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A Meta-model framework for Grinding Simulation

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Abstract

When considering the mechanics of grinding, several physical phenomena have to be modeled, each one having effect on the resulting grinding forces, wheel and workpiece geometry. Depending on the analyzed problem, some dependencies can be neglected to privilege some aspects instead of others. Nevertheless, all models essentially start considering wheel-workpiece engagement and the corresponding material removal (both wheel and workpiece side), deriving the forces by means of energy balances and/or shear mechanics. The meta-model proposed in this paper represents a general framework conceived for providing a time-domain simulation engine based on a dexel representation of wheel and workpiece, capable to "host" all the semi-empirical models existing in literature, where the overall grinding force is the result of the integration of the force contributions associated to the local removal along wheel-workpiece engagement arc. A cascade approach is adopted to solve for forces and displacements the DAEs set describing the dynamic interactions between wheel and workpiece, whereas all the algebraic relationships pertaining to the various specific models are solved in a pre-processing phase, yielding a set of response surfaces that are queried during time integration. Finally, the meta-model framework is instantiated for a model of traverse roll grinding with force-dependent wheel wear.

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Keywords: Grinding; Grinding Wheel; Modelling; Simulation.

Nomenclature

a	actual infeed [mm]
A_α	active grain tip area [mm^2]
b	volume element width [mm]
c_k	coefficients of interpolating functions
d_i	wheel dullness for the *i*-th radial element
ΔT	simulation time step [s]
e_{wh}	wheel nominal specific energy [J/mm^3]
e_{wp}	workpiece nominal specific energy [J/mm^3]
$\mathbf{f}_i$	grinding force vector for the *i*-th volume element [N]
f_{ni}	grinding force normal component for the *i*-th elem. [N]
f_{ti}	grinding force tang. component for the *i*-th elem. [N]
$f_{ti\,s}$	grinding force tangential sliding component [N]
G	grinding ratio
h_{eq}	equivalent chip thickness [mm]
h_i	equivalent chip thickness for the *i*-th radial elem. [mm]
i	index denoting the *i*-th discretization element
$g_j(p_k)$	generic relationship among process parameters
μ	ratio between norm. and tang. force components f_n/f_t
Q'_w	specific removal rate per wheel width unit [mm^2/s]
n	correcting exponent for specific energy
N	number of active grains
p_{ave}	average contact pressure [N/mm^2]
p_k	generic process parameter
r_i	wheel-work engagement for the *i*-th radial elem. [mm]
s_i	wheel radius reduction for the *i*-th radial elem. [mm]
V_i	*i*-th volume element [mm^3]
v_s	wheel tangential velocity [mm/s]
v_t	traverse velocity [mm/s]
v_w	workpiece tangential velocity [mm/s]
w_i	work radius reduction for the *i*-th radial elem. [mm]

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1. Introduction

Several models have been proposed in literature for the dynamic simulation of grinding, whose objective is the representation of the effect of workpiece-wheel relative vibrations.

In [1], a simulation tool was developed for traverse roll grinding, which allows the dynamic simulation of the process in time domain. In this program, the surface of workpiece and grinding wheel are represented by fields of discrete, equal spaced, supporting points. The supporting points enable the calculation conditions of contact in form of the intersection of workpiece and grinding wheel.

Another time-domain dynamic model was presented in [2], which simulates cylindrical plunge grinding processes under general grinding conditions. The model focuses on the prediction of grinding chatter boundaries and growth rates, taking into account several critical issues: the distributed nonlinear force along the contact length, the geometrical interaction between the wheel and workpiece based on their surface profiles, the structure dynamics with multiple degrees of freedom for both the wheel and workpiece, the response delay due to spindle nonlinearities and other effects, and the effect of the motion perpendicular to the normal direction.

A two-dimensional model for the grinding process with two mechanical degrees of freedom is presented in [3]. This model especially deals with the nonlinear grinding contact. It cares for system excitation by stochastically distributed grains as well as for an eccentricity of the wheel due to unbalance. The approach to grinding contact dynamics allows to simulate and study chatter vibrations, workpiece surfaces and wheel wear like they appear in experiment.

