Concept of Vector Multicomponent Physical Quantities, Models and Measurement Method

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Abstract

The paper presents a new view of vector physical quantities as multicomponent quantities. Each of the components of the mentioned multicomponent quantities can carry important and even unique information about the sources and causes of their occurrence. Looking at the vector quantity as the multicomponent quantity led to the need to form the corresponding conception. There are three positions of this conception in this paper, which are formulated as follows: vector multicomponent physical quantities are considered as functions of the set of their constituent information components; the communication functions of the specified information components in the models of multicomponent physical quantities are determined by the laws of vector algebra; information models of vector multicomponent physical quantities allow an alternative representation of information components depending on the selected coordinate system.

The mathematical model of the vector multicomponent physical quantity is presented. This model is fundamental and directly follows from the positions of the conception formulated above. This model can be applied to describe multicomponent displacements and deformations that both simple and complex objects undergo. An example of the complex object can be the manipulator of the universal industrial robot. The space for modeling multicomponent displacements of simple objects was shown in the paper. Information models of vector multicomponent physical quantities allow one to alternatively represent informative components. And the task of constructing such models is complex and ambiguous. Therefore, the formal apparatus for the synthesis of such models, which is based on certain rules and conventions, was proposed in the paper. The theoretical foundations of the method of optical measurements of informative components of multicomponent displacements and deformations of simple objects, which involves the use of multidimensional test objects, are presented.

Keywords: conception, vector multicomponent quantities, multicomponent displacement, models of multicomponent displacement and deformation, method of measurement.

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Концепция векторных многокомпонентных физических величин, модели и метод измерения

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Представлен новый взгляд на векторную физическую величину как на величину многокомпонентную. Каждая из компонентов упомянутых многокомпонентных величин может нести важную и даже уникальную информацию об источниках и причинах их возникновения. Рассмотрение векторной величины как величины многокомпонентной привело к необходимости формирования соответствующей концепции. Представлены три положения концепции, которые заключаются в следующем: векторные многокомпонентные физические величины рассматриваются как функции множества составляющих их информативных компонентов; функции связи названных информативных компонентов в моделях многокомпонентных физических величин определяются законами векторной алгебры; информационные модели векторных многокомпонентных физических величин допускают альтернативное представление информативных составляющих в зависимости от выбранной системы координат.

Представлена математическая модель векторной многокомпонентной физической величины. Данная модель является основополагающей и непосредственно вытекает из сформулированных выше положений концепции. Модель может быть применена при описании многокомпонентных перемещений и деформаций, которые претерпевают и простые, и сложные объекты. Примером сложного объекта может быть модель манипулятора универсального промышленного робота. Показано пространство моделирования многокомпонентных перемещений простых объектов. Информационные модели векторных многокомпонентных физических величин позволяют альтернативно представлять информативные составляющие, а задача построения таких моделей сложна и не однозначна. Поэтому в статье предложен формальный аппарат синтеза таких моделей, который основан на определённых правилах и соглашениях. Представлены теоретические основы метода оптических измерений информативных составляющих многокомпонентных перемещений и деформаций простых объектов, который предполагает использование многомерных тестовых объектов.

Ключевые слова: концепция, векторные многокомпонентные величины, многокомпонентное перемещение, модели многокомпонентных перемещений и деформаций, метод измерения.

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Introduction

The terms "quantity", "physical quantity", "measured physical quantity" are key in the theory of measurements. The term of a physical quantity integrates in self the concepts of a kind, size, measurement unit and value, which make it possible to fill the initial a priori uncertainty in a particular area of knowledge with physical meaning and quantitative content [1]. In this case, the qualitative definiteness of the quantity, which is formulated in the standard [1], is based on the concept of dimension. Physical quantities, for example, the length and diameter, which have the same measurement unit, are recognized as homogeneous. The same standard introduces the concept of a derived quantity as a quantity included in a system of quantities and determined through the basic quantities of this system, each of which has its own dimension. For example, velocity is a derivative of two quantities that are distance and time. Force is a derivative of mass and acceleration, the latter of which is also a derivative quantity. Obviously, the emergence and development of these terms are a natural result of the complication of objects and phenomena that fall within the sphere of human vital interests. Therefore, keeping in mind the dialectics of cognition, one should expect further development in the affected subject area. Indeed, if we consider a physical quantity as a property of a material object or phenomenon, the question arises about the sources of this property, sources that lead to a change in this quantity, but have different reasons and, possibly, a different nature. In this case, the resulting values of quantities integrate the informative components from the action of different sources. Thus, we are talking about a system of homogeneous quantities, which have the same or significantly overlapping spectral range and show their effect in the resulting quantity, which is essentially multicomponent. Informative components of such multicomponent quantities are of interest and therefore must be determined.

Such view of a physical quantity takes it beyond the terminology prescribed by the standard [1]. Based on the needs of practice, we come to the need for new formulations, new models and new methods for measuring such multicomponent quantities.

