Gaussian Tribonacci R-Graceful Labeling of Some Tree Related Graphs

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Abstract

Let r be any natural number. An injective function $\phi: V(G) \to \{0, ki, 1, 1+ki, 2, 2+ki, ..., GT_{q+r-1}\}$ *for all k*, where GT_{q+r-1} is the $(q+r-1)^{th}$ Gaussian Tribonacci number in the Gaussian Tribonacci sequence is said to be Gaussian Tribonacci r-graceful labeling if the induced edge labeling $\phi^*: E(G) \to \{GT_1, GT_2, ..., GT_{q+r-1}\}$ such that $\phi^*(uv) = |\phi(u) - \phi(v)|$ is bijective. If a graph G admits Gaussian Tribonacci r-graceful labeling, then G is called a Gaussian Tribonacci r-graceful graph. A graph G is said to be Gaussian Tribonacci arbitrarily graceful if it is Gaussian Tribonacci r-graceful for all r. In this paper we investigate the Path graph P_n, the Comb graph $P_m \Theta K_1$, the Coconut tree graph $CT(m,n)$, the regular caterpillar graph $P_m \Theta nK_1$, the Bistar graph $B_{m,n}$ and the Subdivision of Bistar graph $S[B_{m,n}]$ are Gaussian Tribonacci arbitrarily graceful.

Keywords: Gaussian Tribonacci sequence, Gaussian Tribonacci graceful labeling, Path graph, Comb graph, Coconut Tree graph, Regular caterpillar graph, Bistar graph and Subdivision of Bistar graph.

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1. Motivation and Main Results

Graphs considered throughout this paper are finite, simple, undirected and nontrivial. Labeling of graph is an assignment of values to vertices and edges or both subject to certain conditions. In 1967, Rosa [6] introduced the concept of graceful labeling. In 1982, Slater [4] introduced the concept of k-graceful labeling of graphs and is defined as follows: Let G be a simple graph with p vertices and q edges. Let k be any natural number. Define an injective mapping $\phi : V(G) \to \{0, 1, 2, ..., q+k-1\}$ that induces bijective mapping $\phi^* : E(G) \to \{k, k+1, ..., q+k-1\}$ where $\phi^*(uv) = |\phi(u) - \phi(v)|$ for all $uv \in E(G)$ and $u, v \in V(G)$, then ϕ is called k-graceful labeling while ϕ^* is called an induced edge k-graceful labeling and the graph G is called k-graceful graph. Graphs that are k-graceful for all k are sometimes called arbitrarily graceful. Yuksel Soykan e tal. [11] introduced the concept of Gaussian Generalised Tribonacci Numbers. In this sequel, we introduce a new concept Gaussian Tribonacci r-graceful labeling of graphs. We follow D. B. West [10] and J. A. Gallian [2], for standard terminology and notations.

Definition 1.1 Let G be a graph with p vertices and q edges. Let r be any natural number. An injective function $\phi : V(G) \to \{0, ki, 1, 1+ki, 2,+ki, ..., GT_{q+r-1}\}$ *for all k* ,where GT_{q+r-1} is the $(q+r-1)^{th}$ Gaussian Tribonacci number in the Gaussian Tribonacci sequence is said to be Gaussian Tribonacci graceful labeling if the induced edge labeling $\phi^* : E(G) \to \{GT_1, GT_2, ..., GT_{q+r-1}\}$ such that $\phi^*(uv) = |\phi(u) - \phi(v)|$ is bijective. If a graph G admits Gaussian Tribonacci graceful labeling, then G is called a Gaussian Tribonacci graceful graph. A graph G is said to be Gaussian Tribonacci arbitrarily graceful if it is Gaussian Tribonacci r-graceful for all r.

Remark 1.1 [11] The Gaussian Tribonacci sequence is obtained as follows:
$GT_0 = 0, GT_1 = 1, GT_2 = 1+i \text{ and } GT_n = GT_{n-1} + GT_{n-2} + GT_{n-3} \ \forall\, n \geq 3$
$ie, \{0, 1, 1+i, 2+i, 4+2i, 7+4i, 13+7i, 24+13i, ...\}$ is the Gaussian Tribonacci sequence.

