Computer Modeling in
Engineering & Sciences

Tech Science Press

DOI: 10.32604/cmes.2022.022878

check for updates

ARTICLE

On Some Novel Fixed Point Results for Generalized F-Contractions in b-Metric-Like Spaces with Application

Kastriot Zoto[1], Ilir Vardhami[2], Dušan Bajović[3], Zoran D. Mitrović[3,*] and Stojan Radenović[4]

[1]Department of Mathematics and Computer Sciences, Faculty of Natural Sciences, University of Gjirokastra, Gjirokastra, 6001, Albania

[2]Department of Mathematics, Faculty of Natural Sciences, University of Tirana, Tirana, 1001, Albania

[3]Faculty of Electrical Engineering, University of Banja Luka, Banja Luka, 78000, Bosnia and Herzegovina

[4]Department of Mathematics, Faculty of Mechanical Engineering, University of Belgrade, Beograd, 11120, Serbia

*Corresponding Author: Zoran D. Mitrović. Email: zoran.mitrovic@etf.unibl.org

Received: 30 March 2022 Accepted: 17 May 2022

ABSTRACT

The focus of our work is on the most recent results in fixed point theory related to contractive mappings. We describe variants of (s, q, ϕ, F)-contractions that expand, supplement and unify an important work widely discussed in the literature, based on existing classes of interpolative and F-contractions. In particular, a large class of contractions in terms of s, q, ϕ and F for both linear and nonlinear contractions are defined in the framework of b-metric-like spaces. The main result in our paper is that (s, q, ϕ, F)-g-weak contractions have a fixed point in b-metric-like spaces if function F or the specified contraction is continuous. As an application of our results, we have shown the existence and uniqueness of solutions of some classes of nonlinear integral equations.

KEYWORDS

(ϕ, F)-contraction; (s, q, ϕ, F)-contraction; b-metric-like space; fixed point

1 Introduction

Fixed point theory has been studied for a long time. Its application relies on the existence of solutions to mathematical problems that are based on the contraction principle. An interesting generalization of the Banach contraction principle was given by Wardowski [1,2] using a different type of contraction called F-contraction and by Karapınar in [3] defining the type of interpolative contractions. These approaches have been extended by weakening the contractive conditions, removing some of the imposed conditions on the used mappings or relaxing axioms of the defined spaces. Starting from these aspects, many researchers have constructed new fixed point theorems in different types of spaces such as metric, b-metric and other generalized metric spaces, as cited in [3–31]. Nevertheless, in the papers of Younis et al. [4,5] the notion of Kanan mappings in the view of F-contraction in the setting of b-metric-like spaces has been expanded and an example related to electrical engineering has been given. In this paper, we introduce general types of (s, ϕ, F) and (s, q, ϕ, F)-contractions, which are variants of Wardowski contractions in the setting of b-metric-like spaces. Using these classes of contractive mappings, we establish unique fixed point theorems that unify and extend recent results on this topic.

2 Preliminaries

In this section, we list some well-known definitions and lemmas in terms of b-metric-like spaces.

Definition 2.1 [9]. Let V be a nonempty set and $s \geq 1$ be a given real number. A mapping $b : V \times V \to [0, +\infty)$ is called a b-metric-like if for all $\gamma, \delta, \nu \in V$ these conditions are satisfied:

(i) $b(\gamma, \delta) = 0$ implies $\gamma = \delta$;

(ii) $b(\gamma, \delta) = b(\delta, \gamma)$;

(iii) $b(\gamma, \delta) \leq s[b(\gamma, \nu) + b(\nu, \delta)]$.

The pair (V, b) is called a b-metric-like space (in the sequel we use $b - m.l.s$ for short).

In a b-metric-like space (V, b), if $\gamma, \delta \in V$ and $b(\gamma, \delta) = 0$, then $\gamma = \delta$. However, the converse need not be true, and $b(\gamma, \gamma)$ may be positive for $\gamma \in V$.

Definition 2.2 [10]. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$, $\{\nu_n\}$ be any sequence in V and $\nu \in V$. Then, the following applies:

(a) The sequence $\{\nu_n\}$ is said to be convergent to ν if $\lim_{n\to+\infty} b(\nu_n, \nu) = b(\nu, \nu)$;

(b) The sequence $\{\nu_n\}$ is said to be a Cauchy sequence in (V, b) if $\lim_{n,m\to+\infty} b(\nu_n, \nu_m)$ exists and is finite;

(c) The pair (V, b) is called a complete $b - m.l.s$ if for every Cauchy sequence $\{\nu_n\} \subset V$, there exists $\nu \in V$ such that $\lim_{n,m\to+\infty} b(\nu_n, \nu_m) = \lim_{n\to+\infty} b(\nu_n, \nu) = b(\nu, \nu)$.