Besides the numerical approaches based on a workpiece-wheel discretization, pure analytical representation of process geometry and grinding forces have been used [4][5]. These analytical models are usually subject to strong assumptions that limit their generality: often they are used to get an insight to a particular phenomenon, but they results insufficient when trying to predict surface quality in real practical cases.

Focusing on the approaches based on wheel and workpiece discretization, it can be noted that the different force and wear models relies on a common "geometrical" kernel devoted to the computation of wheel-workpiece engagement and the consequent wheel wear and workpiece material removal. Thus, a meta-model can be identified, constituted by a set of fundamental relationships that receive in input the reduced set of "geometrical" quantities provided by the simulation engine, yielding in output the grinding force and the updated wheel and workpiece geometry. In particular, the fundamental relationship can be represented by interpolation basis, namely, *response surfaces* (RS), that can be extracted from both analytical expressions and empirical lock-up tables retrievable in literature.

The objective of the present paper is the identification and the formalization of the aforementioned meta-model framework. In Section 2, the framework is described and exemplified for a meaningful case by providing a set of plausible RSs. The reasonableness of the approach will be demonstrated in Section 3 by presenting the results of some numerical simulations, obtained from an implementation in Matlab/Simulink environment of a model of cylindrical traverse grinding process entailing a 1-D dexel representation of wheel and workpiece (inspired by [6]). Discussion and conclusions are developed in Section 4.

2. Modeling approach

When considering the mechanics of grinding, several physical quantities have to be modeled, each one having effect on the resulting grinding forces, wheel and workpiece geometry. Depending on the selected approach, some dependencies can be neglected to privilege some phenomena instead of others. Nevertheless, all models essentially starts considering wheel-workpiece engagement and the corresponding material removal, deriving the forces by means of energy balances and/or shear mechanics. In order to deal with any kind of possible engagement geometry, some approaches are based on wheel/workpiece discretization: in this case the force computation must be referred to each element volume and the overall force is given by a sum of contributions. In this case, the force element is often computed through an energetic approach.

Another essential aspect is represented by the so-called "wheel dullness". Dullness can be interpreted as the loss of sharpness due to grains wear, that impairs the wheel shearing properties: this results in an increased grinding force and a decreased grinding efficiency. In grinding practice, dullness is periodically reset by dressing procedure.

The proposed model represents a general framework conceived for providing a time-domain simulation engine capable to "host" all the approaches based on material removal mechanics, where the overall grinding force is the result of the sum of the force contributions associated to the local removal along wheel-workpiece engagement arc. Surveying the broad literature concerned with grinding process, it can be found that a complete set of input run-time variables sufficient to feed any grinding simulation model includes: wheel/workpiece geometrical engagement, the equivalent chip thickness and the wheel status (dullness).

In the proposed approach, dullness is modeled as an internal state associated to each discretization element, which is subject to an evolution law dependent on grain load.

Once wheel-workpiece engagement is computed, grinding literature proposes models to compute force, wheel wear and workpiece material removal exploiting several parameters/quantities that are sometimes interrelated. In general, a sufficient DAEs set is needed to solve for forces and geometry, where the differential equation represents the dullness evolution ($\dot{d}_i$) and the algebraic equations yield forces, geometry and parameters interrelations. Mathematically,

$$\begin{cases} \mathbf{f}_i = \mathbf{f}_i\left(d_i, r_i, h_i \mid p_k\right), & \text{with } k = 1, 2, \ldots n \\ s_i = s_i\left(d_i, r_i, h_i \mid p_k\right) & \\ w_i = w_i\left(d_i, r_i, h_i \mid p_k\right) & \\ g_j(p_k) = 0, & \text{with } j = 1, 2, \ldots m \\ \dot{d}_i = \dot{d}_i(d_i, r_i, h_i \mid p_k) & \end{cases} \tag{1}$$

