More Functions Related to $\check{S}A^*$ - Open Set in Soft Topological Spaces

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Abstract

In this paper, we introduce some soft functions like Š Stronglyα^* - continuous function, Š Perfectly α^* - continuous function, Š Totally α^* - continuous function. We study the connections of these function with other Š function. Also, we establish the relationships in between the above functions and also investigate various aspects of these functions.

Keywords: soft functions, continuous function

2010 AMS subject classification: 54C05[3]

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[3]Received on June 9th, 2022.Accepted on Sep 5st, 2022.Published on Nov 30th, 2022.doi: 10.23755/rm.v44i0.905. ISSN: 1592-7415. eISSN: 2282-8214.

1. Introduction

Molodtsov introduced the concept of soft sets from which the difficulties of fuzzy sets, intuitionistic fuzzy sets, vague sets, interval mathematics and rough sets have been rectified. Application of soft sets in decision making problems has been found by Maji et al. whereas Chen gave a parametrization reduction of soft sets and a comparison of it with attribute reduction in rough set theory. Further soft sets are a class of special information.

Shabir and Naz introduced soft topological spaces in 2011 and studied some basic properties of them. Meanwhile generalized closed sets in topological spaces were introduced by Levine in 1970 and recent survey of them is in which is extended to soft topological spaces in the year 2012. Further Kannan and Rajalakshmi have introduced soft g – locally closed sets and soft semi star generalized closed sets. Soft strongly g – closed sets have been studied by Kannan, Rajalakshmi and Srikanth. Chandrasekhara Rao and Palaiappan introduced generalized star closed sets in topological spaces and it is extended to the bitopological context by Chandrasekhara Rao and Kannan.

Recently papers about soft sets and their applications in various fields have increased largely. Modern topology depends strongly on the ideas of set theory. Any Research work should result in addition to the existing knowledge of a particular concept. Such an effort not only widens the scope of the concept but also encourages others to explore new and newer ideas. Therefore, in this work we introduce a new soft generalized set called $Š\alpha^*$ open set and its related properties. This may be another starting point for the new soft set mathematical concepts and structures that are based on soft set theoretic operations.

2. Preliminaries

In this section, this project X be an initial universe and Ê be a set of parameters. Let P (X) denote the power set of X and A be a non – empty subset of ξ. A pair ($F^{š}$, A) denoted by $F^{š}{}_{a}$ is called a soft set over X, where Fš is a mapping given by $F^{š}$: A $\rightarrow$ P (X).

Definition2.1.1 [8] For two soft sets ($F^{š}$, A) and (G, B) over a common universe X, we say that ($F^{š}$, A) is a soft subset of (G, B) denoted by ($F^{š}$, A) $\subseteq_s$ (G, B), if

i. A $\subseteq_s$ B and

ii. $F^{š}$ (e) $\subseteq_s$ G (e) for all e $\in \xi$

Definition2.1.2 [8] The complement of a soft set ($F^{š}$, A) denoted by ($F^{š}$, A)^{c,} is defined by (($F^{š}$, A))^{c} = (($F^{šc}$, A), where $F^{šc}$: A $\rightarrow$ P (X) is a mapping given by $F^{šc}$ (e) = X - $F^{š}$ (e), for all e $\in \xi$.

Definition2.1.[8] A Subset of a Štopological space (X, τ_s, ξ) is said to be

1. a Š Semi-Open set
2. if ($F^{š}$, Ê) $\subseteq_s$ ŠCl (Šint ($F^{š}$, Ê) and a Š Semi-Closed set if Š int (Š Cl ($F^{š}$, Ê) $\subseteq_s$ ($F^{š}$, Ê).
3. a Š Pre-Open set [1] if ($F^{š}$, Ê) $\subseteq_s$ Š Int (Š Cl ($F^{š}$, Ê)) and a ŠPre-Closed set if Š Cl (Š

int ($F^{š}$, Ê) ⊆ $_s$ ($F^{š}$, Ê) a Š α-Open set [1] if ($F^{š}$, Ê) $\subseteq_s$ Š In (ŠCl (int($F^{š}$, Ê) and a Š α-Closed set if ŠCl (Šint (ŠCl ($F^{š}$, Ê) $\subseteq_s$ ($F^{š}$, Ê)).

