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An Accelerated Three Term Efficient Algorithm for Numerical Optimization

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تسريع خوارزمية من ثلاث شروط فعاله لتحسين في الامثليية العددية

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ABSTRACT

Background:

A new optimization algorithm is presented. The method is biased with new non monotone line search an accelerated three term conjugate gradient method damped of Quasi Newton method compared to previews method design efficiency in provident of more than one factor for different optimization problem are more dramatic due to the ability of the technique to utile existing data.

Materials and Methods:

New monotone line search, new monotone line search, new modification of Damped Quasi-Newton method, Motivation and New Quasi-Newton Algorithm (MQ) and Global convergence.

Results:

In this work, we have n tendency to compare our new algorithm with same classical strategies like [7] by exploiting of unconstrained nonlinear optimization problem the functions obtained from Andrei [5, 6] Waziri and Sabiu (2015)[10] and La couzetul (2004)[3]. The numerical experiments demonstrate the performance of the proposed method. We selected seven relatively unconstrained problems with the size varies from 10 to 100. We consider the three sizes of each problem so that the total number of problem is 21 test problems. We stop the iteration when $\|g_k\| \leq 10^{-6}$ is satisfied All codes were written in Matlab R2017a and run on a pc with Intel COREi4 with a processor with 4GB of Ram and CPU 2.3GHZ we solved test problems using two different initial starting points.

Conclusion:

In this research article, a project on an accelerated three-term efficient algorithm for numerical optimization has presented the method as completely a derivative-free algorithm with less NOI and NOF and CPU time computed to the existing methods .using classical assumption the global convergence was also proved. Numerical results using the three terms efficient algorithm show that the algorithm is promising.

Keyword:

Global convergence, non-monotone line search, Three-term conjugate gradient method.

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الخلاصة

مقدمة

يتم تقديم خوارزمية تحسين جديدة. الطريقة متحيزة مع بحث خط جديد غير رتيب ، طريقة التدرج المتقارن ذات الثلاثة مصطلحات المتسارعة المخمدة بطريقة شبه نيوتن مقارنة مع طريقة المعاينة. الاستفادة من البيانات الموجودة.

المواد وطرائق العمل

بحث جديد رتيب الخط ، بحث جديد رتيب سطر ، تعديل جديد لطريقة Damped Quasi–Newton ، التحفيز وخوارزمية شبه نيوتن الجديدة ((MQوالتقارب العالمي.

نتائج:

في هذا العمل ، لدينا ميل لمقارنة خوارزمية جديدة مع نفس الاستراتيجيات الكلاسيكية مثل [7] من خلال استغلال مشكلة التحسين غير الخطية غير المقيدة الوظائف التي تم الحصول عليها من [5 Andrei، 6 [10] Waziri and Sabiu (2015) و] La couzetul (2004) [3].التجارب العددية توضح أداء الطريقة المقترحة. لقد اخترنا سبع مشاكل غير مقيدة نسبيًا مع اختلاف الحجم من 10 إلى 100. نحن نأخذ في الاعتبار الأحجام الثلاثة لكل مشكلة بحيث يكون العدد الإجمالي للمشكلة هو 21 مشكلة اختبار. نتوقف عن التكرار عند اقتناع $\| g_k \| \leq 10^{(-6)}$ تمت كتابة جميع الرموز في Matlab R2017a وتشغيلها على جهاز كمبيوتر مزود بـ Intel COREi4 بمعالج بسعة 4 جيجابايت من ذاكرة الوصول العشوائي ووحدة المعالجة المركزية بسرعة 3.2 جيجاهرتز ، قمنا بحل مشكلات الاختبار باستخدام نقطتي بداية مختلفتين.

استنتاج:

في هذه المقالة البحثية ، قدم مشروع على خوارزمية فعالة ثلاثية المدى للتحسين العددي الطريقة كخوارزمية خالية تمامًا من المشتقات مع وقت أقل من NOI و NOF و CPU محسوبة للطرق الحالية. باستخدام الافتراض الكلاسيكي كان التقارب العالمي ثبت أيضا. تظهر النتائج العددية باستخدام الخوارزمية الفعالة ذات المصطلحات الثلاثة أن الخوارزمية واعدة.

الكلمات المفتاحية

التقارب العام ، البحث عن خط غير رتيب ، طريقة التدرج المترافق ثلاثي المصطلحات.