Assuming to be able to find $m=n$ relationships among process parameters p_k, under proper conditions, the corresponding subsystem $g_j(p_k)$, $j=1,...,n$, can be solved w.r.t. p_k and the unknown process parameters canceled out from the other equations. After having proceeded in this manner, the DAEs system of Eq. (1) turns into the following:

$$\begin{cases} \mathbf{f}_i = \mathbf{f}_i\left(d_i, r_i, h_i\right) \\ s_i = s_i\left(d_i, r_i, h_i\right) \\ w_i = w_i\left(d_i, r_i, h_i\right) \\ \dot{d}_i = \dot{d}_i(d_i, r_i, h_i) \end{cases} \tag{2}$$

The $g_j(p_k)$ can be solved numerically and, then, each equation of the (2) can be represented by a proper RS to be queried during simulation at each time step.

In the implementation adopted in this paper (borrowed by [6]), the local engagement between wheel and workpiece r_i is given by the length reduction of 1-D dexel (see Fig. 1).

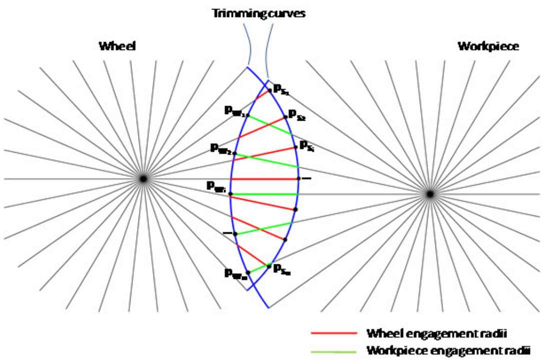


Fig. 1. Wheel-workpiece engagement representation

Whereas dullness property and G-ratio are related to the status of the single grain, the corresponding functions, that usually express average values, hold also locally. The situation is different for the equivalent chip thickness, that is an abstraction based on assumptions regarding workpiece and wheel *overall* engagement. As a matter of fact, the equivalent chip thickness is defined as it follows:

$$h_{eq} \overset{def}{=} \frac{v_w}{v_s} a \tag{3}$$

where a is the infeed, V_w is the workpiece velocity and V_s the wheel tangential velocity. In the pointed relationships considering h_{eq}, the product $V_w \cdot a$ represents the expression of the Material Removal Rate per wheel width unit (*MRR)*; since no hypothesis are made either regarding the MRR entity or geometry (that, for example, changes along with wheel and workpiece diameter), it can be assumed that such relationships hold even for a local MRR, that is substituting h_{eq} with a local equivalent chip thickness given by:

$$h_i \overset{def}{=} \frac{V_i}{\Delta T \cdot v_s \cdot b} \tag{4}$$

In order to show how the aforementioned general framework can be used to bridle other existing models, a meaningful exemplification of RS extraction is provided in the following subsections.

2.1. Wheel wear and workpiece material removal RS

Given wheel-workpiece engagement, the key parameter to split it into wheel wear s_i and workpiece removal w_i is the G-ratio. G-ratio represents the ratio between the volume of material removed from the workpiece and the volume removed from the wheel and it is the most widely used parameter to evaluate wheel efficiency in practice. It can be shown that also w_i/s_i, is given by G-ratio, therefore the w_i and s_i RSs can be traced back to a G-ratio RS.

Assuming a constant MRR, G-ratio behavior can be analyzed through the curve depicted in Fig. 2, which is shared by several literature sources (see, for instance, [7][8]).