4. a Š β-Open set [1] if ($F^{š}$, Ê) $\subseteq_s$ Šcl (Šint (Š cl($F^{š}$, Ê))) and a Š β-Closed set if Š Int(ŠCl(int($F^{š}$, Ê)) $\subseteq_s$ ($F^{š}$, Ê)).

5. aŠ- generalized Closed set(briefly Šgs- Closed) if Š Cl($F^{š}$, Ê) $\subseteq_s$ (G , ξ) whenever ($F^{š}$, Ê)$\subsetneq_s$ (G , ξ)and(G , ξ)is Š Open in (X, τ_s, ξ). The complement of a Š gs-Closed set is called a Šgs-Open set.

6. a Š Semi-generalized Closed set (briefly Š Sg-Closed) if Š Cl ($F^{š}$, Ê) $\subseteq_s$ (G, ξ) whenever ($F^{š}$, Ê) ⊊(G , ξ)and(G , ξ)is Šsemi Open in(X, τ_s, ξ).The complement of aŠSg-Closed set is called a ŠSg-Open set.

7. a generalized ŠSemi-Closed set (briefly gs-Closed) if ŠCl ($F^{š}$, Ê) $\subseteq_s$ (G, ξ) whenever ($F^{š}$, Ê) $\subseteq_s$ (G , ξ) and(G , ξ)is ŠOpenin(X, τ_s,ξ).Thecomplementofa Šgs-Closed setiscalled aŠgs-Open set.

8. aŠ – Closed [9] if Š Cl(F , ξ)⊈(G , ξ) whenever (F , ξ)⊆ $_s$ (G , ξ) and (G , ξ) isŠsemi Openin (X, τ_s, ξ)

9. aŠω-Closed [9] ifŠCl($F^{š}$, Ê) ⊊(G , ξ)whenever(F , ξ)⊊ (G , ξ)and(G , ξ) is Š semi Open

10. αa Š alpha-generalized Closed set (briefly Šαg-Closed) if α Š Cl($F^{š}$, Ê) ⊊ (G , ξ) whenever ($F^{š}$, Ê) ⊊(G , Ê)and(G, Ê) is ŠαOpen in (X, τ_s, Ê).The complement of a Šαg-Closed set is called a Šαg-Open set.

11. a Š generalized alpha Closed set (briefly Šgα-Closed) ifα Š Cl($F^{š}$, Ê) ⊈(G , Ê) whenever ($F^{š}$, Ê) ⊊(G , Ê)and(G , Ê) is Š Open in(X, τ_s, Ê).The complement of aŠ gα-Closed set is called a Šgα-Open set.

12. A Šgeneralized pre Closed set (briefly Šgp-Closed)[1] if p Š Cl($F^{š}$, Ê) ⊊(G , Ê) whenever a Š gp-Open set.

13. aŠgeneralized pre regular Closed set (briefly Šgpr-Closed) [5] if p ŠCl ($F^{š}$, Ê) ⊊ (G , Ê) whenever ($F^{š}$, Ê) ⊈(G , Ê) and (G , Ê) isŠ regular Open in (X, τ_s, Ê). The complement ofa Šgpr-Closedset is called a Š gpr – Open set.

3.1 Strongly Šα*-continuous function

Definition 3.1.1: A Š function f: (X, τ_s, Ê) ⟶ (Ý, τ_s, K) is said to be strongly Šα*-continuous function, if the inverse image of every Š α*- Ô(Ý)in (Ý, τ_s, K) is Š - Ô(X) in (X, τ_s, Ê).

