – 11

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CIRP 25th Design Conference Innovative Product Creation

Validation of an extended approach to multi-robot cell design and motion planning

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Abstract

According to both industrial practice and literature, multi-robot cell design and robot motion planning for vehicle spot welding are two sequential activities, managed by different functional units through different software tools. Due to this sequential computation, the whole process suffers from inherent inefficiency. In this work, a new methodology is proposed, that overcomes the above inefficiency through the simultaneous resolution of design and motion planning problems. Specifically, three mathematical models were introduced that (i) select and positions the resources, (ii) allocate the tasks to the resources and (iii) identify a coordinated robot motion plan. Based on the proposed methodology, we built three ad-hoc cases with the goal to highlight the relations between design, motion planning and environment complexity. These cases could be taken as reference cases so on. Moreover, results on an industrial case are presented.

Peer-review under responsibility of the scientific committee of the CIRP 25th Design Conference Innovative Product Creation
Keywords: Multi-robot cells; Design optimization; Motion planning

1. Introduction

The assembly of the vehicle metal panels and vehicle body-in-white through multi-robot spot-welding cells is generally outsourced by automotive companies to original equipment manufacturers (OEMs). OEMs need to provide the best offer in terms of price per produced unit, while coping with the requests of the clients. These requests include the required production volumes which in turn define the cell cycle time for the execution of a set of welding points and the employment of a predefined body-in-white fixturing systems and transportation device which introduces a set of geometrical constraints. In such a contest, cell design and motion planning are two relevant time-consuming critical activities. Even if the mutual-influence of the multi-robot cell design and motion planning cannot be ignored, current industrial practice is based on the division of these activities and the employment of several methodologies and software tools.

In order to support OEMs to reach these goals, the conceived research focuses on the analysis of design and motion planning problems for multi-robot body-in-white assembly cells. Specifically, this research has led to the development of a methodology able to simultaneously and automatically solve both the problems.

The paper is structured as following: Section 2 presents the state of the art; the approach is described in Section 3 highlighting the innovative aspects in comparison to previous work; Section 4 validates the approach through 3 ad-hoc cases and an industrial case; finally, conclusions and future work are given in Section 5.

2. Literary review

Although multi-robot cell design and off-line motion planning have been investigated for more than two decades, many issues are still open since (i) the complexity of the design and motion planning that represent a barrier for straightforward optimal solution, and (ii) multi-disciplinary activities and research fields are required. Specifically, the integration between the two activities has not been adequately investigated in literature so far. In [1], a 3D optimized layout for assembly

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doi:10.1016/j.procir.2015.01.062

cells is proposed, when resources, tasks and product geometry are given. A similar approach in terms of the sequential execution of the design and motion planning can be found in [2]. This paper proposed an approach for the optimization of the layout of a cell consists of two conveyor belts for part feeding, two manipulators and an assembly station. Once a possible layout is generated, robot trajectories are calculated taking into account the pre-allocated tasks. Similarly, [3] proposed a method for the selection of the most appropriate manipulator systems (combination of a robot arm and positioning table) from a set of candidate systems within the desired calculation time. Location optimization and motion coordination are integrated to derive the task completion time but robot tasks are pre-allocated. A more extended approach for the design of a cooperating robot cell can be found in [4]. Starting from an initial and rough solution, the approach leads to the definition of a final collision-free solution with optimized cycle time. However, the motion planning and collision problems are partially taken into account. A complete off-line programming toolbox for remote laser welding was proposed in [5]. The approach can provide an automated method for computing close-to-optimal robot programs. The approach has been positively tested on real industrial cases. However, the problem of robot positing is not managed.

3. Approach

The approach hereafter proposed and validated is an extension of [6,7]. The approach is based on 3 stages dealing with the motion planning for single robots, the design of the cell and the robot coordination. The simultaneous resolution of cell design and robot coordination is granted by the possibility to iteratively solve the problem till a feasible solution is found. Specifically the provided design is optimal in terms of cell investment costs and feasible in terms of robot motion plans.

