Fair Fuzzy Matching in Middle Fuzzy Graph

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Abstract

A fuzzy matching is a set of edges in which an edge does not incident on a vertex with same membership value. If every vertex of fuzzy graph is ***M***-Plunged then the fuzzy matching is called as fair fuzzy matching. In this paper, we introduce the new concept of fair fuzzy matching in Middle fuzzy graph. We discussed some properties based on these concepts in an Absolute Fuzzy Labeling Graph.

Keywords: Fuzzy Middle Graph; fuzzy fair matching; condensation.

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1. Introduction

The concept of fuzzy set was introduced by Lotfi A. Zadeh in 1965[13]. He developed a mathematical theory to deal with uncertainty. The advantage of replacing the classical sets by Zadeh's fuzzy sets is that it gives greater accuracy. Rosenfeld [8] introduced the notion of fuzzy graphs in the year 1975. Fuzzy matching concept in fuzzy graph is introduced by S.Yahya Mohamed and S. Suganthi [11] and Complete matching domination in fuzzy labelling graph[12].

Middle graph of a fuzzy graph is an unary operation. It is used to extend the graph together with its fair fuzzy matching. Here we consider an absolute fuzzy graph with even number of vertices because fair fuzzy matching exists only for even number of vertices.

2. Preliminaries

Definition 2.1

Let γ be a non-empty set. A ***fuzzy graph*** $G = (\alpha, \beta)$ is a combination of two functions, $\alpha: \gamma \to [0,1]$ and $\beta: \gamma \times \gamma \to [0,1]$ where for all u, v belongs to γ we have $\beta(u,v) \leq \min\{\alpha(u),\ \alpha(v)\}$.

Definition 2.2

Two edges of a fuzzy labeling graph $G = (\alpha, \beta)$ is said to be ***neighbors*** to each other if they incident on a vertex with same membership value.

Definition 2.3

A fuzzy subset M of $\beta(v_i, v_{i+1})$, $1 \leq i \leq n$ is called a ***fuzzy matching M*** in fuzzy labeling graph $G = (\alpha, \beta)$ if its elements are links (neither self loop nor parallel edges) and no two are neighboring in G. The two ends of an edge in M are said to be ***fuzzy tied*** under M.

Example 2.4

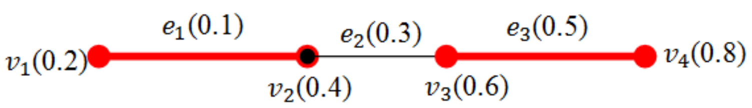

Figure 2.1

In figure 2.1, $\beta = \{e_1(0.1), e_2(0.3), e_3(0.5)\}$, and $f_M = \{e_1(0.1), e_3(0.5)\}$.

Definition 2.5

The vertex v is said to be ***M -fuzzy plunged*** or ***fuzzy plunged by M*** if it belongs to the circumstance of the elements of matching M.

Example 2.6

Consider the following fuzzy graph given in figure 2.2

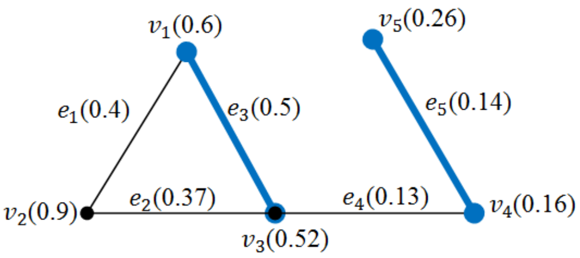

Figure 2.2

In fig 2.2 $M = \{e_3(0.5), e_5(0.14)\}$ is one of the matching in G. Then the vertices $\{v_1(0.6), v_3(0.52), v_4(0.16)$ and $v_5(0.26)\}$ are fuzzy plunged by $\boldsymbol{M}$.

Definition 2.7

Every vertex of the fuzzy labeling graph is fuzzy Mplunged then the fuzzy matching M is said to be ***fuzzy fair matching***. It is denoted byF_{fm}.