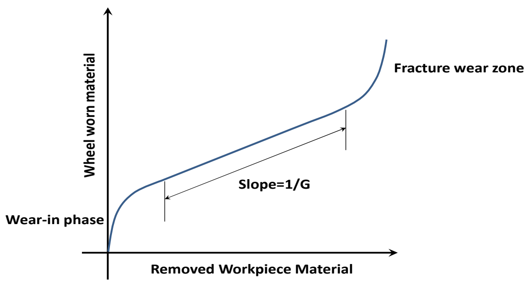


Fig. 2. Relationship between wheel and workpiece removal

For its inherent significance, dullness can be mapped into wheel wear itself, posing that a dullness equal to 0 means that the grain is perfectly sharp (just after dressing), while a dullness equal to 1 means that the wheel is completely worn. Then, whereas G-ratio is the 1st derivative of "removed workpiece material" w.r.t. "wheel worn material", the 2nd derivative of "removed workpiece material" w.r.t. "wheel worn material" (namely, the derivative of G-ratio w.r.t. "wheel worn material") yields the relationship between G-ratio and dullness.

Moreover, G-ratio depends also on equivalent chip thickness. This dependency, for instance, has been pointed out in [7], p. 299, in terms of a negative exponential law, and it has been determined experimentally by Bhattacharyya in [9]. Referring to the experiments presented in this latter reference, the relationship was determined by means of a polynomial interpolation.

Then, assuming that the aforementioned dependencies are not correlated, G-ratio RS can be obtained by simply multiplying the first relationship with the second one and applying a final normalization. The final G-ratio RS is depicted in Fig. 3.

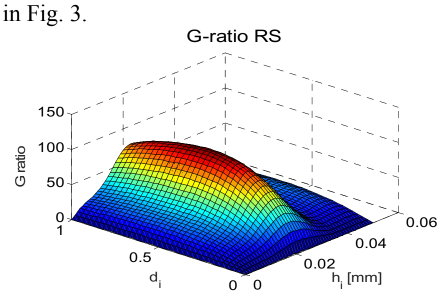


Fig. 3. G-ratio RS

According to Eq.s (2), the RSs requested by the framework refers to w_i (workpiece material removal) and s_i (wheel wear). These RSs can be easily derived by G-ratio and wheel-workpiece engagement r_i as it follows:

$$s_i\left(d_i, r_i, h_i\right) = \frac{r_i}{1 + G(d_i, h_i)} \tag{5}$$

$$w_i\left(d_i, r_i, h_i\right) = \frac{r_i}{1 + 1/G(d_i, h_i)} \tag{6}$$

The final wheel wear and workpiece material removal RSs (at a fixed r_i=0.01mm) are depicted in Fig. 4.

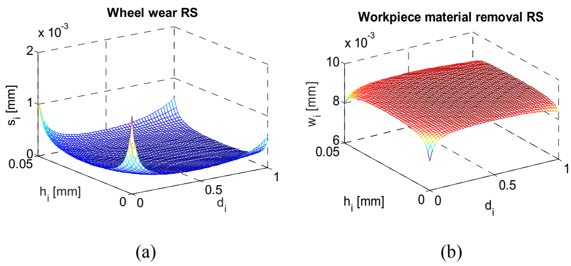


Fig. 4. (a) Wheel wear RS; (b) Workpiece removal

2.2. Dullness Evolution RS

Dullness rate depends on the force that loads the grains [8][12]: the increasing of the force, in fact, increases the friction that wears the grain cutting edge, decreasing its shearing capability. Whereas the literature unanimously considers that the force exerted by each grain increases along with the equivalent chip thickness, dullness rate RS must reflect this property. The same study about G-ratio presented in [9], shows experimental results where wheel life is correlated to equivalent chip thickness; since dullness, by definition, spans from 0 to 1 (as introduced in the previous subsection), the wheel life can be used to calculate dullness rate w.r.t. equivalent chip thickness.

On the other side, assuming that wheel wear can be mapped into dullness (as claimed in the previous subsection), at a fixed workpiece MRR the dullness rate can be mapped into wear rate, namely, in G-ratio. Hence, the same relationship identified in the previous subsection between G-ratio and dullness is used here to describe the relationship between dullness rate and dullness.