The input, output and the three stages of the approach are hereafter briefly described. The differences respect to [6,7] are highlighted and are in each of the stages:

- A different trajectory generation method for stage 2 for a better exploration of the configuration space
- A new objective function for Stage 2 to better condition the identification
- A new mathematical model for the Stage 3

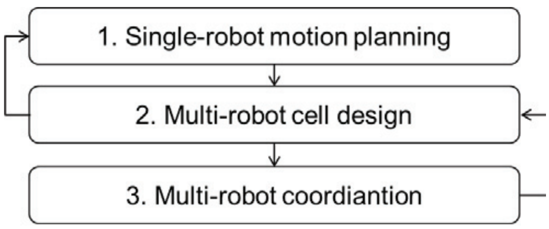


Fig. 1. The approach

3.1. Input & Output

The inputs and the outputs required of the approach have been detailed in Table 1 and Table 2.

Table 1. Model inputs.

Input	Range	Unit	Description
BIW		-	Body-In-White or metal sheets that have to be welded.
WP_{wp}	$1..N^{WP}$	-	Welding Points *WPs*. Position, *mm*, and orientation *deg* in the cell system of the points that have the be welded. N^{WP} denotes the number of possible *WPs* plus a fictitious point that represents the robot initial and ending configuration.
BF		-	Body-in-white Fixturing system of the BIW during the welding process.
BTD		-	BIW transportation device. It transport the BIW in and out of the cell.
RM		-	Robot Model. Type of robot employed.
RSM		-	Robot Support structure Model. System on which the robot are mounted. This system influences the position and the orientation of the robots in the cell.
RPO_{rpo}	$1..N^{RPO}$	-	Possible Robot Position and Orientation *RPOs* in the cell. N^{RPO} denotes the number of possible *RPOs*.
WGM_{wgm}	$1..N^{WGM}$	-	Welding gun models *WGMs* to be allocated to the robots. N^{WGM} denotes the number of possible *WGMs*.
RCT	R	s	Required cycle time. Imposed by the client, it represent the maximum cycle time of the cell.
NC^{RM}	N	-	Number of already aCquired *RM*.
NC^{RSM}	N	-	Number of already aCquired *RSM*.
NC^{WGM}_{wgm}	N	-	Number of already aCquired WGM_{wgm}.
$COST^{RM}$	R	€	Cost of *RM*.
$COST^{RSM}$	R	€	Cost of *RSM*.
$COST^{WGM}$	R	€	Cost of WGM_{wgm}.
WT_{wp}	R	s	Welding time for each WP_{wp}

Table 2. Model outputs.

Output	Range	Unit	Description
COST	R^+	€	Cell investment cost
TN^{RM}	N	-	Total Number of required *RM*.
TN^{RSM}	{0,1}	-	Total Number of required *RSM*.
TN^{WGM}_{wgm}	N	-	Total Number of required WGM_{wgm}.
NA^{RM}	N	-	Number of *RM* to be Acquired.
NA^{RSM}	{0,1}	-	Number of *RSM* to be Acquired.
NA^{WGM}_{wgm}	N	-	Number of required WGM_{wgm} to be Acquired.
$RGP_{wgm,rpo}$	{0,1}	-	Allocation of the welding guns to the robots – Equal to 1 if robot mounting WGM_{wgm} is in RPO_{rpo}.
$WPA_{rpo,wp}$	{0,1}	-	Equal to 1 if the welding point WP_{wp} is allocated to RPO_{rpo}.
$MP_{wgm,rpo,wp1,wp2}$	{0,1}	-	Motion plan for robot in RPO_{rpo} with WGM_{wgm} from WP_{wp1} to WP_{wp2} – Equal to 1 if robot in RPO_{rpo} processes WP_{wp2} immediately after WP_{wp1}.
$MTT_{wgm,rpo,wp1,wp2}$	R^+	s	Time necessary to robot in RPO_{rpo} mounting WGM_{wgm} to move from WP_{wp1} to WP_{wp2} and weld WP_{wp2}.
$C_{wgm,rpo,wp1,wp2}$	R^+	s	Completion time for robot in RPO_{rpo} mounting WGM_{wgm} to move from WP_{wp1} to WP_{wp2} and weld WP_{wp2} [s].
$I_{wgm,rpo,wp1,wp2}$	R^+	s	Starting time for robot in RPO_{rpo} mounting WGM_{wgm} to move from WP_{wp1} to WP_{wp2} and weld WP_{wp2} .
$D_{wgm,rpo,wp1,wp2}$	R^+	s	Temporal delay for robot in RPO_{rpo} mounting WGM_{wgm} to move from WP_{wp1} to WP_{wp2} and weld WP_{wp2}.
OCT_{rpo}	R^+	s	Obtained cycle time for robot in RPO_{rpo}.
MAXOCT	R^+	s	Obtained cell cycle time.