A comprehensive analysis of the problem and an excursion into a number of technical applications led to the concept of vector multicomponent physical quantities [2, 3], which, for example, finds application in the problems of determining the informative components of complex multicomponent displacements and deformations of objects of varying degrees of complexity.

The conception of the vector multicomponent physical quantity

The first ideas that formed the basis of the conception originated in the process of analyzing the test results of aircraft gas turbine engines. Engines elements under various operating conditions undergo complex displacements and deformations. At the same time, the sources of these displacements and deformations often have a different nature and significance, and in the aggregate they are reflected in the resulting technical and economic indicators.

The presence of such sources is explained by the structural complexity and energy saturation of gas turbine engines. Any gas turbine engine is a mobile system, which is affected by transient processes when changing modes of its operation, vibration, structural defects of components and assemblies, and much more. The factors influencing the condition of the engines are temperature and revolutions changes, unsteady heat transfer, clearances in the blade locks, rotors precession, pressure drops and much more. Revealing the contribution of each source to the resulting displacements of constructional elements at the appropriate time and at selected points is of fundamental importance for assessing and identifying design errors, and developing promising technical solutions.

Thus, the resulting vector quantity, in this case displacement, is essentially multicomponent, each component of which carries information about the sources and reasons for their appearance and change.

The view of the vector quantity as the multicomponent quantity, which in an integral form reflects the variety of processes that lead to complex displacements and deformations of both complex and simple objects, led to the realization of the need to form the appropriate conception.

In papers [2, 3] the following definition of the conception is given: if the controlled objects and the processes associated with them have a complex character and (or) structure, then the movements that are their consequence are themselves characterized by a certain structure, the elements of which are interconnected in some way, are in interaction, have a mutual influence on each other and carry additional information about the process or object.

The conception of vector multicomponent physical quantities is based on the following three positions [2–4]:

– Vector multicomponent physical quantities are considered as functions of the set of their constituent informative components;

– The above-mentioned informative component's communication functions in models of multicomponent physical quantities are determined by the laws of vector algebra;

– Information models of vector multicomponent physical quantities allow alternatives representation of informative components, depending on the selected coordinate system.

Then the mathematical model of a vector multicomponent physical quantity, which includes informative components of the same dimension and reflects complex processes occurring with the object, is represented in the expansion along the axes of the Cartesian coordinate system in the following form:

$$\left.\begin{aligned}\mathbf{X}_x(\mathbf{r},\tau)&=\mathbf{F}\left(\mathbf{x}_{1x}(\mathbf{r},\tau),...,\mathbf{x}_{px}(\mathbf{r},\tau)\right);\\ \mathbf{X}_y(\mathbf{r},\tau)&=\mathbf{F}\left(\mathbf{x}_{1y}(\mathbf{r},\tau),...,\mathbf{x}_{py}(\mathbf{r},\tau)\right);\\ \mathbf{X}_z(\mathbf{r},\tau)&=\mathbf{F}\left(\mathbf{x}_{1z}(\mathbf{r},\tau),...,\mathbf{x}_{pz}(\mathbf{r},\tau)\right),\end{aligned}\right\}, \tag{1}$$

where $\mathbf{X}_x(\mathbf{r},\tau), \mathbf{X}_y(\mathbf{r},\tau), \mathbf{X}_z(\mathbf{r},\tau)$ are the projections of multicomponent displacements on the coordinate axes of the Cartesian coordinate system; $\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau)$ are projections of informative components onto the k-th coordinate axis $(k\in\{x,y,z\})$ of multicomponent displacement $\mathbf{X}$; $\mathbf{r}$, τ are spatial and temporal coordinates; $\mathbf{F}$ is communication function, which are determined by the physics of the investigated object or process.

This model follows from the provisions of the conception formulated above directly and is fundamental.

The model (1) can be concretized with the presentation of the resulting value as a vector sum of the corresponding informative components in accordance with the second position of the conception:

$$\left.\begin{aligned}\mathbf{X}_x(\mathbf{r},\tau)&=\sum_{j=1}^{p}\mathbf{x}_{jx}(\mathbf{r},\tau);\\ \mathbf{X}_y(\mathbf{r},\tau)&=\sum_{j=1}^{p}\mathbf{x}_{jy}(\mathbf{r},\tau);\\ \mathbf{X}_z(\mathbf{r},\tau)&=\sum_{j=1}^{p}\mathbf{x}_{jz}(\mathbf{r},\tau),\end{aligned}\right\}, \tag{2}$$

where $\sum_{j=1}^{p}\mathbf{x}_{jx}(\mathbf{r},\tau)$, $\sum_{j=1}^{p}\mathbf{x}_{jy}(\mathbf{r},\tau)$, $\sum_{j=1}^{p}\mathbf{x}_{jz}(\mathbf{r},\tau)$ are vector sums of p informative components of the coordinate components of the quantity $\mathbf{X}$.