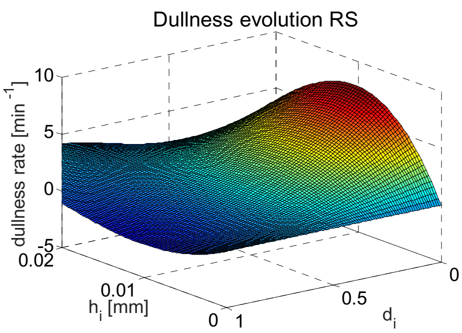


Fig. 5. Dullness evolution RS (section d_i-h_i)

Then, assuming that the identified dependencies are not correlated, the final RS can be obtained by multiplying the first relationship with the second and applying a final normalization (see Fig. 5). It is worthwhile to be noted that dullness rate can assume also negative values: from the technological point of view it means that the model foresees the possibility to have wheel self-sharpening.

2.3. Forces RSs

During grinding operations energy is spent both in removing material from the workpieces (that is the process aim) and in wear the wheel (an avoidable drawback). Then, the power balance can be expressed in terms of the so-called “specific energy”, that is the energy required to remove a material unit volume, both for workpiece and for wheel (see [10]).

Another common modeling concept used to define grinding force is the ratio between the tangential component (aligned with the cutting velocity) and the normal component (normal to cutting velocity). Without presenting details, it can just be observed that this μ-ratio usually depends on ground material, wheel type, dullness (see, for instance, [11]) and depth of cut, that can be mapped in the equivalent chip thickness (see, for instance, [14]).

Specific energies are affected by dulling occurrence. In [7] Malkin claims that, even though metal removal occurs mostly by chip formation, it would seem that some grinding energy must be expended by mechanisms other than chip formation. These mechanisms involve the sliding of dulled flattened tips of the abrasive grains against the workpiece surface without removing any material. During grinding, the wear flats may become glazed and further enlarged through attritious wear and adhesion of metal particles from the workpiece. The grows of wear flats is offset to a greater or lesser degree, by “self-sharpening” due to bulk wear of the wheel, whereby some flats are wholly or partially removed by grit fracture or grit dislodgement from the binder. Hence, dulling effect influence directly the specific energies.

Considering, now, the equivalent chip thickness, it is well-known that specific energy tends to increase as chip thickness decreases, due to the fact that cutting force components decrease, while the components due to friction decrease much lightly.

On these premises, the overall dependencies can be identified by considering two nominal specific energies for wheel and workpiece, associated to the force needed to shear the material (given the equivalent chip thickness), together with the dependence on dullness, which determine the *attritious* (or *sliding*) force component. In formulas:

$$f_{t_i} = \left(d_i, h_i \mid e_{wh}, e_{wp}\right) \tag{7}$$

$$f_{n_i} = \left(d_i, h_i \mid e_{wh}, e_{wp}, \mu\left(d_i, h_i\right)\right) \tag{8}$$

On the basis of [7] and [12] , the Eq. (7) and the Eq. (8) can be written as it follows:

$$f_{t_i} = e_{wh}\frac{h_i}{1 + G(d_i, h_i)} + e_{wp}\frac{h_i}{1 + 1/G(d_i, h_i)} + f_{t_i\,s}(d_i) \tag{9}$$

$$f_{n_i} = f_{t_i} \cdot \mu\left(d_i, h_i\right) \tag{10}$$

The expression of μ has been exemplified by interpolating a multivariate function exploiting the experimental data presented in [11] and [14]. Regarding the specific energies, when the sliding component is not treated separately, the proposed formulation can be approximated with the usual exponential correction (very widespread in literature [13]):

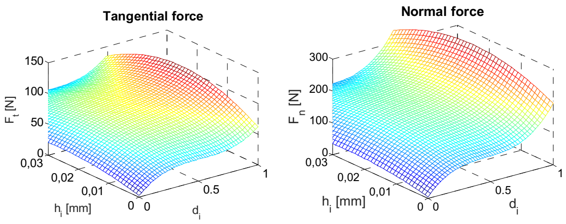
$$e_{wh(wp)} \propto h_i^{-n} \quad (11)$$

On the other side, the effect of dullness on the sliding component can be traced back to the formulation proposed, for instance, by Lichun [12]:

$$f_{t_{i\,s}} = NA_{\alpha}\, p_{ave} \quad (12)$$

Assuming that the number of active grains and the average contact pressure are constant (this latter is sometimes likened to the plasticity stress), the grain tip area can be mapped into the dullness and Eq. (12) is exploited to compute the sliding force component.