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INTRODUCTION

This paper is concerned with damped Quasi-Newton methods for finding a local minimum of the unconstraint optimization problem [4, 7 and 17].

$$\min_{x \in R^n} f(x) \tag{1}$$

Under line search algorithms with the basic iteration

$$x_{k+1} = x_k + \propto_k d_k \tag{2}$$

Where x_k is given here, $\propto_k$denoted the step length and d_k is the search direction defined by

$$d_1 = \propto^{-1} g_1 \ \ if \ k = 1 \tag{3}$$

$$D_k = - B_k^{-1} g_k \tag{4}$$

Where$g_k$denoted the gradient of $fatx_k$ the matrix D and $B_k fork \geq 1$ approximate the Hessian of f at $x_1$and x_k respectively. Since the matrix B_1 is used to initiate a Quasi Newton update at the end of the first iteration, certain new information may be used to define this matrix not necessarily equal to the user-supplied D, though usually $B_1 = D$ at each iteration, a new Hessan approximation $B_{k+1}$is calculated updating $B_k$using a damped of Quasi-Newton method.

It is well known that sufficient descent condition, conjugate condition, and minimizing condition number are important factors to accelerate iteration. [16-18] accelerate the iteration and eliminated the round off error a dynamical compensation in our proposed dumped of Quasi-Newton method is suggested that is satisfied as much as possible. which aim to consider accelerating the iteration when the second derivation approximation of the objective function is not as satisfying the damped Quasi Newton equation proposed, these methods are mostly used when the second derivative matrix of the objective function is either unavailable or too costly to compute, there are very similar to newton's method avoid the need to f computing Hessian matrices by recurring from iteration to iteration [15].

A general class of Quasi-Newton update was proposed by Broden [1, 2, and 14].

$$H_{k+1} = H_k - Q \tag{5}$$

Where

$$Q = \frac{H_k Y_k X_k^T H_k}{Y_k^T H_k Y_k} + \frac{V_k V_k^T}{V_k^T Y_k} + \phi_k (Y_k^T H_k Y_k) Y_k \tag{6}$$

$$Y_k = \frac{V_k V_k^T}{V_k^T Y_k} - \frac{H_k Y_k}{Y_k^T H_k Y_k} \quad , \; V_k = \frac{V_k V_k^T}{V_k^T Y_k} - \frac{H_k Y_k}{Y_k^T H_k Y_k} \tag{7}$$

$$W_k = \frac{Y_k^T H_k Y_k}{V_k^T Y_k} \quad , S_k = \frac{V_k \beta_k V_k^T}{V_k^T Y_k} \tag{8}$$

Al-Baali [4 and 18] and Gradient (2009) [13, 15-17] show that the performance of the BFGS method can be improved if Y_k modified before updating to the damped technique.

$$\hat{Y}_k = \phi_k Y_k + (1 - \phi_k)\beta_k S_k \tag{9}$$

Where Q_k is a parameter chosen appropriately large in the interval (0, 1], the resulting damped (D)-BFGS method is proposed by Powell [9, 18] for the Lagrangian function in constrained optimization and used many times with only values of $Q_k \geq 0.8$ see for example (Feltcher 1987, Nocedal and wright 1999)[7] the aim of this paper is to the new damped technique with small values of $\phi_k$can be used to modify the BFGS method for unconstrained optimization. Illustrated this possibility for several numerical optimization problems using non-monotone line search.

Materials and Methods

- **New monotone line search**

The order to analyze the convergence of our algorithm we need the following assumptions

H1: The objective function $f(x)$ is continuously differentiable and has a lower bound on R^n ,

H2: the gradient $g(x) = \nabla f(x)$ of $f(x)$ is Lipschitz continuously differentiable on an open convex set B that contains the level set $L(x_0) = \{x \in R^n \,; f(x) \leq f(x_0)\}$with $x_0$given i.e there exist an $L > 0$ such that

$$\|g(x) - g(x_0)\| \leq L\|x - y\| \quad , \forall\, x, y \in B \tag{10}$$

Since L is usually not known a priori in practical computation but it plays an important role in algorithm design. We need to estimate it for the new non-monotone line search same approach for estimating L as proposed [11-13].