The third position of the concept is due to the relativity of movement. Informative components in models (1) and (2) can be expressed in different ways depending on the position of the base coordinate system and the selected point of the controlled object. However, this does not affect the reliability and reproducibility of informative components in models when moving from one basic coordinate system to another. The transition from one model to another is carried out by an unambiguous recalculation using the corresponding homogeneous transition matrix [5].

The ambiguity and multivariance of the models are due to another significant circumstance.

Analysis of practical problems, which differ in the complexity of objects and their trajectories in real space, made it possible to structure the modeling area:

– Models of complex multicomponent displacements that simple objects undergo;

– Models of multicomponent simple displacements that complex objects undergo;

– Models of multicomponent complex displacements that complex objects undergo.

Another example of the complex object that undergoes complex multicomponent movements is the manipulator of the universal industrial robot. Solutions of direct and inverse kinematic tasks of the robot manipulator using matrices of rotation and displacement of the links allows one to select the informative components of resulting displacements of the flange of the last link of the manipulator [5].

Obviously, multicomponent displacements in models (1) and (2) describe the displacement of a point of an object, which can belong to both a complex and a simple object. Therefore, these models are applicable in all these cases.

Before proceeding to the solution of the following problems, it is necessary to clarify the terminology. Many papers use the notion of multidimensional displacement, which reflects the movement of objects in three-dimensional space [6]. This term is not equivalent or identical to the term of multicomponent movement, which is discussed in this article. Multicomponent displacements, which correspond to the positions of the formulated concept, can be one-dimensional and multidimensional.

The space for modeling multicomponent displacements of simple objects can be represented in the following view (Figure 1).

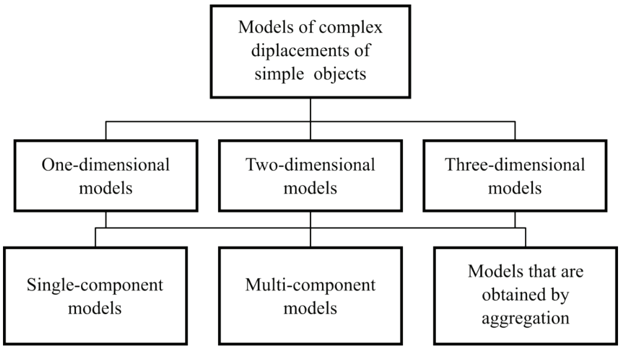

Figure 1 – Classification of mathematical models of multi-component displacements of mobile objects

Thus, even for simple objects, the modeling space provides significant scope for creativity. In addition, information models of vector multicomponent physical quantities allow one to alternatively represent informative components. This is directly noted in the third position of the conception. That is, the task of building such models is complex and ambiguous. Therefore, a formal apparatus for the synthesis of such models is needed, which is based on certain rules and agreements.

Formal synthesis of models of multicomponent displacements of moving objects

Let us take as a basis the model in the form of the system of equations (2) and pass to the scalar form. To do this, we introduce the coefficients $\eta_{i,j,k} \in [0,1]$, $\varsigma_{i,j,k} \in \{0,1,-1\}$, which are determined by the following agreements:

$$\eta_{i,j,k} = \begin{cases} 0, \text{if the information component } \mathbf{x}_{ijk}(\mathbf{r},\tau) \text{ is missing;} \\ (0,1], \text{if the information component } \mathbf{x}_{ijk}(\mathbf{r},\tau) \text{ is present,} \end{cases} \quad (3)$$

$$\varsigma_{i,j,k} = \begin{cases} +1, \text{if the projections of the vectors } \mathbf{x}_{i,j,k} \text{ coincide with} \\ \text{the directions of the corresponding coordinate axis;} \\ -1, \text{if the projections of the vectors } \mathbf{x}_{i,j,k} \text{ do not coincide} \\ \text{with the directions of the corresponding coordinate axis;} \\ \ \ 0, \text{if the corresponding information component} \\ \text{is missing,} \end{cases} \quad (4)$$

where i is serial number of the models; $k \in \{x, y, z\}$ determines the dimension of the models; j is serial number of informative components.

Then model (2) will be written in the following form:

$$\left.\begin{aligned} X_{ix}(\mathbf{r},\tau) &= \sum_{j=1}^{p} \eta_{ijx}\varsigma_{ijx}x_{ijx}(\mathbf{r},\tau); \\ X_{iy}(\mathbf{r},\tau) &= \sum_{j=1}^{p} \eta_{ijy}\varsigma_{ijy}x_{ijy}(\mathbf{r},\tau); \\ X_{iz}(\mathbf{r},\tau) &= \sum_{j=1}^{p} \eta_{ijz}\varsigma_{ijz}x_{ijz}(\mathbf{r},\tau). \end{aligned}\right\} \quad (5)$$

Model (5) is a combination of one-dimensional models, in which the informative components $x_{ijx}(\mathbf{r}, \tau)$, $x_{ijy}(\mathbf{r}, \tau)$, $x_{ijz}(\mathbf{r}, \tau)$ are projections of the informative components of the vector quantity $\mathbf{X}_i$ on the axes of the Cartesian coordinate system, and the direction of these projections along the corresponding axes determined by the signs that are established by agreement (4).