3.2. Approach stages

During single robot motion planning (Stage 1), a motion plan is defined for each robot position and orientation and each welding gun, i.e. for each couple $\{RPO_{rpo}, WGM_{wgm}\}$. The Stage has been carefully described in [6,7] in terms of strategy, employed motion planner [8] and collision detection algorithms [9]. Respect to [6,7], trajectory generation exploits probabilistic roadmap techniques with lazy collision [10,11]. The employment of lazy collision allow the generation of extended roadmap and the simultaneous reduction of the computational time. Moreover, a criterion based on the minimization of the joint movements is selected. The idea is to evaluate the distance between the joints in the joint space. This criterion limits the unnecessary movements of the robots in the workspace.

During multi-robot cell design (Stage 2), the design of the cell is identified through the selection of the necessary resources in terms of robot, their position/orientation in the cell, allocation of the welding guns to the robots. Together with the cell design, a first motion plan solution is generated for each robot. Thus, welding points are allocated to the robots and a welding sequence is generated. This motion plan do not take into account the possible collision among the robots and will be revised during the Stage 3 of the approach. Moreover, the allocation of the welding points to the robot is not unique: a welding point can be allocated to more than one robot. Multi-robot cell design is based on a mathematical mixed-integer linear mathematical model aiming at the minimization of the cell investment costs. In comparison to [6,7], the objective function has been modified eliminating the penalties for the obtainment of a cycle time greater than the required cycle time is not present. Indeed, *RCT* is a specific client request that have to be necessarily and correctly answered.

Minimize:

$$COST = \begin{Bmatrix} NA^{RM} COST^{RM} + \sum_{wgm} \left(NA^{WGM_{wgm}} COST^{WGM_{wgm}} \right) + \\ NA^{RSM} COST^{RSM} \end{Bmatrix} \quad (1)$$

Subject to:
9 Resource constraints
13 Motion plan constraints
2 Cycle time constraints

Stage 3 of the approach aims at coordinating the robots on the basis of the cell design produced by the Stage 2. Robot coordination is actually based on 3 sub-stages, making the here-proposed approach a decoupled approach [10,12]. Stage 3.1 takes into account the cell design proposed by Stage 2 and provides the final allocation of the welding points to the robots and a final motion plan for each robot. Then, Stage 3.2 evaluates for each couple of trajectories belonging to the identified motion plan possible collisions. Potential collisions will be avoided though the cell motion plan scheduling in Stage 3.3. Specifically, Stage 3.1 and 3.3 are based on two mixed-integer mathematical models aiming at minimizing the cell cycle time (Eq. 2 and Eq. 3). In comparison to [6,7], the mathematical model of Stage 3.3 was modified eliminating from Eq. 3 unnecessary terms and reformulating several constraints. On the contrary, Stage 3.2 is based on a volume-swept-like algorithm, not present in [6,7].

Minimize:

$$MAXOCT = \max_{rpo} \{ OCT_{rpo} \} \quad (2)$$

Subject to:
13 Motion plan constraints
2 Cycle time constraints

Minimize:

$$MAXOCT = \max_{rpo} \{ OCT_{rpo} \} \quad (3)$$

Subject to:
8 Motion plan constraints
2 Cycle time constraints

4. Approach Validation

The proposed approach has been validated on 3 ad-hoc cases. These cases are hereafter described in order to be easily replicated and employed as reference cases. Finally, an industrial test case is shortly presented.

4.1. Case 1

Case 1 (Table 3) aims to solve the cell design and motion planning for the welding of 8 *WPs* (Table A1) with 2 possible WGMs (Table A2, Fig. A1) and with the robot "COMAU Smart NJ4-175-2.2" to be placed in 6 possible *RPOs* (Table A4, Fig. 2-3). The robot D-H parameters are described in Table A3. *BIW*, *BF*, *BTD* and *RSM* are represented by 13 simplified obstacles: 8 cubic obstacles and 5 parallelepipeds (Fig. 2). The obstacles position (randomly generated) and orientation are described in Table A5 through rototranslation matrices referring to the cell system. The number and cost of available resources are depicted in Table 3.