If L is a Lipschitz constant. However, a very large Lipschitz constant can lead to a very small step size and makes damped Quasi-Newton methods with the new non-monotone line search converge very slowly, therefore we should see Lipschitz constants that are as small as possible in practical computation, in the kth iteration we take respectively the approximate Lipschitz constant as

$$L_k = \max\left(L_0, min\left(\frac{\|\delta_{k-1}^T Y_{k-1}\|}{\|\delta_{(k-1)}\|^2}, M\right)\right) \tag{11}$$

Where

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$$\delta_{k-1} = x_k - x_{k-1} \text{ and } Y_k = g_k - g_{k-1} \tag{12}$$

With $L_0 > 0$ and M_1being a large positive number in which the new non-monotone line search is used in the practical computation. Their global convergence and convergence rate will be given in the subsequent section. New non-monotone line search. Given μ ∈ (0,0.5), $\rho \in (0,1)$, $c \in (0.5,1)$, and $u \in [0,1]$, set $s_k = \frac{1-c}{L_k}, \frac{(1-u)\|Y_k\|^2 - 4Y_k^T d_k}{\|d_k\|^2}$ and α_k is the largest α in $\{s_k , s_k\rho , s_k\rho^2, \dots\}$ such that

$$\max[f_{k-j}] - f(x_k + \alpha d_k) \geq -\alpha\mu\left[Y_k^T d_k + \frac{1}{2}\alpha L_k\|d_k\|^2\right] \tag{13}$$

And

$$Y_k^T d_{k+1} \leq -c\|Y_{k+1}\|^2 \tag{14}$$

Where

$$d_k = -Y_k + \frac{Y_k^T d_k}{(1-c)\|Y_k\|^2 - uY_k^T d_k} d_k \tag{15}$$

and L_k is estimated by (11) respectively.

- **New modified of the Damped Quasi-Newton Method**

Quasi-Newton (QN) methods are recognized today as one of the most efficient ways to solve nonlinear unconstrained optimization problems these methods are mostly used when the second derivative matrix of the objective function is either unavailable or too costly to compute, a general class of Quasi-Newton update was proposed by Broden[2 and 14].

$$H_{k+1} = H_k - Q \tag{16}$$

$$Q = \frac{H_k Y_k Y_k^T H_k}{Y_k^T H_k Y_k} + \frac{V_k V_k^T}{V_k^T Y_k} + \varphi_k\left(Y_k^T H_k Y_k\right)\gamma_k \tag{17}$$

Where

$$\gamma_k = \frac{V_k V_k^T}{V_k^T Y_K} - \frac{H_k Y_K}{Y_k^T H_k Y_k}$$

$$\omega_k = \frac{Y_k^T H_k Y_k}{V_k^T Y_k}, \delta_k = \frac{V_k^T B_k V_k}{V_k^T Y_k}$$

$$\varphi_k = \frac{1-\theta_k}{1+\theta_k(\omega_k\delta_k - 1)}$$

Where $\theta_k \in R^1$is a parameter, There are three popular choices of θ_k[9, 8 and 14].

To improve the performance of the QN update, Biggs [7] proposed to choose H_{k+1} to satisfy the following modified equation

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$$H_{k+1}Y_k = \epsilon_k V_k \tag{18}$$

Where $\epsilon_k > 0$ is a scaling parameter.

• Motivation and New Quasi-Newton Algorithm (MQ)

Now we describe the algorithm of the proposed method as:

Step 1: choose $x_0 \in R^n$, $U[0,1]$ and set $K = 1, H_0 = I$,

Step 2: if $\|g_k\| = 0$ then stop else $d_k = -H_k g_k$,

Step 3: set $x_{k+1} = x_k + \alpha_k d_k$ where $d_k = -H_k g_k$ is defined by $V_k = \alpha_k d_k$, $Y_k = g_{k+1} - g_k$, $H_{k+1} = H_k + Q_k$ and α_k is defined by the new non-monotonic line search (13),

Step 4: set $k = k + 1$ go to step 3.