Model (5) can be written in the generalized form:

$$X_{ik}(\mathbf{r},\tau) = \sum_{j=1}^{p} \eta_{ijk}\varsigma_{ijk}x_{ijk}(\mathbf{r},\tau), \quad k \in \{x, y, z\}. \quad (6)$$

The combination of variants from (6) makes it possible to build models in one-dimensional, two-dimensional and three-dimensional spaces, which corresponds to the classification presented in Figure 1.

In accordance with the original formulation of the problem the informational components $x_{ijk}(\mathbf{r},\tau)$, $j \in \{1,..., p\}$ are of interest.

The task of defining informative components can be illustrated by the following structure (Figure 2):

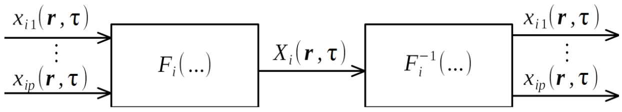

Figure 2 – Illustration of the task of determining informative components for a one-dimensional model of a multicomponent quantity

Here $F_i(...)$ is the function of communication of informative components: $x_{i1}(\mathbf{r}, \tau)$,…, $x_{ip}(\mathbf{r}, \tau)$ in a multicomponent physical quantity $X_i(\mathbf{r}, \tau)$, which can be described by model (6); $F_i^{-1}(...)$ is the inverse function, which should provide the transition from $X_i(\mathbf{r}, \tau)$ to $x_{i1}(\mathbf{r}, \tau)$,…, $x_{ip}(\mathbf{r}, \tau)$.

The problem is that a single-channel (one-dimensional) structure built on the basis of model (2) does not allow the transition from $X_i(\mathbf{r}, \tau)$ to $X_i(\mathbf{r}, \tau)$, which is due to the incorrectness of such a task. The way out of this situation is to combination the models. The generalized structure of the system that implements the combination of one-dimensional models is shown in Figure 3.

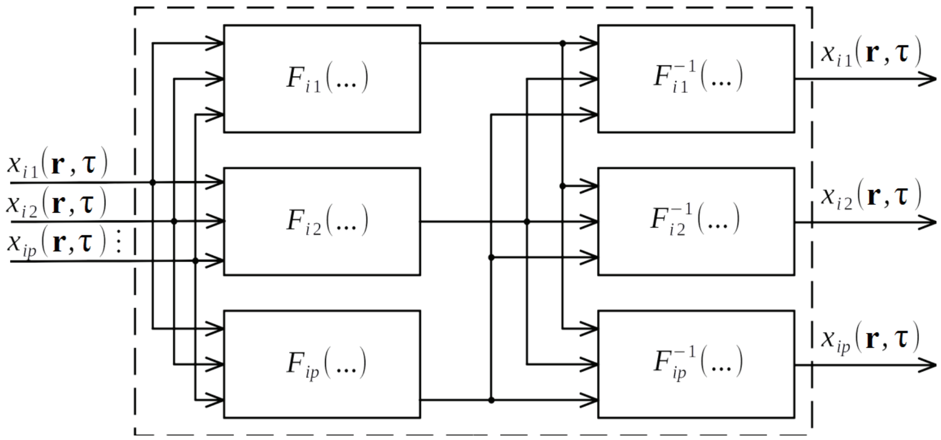

Figure 3 – Structure based on the combination of one-dimensional models that implements information redundancy

This structure illustrates information redundancy, which can be realized, for example, by several measurement channels and, accordingly, by composing a system of equations that can be solved with respect to each of the informative components.

Plane models of multicomponent displacements of moving objects

Each of the equations included in systems (1), (2) or (5) is obtained by projecting the simulated multicomponent quantity onto the corresponding coordinate axis of the Cartesian coordinate system. That is, each of the projections is a model of the multicomponent displacement of the object point in the projection onto a one-dimensional space. The transition from three-dimensional models of multicomponent displacements to flat models allows the latter to be combined with images on the plane of video cameras. This is important question for the development of the method for measuring these informative components.

If we take into account the zoom coefficient of the optical system, then in accordance with (2) we can write:

$$\left.\begin{aligned} \mathbf{Y}_x(\mathbf{r},\tau) &= \sigma\left\{\sum_{j=1}^{p}\mathbf{x}_{jx}(\mathbf{r},\tau)\right\}; \\ \mathbf{Y}_y(\mathbf{r},\tau) &= \sigma\left\{\sum_{j=1}^{p}\mathbf{x}_{jy}(\mathbf{r},\tau)\right\}, \end{aligned}\right\} \qquad (7)$$

where $Y_x(\mathbf{r}, \tau)$, $Y_y(\mathbf{r}, \tau)$ are images of multicomponent displacements of some point of the object in the coordinate system of the video camera; σ is zoom coefficient of the optical system.