Definition 1.2 [9] The Comb graph $P_n \Theta K_1$ is obtained by joining a single pendent edge to each vertex of the path P_n.

Definition 1.3 [3] A regular caterpillar graph $P_m \Theta nK_1$ is obtained from the path P_m by joining nK_1 vertices to each vertices of the path P_m.

Definition 1.4 [8] The Bistar graph $B_{m,n}$ is obtained from K_2 by attaching m pendent edges to one end of K_2 and n pendent edges to the other end of K_2.

Definition 1.5 [10] The Subdivision of Bistar graph $S(B_{m,n})$ is obtained by subdividing each edge of a Bistar graph $B_{m,n}$.

Definition 1.6[9] A Coconut Tree graph CT(m,n) is obtained from the path P_n by appending n new pendent edges at an end vertex of P_n.

2. Main Results

Theorem 2.1 The path graph P_n is Gaussian Tribonacci arbitrarily graceful for all $n \geq 2$.

Proof.

Let P_n be a path graph of length $n-1$ with vertex set $V(P_n) = \{u_i / 1 \leq i \leq n\}$ and edge set $E(P_n) = \{u_i u_{i+1} / 1 \leq i \leq n-1\}$ such that $|V(P_n)| = p = n \; and \; |E(P_n)| = q = n-1$

Define $\phi : V(P_n) \to \{0, ki, 1, 1+ki, 2, 2+ki, ..., GT_{q+r-1}\} \; for \; all \; k$ by

$\phi(u_1) = T_0$

$\phi(u_i) = \phi(u_{i-1}) + (-1)^i GT_{q+r-i+1}, 2 \leq i \leq n, r \geq 1$

Thus ϕ admits Gaussian Tribonacci graceful labeling for all r .

Hence the path graph P_n is Gaussian Tribonacci arbitrarily graceful for all $n \geq 2$.

Example 2.1

The Gaussian Tribonacci 2-graceful labeling of path graph P_5 is given in Figure 2.1

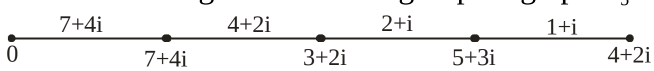

Figure 2.1

Theorem 2.2 The Comb graph $P_n \Theta K_1$ is Gaussian Tribonacci arbitrarily graceful for all $n \geq 2$.

Proof.

Let $u_i, 1 \leq i \leq n$ be the vertices of the path P_n and let $v_i, 1 \leq i \leq n$ be the vertices which are joined to each vertices of the path P_n. The resultant graph $P_n \Theta K_1$ whose vertex set is $V(P_n \Theta K_1) = \{u_i, v_i / 1 \leq i \leq n\}$ and edge set is $E(P_n \Theta K_1) = \{\{u_i v_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n-1\}\}$ such that $|V(P_n \Theta K_1)| = p = 2n$ and $|E(P_n \Theta K_1)| = q = 2n-1$

Case 1

For $n = 2$

Define $\phi : V(P_n) \to \{0, ki, 1, 1+ki, 2, 2+ki, ..., GT_{q+r-1}\}$ *for all k* by

$\phi(u_1) = GT_0$, $\phi(u_2) = GT_{r+1}, r \geq 1$,

$\phi(v_1) = GT_{q+r-1}$, $r \geq 1$, $\phi(v_2) = \phi(u_2) - GT_r$, $r \geq 1$

Case 2

For $n \geq 3$

Define $\phi : V(P_n) \to \{0, ki, 1, 1+ki, 2, 2+ki, ..., GT_{q+r-1}\}$ *for all k* by

$\phi(u_1) = GT_0$, $\phi(u_i) = \phi(u_{i-1}) + (-1)^i GT_{q+r-i+1}$, $2 \leq i \leq n, r \geq 1, \phi(v_1) = GT_r, r \geq 1$

$\phi(v_i) = \phi(u_i) - GT_{r+i-1}$, $2 \leq i \leq n$, $r \geq 1$

Thus ϕ admits Gaussian Tribonacci graceful labeling for all r.

