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Group Decision Making in Conditions of Uncertainty using Fermat's Weak Fuzzy Graphs and Beal's Weak Fuzzy Graphs

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Abstract

Decision making is a process of solving problems for choosing the best alternative. The best way to illustrate the alternatives and relation between them is a graph. Developing a fuzzy graph is the convenient way of illutration if there is uncertainty in alternatives or in their relation. In group decision making problems, according to a group of experts, the relation between alternatives involves measure of preference and non preference. Intuitionistic fuzzy graph has limitations to model such problems. In n- Pythagorean fuzzy graphs the hesitancy degree and other decision tools are restricted to second degree.To overcome the flaws of intuitionistic fuzzy graphs and n- Pythagorean fuzzy graphs, we introduced Fermat's Fuzzy Graphs in 2022. In this paper the decision tools are generalized for Fermat's Fuzzy Graphs. A practical example of selection of investement scheme is illustrated. Finally, Beal's Fuzzy graphs is developed as generalization of Fermat's Fuzzy Graphs.

Keywords: Fuzzy Graph; Weak Fuzzy Graph; Fermat's Fuzzy Graph; Fermat's Weak Fuzzy Graphs; Beal's Fuzzy Graph; Beal's Weak Fuzzy Graphs.

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1. Introduction

L A Zadeh in 1965[1] introduced fuzzy sets to describe the vagueness phenomena in real world problems.In 1975[2], Azriel Rosenfeld introduced fuzzy graphs. A.Prasanna and T M Nishad introduced weak fuzzy graphs in 2021 [3]. In 2009, Hongmei and Lianhua defined Interval Valued Fuzzy Graph (IVFG) [4] and in 2013 Talebi and Rashmanlou studied properties of isomorphism and complement of an IVFG[5]. To overcome the flaws of Intuitionistic Fuzzy Graph in simulation, Muhammed Akram, Amna Habib, etc. discussed specific types of Pythagorean Fuzzy Graphs and applications to decision making in 2018[6]. Fermat's Weak Fuzzy graph and hesitancy degree in general scale are discussed by B.M Harif and T.M Nishad in 2022[7]. The American Banker and amateur mathematician Mr.Daniel Andrew Beal formulated the Beal's conjecture in1993 [8] as a generalization of Fermat's Conjecture. The contents of this article are as follows.

In section 2 some fundamental concepts of Fuzzy Graphs and Fermat's Fuzzy Graphs are reviewed.Section 3 illustrates the mathematical model of a group decision making problem using Fermat's weak Fuzy Graph. Section 4 describes the generalized decision tools for Fermat's Fuzzy Graphs. A practical example of selection of investment scheme is illustrated in section 5. In section 6 , the fundamental concepts of Beal's Fuzzy Graph and some theorems are developed.The whole article is concluded in section 7.

2.Some Fundamental concepts of Fuzzy Graphs

A mapping $m: A \to [0,1]$ from a non empty set A is a fuzzy subset of A. A fuzzy relation r on the fuzzy subset m , is a fuzzy subset of $A \times A$. A is assumed as finite non empty set.

Definition 2.1: Suppose A is the underlying set. A fuzzy graph is a pair of functions G : (m, r) where fuzzy subset $m: A \to [0,1]$, the fuzzy relation r on m is denoted by $r : A \times A \to [0,1]$, such that for all $u, v \in A$, we have $r(u,v) \le m(u) \wedge m(v)$ where $\wedge$ stands minimum.

G^*: (m^*, r^*) denotes the underlying crisp graph of a fuzzy graph G : (m, r) where $m^* = \{u \in A / m(u) > 0\}$ and $r^* = \{(u,v) \in A \times A / r(u,v) > 0\}$. The nodes u and v are known as neighbours if $r(u,v) > 0$.

Definition 2.2.: A fuzzy graph G:(m, r) is a strong fuzzy graph if $r(u,v) = m(u) \wedge m(v), \forall (u,v) \in r^*$.

Definition 2.3: A fuzzy graph G :(m, r) is a weak fuzzy graph if $r(u,v) < m(u) \wedge m(v)$for all $(u,v) \in r^*$.

Definition 2.4: A Fermat's Fuzzy Set (FFS) on a universal set A is a set of 3 tuples of the form F={(u, $I_F(u)$, $O_F(u)$)} where $I_F(u)$ and $O_F(u)$ represents the membership and non membership degrees of $u \in A$ and $I_F(u)$, $O_F(u)$ satisfy $0 \leq I_F^n(u) + O_F^n(u) \leq 1$ for all $u \in A$, $n \in$ N={1,2,3,..} .