Theorem 3.1.2: Let f : (X, τ_s, Ê) ⟶ (Ý, τ_s, K) be strongly Šα*-continuous function, then it isŠ-continuous function.

Proof:

Let ($F^{š}$, Ê) be Š - Ô(X) in (Ý, τ_s, K). Since every Š - Ô(X) is Š α*- Ô(X), then ($F^{š}$, Ê) is Š α*- Ô(X) in (Ý, τ_s, K). Since, f is strongly Šα*-continuous function, f^{-1}($F^{š}$, Ê) is Š - Ô(X) in (X, τ_s, Ê).Therefore, f is Š-continuous.

Remark 3.1.3: The converse of the above theorem need not be true.

Example 3.1.4: Let X = Ý = { x_1, ,x_2}, τ_s = {$F^š_1$, $F^š_2$, $F^š_3$,$F^š_{15}$,$F^š_{16}$},and σ_s = {$F^š_3$, $F^š_{11}$, $F^š_{12}$,$F^š_{15}$,$F^š_{16}$},
Š α*- Ô(Ý)= {$F^š_1$,$F^š_2$, $F^š_3$,$F^š_7$,$F^š_8$,$F^š_9$,$F^š_{10}$,$F^š_{11}$,$F^š_{12}$,$F^š_{13}$,$F^š_{14}$,$F^š_{15}$,$F^š_{16}$ }. Let f : (X, τ_s, Ê) → (Ý, τ_s, K)be defined by f($F^š_1$) = $F^š_3$, f($F^š_2$) = $F^š_{11}$,f($F^š_3$) = $F^š_{12}$,f($F^š_4$) = $F^š_1$,f($F^š_5$) = $F^š_2$,f($F^š_6$) = $F^š_{13}$,f($F^š_7$) = $F^š_4$f($F^š_8$)= $F^š_{14}$,f($F^š_9$) = f($F^š_{10}$) = f($F^š_{11}$)= f($F^š_{12}$)= $F^š_2$,f($F^š_{13}$) = $F^š_9$,f($F^š_{14}$) = $F^š_6$,f($F^š_{15}$) = $F^š_{15}$,f($F^š_{16}$) = $F^š_{16}$. Clearly f is Š– continuous but not strongly Šα*-continuous function, because f^{-1}($F^š_1$) = $F^š_4$is not Š - Ô(X) in (X, τ_s, Ê).

Theorem 3.1.5: Let f : (X, τ_s, Ê) → (Ý, τ_s, K)be strongly Šα*-continuous function iff the inverse image of every Š α* - Ç(Ý) in (Ý, τ_s, K) is Š - Ç(X) in (X, τ_s, Ê)
Proof:
Assume that f is strongly Šα*-continuous function. Let ($F^š$, ξ) be any Š α* - Ç(X) in (Ý, τ_s, K). Then, ($F^š$, Ê)^{c}is Š α*- Ô(X) in (Ý, τ_s, K). Since f is strongly Šα*-continuous function.
f^{-1}(($F^š$, Ê) ^{c}) is Š - Ô(X) in (X, τ_s, Ê). But f^{-1}(($F^š$, ξ) ^{c}) = X - f^{-1}(($F^š$, ξ) is Š - Ô(X) in (X, τ_s, Ê)⇒ f^{-1}(($F^š$, Ê) is Š - Ç(X) in (X, τ_s, Ê). Conversely, assume that the inverse image of every Š α*- Ç(X) in (Ý, τ_s, Ê) is Š - Ç(X) in (X, τ_s, Ê). Let ($F^š$, Ê) be any Š α*- Ô(X) in (Ý, τ_s, K). Then, ($F^š$, Ê)^{c}is Š α*- Ç(X) in (Ý, τ_s, K). By assumption, f^{-1}(($F^š$, Ê) ^{c}) is Š - Ç(X) in (X, τ_s, Ê). But f^{-1}(($F^š$, Ê) ^{c}) = X - f^{-1}(($F^š$, Ê) is Š - Ç(X) in (X, τ_s, Ê)⇒ f^{-1}(($F^š$, Ê) is Š - Ô(X) in (X, τ_s, Ê). Hence f is strongly Šα*-continuous function.