Hence the comb graph $P_n \Theta K_1$ is Gaussian Tribonacci arbitrarily graceful for all $n \geq 2$.

Example2.2

The Gaussian Tribonacci 3-graceful labeling of Comb graph $P_2 \Theta K_1$ is given in Figure 2.2

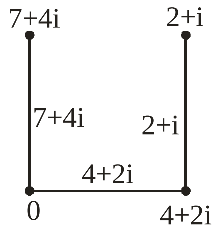

Figure 2.2

The Gaussian Tribonacci 2-graceful labeling of Comb graph $P_5 \Theta K_1$ is given in Figure 2.3

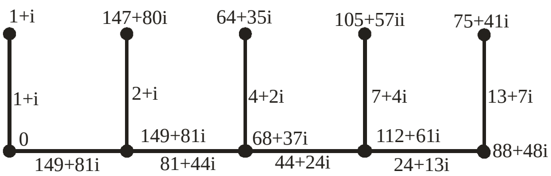

Figure 2.3

Theorem 2.3 The Coconut Tree graph CT (m, n) is Gaussian Tribonacci arbitrarily graceful for all $m, n \geq 2$.

Proof. Let $u_i, 1 \leq i \leq m$ be the vertices of the path P_m and let $v_i, 1 \leq i \leq n$ be the new vertices which are attached to the m^{th} vertex of the path P_m. The resultant graph is CT (m, n) whose vertex set is $V[CT(m,n)] = \{\{u_i / 1 \leq i \leq m\} \cup \{v_i / 1 \leq i \leq n\}\}$ and edge set is $E[CT(m,n)] = \{\{u_i u_{i+1} / 1 \le i \le m-1\} \cup \{u_m v_i / 1 \le i \le n\}\}$ such that $|V[CT(m,n)]| = p = m+n$ and $|E[CT(m,n)]| = q = m+n-1$

Define $\phi : V(P_n) \to \{0, ki, 1, 1+ki, 2, 2+ki, ..., GT_{q+r-1}\}$ *for all k* by

$\phi(u_1) = GT_0,$

$\phi(u_i) = \phi(u_{i-1}) + (-1)^i GT_{q+r-i+1}, 2 \le i \le m, r \ge 1$

$\phi(v_i) = \phi(u_m) - GT_{r+i-1}, 1 \le i \le n, r \ge 1$

Thus ϕ admits Gaussian Tribonacci graceful labeling for all r.

Hence the Coconut Tree graph CT(m,n) is Gaussian Tribonacci arbitrarily graceful for all $m, n \ge 2$.

Example 2.3

The Gaussian Tribonacci1-graceful labeling of Coconut Tree graph $CT(5,5)$ is given in Figure 2.4

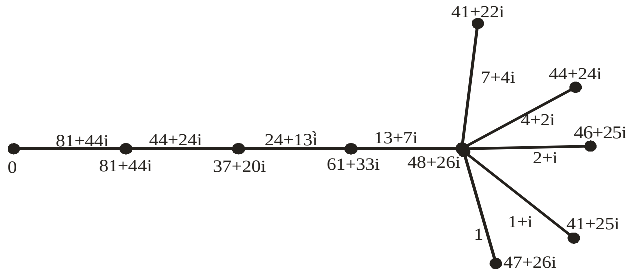

Figure 2.4

Theorem 2.4 The regular caterpillar graph $P_m \Theta\, nK_1$ is Gaussian Tribonacci arbitrarily graceful for all $m, n \ge 2$

Proof.