Definition 2.5: A Fermat's fuzzy relation (FFR) R on $A \times A$ is a set of 3 tuples of the form R = { (uv, $I_R(uv)$, $O_R(uv)$ } where $I_R(uv)$, and $O_R(uv)$ represents the membership degree and non membership degree of uv in R and $I_R(uv), O_R(uv)$ satisfy $0 \leq I_R{}^n(uv) + O_R{}^n(uv) \leq 1$ for all $uv \in A \times A$. FFR need not be symmetric. Hence $I_R(uv)$ need not be equal to $I_R(vu)$.

Definition 2.6: A Fermat's fuzzy graph (FFG(n)) on a non empty set A is a pair G : (σ, μ) with σ as FFS on A and μ as FFR on A such that
$I_\mu(uv) \leq I_\sigma(u) \wedge I_\sigma(v)$, $O_\mu(uv) \geq O_\sigma(u) \vee O_\sigma(v)$ and
$0 \leq I_\mu{}^n(uv) + O_\mu{}^n(uv) \leq 1$ for all $u,v \in A$, $n \in$ N={1,2,3,..} .where
I_μ : $A \times A \to [0,1]$ and O_μ: $A \times A \to [0,1]$ represents the membership and non membership functions of μ respectively.

Definition 2.7: A Fermat's fuzzy preference relation (FFPR) on the set of nodes $N = \{x_1, x_2, \ldots x_n\}$ is represented by a matrix $M = (m_{ij})_{nxn}$, where $m_{ij} = (x_i x_j, I(x_i x_j), O(x_i x_j))$ for all $i,j = 1,2,3..n$. Let $m_{ij} = (I_{ij}, O_{ij})$ where I_{ij} indicates the degree to which the node x_i is preferred to node x_j and O_{ij} denotes the degree to which the node x_i is not preferred to the node x_j and $\pi_{ij} = \sqrt[n]{1 - I_{ij}^n - O_{ij}^n}$ is interpreted as hesitancy degree ,with the conditions, $I_{ij}, O_{ij} \in [0,1]$, $0 \leq I_{ij}^n + O_{ij}^n \leq 1$, $I_{ij} = O_{ji}$, $I_{ii} = O_{ii} = 0.5$ for all $i,j = 1,2,3..n$.

Definition 2.8: A Fermat's fuzzy graph G : (σ, μ) is said to be Fermat's Strong fuzzy Graph FSFG(n) with underlying crisp graph G*: (σ^*, μ^*) if $I_\mu(uv) = I_\sigma(u) \wedge I_\sigma(v)$, $O_\mu(uv) = O_\sigma(u) \vee O_\sigma(v)$ for all $uv \in \mu^*$

Definition 2.9: A Fermat's fuzzy graph G : (σ, μ) is said to be Fermat's Weak Fuzzy Graph FWFG(n) with underlying crisp graph G*: (σ^*, μ^*) if $I_\mu(uv) < I_\sigma(u) \wedge I_\sigma(v)$, $O_\mu(uv) > O_\sigma(u) \vee O_\sigma(v)$ for all $uv \in \mu^*$

Definition 2.10 : A Fermat's fuzzy graph G : (σ, μ) is said to be complete FFG with underlying crisp graph G*: (σ^*, μ^*) if $I_\mu(uv) = I_\sigma(u) \wedge I_\sigma(v)$, $O_\mu(uv) = O_\sigma(u) \vee$

$O_\sigma(v)$ for all $u,v \in \sigma^*$.

3.Modeling of Group Decision Making Problem

Example 3.1: Mr. X from India wish to invest money in any of the following 5 schemes that helps him better financial security in future.

1. Public Provident Fund □ S1□
2. National Saving Certificate □ S2□
3. Atal Pension Yojana □ S3□
4. National Pension Scheme □ S4 □
5. Sovereign Gold Bonds □ S5□

He consulted with 4 experts and they advised the merits and demerits of each particular scheme comparing with other. The aggregate of information FFPR is prepared as relation matrices.How can he select the best Scheme?