Theorem 3.1.6: Let f : (X, τ_s, Ê) → (Ý, τ_s, K)be strongly Š -continuous function then it is strongly Šα*-continuous function.
Proof: Let ($F^š$, Ê) be any Š - Ô(X) in (Ý, τ_s, K). Since every Š - Ô(X) is Š α*- Ô(X), Since f is strongly Š - continuous function, then f^{-1}(($F^š$, Ê) is both Š - Ô(X) and Š - Ç(X) in (X, τ_s, Ê).
⇒f^{-1}(($F^š$, Ê) is Š - Ô(X) in (X, τ_s, Ê). Hence f is strongly Š α*- continuous function.

Remark 3.1.7: The converse of the above theorem need not be true.

Example 3.1.8: Let X = Ý = { x_1, ,x_2}, τ_s = {$F^š_1$,$F^š_2$, $F^š_3$, $F^š_5$,$F^š_7$,$F^š_8$,$F^š_9$,$F^š_{10}$,$F^š_{12}$,$F^š_{13}$,$F^š_{14}$,$F^š_{15}$,$F^š_{16}$ }, τ_s^c = {$F^š_1$,$F^š_2$, $F^š_4$, $F^š_6$,$F^š_7$,$F^š_8$,$F^š_9$,$F^š_{10}$,$F^š_{13}$,$F^š_{14}$,$F^š_{15}$,$F^š_{16}$ }, and σ_s = {$F^š_1$, $F^š_{13}$$F^š_{15}$,$F^š_{16}$},
Š α*- Ô(Ý)= {$F^š_1$, $F^š_3$,$F^š_7$,$F^š_8$,$F^š_{11}$,$F^š_{12}$,$F^š_{13}$,$F^š_{15}$,$F^š_{16}$ }. Let f : (X, τ_s, Ê) → (Ý, τ_s, K)be defined by f($F^š_1$) = $F^š_5$, f($F^š_2$) = $F^š_2$, f($F^š_3$) = $F^š_3$,f($F^š_4$) = $F^š_4$f($F^š_5$) = $F^š_5$, f($F^š_6$) = $F^š_6$f($F^š_7$) = $F^š_7$,f($F^š_8$)= $F^š_8$f($F^š_9$) =$F^š_9$,f($F^š_{10}$) = $F^š_{10}$,f($F^š_{11}$) =$F^š_{11}$,f($F^š_{12}$) = $F^š_{12}$,f($F^š_{13}$) = $F^š_{13}$,f($F^š_{14}$) = $F^š_4$,f($F^š_{15}$) = $F^š_{15}$,f($F^š_{16}$) = $F^š_{16}$. Clearly f is strongly Šα*-continuous function but not strongly Š-continuous function,Sincef $^{-1}$($F^š_1$) = $F^š_5$is Š - Ô(X) but not Š - Ç(X)

Theorem 3.1.13: Let f : (X, τ_s, Ê) → (Ý, τ_s, K)be strongly Šα*-continuous function and g: (X, σ_s, Ê) → (ž, η_s, K) be Šα*-continuous function, then

go f : (X, τ_s, Ê) ⟶ (ž, η_s, K) is Šα*-continuous.
Proof: Let(F^š, Ê) be any Š - Ô(X) in (ž, η_s, K). Since g is Šα*-continuous, then g^{-1}((F^š, Ê)is Š α*- Ô(X) in (Ý, τ_s, K). Since f is strongly Šα*-continuous function, then f^{-1}(g $^{-1}$((F^š, ξ)) is Š - Ô(X) in (X, τ_s, Ê)⇒(go f) $^{-1}$(F^š, Ê) is Š - Ô(X) in (X, τ_s, Ê).)⇒(go f) $^{-1}$(F^š, Ê) is Šα* - Ô(X) in (X, τ_s, Ê). Hence gof isŠ α*–- continuous.