Let $v_i, 1 \le i \le m$ be the vertices of the path P_m and let $v_{ij}, 1 \le i \le m, 1 \le j \le n$ be the vertices attached to each vertices of the path P_m. The resultant graph is $P_m \Theta nK_1$ whose vertex set is $V[P_m \Theta\, nK_1] = \{\{v_i / 1 \le i \le m\} \cup \{v_{ij} / 1 \le i \le m, 1 \le j \le n\}\}$ and edge set is $E[P_m \Theta\, nK_1] = \{\{v_i v_{i+1} / 1 \le i \le m-1\} \cup \{v_i v_{ij} / 1 \le i \le m, 1 \le j \le n\}\}$ such that $|V[P_m \Theta\, nK_1]| = p = m + mn$ *and* $|E[P_m \Theta\, nK_1]| = q = m + mn - 1$

Define a function $\phi : V(P_n) \to \{0, ki, 1, 1+ki, 2, 2+ki, ..., GT_{q+r-1}\}$ *for all k* by

$\phi(v_1) = GT_0, \phi(v_i) = \phi(v_{i-1}) + (-1)^i GT_{q+r-i+1}, 2 \le i \le m, r \ge 1$

$\phi(v_{1j}) = GT_{q+r-1-j-(m-2)}, 1 \le j \le n, r \ge 1$

$\phi(v_{ij}) = \phi(v_i) - GT_{q+r-1-(i-1)n-j-(m-2)}, 2 \le i \le m, 1 \le j \le n, r \ge 1$

Thus ϕ admits Gaussian Tribonacci r-graceful labeling for all r .

Hence the regular caterpillar graph $P_m \Theta n K_1$ are Gaussian Tribonacci arbitrarily graceful for all $m, n \geq 2$.

Example 2.4 The Gaussian Tribonacci 2-graceful labeling of Regular caterpillar graph $P_4 \Theta 2K_1$ is given in Figure 2.5

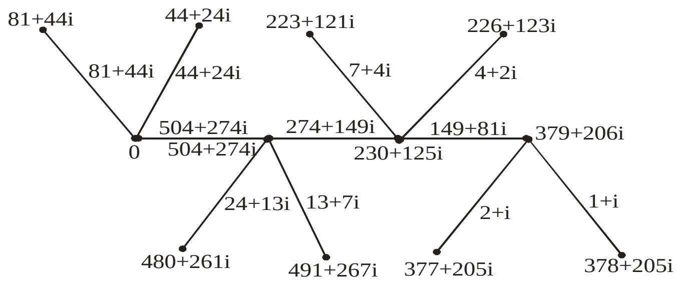

Figure 2.5

Theorem 2.5 The Bistar graph $B_{m,n}$ is Gaussian Tribonacci arbitrarily graceful for all $m, n \geq 2$.

Proof.

Let u, v be the vertices of K_2 and let $u_i, 1 \leq i \leq m$ be the m vertices attached to one end of K_2 and $v_j, 1 \leq j \leq n$ be the n vertices attached to the other end of K_2. The resultant graph is $B_{m,n}$ whose vertex set is $V(B_{m,n}) = \{u_i, v_j, u, v / 1 \leq i \leq m, 1 \leq j \leq n\}$ and edge set is $E(B_{m,n}) = \{ \{uu_i / 1 \leq i \leq m\} \cup \{vv_j / 1 \leq j \leq n\} \cup \{uv\}\}$ such that $|V(B_{m,n})| = p = m + n + 2$ and $|E(B_{m,n})| = q = m + n + 1$

Define a function $\phi : V(P_n) \to \{0, ki, 1, 1 + ki, 2, 2 + ki, ..., GT_{q+r-1}\}$ *for all k* by

$\phi(u) = GT_0, \phi(v) = GT_{q+r-1}, \phi(v_i) = \phi(v) - GT_{q+r-i-1}, 1 \leq i \leq n, r \geq 1$

$\phi(u_i) = GT_{q+r-n-1-i}, 1 \leq i \leq m, r \geq 1$

Thus ϕ admits Gaussian Tribonacci r-graceful labeling for all r.

Hence $B_{m,n}$ is Gaussian Tribonacci arbitrarily graceful for all $m, n \geq 2$.