Table 3. Input – Case 1.

Input	Description
BIW, BF, BTD, RSM	Represented by 13 obstacles
N^{WP}	8
N^{RPO}	6
N^{WGM}	2
RCT	30*s*
NC^{RM} NC^{RSM}	0 0
$NC^{WGM}{}_{wgm}$	0, 0
$COST^{RM}$ $COST^{RSM}$	25680€ 150000€
$COST^{WGM}$	9255€, 9655€
WT_{wp}	0.8*s*

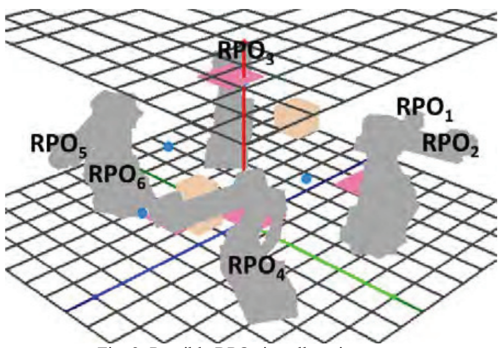


Fig. 2. Possible RPOs in cell environment

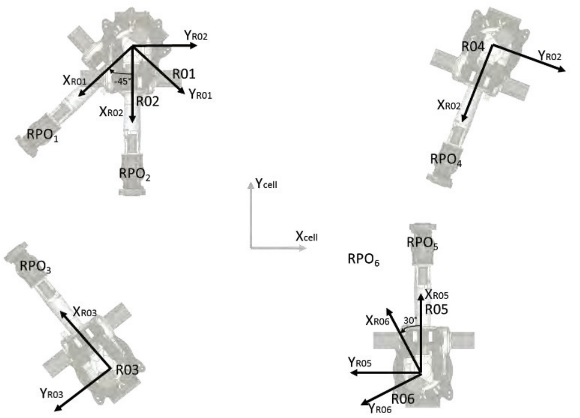


Fig. 3. Robot position and orientations (not considering robot initial orientation) – Case 1

The resolution of the problem took 48 hours, mainly for Stage 1. The final solution (Fig. 4) is characterized by the selection of 3 robots in RPO_2, RPO_3, RPO_4. The welding gun model WGM_2 is allocated to the robot in RPO_2 and RPO_4, while WGM_1 is allocated to the robot in RPO_3. Moreover, RPO_2 is responsible for the welding of WP_5, WP_7, and WP_8 (sequence 8→7→5); RPO_3 is responsible for the welding of WP_1 and WP_3 (sequence 1→3); RPO_4 is responsible for the welding of WP_2, WP_4 and WP_6 (sequence 4→2→6). The cell cycle time is equal to 20.8 s. thus coping with the *RCT*. Finally, the cell cost is equal to 255605 €. From the robot selection point of view, this cost is minimized since the minimum number of necessary robots is found (none combination of only 2 *RPO*/*WGM* grants the machinability of all the *WPs* – Table 4). From the welding gun point of view, WGM_2 is selected twice even if it is more expensive than WGM_1. This result can be easily explained observing Table 4: WGM_1 independently from the robot position present is able to reach a limited number of *WPs* (5 out of 8 against the 8 out of 8 of WGM_2).

Table 4. WP reachability – Case 1.

WP_{wg}	RPO_{rpo}/WGM_{wgm} able to reach WP_{wp}
WP_1	RPO_1/WGM_2; RPO_2/WGM_2; RPO_3/WGM_1; RPO_4/WGM_2
WP_2	RPO_4/WGM_2; RPO_5/WGM_2; RPO_6/WGM_2
WP_3	RPO_3/WGM_1; RPO_5/WGM_1; RPO_6/WGM_1
WP_4	RPO_1/WGM_2; RPO_2/WGM_2; RPO_4/WGM_2
WP_5	RPO_1/WGM_1; RPO_1/WGM_2; RPO_2/WGM_2; RPO_3/WGM_1; RPO_4/WGM_2
WP_6	RPO_4/WGM_2; RPO_6/WGM_2
WP_7	RPO_1/WGM_2; RPO_2/WGM_2; RPO_3/WGM_1; RPO_5/WGM_1; RPO_5/WGM_2; RPO_6/WGM_1
WP_8	RPO_1/WGM_2; RPO_2/WGM_2; RPO_3/WGM_2;