For convenience, let's move on to the scalar form of the model. To do this, we use the agreements (3) and (4):

$$\left.\begin{aligned} Y_{ix}(\mathbf{r},\tau) &= \sigma\left\{\sum_{j=1}^{p}\eta_{ijx}\varsigma_{ijx}x_{ijx}(\mathbf{r},\tau)\right\}; \\ Y_{iy}(\mathbf{r},\tau) &= \sigma\left\{\sum_{j=1}^{p}\eta_{ijy}\varsigma_{ijy}x_{ijy}(\mathbf{r},\tau)\right\}. \end{aligned}\right\} \qquad (8)$$

The presence of combination coefficients: $\eta_{i,j,k}$, $\varsigma_{i,j,k}$ makes it possible to automatically build models that will reflect different processes and phenomena in the object. Models (7) can be used in technical vision systems to determine the informative components of displacements of points of an object in three-dimensional space. However, restoring the real coordinates of points of moving objects in three-dimensional space from their images on a plane is incorrect task [5]. This problem can be solved using binocular vision. But we will consider another method, which had called the "method of multidimensional test objects".

A method based on the use of multidimensional test objects

The problem of reconstructing the informative components of complex multicomponent displacements of objects points in three-dimensional space from their flat images is complicated additionally by the fact that the known, including optical, measurement methods are not selective to the mentioned informative components. For this reason the purpose of the method is to solve the illposed problem of reconstructing the real values of the informative components of the displacements of moving objects in three-dimensional space from their flat images with no selectivity of the known measurement methods and instruments to these components.

Almost all known methods of improving the quality of measuring systems, except for conservative ones, are based on the use of information redundancy of system [7–9]. Here it is proposed to use multidimensional test objects with a priori known parameters at the input of the optical measuring system to ensure information redundancy.

Since we are talking about restoring the components of multicomponent displacements of objects in three-dimensional space, which are vector quantities, then the parameters of multidimensional test objects that should be included in the model also must be vectored quantities. The type and number of multidimensional test objects parameters are determined by the multidimensionality of controlled displacements and are functionally linked to them in the models of multicomponent displacements:

$$\left.\begin{aligned} \mathbf{X}_x(\mathbf{r},\tau) &= \mathbf{F}\left(\mathbf{x}_{1x}(\mathbf{r},\tau),...,\mathbf{x}_{px}(\mathbf{r},\tau),\,\mathbf{L}_{1x},...,\mathbf{L}_{qx}\right);\\ \mathbf{X}_y(\mathbf{r},\tau) &= \mathbf{F}\left(\mathbf{x}_{1y}(\mathbf{r},\tau),...,\,\mathbf{x}_{py}(\mathbf{r},\tau),\,\mathbf{L}_{1y},...,\,\mathbf{L}_{qy}\right);\\ \mathbf{X}_z(\mathbf{r},\tau) &= \mathbf{F}\left(\mathbf{x}_{1z}(\mathbf{r},\tau),...,\,\mathbf{x}_{pz}(\mathbf{r},\tau),\,\mathbf{L}_{1z},...,\,\mathbf{L}_{qz}\right), \end{aligned}\right\} \quad (8)$$

where $\mathbf{L}_{1k}, \ldots, \mathbf{L}_{qk}$ are the parameters of the k-th coordinate component $\mathbf{L}_k$ of the multivariate test $\mathbf{L}$; q is the number of components of the k-th coordinate component $\mathbf{L}_k$ of the multivariate test $\mathbf{L}$.

So, the principal feature of the model (8) is the introduction into it as known informative components of the test objects parameters $\mathbf{L}_{1k}, \ldots, \mathbf{L}_{qk}$ that are set in vector form.

Figure 4 shows the two-dimensional test object that is obtained by combining two one-dimensional ones.

The test object ABCD is located in plane $O_0X_0Y_0$ and has the following known parameters:

$AO_i = nL_{ABx}$ and $BO_i = (1–n)L_{ABx}$, (n = 0.5);

$CO_i = nL_{CDy}$ and $DO_i = (1–n)L_{CDy}$, (n = 0.5);

$EB = (1–n)L_{ABx}$ and $FD = (1–n)L_{CDy}$, (n = 0.75).

These parameters in the models (8) are used in vector form, for example:

$\mathbf{AO}_i = n\mathbf{L}_{ABx} = nL_{ABx} \cdot \mathbf{i}$, ($n$ = 0.5);

$\mathbf{BO}_i = (1–n)\mathbf{L}_{ABx} = (1–n)\mathbf{L}_{ABx} \cdot \mathbf{i}$, ($n$ = 0.5);

$\mathbf{CO}_i = n\mathbf{L}_{CDy} = nL_{CDy} \cdot \mathbf{j}$, ($n$ = 0.5);

$\mathbf{DO}_i = (1–n)\mathbf{L}_{CDy} = (1–n)\mathbf{L}_{CDy} \cdot \mathbf{j}$, ($n$ = 0.5);

$\mathbf{EB} = (1–n)L_{ABx} = (1–n)L_{ABx} \cdot \mathbf{i}$, ($n$ = 0.75);

$\mathbf{FD} = (1–n)L_{CDy} = (1–n)L_{CDy} \cdot \mathbf{j}$, ($n$ = 0.75),

where **i** and **j** are the basis vectors whose direction coincides with the direction of the axes O_0X_0 and O_0Y_0.