Definition 2.3 [10]. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and f be a self-mapping on V. We say that the function f is continuous if and only if $\lim_{n\to+\infty} b(f\nu_n, f\nu) = b(f\nu, f\nu)$, for each sequence $\{\nu_n\} \subset V$, which satisfies $\lim_{n\to+\infty} b(\nu_n, \nu) = b(\nu, \nu)$.

Note that in a $b - m.l.s$ with parameter $s \geq 1$, if $\lim_{n,m\to+\infty} b(\nu_n, \nu_m) = 0$ then the limit of the sequence $\{\nu_n\}$ is unique if it exists.

Lemma 2.1 [11,12]. Let (V, b) be a complete $b - m.l.s$ with parameter $s \geq 1$ and $\{\nu_n\}$ be a sequence such that $b(\nu_n, \nu_{n+1}) \leq \lambda b(\nu_{n-1}, \nu_n)$, for all $n \in \mathbb{N}$, where $\lambda \in [0, 1)$. Then $\{\nu_n\}$ is a b-Cauchy sequence such that $\lim_{n,m\to+\infty} b(\nu_n, \nu_m) = 0$.

Lemma 2.2. [9]. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and suppose that $\{\nu_n\}$ converges to a ν and $b(\nu, \nu) = 0$. Then

$$s^{-1}b(\nu, z) \leq \liminf_{n\to+\infty} b(\nu_n, z) \leq \limsup_{n\to+\infty} b(\nu_n, z) \leq sb(\nu, z),$$

for all $z \in V$.

Lemma 2.3. [9]. Let (V, b), be a $b - m.l.s$ with parameter $s \geq 1$. Then, the following applies:

(a) If $b(\gamma, \delta) = 0$, then $b(\gamma, \gamma) = b(\delta, \delta) = 0$;

(b) If $\{\nu_n\}$ is a sequence such that $\lim_{n\to+\infty} b(\nu_n, \nu_{n+1}) = 0$, then we have

$$\lim_{n\to+\infty} b(\nu_n, \nu_n) = \lim_{n\to+\infty} b(\nu_{n+1}, \nu_{n+1}) = 0;$$

(c) If $\gamma \neq \delta$, then $b(\gamma, \delta) > 0$.

Lemma 2.4. [13]. Let (V, b) be a complete $b - m.l.s$ with parameter $s \geq 1$. Let $\{\nu_n\} \subset V$ be a sequence such that $\lim_{n\to+\infty} b(\nu_n, \nu_{n+1}) = 0$. If for sequence $\{\nu_n\}$ holds $\lim_{n,m\to+\infty} b(\nu_n, \nu_m) \neq 0$, then there exist $\varepsilon > 0$ and sequences $\{m_k\}_{k=1}^{+\infty}$ and $\{n_k\}_{k=1}^{+\infty}$ of natural numbers with $n_k > m_k > k$, such that

$$b(\nu_{m_k}, \nu_{n_k}) \geq \varepsilon, \quad b(\nu_{m_k}, \nu_{n_k-1}) < \varepsilon,$$

$$\frac{\varepsilon}{s^2} \leq \limsup_{k\to+\infty}, b(\nu_{m_k-1}, \nu_{n_k-1}) \leq \varepsilon s,$$
$$\frac{\varepsilon}{s} \leq \limsup_{k\to+\infty}, b(\nu_{n_k-1}, \nu_{m_k}) \leq \varepsilon s^2 \text{and}$$
$$\frac{\varepsilon}{s} \leq \limsup_{k\to+\infty}, b(\nu_{m_k-1}, \nu_{n_k}) \leq \varepsilon s^2.$$

3 Results

We begin the main section with a definition that is an expanding outlook of Wardowski type (ϕ, F)-contractions in the frame of a generalized metric space such as $b - m.l.s$.

Definition 3.1. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and f be a self-mapping on V. We say that f is a (s, q, ϕ, F)-contraction if there exist the functions $F : (0, +\infty) \to \mathbb{R}$ and $\phi : (0, +\infty) \to (0, +\infty)$ such that

(a) F is strictly increasing;

(b) $\liminf_{r\to t^+} \phi(r) > 0$ for all $t > 0$;

(c) For all $\nu, \delta \in V$ with $f\nu \neq f\delta$, and for some $q > 0$

$$\phi(b(\nu, \delta)) + F(s^q b(f\nu, f\delta)) \leq F(b(\nu, \delta)). \tag{1}$$

Remark 3.1. In the above definitions property F and conditions (1) yield

$$b(f\nu, f\delta) \leq s^q b(f\nu, f\delta) < b(\nu, \delta).$$

The continuity of the mapping f follows from the inequality $b(f\nu, f\delta) < b(\nu, \delta)$.