Modeling: Suppose the 5 schemes are S1,S2,S3,S4 and S5. Consider the discrete set of alternatives A = {S1,S2,S3,S4,S5}.Since the alternatives are present , assign the membership degree as 1 and non membership degree 0 to each alternatives. Consider the set of experts as {E1,E2,E3,E4}. Since each experts gives the acceptance and rejection reasons comparing every pair of alternatives, the aggregate of information FFPR can be represented as relation matrices. This data represents a FFG(n). If in the given FFPRs , all the membership values are in (0,1) and non membership values are greater than 0 then the given FFG(n) will be FWFG(n).

4. Decision tools for Fermat's Fuzzy Graph

In decision making, the Optimal Score having maximum rank is considered as best choice. The scores to rank the alternatives can be calculated using score function $S\left(p_i\right)$. Here p_i is the collective Fermat's Fuzzy Element which can be obtained using Fermat's Fuzzy Weighted Averaging Operator FFWA. The weight of each expert can be obtained using deviations of each experts and the deviations can be calculated from difference matrices. The entries in difference matrix is calculated using Fermats Fuzzy Hamming distance between Fermat's Fuzzy Elements.

4.1 Fuzzy Averaging operator FFA

$$FFA\left(p_{i1}^{(k)}, p_{i2}^{(k)}, ..., p_{im}^{(k)}\right) = \left(\sqrt[n]{1-\left(\prod_{j=1}^{m}\left(1-I_{\mu_{ij}}^{n}\right)\right)^{1/m}}, \left(\prod_{j=1}^{m}\left(O_{\mu_{ij}}\right)\right)^{1/m}\right), i = 1,2,3,..,m.$$

Fermat's Fuzzy Element (FFE) $p_i^{(k)}$ indicates preference of each expert E_k over each pair of alternatives.

It is determined using Fermat's Fuzzy Averaging operator FFA

$p_i^{(k)} = FFA\left(p_{i1}^{(k)}, p_{i2}^{(k)}, ..., p_{im}^{(k)}\right), i = 1, 2, 3, .., m.$ FFE is used in calculation of FFWA.

4.2 Fermat's Fuzzy Hamming distance between FFEs.

From the given FFPR,

$$d\left(p_{ij}^{(l)}, p_{ij}^{(k)}\right) = \frac{1}{2}\left(\left|I^n_{\mu_{p_{ij}^{(l)}}} - I^n_{\mu_{p_{ij}^{(k)}}}\right| + \left|O^n_{\mu_{p_{ij}^{(l)}}} - O^n_{\mu_{p_{ij}^{(k)}}}\right| + \left|\pi^n_{p_{ij}^{(l)}} - \pi^n_{p_{ij}^{(k)}}\right|\right),$$ where

$\pi_{p_{ij}^{(l)}} = \sqrt[n]{1 - I^n_{\mu_{p_{ij}^{(l)}}} - O^n_{\mu_{p_{ij}^{(l)}}}}, \pi_{p_{ij}^{(k)}} = \sqrt[n]{1 - I^n_{\mu_{p_{ij}^{(k)}}} - O^n_{\mu_{p_{ij}^{(k)}}}}$ are hesitancy degree.

4.3 Difference matrix

$$D_{lk} = \left(d_{ij}^{(lk)}\right)_{mxm} = d\left(p_{ij}^{(l)}, p_{ij}^{(k)}\right)_{mxm}$$

4.4 Average Values of Difference Matrix

The equation to determine average values of difference matrix $\overline{d_{lk}} = \frac{1}{m^2}\sum_{i=1}^{m}\sum_{j=1}^{m} d_{ij}^{(lk)}$

4.5 Deviation of expert E_r from remaining experts

The equation to determine deviation of expert E_r from remaining experts $d_r = \sum_{k=1, k\neq r}^{s} \overline{d_{rk}}$

4.6 Weight of experts w_r.

The equation to determine weight of experts w_r. $w_r = \frac{(d_r)^{-1}}{\sum_{r=1}^{s}(d_r)^{-1}}, r = 1, 2, .., s.$

4.7 Fermat's Fuzzy Weighed Averaging operator FFWA

Fermat's Fuzzy Weighed Averaging operator FFWA to compute collective Fermat's Fuzzy Element p_i over other alternatives is

$$p_i = FFWA\left(p_i^{(1)}, p_i^{(2)}, ..., p_i^{(s)}\right) = \left(\sqrt[n]{1 - \prod_{k=1}^{s}\left(1 - I^n_{\mu_{p_i^{(k)}}}\right)^{w_k}}, \prod_{k=1}^{s}\left(O_{\mu_{p_i^{(k)}}}\right)^{w_k}\right) = \left(I_{p_i}, O_{p_i}\right)$$

4.8 Score Function

Score function $S(p_i)$ to rank the alternatives $u_i, i = 1, 2, ..., m$,

$$S(p_i) = \left(I^n_{p_i} - O^n_{p_i}\right)\left(I_\sigma(u_i) - O_\sigma(u_i)\right), \ u_i, i = 1, 2, ..., m.$$

5. Illustration of a Practical Example

In example 3.1, Suppose the aggregate of information FFPR is given as following relation matrices.