Theorem 3.1.14: Let f : (X, τ_s, Ê) ⟶ (Ý, σ_s, K)be strongly Š α*-continuous function and g: (Ý, σ_s, K) ⟶ (ž, η_s, K) be Šα*-irresolute, then
go f : (X, τ_s, Ê) ⟶(ž, η_s, K) is strongly Šα*– continuous.
Proof: Let (F^š, ξ) be any Šα*- Ô(X) in (ž, η_s, K). Since g is Šα*-irresolute, then g^{-1}((F^š, Ê)is Š α*- Ô(X) in (Ý, σ_s, K). Since f is strongly Šα*-continuous function, then f^{-1}(g $^{-1}$((F^š, Ê)) is Š - Ô(X) in (X, τ_s, Ê)⇒(go f) $^{-1}$((F^š, Ê) is Š - Ô(X) in (X, τ_s, Ê). Hence gof is strongly Šα*– continuous.

Theorem 3.1.15: Let f : (X, τ_s, Ê) ⟶ (Ý, σ_s, K)be Šα*-continuous and
g: (Ý, σ_s, K) ⟶ (ž, η_s, K) be strongly Š α*-continuous function, then
go f : (X, τ_s, Ê) ⟶ (ž, η_s, K) is Š α*–irresolute.
Proof: Let (F^š, Ê) be any Šα*- Ô(X) in (ž, η_s, K). Since g is strongly Šα*-continuous, then g $^{-1}$((F^š, Ê)isŠ - Ô(X) in (Ý, τ_s, K). Since f is Šα*-continuous function, then f^{-1}(g $^{-1}$((F^š, Ê)) is Šα*- Ô(X) in (X, τ_s, Ê)⇒(go f) $^{-1}$((F^š, Ê) is Šα*- Ô(X) in (X, τ_s, Ê). Hence gof is Š α*– irresolute.

Theorem 3.1.16: Let f : (X, τ_s, Ê) ⟶ (Ý, τ_s, K)be strongly Š α*-continuous function and g: (X, σ_s, Ê) ⟶ (ž, η_s, K) be strongly Šα*-continuous function, then
go f : (X, τ_s, Ê) ⟶ (ž, η_s, K) is strongly Š α*– continuous.
Proof: Let (F^š, Ê) be any Šα* - Ô(X) in (ž, η_s, K). Since g is strongly Šα*-continuous, then
(g^{-1}(F^š, Ê))is Š Ô(X) in (Ý, τ_s, K). Since f is strongly Šα*-continuous function, then f^{-1}(g $^{-1}$((F^š, Ê)) is Š - Ô(X) in (X, τ_s, Ê)⇒(go f) $^{-1}$((F^š, Ê) is Š - Ô(X) in (X, τ_s, Ê). Hence gof isstrongly Š α*– continuous.

Theorem 3.1.17: Let f : (X, τ_s, Ê) ⟶ (Ý, σ_s, K)be Š -continuous function and
g: (Ý, σ_s, K) ⟶(ž, η_s, K) be strongly Šα*-continuous function, then
go f : (X, τ_s, Ê) ⟶ (ž, η_s, K) is strongly Šα*– continuous.
Proof: Let (F^š, Ê) be any Šα*- Ô(X) in (ž, η_s, K). Since g is strongly Šα*-continuous, then
g^{-1}((F^š, Ê)is Š- Ô(X) in (Ý, τ_s, K). Since f is Š-continuous function, then f^{-1}(g $^{-1}$((F^š, ξ)) is Š - Ô(X) in (X, τ_s, Ê)⇒(go f) $^{-1}$((F^š, Ê) is Š - Ô(X) in (X, τ_s, Ê). Hence gof is strongly Š α*– continuous.

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