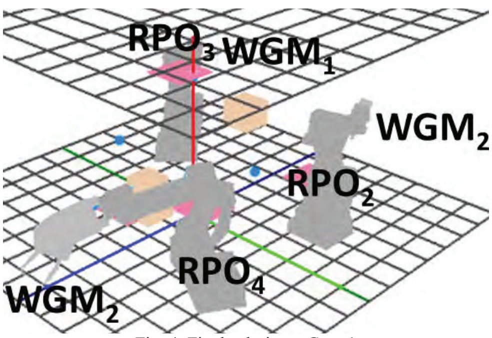


Fig. 4. Final solution – Case 1

4.2. Case 2

Case 2 is defined as an extension of Case 1. The considered set of input is unchanged apart from the number and position of obstacles in the cell. Specifically, the number of obstacles is doubled (26 obstacles). New obstacles positions are presented in Table A6 and in Fig. 5.

Results (Fig. 6) partially confirm the cell design identified in Case 1, since robot in RPO_1 is selected instead of robot in RPO_2. Moreover, a different motion plan is generated. Specifically, RPO_1 visits WP_1, WP_4, WP_5, WP_7, and WP_8 (sequence 1→5→4→7→8); RPO_3 welds WP_3; RPO_4 is responsible for the welding of WP_2 and WP_6 (sequence 2→6). The cell cycle time is equal to 23.36 s. Since the *WP* reachability in Table 4 is still valid for Case 2, the different *WP* allocation is due to the presence of obstacles that lead to the definition of complex path.

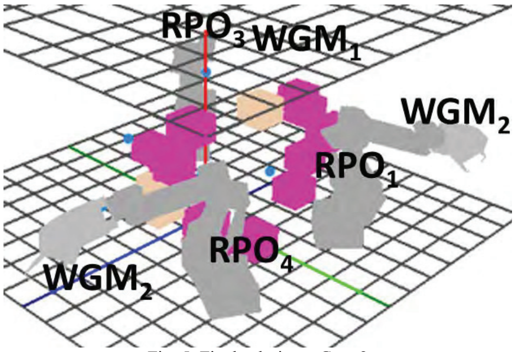


Fig. 5. Final solution – Case 2

4.3. Case 3

As for Case 2, Case 3 present an increased complexity of the environment. The number of obstacles reaches 38. Table A7 presents the positions of the new obstacles.

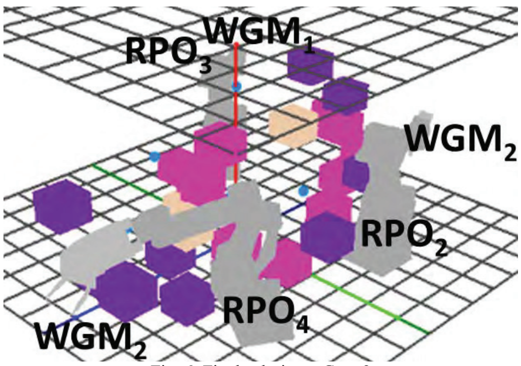


Fig. 6. Final solution – Case 3

Once again, the cell design solution is confirmed (Fig. 6) as well as the welding point allocation and sequence. However, because of the increasing number of obstacles, the trajectories generated for Case 3 by Stage 1 are mainly different from the trajectories generated in Case 2. Moreover, the final cycle time requires 2 seconds more (25.72 s). Thus, it seems that the time for the final motion plan increases with the complexity of the environment.

4.4. Industrial case

The presented approach have been tested on a real industrial case. The case was provided by an Italian overall equipment manufacturer. The multi-robot cell is composed by 5 robots SMART-5 NJ4-175-2.2 (Fig. 7) mounted on a bridge support structure, 3 welding gun models (Fig. 8), a fixturing systems composed by 34 elements. The bridge structure presents 6 possible positions for the robots and 3 possible orientations for each position, for a total of 18 possible *RPOs*. The first robot (R01) mounts the WGM_1 and welds 5 points in 25.9*s*, imposing the cell cycle time. The second robot (R02) mounts the same welding gun as in R01 robot but is responsible for 4 welding points with a cycle time of 24.74*s*. The third robot (R03) mounts the WGM_2. Its cycle time for execution of 4 welding points is 22.98*s*. Finally, fourth (R04) and fifth (R05) robots have a cycle time of 22.33*s* and 20.99*s*, respectively, for the execution of 4 *WPs* with the WGM_3. WGM_1, WGM_2 and WGM_3 present a different spatial occupancy and costs and grant a different accessibility.