Data 1. The information from E_1 in the form of Relation Matrix.

$$\begin{array}{c} \\ C1 \\ C2 \\ C3 \\ C4 \\ C5 \end{array} \begin{array}{c} \begin{array}{ccccc} C1 & C2 & C3 & C4 & C5 \end{array} \\ \begin{pmatrix} (0.5,0.5) & (0.6,0.8) & (0.7,0.9) & (0.6,0.9) & (0.3,0.9) \\ (0.8,0.6) & (0.5,0.5) & (0.5,0.8) & (0.6,0.7) & (0.7,0.9) \\ (0.9,0.7) & (0.8,0.5) & (0.5,0.5) & (0.4,0.9) & (0.8,0.4) \\ (0.9,0.6) & (0.7,0.6) & (0.9,0.4) & (0.5,0.5) & (0.8,0.5) \\ (0.9,0.3) & (0.9,0.7) & (0.4,0.8) & (0.5,0.8) & (0.5,0.5) \end{pmatrix} \end{array}$$

Data 2. The information from E_2 in the form of Relation Matrix.

$$\begin{array}{c} \\ C1 \\ C2 \\ C3 \\ C4 \\ C5 \end{array} \begin{array}{c} \begin{array}{ccccc} C1 & C2 & C3 & C4 & C5 \end{array} \\ \begin{pmatrix} (0.5,0.5) & (0.6,0.7) & (0.7,0.8) & (0.8,0.8) & (0.3,0.8) \\ (0.7,0.6) & (0.5,0.5) & (0.5,0.8) & (0.6,0.7) & (0.7,0.8) \\ (0.8,0.7) & (0.8,0.5) & (0.5,0.5) & (0.4,0.9) & (0.8,0.5) \\ (0.8,0.8) & (0.7,0.6) & (0.9,0.4) & (0.5,0.5) & (0.8,0.5) \\ (0.8,0.3) & (0.8,0.7) & (0.5,0.8) & (0.5,0.8) & (0.5,0.5) \end{pmatrix} \end{array}$$

Data 3. The information from E_3 in the form of Relation Matrix.

$$\begin{array}{c} \\ C1 \\ C2 \\ C3 \\ C4 \\ C5 \end{array} \begin{array}{c} \begin{array}{ccccc} C1 & C2 & C3 & C4 & C5 \end{array} \\ \begin{pmatrix} (0.5,0.5) & (0.5,0.8) & (0.6,0.9) & (0.7,0.9) & (0.4,0.9) \\ (0.8,0.5) & (0.5,0.5) & (0.5,0.8) & (0.6,0.7) & (0.5,0.9) \\ (0.9,0.6) & (0.8,0.5) & (0.5,0.5) & (0.4,0.9) & (0.6,0.4) \\ (0.9,0.7) & (0.7,0.6) & (0.9,0.4) & (0.5,0.5) & (0.8,0.5) \\ (0.9,0.4) & (0.9,0.5) & (0.4,0.6) & (0.5,0.8) & (0.5,0.5) \end{pmatrix} \end{array}$$

Data 4. The information from E_4 in the form of Relation Matrix.

$$\begin{array}{c} \\ C1 \\ C2 \\ C3 \\ C4 \\ C5 \end{array} \begin{array}{c} \begin{array}{ccccc} C1 & C2 & C3 & C4 & C5 \end{array} \\ \begin{pmatrix} (0.5,0.5) & (0.6,0.8) & (0.7,0.9) & (0.7,0.9) & (0.3,0.9) \\ (0.8,0.6) & (0.5,0.5) & (0.5,0.8) & (0.7,0.7) & (0.7,0.9) \\ (0.9,0.7) & (0.8,0.5) & (0.5,0.5) & (0.4,0.8) & (0.8,0.4) \\ (0.9,0.7) & (0.7,0.7) & (0.8,0.4) & (0.5,0.5) & (0.8,0.5) \\ (0.9,0.3) & (0.9,0.7) & (0.4,0.8) & (0.5,0.8) & (0.5,0.5) \end{pmatrix} \end{array}$$