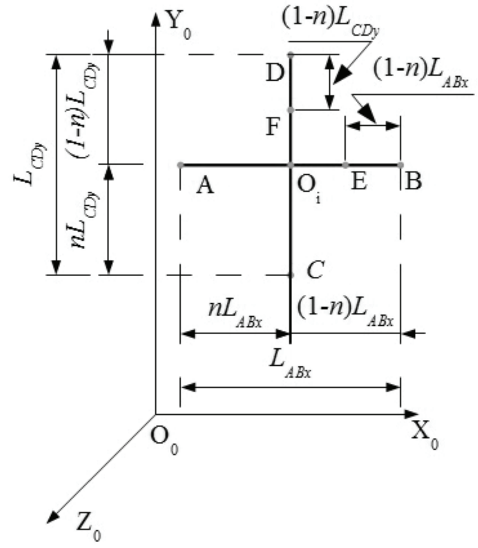

Figure 4 – Two-dimensional test object in the form of the cross

The task of classifying or displaying the variety of test objects is not posed in this article. Here, only the fundamental requirements imposed on them are formulated.

The general methodology for the formation of multidimensional tests and the functions of the connection of their components with multicomponent quantities in the models are determined by the main provisions of the concept of vector multicomponent physical quantities and are formulated as follows:

– Multidimensional multicomponent tests are considered as functions of the set of their constituent components;

– The above-mentioned test component's communication functions in the multicomponent test models are determined by the laws of vector algebra;

– Models of vector multidimensional multicomponent tests allow an alternative representation of test components depending on the problem being solved.

Based on the above provisions on the multidimensional test object, let us determine the form of the connection function **F** of informative components $\mathbf{x}_{1k}(\mathbf{r},\tau)$, ..., $\mathbf{x}_{pk}(\mathbf{r},\tau)$ and components $\mathbf{L}_{1k}$, ..., $\mathbf{L}_{qk}$ of the k-th coordinate component $\mathbf{L}_k$ of the multidimensional test $\mathbf{L}$ in model (8):

$$\mathbf{F}_{ik}\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\}=\sum_{k}^{\{x,y,z\}}\sum_{u=1}^{q}v_{iuk}\mathbf{L}_{iuk}+\sum_{k}^{\{x,y,z\}}\sum_{j=1}^{p}\eta_{ijk}\mathbf{x}_{ijk}(\mathbf{r},\tau), \tag{9}$$

where i is the serial number of the communication function; $k\in\{x,y,z\}$ is set of coordinate components; u is the serial number of the components of the multicomponent test $\mathbf{L}_{iuk}$; j is the serial number of the informative components of the k-th coordinate component of the multicomponent displacement $\mathbf{X}_k(\mathbf{r},\tau)$; $v_{iuk}\in[0,1]$ are the weight coefficients that reflect the absence – 0 – or the presence of – (0, 1] – of the corresponding component of the multicomponent test $\mathbf{L}_{iuk}$ in the model (8); $\eta_{ijk}\in[0,1]$ are the weight coefficients that reflect the absence – 0 – or the presence of – (0, 1] – of the corresponding informative component $\mathbf{x}_{ijk}(\mathbf{r},\tau)$ in the model (8).

This representation of the model (8) provides a mechanism for its adaptation to specific practical problems by varying the coefficients: $v_{iux}\in[0,1]$, $v_{iuy}\in[0,1]$, $v_{iuz}\in[0,1]$, $\eta_{ijx}\in[0,1]$, $\eta_{ijy}\in[0,1]$ $\eta_{ijz}\in[0,1]$ in the field of their definition.

Models (9), by analogy with (6), can be represented in scalar form. In order to do this, additional conventions should be introduced, which we will return to directly when describing the measurement method.

Thus, we have constructed models of vector multicomponent displacements in vector form and when certain conventions are met in scalar form. We understand the need for redundancy of the information entering the system to ensure the possibility of measuring the information components sewn in these models. And we propose the method for organizing information redundancy, which is based on the use of multidimensional test objects. Multidimensional test objects are rigidly connected to a moving object and move with it. So, the information redundancy is a necessary condition. Let's move on to the method.