Elaborating some experimental data presented in [12] and adopting tabulated nominal specific energies, Eq. (9) can be evaluated for a mesh of h_i and d_i, yielding the final forces RSs that are depicted in Fig. 6 (normalized for a wheel width of 10 mm).



(a) (b)

Fig. 6. Forces RSs: (a) tangential, (b) normal

3. Simulation results and qualitative experimental assessment

Following the present approach, a traverse grinding simulation model has been implemented in Matlab/Simulink environment, based on a description of wheel and workpiece by means of 1-D dexel models (Fig. 7). The model allows the coupling with wheel-workpiece relative dynamics, enabling the prediction on vibrations onset.

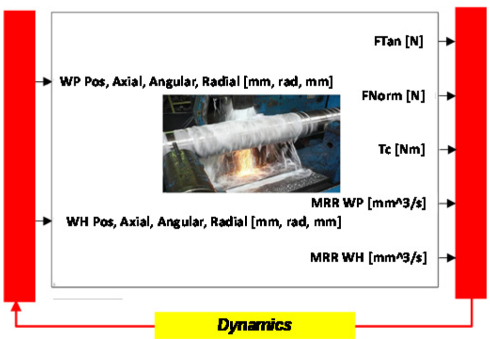


Fig. 7. Grinding simulation model block in Matlab/Simulink

A preliminary simulation has been launched, exploiting the RSs computed starting from some reference data found in literature (see the previous sections). Due to the inconsistency of the data taken from different articles, that deal with different grinding operations and experimental set-ups, the simulation results are not requested to match with a real grinding case, but they should just demonstrate to have captured some relevant phenomena encounterable in common grinding practice.

Whereas vibration issues concerned with dynamics are out of the scope of the present study, the wheel-workpiece relative compliance has been considered null, so that a purely kinematic simulation is provided. Besides the RSs, the other grinding parameters, referred to a single-pass traverse roll grinding operation, are listed in Table 1.

Table 1. Grinding simulation parameters.

Wheel diam. [mm]	Wheel width [mm]	Work diam. [mm]	Work length [mm]	v_s [m/s]	v_w [m/s]	v_t [m/s]	End infeed [mm]	Infeed veloc. [mm/s]
200	100	200	200	15	0.314	0.025	0.04	0

In Fig. 8 the final wheel and workpiece geometries are depicted with an amplification of the deviation from the average geometry (x 5000). It can be noted that the final geometry of the cylinder is characterized by a significant taper, due to the progressive wheel wear which provokes a decrease of the actual infeed along cylinder axis (usually this error is compensated by a proper continuous infeed velocity). A spiral of unremoved material is also evident on the cylinder surface due to the uneven wheel wear: these spirals are usually denominated “feed marks” and are quite typical in roll grinding practice: they are usually eliminated by subsequent finishing passes. Regarding the wheel, a wear step is clearly noticeable of its surface: it is due to the overlap of the grinding stripes occurring in one cylinder revolution. The stepwise wheel wear is well-known in literature [1].

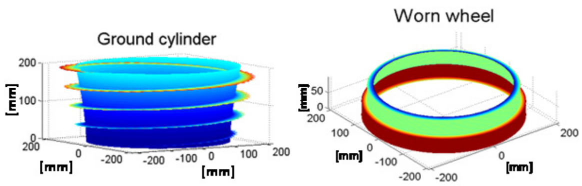


Fig. 8. Workpiece and wheel final geometry

In Fig. 9 the grinding forces behavior is depicted. It is clearly visible the transient behavior related to wheel engagement and disengagement, together with a slight increasing trend of the central steady part (above all for the tangential component). It is worthwhile to be highlighted that the increasing trend exists even if the actual infeed is decreasing due to wheel wear (the reason will be explained in the following).