• Global Convergence

Lemma a: Assume that (H1) and (H2) hold and algorithm MQ with the new non-monotone line search generated on an infinite sequence $\{x_k\}$, then there exist $m_0 > 0$ and $M_0 > 0$ such that

$$M_0 \leq l_k \leq m_0 \tag{19}$$

Proof: Obviously $L_k \geq L_0$ and we can take $m_0 = L_0$ for (11), we have

$$L_k = \max\left(L_0, \min\left(\frac{\|Y_{k-1}\|^2}{\delta_{k-1}^T Y_{k-1}}, M_1\right)\right) \leq \max(L_0, M_1) \tag{20}$$

By letting $M_0 = \max(L_0, L, M)$ we complete the proof ■

Lemma b: Assume that (HH1) and (H2) holds, and algorithm MQ with the new non-monotone line search generates an infinite sequence $\{x_k\}$ if $g_k^T d_k < 0$ and

$$\alpha_k \leq \frac{L-c}{L} \frac{(1-u)\|g_k\|^2 - u g_k^T d_k}{\|d_k\|^2} \tag{21}$$

Then $g_{(k+1)^T} d_{k+1} \leq -c\|g_{k+1}\|^2$

Proof: by the two inequalities and Cauchy – Schwartz inequality, we have

$$-(1-c)\left[(1-u)\|g_k\|_k^2 - v g_k^T d_k\right] \geq \alpha_k L \, \|d_k\|^2 \tag{22}$$

$$= \alpha_k L \frac{\|g_{k+1}\|.\|d_k\|}{\|g_{k+1}\|^2} \|g_{k+1}\|.\,\|d_k\| \tag{23}$$

$$\geq \frac{\|g_{k+1}\|.\|g_{k+1} - g_k\|}{\|g_{k+1}\|^2} . \left|g_{k+1}^T d_k\right| \tag{24}$$

$$\geq \frac{\left|g_{k+1}^T . (g_{k+1} - g_k)\right|}{\|g_{k+1}\|^2} . \left|g_{k+1}^T d_k\right| \tag{25}$$

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$$\geq \frac{g_{k+1}^T.(g_{k+1}-g_k)}{(1-u)\|g_{k+1}\|^2-ug_k^Td_k}.\frac{(1-u)\|g_{k+1}\|^2-ug_k^Td_k}{\|g_{k+1}\|^2}.g_{k+1}^Td_k \quad (26)$$

$$= \beta_{k+1}.\frac{(1-u)\|g_{k+1}\|^2-ug_k^Td_k}{\|g_{k+1}\|^2}.g_{k+1}^Td_k \quad (27)$$

Therefore

$$(1-c)\|g_{k+1}\|^2 \geq \beta_{k+1}g_{k+1}^Td_k \quad (28)$$

And thus

-c$\|g_k\|^2 \geq -\|g_{k+1}\|^2 + \beta_{k+1}g_{k+1}^Td_k = g_{k+1}^Td_{k+1}$ ■

Numerical results and comparisons

In this work, we have n tendency to compare our new algorithm with the same classical strategies like [7] by exploiting unconstrained nonlinear optimization problems the functions obtained from Andrei [5 and 6] Waziri and Sabiu (2015)[10], and La couzetul (2004)[3]. The numerical experiments demonstrate the performance of the proposed method. We selected seven relatively unconstrained problems with the size varies from 10 to 100. We consider three sizes of each problem so that the total number of problem is 21 test problems. We stop the iteration when $||g_k|| \leq 10^{-6}$ is satisfied All codes were written in Matlab R2017a and run on a pc with Intel COREi4 with a processor with 4GB of Ram and CPU 2.3GHZ we solved test problems using two different initial starting points.

PROBLEMS

Generally, each problem should be declared and fully stated:

Problem (1) the strictly convex function

$$F_{i(x)} = e^{x_i - 1}\,; i = 2,3,\dots,n$$

Problem (2) the exponential function

$$F_i(x) = \frac{i}{10}\left(1 - x_i^2 - e^{-x_i^2}\right); i = 1,2,\dots,n$$

$$f_n = \frac{n}{10}\left(1 - e^{-x_n^2}\right)$$

Problem (3) The Tridiagonal system

$$f_{1(x)} = 4(x_1 - x_2^2)$$

$$f_{i(x)} = 8!\left(x_i^2 - x_{i-1}\right) - 2(1 - x_i) + 4\left(x_i - x_{i+1}^2\right), for\ i = 2,3,\dots,n-1$$