Example 2.8

Consider the following fuzzy graph given in figure 2.3

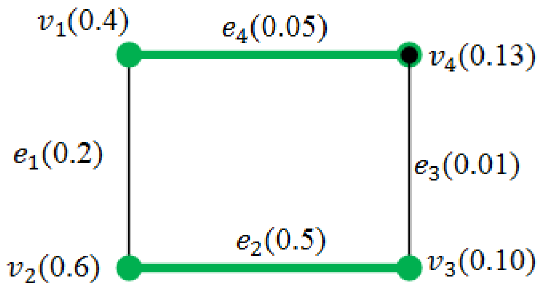

Figure 2.3

In fig 2.3, $M = \{e_2(0.5), e_4(0.05)\}$ is one of the fuzzy matching in fuzzy labeling graph G in which all vertices are fuzzy plunged by M. Hence, M is a fuzzy fair matching.

Definition 2.9

Let M be a fuzzy matching in a fuzzy labeling graph $G = (\alpha, \beta)$. Then M −fuzzy interchanging path ($MFIP$) is a path whose edges appear alternatively in $(\beta - M)$ and M.

Definition 2.10

An Absolute fuzzy labeling graph $G = (\alpha, \beta)$ is a fuzzy graph G in which $\beta(v_i, v_j) > 0$ for all $v_i, v_j \in V$ and $\beta(v_i, v_j) = \min\{\alpha(v_i), \alpha(v_i)\}$ for all $v_i, v_j \in V$. It is denoted by *AFG*.

Example 2.11

Consider the following fuzzy graph given in figure 2.4

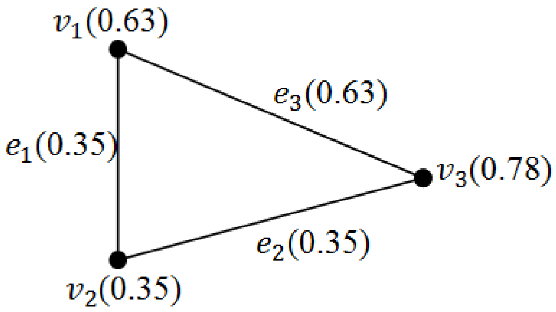

Figure 2.4

3. Main Results

Definition 3.1

Let $G = (\alpha, \beta)$ be a fuzzy graph and $\boldsymbol{M}$ be a fair fuzzy matching in $G = (\alpha, \beta)$.Then the condensation of $\boldsymbol{M}$ is defined as each fair fuzzy matching convert into a single vertex.

Definition 3.2

The minimum number of fair matching to cover all the edges of a fuzzy graph is called the condensation number.

Definition 3.3

Let $G = (\alpha, \beta)$ be a fuzzy graph and $\boldsymbol{M}$ be a fair fuzzy matching in $G = (\alpha, \beta)$.The Middle fuzzy graph $\boldsymbol{M}(\boldsymbol{G})$ contains

(i) The vertex set as the union of all vertices in G and the condensation of fair matching. (ie.,) $V(G) \cup C(M)$.

(ii) The edge set as there exists an edge between v_i and v_j with $\beta(v_i, v_j) > 0\ for\ all\ v_i,\ v_j \in G$ and also each $C(M)$ has an edge $\beta(v_i, v_j) > 0\ for\ all\ v_i, v_j \in G, \beta(v_i, v_j) = 0\ for\ all\ v_i, v_j \in C(M)$.

Theorem 3.4

Let $G = (\alpha, \beta)$ be a fuzzy absolute labelling graph with fuzzy fair matching M. Then the number of edges in $M(G)$ is $3n(n-1)/2$ where n is even.

Proof:

Consider $G = (\alpha, \beta)$ be a fuzzy absolute labelling graph with fuzzy fair matching M.

To find the number of edges in Middle fuzzy graph of G.