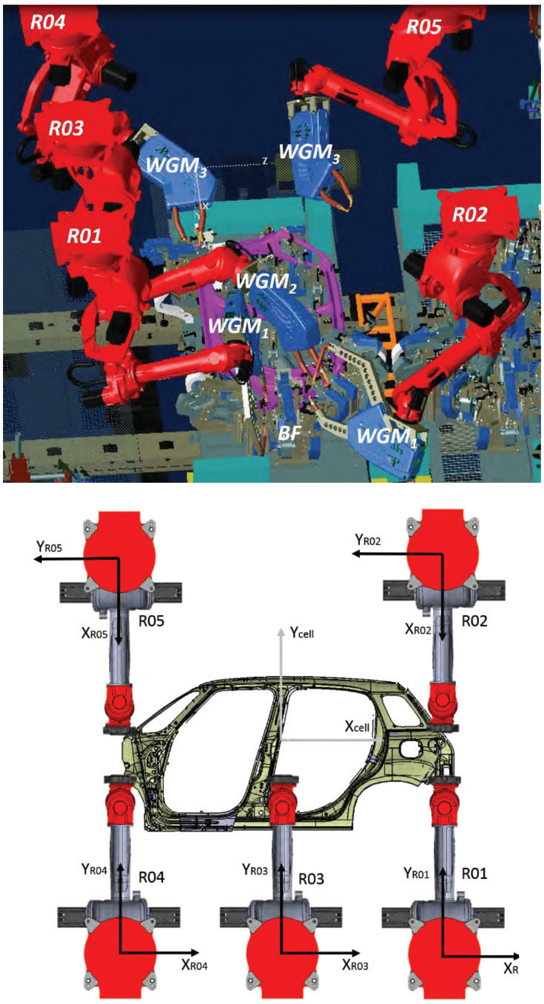


Fig. 7. Welding guns – Industrial case

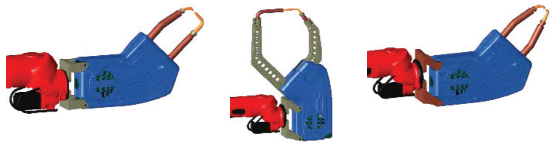


Fig. 8. Welding guns – Industrial case

Up to now, the industrial case have been employed for the analysis of Stage 1 and 3, i.e. for the definition of the motion plan. A coordinated motion plan was successfully obtained. However, the current obtained cycle time is equal to 42 s. Indeed, some automatically path presents unnecessary motion that make complex the subsequent coordination of the robots. Then, current studies are focusing on the improvement of the techniques exploited for the definition of single-robot motion planning.

5. Conclusions and future work

The proposed approach is able to simultaneously solve the design and motion planning problems for multi-robot spot-welding cells for body-in-white assembly. The approach represents a precious software tool to support human operators during the resolution of these problems. The paper presents the novelties respect to previous works and demonstrate the approach feasibly on three ad-hoc cases to be employed as reference cases. Moreover, the results on an industrial test case are shown.

Appendix A. Test case data

Hereafter the detailed data of case 1, 2 and 3 are presented.

Table A1.Welding points – Case 1.

WP_{wp}	Position *mm* in cell reference system	Orientation *deg* in cell reference system – Z'Y''Z''	Rot range *deg* along ZGi sys
WP_1	-689.36 -784.45 +257.2	-166.36 92.54 77.26	-10 15 5
WP_2	1201.99 -1098.56 195.64	-59.71 108.49 -60.62	0 20 5
WP_3	601.99 -1098.56 1165.64	-139.71 108.49 -80.62	-30 20 5
WP_4	-659.36 -784.45 +157.2	-156.36 62.54 37.26	0 0 0
WP_5	-389.36 -284.45 +457.2	-166.36 92.54 77.26	0 20 5
WP_6	701.99 -798.56 395.64	-59.71 108.49 -60.62	-20 0 5
WP_7	-998.07 427.73 566.75	-87.68 96.95 27.24	-15 45 5
WP_8	0.0 0.0 2400.0	0.0 0.0 0.0	-20 0 5

Table A2.Welding guns – Case 1.