$$f_n = 8\, x_n(x_n^2 - x_{n-1}) - 2(1 - x_n)$$

Problem (4) The Generalized function of Rosenbrock

$$f_{1(x)} = x_1 - e^{\frac{\cos(x_1+x_2)}{2}}$$

$$f_{i(x)} = x_i - e^{\cos(x_{i+1}+x_i)}$$

$$f_{n(x)} = x_n - e^{\frac{\cos(x_{n-1}+x_n)}{n+1}}$$

Problem (5) generalized Oren and predicate function

$$f_{1(x)} = x_1^4$$

$$f_{i(x)} = \left[\sum_{i=1}^{n} i x_i^2\right]^2$$

$$f_n(x) = [x_1^2 + 2x_2^2 + \cdots + n x_n^2]^2\,;\, x_0 = (1,1,\ldots,1)^T$$

Problem (6) the variable bond function

$f_1(x) = -2x_1^2 + 3x_1 - 2x_2 + 0.5x\_3 + 1$

$f_{i(x)} = -2x_i^2 + 3x_i - x_{i-1} - 1.5\, x_{i+1} + 1\,;\, for\ i = 1,2,3, \ldots n-1$

$, f_{n(x)} = -2x_n^2 + 3x_n - 0.5x_{n-1} + 1$

Problem (7) the General Penall Function

$$f_1(x) = (x-1)^2 + eps(\, x_1^2 - 0.25)^2$$

$$f_i(x) = \sum_{i=1}^{n} (x_i - 1)^2 + eps\,\left(x_i^2 - 0.25\right)^2$$

$$f_n(x_n) = (x_1 - 1)^2 + (x_2 - 1)^2 + \cdots + (x_n - 1)^2 + eps\,(x_n^2 - 0.25)^2$$
$$x_n = (1,2,\ldots,n)^T, eps = 1.e - 5$$

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Table 1. Numerical comparison of the other algorithm and new algorithm

problem	New algorithm (MQ)				Other		
	N	NOI	Time	NOF	NOI	Time	NOF
1	10	9	0.0133	18	34	0.1562	69
	50	12	0.0123	23	40	0.0875	82
	100	14	0.0266	28	44	0.1397	94
2	10	21	0.0247	48	28	0.0331	56
	50	18	0.0194	36	15	0.1768	32
	100	14	0.0015	28	14	0.0026	29
3	10	673	1.1453	1352	-	-	-
	50	560	1.0348	1120	1530	3.2361	3160
	100	761	1.3753	1682	1517	3.5601	3034
4	10	21	0.0471	42	30	0.0416	67
	50	66	0.0638	120	42	0.0794	89
	100	21	0.0395	44	44	0.0729	33
5	10	71	0.1526	152	-	-	-
	50	82	0.2893	174	-	-	-
	100	114	0.5643	226	-	-	-
6	10	26	0.0304	52	78	0.1840	156
	50	42	0.5607	86	92	1.4105	184
	100	38	0.347	76	100	8.9049	202
7	10	25	0.4078	50	7	0.0227	14
	50	108	0.1307	116	-	-	-
	100	32	0.0758	64	16	0.0784	32

The tables above represented the numerical comparison of the other algorithm[10] and the New algorithm (MQ) in terms of NOI and NOF and CPU time in seconds, however from Table (1) the new algorithm has less NOI, NOF, and CPU time in most of the problems, this is due to the good selection of the line search.

Table 2. Numerical Comparison of the probability of other algorithms and probability of new algorithm					
Probability of New algorithm (MQ)			Probability of other algorithms		
Probability of NOI	Probability of time	Probability of NOF	Probability of NOI	Probability of time	Probability of NOF
0.003299	0.00209	0.003251	0.009364	0.008589	0.00941
0.004399	0.001933	0.004154	0.011016	0.004811	0.011182
0.005132	0.004181	0.005057	0.012118	0.007682	0.012819
0.007698	0.003882	0.008669	0.007711	0.00182	0.007637
0.006598	0.003049	0.006502	0.004131	0.009721	0.004364
0.005132	0.000236	0.005057	0.003856	0.000143	0.003955
0.246701	0.180016	0.244176	0	0	0
0.205279	0.162648	0.202276	0.421372	0.17794	0.430929
0.278959	0.216167	0.303775	0.417791	0.195755	0.413746
0.007698	0.007403	0.007585	0.008262	0.002287	0.009137
0.024194	0.010028	0.021672	0.011567	0.004366	0.012137
0.007698	0.006209	0.007947	0.012118	0.004008	0.0045
0.026026	0.023985	0.027452	0	0	0
0.030059	0.045472	0.031425	0	0	0
0.041789	0.088696	0.040816	0	0	0
0.009531	0.004778	0.009391	0.021482	0.010117	0.021274
0.015396	0.08813	0.015532	0.025337	0.077558	0.025092
0.01393	0.054541	0.013726	0.027541	0.489643	0.027547
0.009164	0.064097	0.00903	0.001928	0.001248	0.001909
0.039589	0.020543	0.02095	0	0	0
0.01173	0.011914	0.011559	0.004406	0.004311	0.004364

Moreover, Figures (1) and (2) are comparisons using the performance profile of the new algorithm (MQ) both in the estimate of NOI and NOF and the CPU time as the dimension increases this also shows the advantages of the three-term combination and also the logical selection of the parameter.