The number of edges in $M(G)$

$$= \beta(v_i , v_j) > 0 \; for \; all \; v_i, v_j \in G$$

+ number lines joining $C(M)$ to all vertices in G

$$= n(n-1)/2 + |C(M)| \, |V(G)|$$

$$= \frac{n(n-1)}{2} + (n-1)n$$

$$= n(n-1)[1/2 + 1]$$

Hence the number of edges in $M(G)$ is $3n(n-1)/2$.

Theorem 3.5

Every middle graph of an absolute fuzzy graph with n vertices has $(n-1)$ condensation number. Here n is even.

Proof:

Let $G = (\alpha, \beta)$ be an absolute fuzzy graph and M be the fuzzy fair matching in G.

Now, we can construct the fair matching to cover all edges of G.

The $E - count$ of each fair matching is $n/2$ and the number of edges in an absolute fuzzy graph is $n(n-1)/2$.

For $n = 4$, three distinct fair matching need to cover all edges of G and $E - count$ of each fair matching is 2. Hence $C(M)$ contains 3 points.

For $n = 6$, $E - count$ of each fuzzy fair matching is 3 and five distinct fair matching cover all edges of G. Then these fair matching are converted into five points. Hence the condensation number is 5.

Similarly for $n = 8$, seven fair matching need to cover all edges of G.Then these fair matching are converted into seven points .Hence the condensation number is 7.

In general, an absolute fuzzy labelling graph with n vertices has $(n-1)$ condensation number.

Theorem 3.6

Let $G = (\alpha, \beta)$ be an absolute fuzzy graph with even number of vertices and M be a fair matching in G. Then middle graph of G does not contain fair fuzzy matching.

Proof:

Let $G = (\alpha, \beta)$ be an absolute fuzzy graph with even number of vertices and M be a fair matching in G.

An absolute fuzzy graph with n vertices has $(n-1)$ condensation point. Now we construct the middle graph of G. Also by the definition of middle graph these $(n-1)$ points adjacent to all vertices in G.

Hence $M(G)$ contains the vertex set as union of $V(G) \; and \; C(M)$.

Therefore, $M(G)$ contains (even + odd) number of vertices. Always $M(G)$ contains odd number of vertices. Also $M(G)$ is not an absolute graph.
Then we can find a fuzzy matching which does not cover all vertices of $M(G)$ because $C(M)$ adjacent to all vertices in G. Also there exist $\beta(v_i, v_j) > 0 \ \forall v_i \in C(M)$ and $v_j \in G$. So that the required matching is not fair. Hence $M(G)$ does not have fair fuzzy matching.

Definition 3.7

The number of edges incident to $\boldsymbol{v}$ is called $\boldsymbol{grade\ of\ v}$.

Theorem 3.8

Let $G = (\alpha, \beta)$ be an absolute fuzzy graph with even number of vertices and M be a fair matching in G. Then maximum grade is $2(n-1)$ and minimum grade is n.

Proof:

Let $G = (\alpha, \beta)$ be an absolute fuzzy graph with even number of vertices and M be a fair matching in G.
By the definition, The middle fuzzy graph $M(G)$ contains
(i) The vertex set as the union of all vertices in G and the condensation of fair matching (ie.,) $V(G) \cup C(M)$.
(ii) The edge set as there exists an edge between v_i and v_j with $\beta(v_i, v_j) > 0$ $for\ all\ v_i$, $v_j \in G$ and also each $C(M)$ has an edge $\beta(v_i, v_j) > 0$ $for\ all\ v_i, v_j \in G$, $\beta(v_i, v_j) = 0$ $for\ all\ v_i, v_j \in C(M)$. But $\beta(v_i, v_j) = 0 \ \forall v_i, v_j \in C(M)$.
Hence the maximum grade of $M(G)$ is $2(n-1)$ also the minimum grade of $M(G)$ is n.

4 Conclusions

In this paper, we introduced the new concept of fair fuzzy matching in an Middle fuzzy graph. Also we defined condensation of fair fuzzy matching and condensation point. In Future, we will find total fuzzy graph, central fuzzy graph using fuzzy matching and fair fuzzy matching.

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