WGM_{wgm}	Rototranslation matrix from gun system (Gi) to robot tool system (R0iT)	Rototranslation matrix from gun system control point (GiCP) to gun system (Gi)
WGM_1	[0 -1 0 0; -1 0 0 0; 0 0 -1 0; 0 0 0 1]	[0 -1 0 0; 0 0 1 -1277; -1 0 0 -435; 0 0 0 1]
WGM_2	[0 0 -1 0; 1 0 0 0; 0 -1 0 0; 0 0 0 1]	[0 1 0 0.89; -0.707 0 -0.707 -1180.16; -0.707 0 0.707 -557.84; 0 0 0 1]

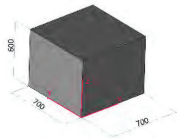
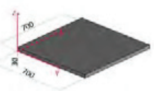
Table A3. D-H for COMAU Smart NJ4-175-2.2 – Case 1.

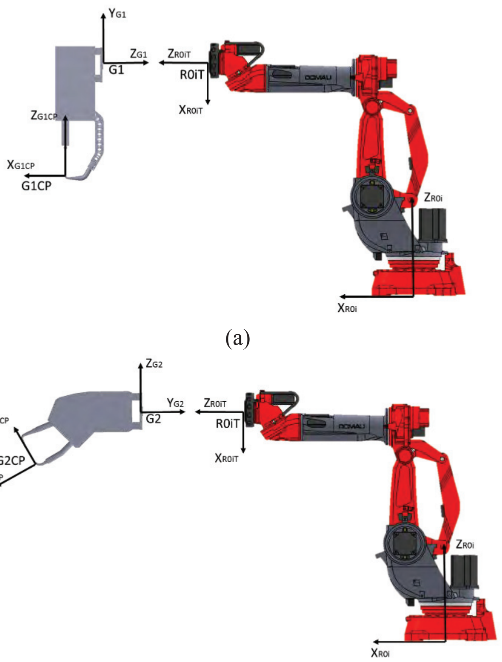
Input	Description
a mm	350 750 250 0 0 0
α rad	π/2 π -π/2 2π/3 -2π/3 0
d mm	-830 0 0 -1097 150 -198
θ *rad*	θ_1 $\theta_2-\pi/2$ θ_3 $\theta_4+\pi/2$ θ_5 θ_6
base frame	[1 0 0 0; 0 -1 0 0; 0 0 -1 0; 0 0 0 1]
tool frame	[0 1 0 0; 1 0 0 0; 0 0 -1 0; 0 0 0 1]

Table A4. Robot position and orientation – Case 1.

RPO_{rpo}	Rototranslation matrix in cell reference system. Position in *mm*	Rotation *deg* along robot Z dir	Robot initial joint conf *rad*
RPO_1	[0 1 0 -1000; -1 0 0 1800; 0 0 1 0; 0 0 0 1]	-45	1.3472 -0.3491 -1.9199 0 0.3491 0
RPO_2	[0 1 0 -1000; -1 0 0 1800; 0 0 1 0; 0 0 0 1]	0	1.3472 -0.3491 -1.9199 0 0.3491 0
RPO_3	[0 -1 0 -1400; 1 0 0 -1800; 0 0 1 0; 0 0 0 1]	45	-1.3472 -0.3491 -1.9199 0 0.3491 0
RPO_4	[0 1 0 1400; -1 0 0 1800; 0 0 1 0; 0 0 0 1]	-30	-1.3472 -0.3491 -1.9199 0 0.3491 0
RPO_5	[0 -1 0 1000; 1 0 0 -1800; 0 0 1 0; 0 0 0 1]	0	1.3472 -0.3491 -1.9199 0 0.3491 0
RPO_6	[0 -1 0 1000; 1 0 0 -1800; 0 0 1 0; 0 0 0 1]	30	1.3472 -0.3491 -1.9199 0 0.3491 0

Table A5. Obstacles position and orientation – Case 1.