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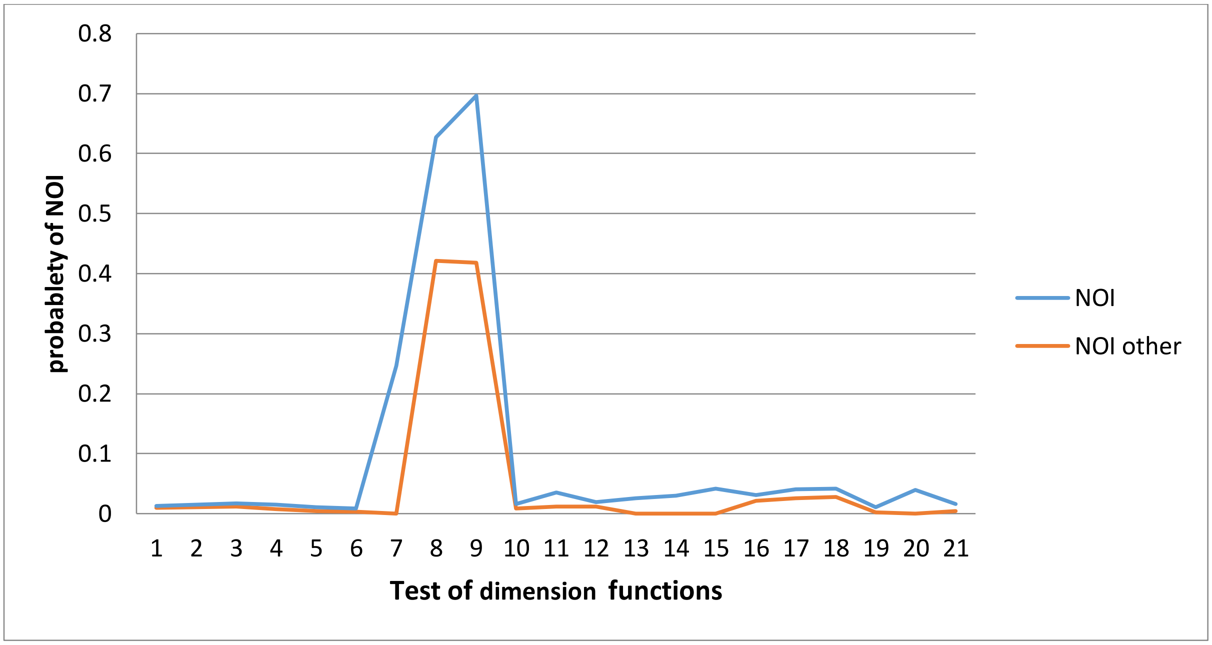

Figure 1. Comparison of NOI for 7 problems at dimensions 10, 50, and 100.