The above data represents a FFG(n). Among the relations,0.8+0.8= 0.9+0.7= 0.7+0.9 = 1.6 is the maximum sum among measures of acceptance and corresponding rejection. Since there is more than one pair with the same sum, we break the tie by comparing the sum of powers and selecting the pair that brings maximum sum.Here $0.8^2+0.8^2 = 1.28 <$

$0.9^2+0.7^2=1.3$. So we consider sum of higher powers of 0.9 and 0.7 till we get a sum ≤ 1. Note that $0.9^3+0.7^3=1.072>1$.But $0.9^4+0.7^4=0.8962<1$ .So the FFG(n) is FFG(4).Since alternatives are present , the membership value 1 and non membership value 0 have to be assigned to each alternatives. In the given FFRs , all the membership and non membership values are in (0,1). Hence the given FFG(4) is FWFG(4).

Fermat's Fuzzy Eelements are

$P_1^{(1)} = (0.5854,0.7816)$, $P_2^{(1)} = (0.6624,0.6853)$, $P_3^{(1)} = (0.7732,0.5753)$,

$P_4^{(1)} = (0.8196,0.5144)$, $P_5^{(1)} = (0.7786,0.5827)$

$P_1^{(2)} = (0.6544,0.7090)$, $P_2^{(2)} = (0.6231,0.6694)$, $P_3^{(2)} = (0.7301,0.6015)$,

$P_4^{(2)} = (0.7896,0.5448)$, $P_5^{(2)} = (0.6855,0.5827)$

$P_1^{(3)} = (0.5725,0.7816)$, $P_2^{(3)} = (0.6304,0.6608)$, $P_3^{(3)} = (0.7435,0.5578)$,

$P_4^{(3)} = (0.8196,0.5305)$, $P_5^{(3)} = (0.7786,0.5448)$

$P_1^{(4)} = (0.6129,0.7816)$, $P_2^{(4)} = (0.6803,0.6853)$, $P_3^{(4)} = (0.7732,0.5619)$,

$P_4^{(4)} = (0.7896,0.5471)$, $P_5^{(4)} = (0.7786,0.5827)$

From the difference matrices and the average values of difference matrices we get the deviations $d_1= 0.196528$, $d_2= 0.341216$, $d_3= 0.261024$ and $d_4= 0.242128$. Then the weights of experts are $w_1= 0.31842$, $w_2= 0.18340$, $w_3= 0.23974$ and $w_4= 0.25845$.

Now the collective Fermat's Fuzzy Elements are $p_1= (I_{p1}, O_{p1}) = (0.60466,0.76775)$,

$p_2 = (I_{p2}, O_{p2}) = (0.65365,0.67642)$, $p_3 = (I_{p3}, O_{p3}) = (0.75922,0.57224)$, $p_4 = (I_{p4}, O_{p4}) = (0.80715,0.53211)$ and $p_5 = (I_{p5}, O_{p5}) = (0.76520,0.57338)$.

The corresponding score function gives the following scores

$S(p_1) = -0.21377$, $S(p_2) = -0.02680$, $S(p_3) = 0.22503$, $S(p_4) = 0.34427$ and $S(p_5) = 0.23476$

Since $S(p_4)$ is the maximum score ,the best Choice is S4, the National Pension Scheme.

6. Beal's Fuzzy Graph BFG(m,n)

If the membership value of acceptance (or rejection) is given a limit (say α) then the membership value (say β) of rejection (or acceptance) is assumed to be governed by the in equation $\beta^n \leq 1-\alpha^m$ for some $m,n \in N=\{1,2,3,..\}$ .Therefore the generalization of FFG(n) has importance.

Definition 6.1: A Beal's Fuzzy Set (BFS) on a universal set A is a set of 3 tuples of the form F={(u, $I_F(u)$, $O_F(u)$)} where $I_F(u)$ and $O_F(u)$ represents the membership and non membership degrees of $u \in A$ and $I_F(u)$, $O_F(u)$ satisfy $0 \leq I_F^m(u) + O_F^n(u) \leq 1$ for all $u \in A$, $m,n \in$ N={1,2,3,..} .