Necessary and sufficient conditions for the physical realization of the optical measurement method based on the use of multidimensional test objects

1. The ability to form a system of n equations that are asymmetric with respect to informative components $\mathbf{x}_{1k}(\mathbf{r},\tau)$, ..., $\mathbf{x}_{pk}(\mathbf{r},\tau)$ ($k\in\{x,y,z\}$ is the set of coordinate components) of displacements of the corresponding points of the test object image:

$$\left.\begin{array}{l}\mathbf{Y}_1(\mathbf{r},\tau)=\Psi_1\left\{\mathbf{F}_1\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\}\right\};\\ \dots\dots\dots\dots\dots\dots\\ \dots\dots\dots\dots\dots\dots\\ \mathbf{Y}_n(\mathbf{r},\tau)=\Psi_n\left\{\mathbf{F}_p\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\}\right\},\end{array}\right\}(n\ge p\ge 2), \tag{10}$$

$$\mathbf{F}_1\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\}\ne...\ne\\ \ne\mathbf{F}_p\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\}, \tag{11}$$

where $\mathbf{Y}_1(\mathbf{r},\tau)$, ..., $\mathbf{Y}_n(\mathbf{r},\tau)$ are functions of displacement of the corresponding points of the image of the object being monitored relative to the selected on the image of the points of reference; $\mathbf{F}_1\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\},...,$ $\mathbf{F}_p\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\}$ are vector functions of the set of informative components $\mathbf{x}_{1k}(\mathbf{r},\tau)$, ..., $\mathbf{x}_{pk}(\mathbf{r},\tau)$ and components $\mathbf{L}_{1k}$, ..., $\mathbf{L}_{qk}(\mathbf{r},\tau)$ of the coordinate component $\mathbf{L}_k$ of a multidimensional test object (of the multidimensional test) $\mathbf{L}$.

2. Realizability of special measuring and computing algorithms:

$$\left.\begin{array}{l}\mathbf{x}_{1k}(\mathbf{r},\tau)=f_1\left\{\mathbf{Y}_1(\mathbf{r},\tau),...,\mathbf{Y}_n(\mathbf{r},\tau)\right\};\\ \dots\dots\dots\dots\dots\dots\\ \dots\dots\dots\dots\dots\dots\\ \mathbf{x}_{pk}(\mathbf{r},\tau)=f_p\left\{\mathbf{Y}_1(\mathbf{r},\tau),...,\mathbf{Y}_n(\mathbf{r},\tau)\right\},\end{array}\right\} \tag{12}$$

whose existence condition, with continuity and differentiability $\mathbf{Y}_1(\mathbf{r},\tau)$, ..., $\mathbf{Y}_n(\mathbf{r},\tau)$ in the whole range of measurement, is that the Jacobian does not become zero:

$$\det\left[\frac{\partial\,\mathbf{Y}_i(\mathbf{r},\tau)}{\partial\,\mathbf{x}_{jk}(\mathbf{r},\tau)}\right]\ne 0;\;\; i=\overline{1,n};\;\; j=\overline{1,p}. \tag{13}$$

The condition (13) is ensured by the implementation of the "asymmetry" of the values $\mathbf{Y}_1(\mathbf{r},\tau)$, ..., $\mathbf{Y}_n(\mathbf{r},\tau)$ relative to their constituent components $\mathbf{x}_{1k}(\mathbf{r},\tau)$, ..., $\mathbf{x}_{pk}(\mathbf{r},\tau)$ and $\mathbf{L}_{1k}$, ..., $\mathbf{L}_{qk}(\mathbf{r},\tau)$, which is expressed by the inequality (11).

Obviously, when using a single-channel optical system, the functions ψ_1, ..., ψ_n are the identical. If we use the transmission coefficient σ of the optical converter, then the system of equations (10) can be rewritten as follows:

$$\left.\begin{array}{l} \mathbf{Y}_1(\mathbf{r},\tau)=\sigma\left\{\mathbf{F}_1\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\}\right\}; \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ \mathbf{Y}_n(\mathbf{r},\tau)=\sigma\left\{\mathbf{F}_p\left\{\mathbf{x}_{1k}(\mathbf{r},\tau),...,\mathbf{x}_{pk}(\mathbf{r},\tau),\mathbf{L}_{1k},...,\mathbf{L}_{qk}\right\}\right\}, \end{array}\right\}(n\ge p\ge 2). \quad (14)$$