Example 2.5

The Gaussian Tribonacci 1-graceful labeling of Bistar graph $B_{2,4}$ is given in Figure 2.6

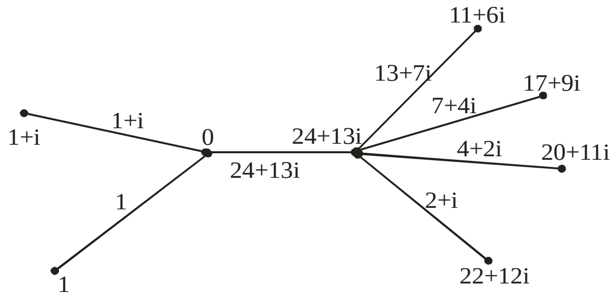

Figure 2.6

Theorem 2.6 The subdivision of the bistar graph $S(B_{m,n})$ is Gaussian Tribonacci arbitrarily graceful for all $m,n \geq 1$

Proof.

Let u,v be the central vertices of the Bistar graph $B_{m,n}$ and let $u_i, 1 \leq i \leq m$ *and* $v_i, 1 \leq i \leq n$ be the vertices joined with u and v respectively. Let $s, u_i^1, 1 \leq i \leq m$ *and* $v_i^1, 1 \leq i \leq n$ be the new vertices obtained by subdividing the edges $uv, uu_i, 1 \leq i \leq m$ *and* $vv_i, 1 \leq i \leq n$ respectively. The resulting graph is $S(B_{m,n})$ whose vertex set is $V[S(B_{m,n})] = \{ \{ u_i, u_i^1 / 1 \leq i \leq m\} \cup \{v_i, v_i^1 / 1 \leq i \leq n\} \cup \{u, v\} \cup \{s\}\}$ and edge set is $E[S(B_{m,n})] = \{ \{ u_i^1 u_i, uu_i^1 / 1 \leq i \leq m\} \cup \{us\} \cup \{sv\} \cup \{ vv_i^1, v_i^1 v_i / 1 \leq i \leq n\}\}$ such that $|V[S(B_{m,n})]| = p = 2(m+n)+3$ and $|E[S(B_{m,n})]| = q = 2(m+n)+2$

Define a function $\phi : V(P_n) \to \{0, ki, 1, 1+ki, 2, 2+ki, ..., GT_{q+r-1}\}$ *for all* k by

$\phi(s) = 0, \phi(v) = GT_{q+r-1}, r \geq 1, \phi(u) = GT_{q+r-2}$ $\phi(u_i^1) = \phi(u) - GT_{r+i-1}, 1 \leq i \leq m, r \geq 1$

$\phi(u_i) = \phi(u_i^1) - GT_{2m+r-i}, 1 \leq i \leq m, r \geq 1, \phi(v_i^1) = \phi(v) - GT_{q+r-2-i}, 1 \leq i \leq n, r \geq 1$

$\phi(v_i) = \phi(v_i^1) - GT_{2m+i+r-1}, 1 \leq i \leq n, r \geq 1$

Thus ϕ admits Tribonacci r-graceful labeling for all r.

Hence the subdivision of the bistar graph $S(B_{m,n})$ is Gaussian Tribonacci arbitrarily graceful for all $m,n \geq 1$.

Example 2.6

The Gaussian Tribonacci 1-graceful labeling of Subdivision of Bistar graph S($B_{2,2}$) is given in Figure 2.7

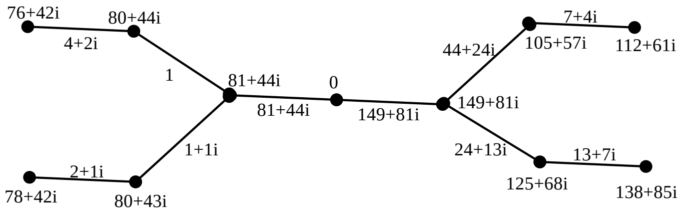

Figure 2.7

3. Conclusion

In this paper, we investigate the path graph, the Comb graph, the Coconut Tree graph, the Regular caterpillar graph, the Bistar graph and the Subdivision of Bistar graph are Gaussian Tribonacci arbitrarily graceful. In future, we investigate Gaussian Tribonacci arbitrarily graceful labeling of cycle related graphs.

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