Definition 6.2: A Beal's fuzzy relation (BFR) R on $A \times A$ is a set of 3 tuples of the form R = { (uv, $I_R(uv)$, $O_R(uv)$ } where $I_R(uv)$, and $O_R(uv)$ represents the membership degree and non membership degree of uv in R and $I_R(uv), O_R(uv)$ satisfy $0 \leq I_R^m(uv) +) O_R^n(uv) \leq 1$ for all $uv \in A \times A$. BFR need not be symmetric. Hence $I_R(uv)$ need not be equal to $I_R(vu)$.

Definition 6.3:A Beal's fuzzy graph BFG(m,n) on a non empty set A is a pair G : (σ, μ) with σ as BFS on A and μ as BFR on A such that

$I_\mu(uv) \leq I_\sigma(u) \wedge I_\sigma(v)$, $O_\mu(uv) \geq O_\sigma(u) \vee O_\sigma(v)$ and $0 \leq I_\mu^m(uv) +) O_\mu^n(uv) \leq 1$ for all $u,v \in A$, $m,n \in$ N={1,2,3,..} .where

$I_\mu : A \times A \to [0,1]$ and $O_\mu : A \times A \to [0,1]$ represents the membership and non membership functions of μ respectively.

Definition 6.4: A Beal's fuzzy graph G : (σ, μ) is said to be Beal's Strong fuzzy Graph BSFG(m,n) with underlying crisp graph G*: (σ^*, μ^*) if $I_\mu(uv) = I_\sigma(u) \wedge I_\sigma(v)$, $O_\mu(uv) = O_\sigma(u) \vee O_\sigma(v)$ for all $uv \in \mu^*$.

Definition 6.5: A Beal's fuzzy graph G : (σ, μ) is said to be Beal's Weak Fuzzy Graph BWFG(m,n) with underlying crisp graph G*: (σ^*, μ^*) if $I_\mu(uv) < I_\sigma(u) \wedge I_\sigma(v)$, $O_\mu(uv) > O_\sigma(u) \vee O_\sigma(v)$ for all $uv \in \mu^*$

Definition 6.6 : A Beal's fuzzy graph G : (σ, μ) is said to be complete BFG with underlying crisp graph G*: (σ^*, μ^*) if $I_\mu(uv) = I_\sigma(u) \wedge I_\sigma(v)$, $O_\mu(uv) = O_\sigma(u) \vee O_\sigma(v)$ for all $u,v \in \sigma^*$.

Theorem 6.1: When m = n, BFG (m,n) $\Rightarrow$ FFG(n) and BFG (1,1) $\Rightarrow$ FFG(1) which is an intuitionistic fuzzy graph.

Proof. Directly follows from the definitions.

i.e, Beal's Fuzzy graph is generalization of Fermat's Fuzzy Graph and Fermat's Fuzzy Graph is generalization of Intuitionistic Fuzzy Graph.

Theorem 6.2: BWFG (m-1,n-1) $\Rightarrow$ BWFG(m,n) but the converse is not true.

Proof. Let G : (σ, μ) be a BWFG (n-1) with σ as BFS on A and μ as BFR on A .

Since $I_\mu(uv) < 1$, $I_\mu^{m-1}(uv) < 1 \Rightarrow I_\mu^m(uv) < I_\mu^{m-1}(uv) < 1, for\ all\ m \in N \to (1)$

Similarly since $O_\mu(uv) < 1$, $O_\mu^{n-1}(uv) < 1 \Rightarrow O_\mu^n(uv) < O_\mu^{n-1}(uv)$

Therefore $I_{\mu}^{m-1}(uv)\ +)O_{\mu}^{n-1}(uv) \leq 1 \Rightarrow I_{\mu}^{m}(uv) + O_{\mu}^{n}(uv) \leq 1$.
Hence BWFG(m-1,n-1) $\Rightarrow$ BWFG(m,n).
It is obvious from equation (1) that the converse is not true.

7 Conclusion

In this article some fundamental concepts of Fuzzy Graphs and Fermat's Fuzzy Graphs are reviewed.The decision tools are generalized for Fermat's Fuzzy Graphs. Application of Fermats Weak Fuzzy Graph in modeling group decision making problem is illustrated with a practical example. The fundamental concepts of Beal's Fuzzy Graph are developed. The applications of FFG(n) and BFG(m,n) in various fields of science, social science and engineering are under research.

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