Obstacles type	Rototranslation matrix in cell reference system. Position in *mm*
Cube 	[1 0 0 -800; 0 1 0 -700; 0 0 1 1200; 0 0 0 1] [1 0 0 1000; 0 1 0 50; 0 0 1 600; 0 0 0 1] [1 0 0 -900; 0 1 0 300; 0 0 1 0; 0 0 0 1] [1 0 0 -1000; 0 1 0 150; 0 0 1 1500; 0 0 0 1] [1 0 0 -1500; 0 1 0 -300; 0 0 1 900; 0 0 0 1] [1 0 0 -100; 0 1 0 -1200; 0 0 1 0; 0 0 0 1] [1 0 0 200; 0 1 0 0; 0 0 1 100; 0 0 0 1] [1 0 0 -200; 0 1 0 -400 0 0 1 600; 0 0 0 1]
Plate 	[1 0 0 0; 0 1 0 0; 0 0 1 600; 0 0 0 1] [1 0 0 -2100; 0 1 0 700; 0 0 1 400; 0 0 0 1] [1 0 0 1000; 0 1 0 -400; 0 0 1 600; 0 0 0 1] [1 0 0 200; 0 1 0 400; 0 0 1 620; 0 0 0 1] [1 0 0 -300; 0 1 0 -600; 0 0 1 2400; 0 0 0 1]



(a)

(b)

Fig. A1. Welding gun position and orientation – Case 1: (a) WGM_1, (b) WGM_2

Table A6. Obstacles position and orientation – Case 2.

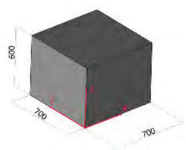
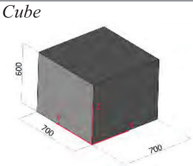
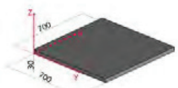
Obstacles type	Rototranslation matrix in cell reference system. Position in *mm*
Cube 	[1 0 0 400; 0 1 0 -300; 0 0 1 1000; 0 0 0 1] [1 0 0 500; 0 1 0 200; 0 0 1 1700; 0 0 0 1] [1 0 0 400; 0 1 0 400; 0 0 1 200; 0 0 0 1] [1 0 0 -700; 0 1 0 -2000; 0 0 1 200; 0 0 0 1] [1 0 0 0; 0 1 0 1000; 0 0 1 0; 0 0 0 1] [1 0 0 -1600; 0 1 0 400; 0 0 1 0; 0 0 0 1] [1 0 0 -2600; 0 1 0 -200; 0 0 1 0; 0 0 0 1] [1 0 0 -2700; 0 1 0 -500; 0 0 1 1000; 0 0 0 1] [1 0 0 -3000; 0 1 0 -100; 0 0 1 300; 0 0 0 1] [1 0 0 -3000; 0 1 0 100; 0 0 1 500; 0 0 0 1] [1 0 0 -2400; 0 1 0 -100; 0 0 1 1000; 0 0 0 1]

Table A7. Obstacles position and orientation – Case 3.

Obstacles type	Rototranslation matrix in cell reference system. Position in *mm*
Cube 	[1 0 0 600; 0 1 0 0; 0 0 1 1000; 0 0 0 1] [1 0 0 2000; 0 1 0 -600; 0 0 1 0; 0 0 0 1] [1 0 0 1100; 0 1 0 -200; 0 0 1 100; 0 0 0 1] [1 0 0 1700; 0 1 0 800; 0 0 1 100; 0 0 0 1] [1 0 0 0; 0 1 0 1700; 0 0 1 700; 0 0 0 1] [1 0 0 2800; 0 1 0 -200; 0 0 1 1500; 0 0 0 1] [1 0 0 2900; 0 1 0 1100; 0 0 1 900; 0 0 0 1] [1 0 0 -2200; 0 1 0 600; 0 0 1 500; 0 0 0 1] [1 0 0 -2200; 0 1 0 -600; 0 0 1 1900; 0 0 0 1] [1 0 0 -2600; 0 1 0 -200; 0 0 1 1400; 0 0 0 1] [1 0 0 -2400; 0 1 0 -100; 0 0 1 1000; 0 0 0 1]
Plate 	[1 0 0 -2400; 0 1 0 -100; 0 0 1 1000; 0 0 0 1] 1 0 0 -1000; 0 1 0 -1300; 0 0 1 1600; 0 0 0 1] [1 0 0 500; 0 1 0 500; 0 0 1 500; 0 0 0 1]

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