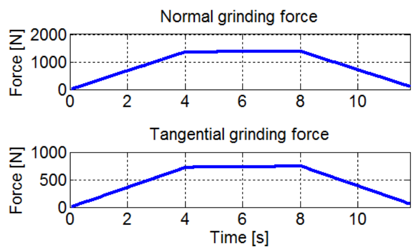


Fig. 9. Simulated grinding forces

In Fig. 10, the power and the actual infeed measured during a traverse roll grinding operation (analogous to one of Table 1) are plotted in time, for different grinding passes (the cylinder is about 4000 mm long). It can be shown that paradoxically the highest power demand does not correspond to the highest infeed, even if all the other grinding parameters are constant. A possible explanation of this phenomenon involves the fact that the high grinding force could have provoked an important increase of G-ratio, with the consequent "self-sharpening" of the wheel and a decrease of the grinding force. Indeed, the RSs of Fig. 4 shows how, for very high h_i and d_i this trend is possible, coherently with simulation results. Moreover, it can be noted that this trend is a little bit accentuated for the tangential components of grinding force, that is the component involved in power consumption.

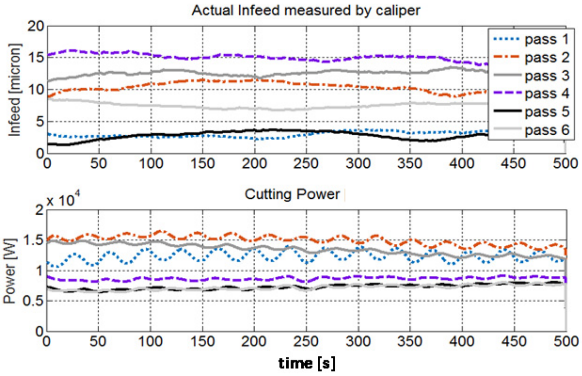


Fig. 10. Experimental grinding power vs infeed

Indeed, the actual μ-ratio coming from the simulation is far to be constant (see Fig. 11): it is higher at the beginning of the simulation, that is when the actual infeed is higher because the wheel is still unworn and the wear affect mostly force tangential component. Dullness evolution (see Fig. 8), confirms that force behavior is strictly related to a progressive wheel wear.

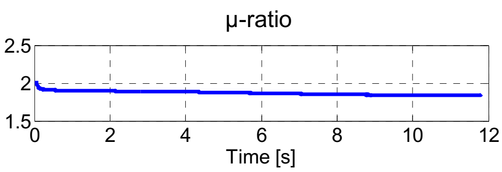


Fig. 7. μ-ratio evolution

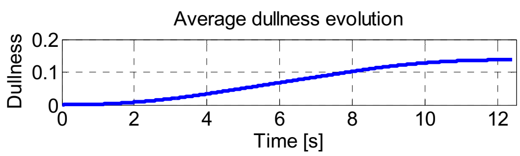


Fig. 11. Average dullness evolution

Conclusions

A meta-model simulation framework for grinding process simulation has been proposed, to be used as a base for instantiating easily all the modeling approaches elaborated in grinding literature regarding material removal mechanism for workpiece and wheel. A preliminary literature survey has demonstrated that the identified reduced set of input variables (to be computed run-time in the simulation environment) can be adequate to incorporate a first set of relevant phenomena, described by a corresponding set of published researches.

Then a simulation example is provided where the wheel wear, depending on dullness and grinding force itself, produces an uneven grinding force behavior. This latter is qualitatively compatible with the results of some traverse roll grinding tests performed on a large commercial grinder.

This work has to be regarded as a feasibility study prone to generate a set of usable simulation models showing effective prediction capabilities. In order to achieve this result, a methodology must be elaborated to extract the necessary RSs directly from a proper experimental campaigns. Further activities are foreseen in this direction.

Acknowledgements

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