If we use the formal mechanism of synthesis of models and the agreements (3) and (4), which were discussed earlier, we can proceed to the scalar form of the system of equations (14):

$$\left.\begin{array}{l} \gamma_1 Y_1(\tau)=\sigma\left\{\sum\limits_{k}^{\{x,y,z\}}\sum\limits_{u=1}^{q}\xi_{1uk}v_{1uk}L_{uk}+\sum\limits_{k}^{\{x,y,z\}}\sum\limits_{j=1}^{p}\varsigma_{1jk}\eta_{1jk}x_{jk}(\tau)\right\}; \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ \gamma_n Y_n(\tau)=\sigma\left\{\sum\limits_{k}^{\{x,y,z\}}\sum\limits_{u=1}^{q}\xi_{nuk}v_{nuk}L_{uk}+\sum\limits_{k}^{\{x,y,z\}}\sum\limits_{j=1}^{p}\varsigma_{njk}\eta_{njk}x_{jk}(\tau)\right\}, \end{array}\right\}(n\ge p\ge 2), \quad (15)$$

where $Y_1(\tau)$, ..., $Y_n(\tau)$ are distances from the starting points (marks) which had selected on the sensitive plane of the image receiver to the i-th points of the image of the controlled object; σ is optical transducer transmission coefficient; the coefficients γ_i take on values according to the following agreement:

$$\gamma_i=\begin{cases} +1, \text{ if the projections of the vectors } \mathbf{Y}_i(\mathbf{r},\tau) \text{ coincide with} \\ \quad \text{the directions of the corresponding coordinate axes;} \\ -1, \text{ if the projections of the vectors } \mathbf{Y}_i(\mathbf{r},\tau) \text{ are opposite} \\ \quad \text{to the directions of the corresponding coordinate axes,} \end{cases} \quad (16)$$

where i corresponds to the serial number of the equation.

Since $\gamma_i \in \{+1,-1\}$ the system of equations (15) can be written in the following form:

$$\left.\begin{array}{l} Y_1(\tau)=-\gamma_1\sigma\left\{\sum\limits_{k}^{\{x,y,z\}}\sum\limits_{u=1}^{q}\xi_{1uk}v_{1uk}L_{uk}+\sum\limits_{k}^{\{x,y,z\}}\sum\limits_{j=1}^{p}\varsigma_{1jk}\eta_{1jk}x_{jk}(\tau)\right\}; \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ Y_n(\tau)=-\gamma_n\sigma\left\{\sum\limits_{k}^{\{x,y,z\}}\sum\limits_{u=1}^{q}\xi_{nuk}v_{nuk}L_{uk}+\sum\limits_{k}^{\{x,y,z\}}\sum\limits_{j=1}^{p}\varsigma_{njk}\eta_{njk}x_{jk}(\tau)\right\}, \end{array}\right\}(n\ge p\ge 2). \quad (17)$$

The agreement (16) is preserved for the system of equations (17). And condition (13) for the existence of the corresponding measuring and computational algorithms that follow from the solution from the system of equations (17) will look as follows:

$$\det\left[\frac{\partial Y_i(\tau)}{\partial x_{jk}(\tau)}\right]\ne 0,\ \ i=\overline{1,n},\ \ j=\overline{1,p}. \quad (18)$$

Having solved the system of equations (18) with respect to $x_{1k}(\tau)$, ..., $x_{pk}(\tau)$, we obtain the corresponding measuring and computational algorithms:

$$\left.\begin{array}{l} x_{1k}(\tau)=f_1\{Y_1(\tau),...,Y_n(\tau)\}; \\ \dots\dots\dots\dots \\ \dots\dots\dots\dots \\ x_{pk}(\tau)=f_p\{Y_1(\tau),...,Y_n(\tau)\}. \end{array}\right\} \quad (19)$$

The appearance of the “-” sign in front of the value of the corresponding displacement component indicates a direction that is opposite to the direction of the corresponding coordinate axis.

Conclusion

The concept of vector multicomponent physical quantities and the method of multidimensional test objects in optical measurements, which are presented in this paper, are the theoretical basis for constructing measuring systems for determining the informative components of complex multicomponent displacements and deformations of moving objects. The possibility of solving the incorrect problem of restoring the real coordinates of a moving object from a sequence of its flat images is an additional advantage of the method. Some particular implementations of the presented method are shown in [10, 11]. There are other patented examples, but they do not exhaust all the possibilities of the theory presented here, which, regardless of the presence of certain patents, has independent significance.

The areas of practical applications of the presented concept and the method of optical measurements of multicomponent displacements and deformations are apparently beyond the limits of the author’s imagination. If we consider complex mechanical systems, then the optical measurement method can be used to determine the components of complex displacements and deformations of structural elements in the process of finetuning and testing gas turbine engines. As noted in the abstract, the informative components of displacements and deformations are displacements and deformations of the links of manipulators of universal industrial robots. At the same time, it is known that a universal industrial robot becomes technologically complete equipment after performing a calibration operation, which requires highprecision measurement of displacements and orientations of the manipulator flange in the working area. The most important practical advantage of the considered measurement method is the optimization of the amount of additional information at the input of the optical system. This makes it possible to obtain fast information processing algorithms in vision systems designed to work as part of real-time control systems.

Despite the quite strict and fairly consistent presentation, all even theoretical issues have not been exhausted in the work. For example, it has already been written about this, it is necessary to separately consider the problem of generating multidimensional tests, to investigate the effects of their type, number and optimization of tests components on the quality of the measuring system both in the general theoretical aspect and to solve specific application problems. Therefore, a number of such questions have yet to be considered and built into the body of the presented theory.

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