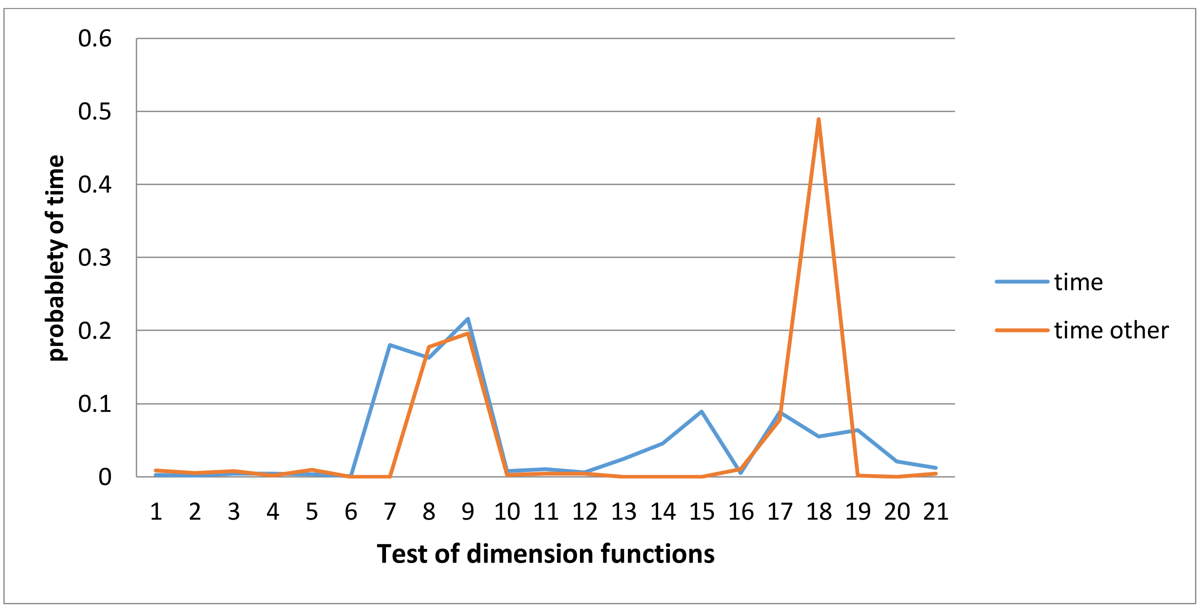

Figure 2. Comparison of time for 7 problems at dimensions 10, 50, and 100.

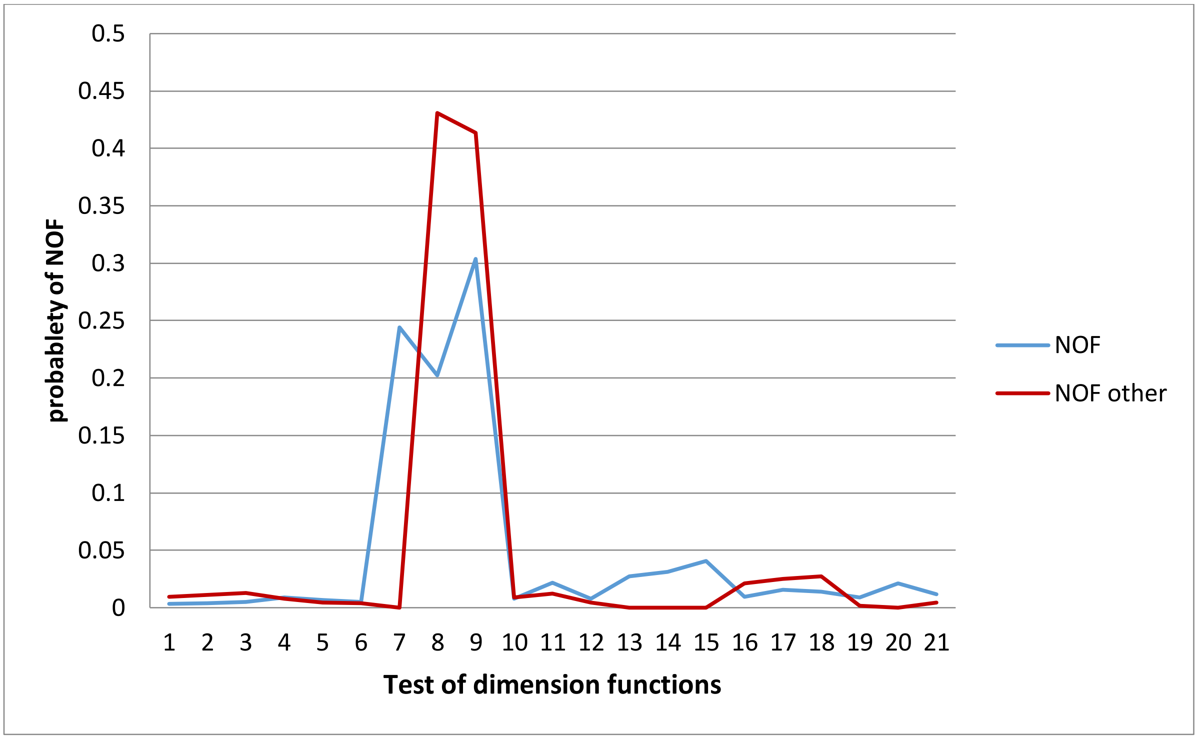

Figure 3. Comparison of NOF for 7 problems at dimensions 10, 50, and 100.

Conflict of interests.

There are non-conflicts of interest.

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