Remark 3.2.

- Our definition generalizes the previous definitions given in [14–16]. It contains a reduced number of conditions compared with the previous definitions.
- The definition of (s, q, F)-contraction is an immediate consequence of Definition 2.1, if we take $\phi : (0, +\infty) \to (0, +\infty)$ to be a constant function.
- If $s = 1$ we get the definition of Wardowski in [1,2] in the case of metric spaces.
- For $s = 1$ the definition is valid in the framework of a metric space.

The following is the first fixed point theorem for (s, q, ϕ, F)-contraction type mapping.

Theorem 3.1. Let (V, b) be a complete $b - m.l.s$ with parameter $s \geq 1$. If f is a (s, q, ϕ, F)-contraction on V, then the function f has a unique fixed point in V.

Proof. Let be $\nu_0 \in V$ and the Picard iterative sequence $\{\nu_n\}$ defined by $\nu_{n+1} = f(\nu_n)$ for $n \in \{0, 1, 2, \ldots\}$. The proof is clear in the case that there exists $n_0 \in \mathbb{N}$, with $\nu_{n_0+1} = \nu_{n_0}$. So, we will assume that $\nu_{n+1} \neq \nu_n$, which implies $f\nu_n \neq f\nu_{n-1}$ and $b(f\nu_n, f\nu_{n-1}) > 0$, for all $n \in \mathbb{N} \cup \{0\}$. Using inequality (1), we have

$$\begin{aligned} F(s^q b(\nu_n, \nu_{n+1})) &\leq \phi(b(\nu_{n-1}, \nu_n)) + F(s^q b(\nu_n, \nu_{n+1})) \\ &= \phi(b(\nu_{n-1}, \nu_n)) + F(s^q b(f\nu_{n-1}, f\nu_n)) \\ &\leq F(b(\nu_{n-1}, \nu_n)). \end{aligned} \tag{2}$$

Further from inequality (2), we get

$$s^q b(\nu_n, \nu_{n+1}) < b(\nu_{n-1}, \nu_n),$$

which implies

$$b(\nu_n, \nu_{n+1}) < \frac{1}{s^q} b(\nu_{n-1}, \nu_n).$$

In view of Lemma 2.1, the corresponding Picard sequence $\{\nu_n\}$ with the initial point ν_0 is a Cauchy sequence such that $\lim_{n,m\to+\infty} b(\nu_n . \nu_n) = 0$. Since (V, b) is a complete b-metric-like space, we conclude that there exists $\nu \in V$ such that

$$\lim_{n\to+\infty} b(\nu_n, \nu) = b(\nu, \nu) = \lim_{n,m\to+\infty} b(\nu_n, \nu_m) = 0. \tag{3}$$

According to the (1), it follows:

$$F(s^q b(f\nu, f\nu_n)) \leq \phi(b(\nu, \nu_n)) + F(s^q b(f\nu, f\nu_n)) \leq F(b(\nu, \nu_n)),$$

that from property of F we get

$$s^q b(f\nu, f\nu_n) \leq b(\nu, \nu_n). \tag{4}$$

From triangular property and (4), we have

$$b(f\nu, f\nu) \leq 2sb(f\nu, f\nu_n) \leq 2s^q b(f\nu, f\nu_n) \leq 2b(\nu, \nu_n).$$

Since f is continuous and using (3), (4) we obtain

$$b(f\nu, f\nu) = \lim_{n\to+\infty} b(f\nu_n, f\nu) \leq \lim_{n\to+\infty} 2b(\nu_n, \nu) = 2b(\nu, \nu) = 0. \tag{5}$$

Since $b(\nu, f\nu) \leq s[b(\nu, f\nu_n) + b(f\nu, f\nu_n)]$, as $n \to +\infty$ we obtain that $b(\nu, f\nu) = 0$. Thus $f\nu = \nu$ and so f has a fixed point. Also from (3), we have $b(\nu, \nu) = 0$. To prove the uniqueness of the fixed point, suppose that $u \in V$ is another different fixed point. From $u \neq \nu$ follows $fu \neq f\nu$, then

$$F(s^q b(u, \nu)) = F(s^q b(fu, f\nu)) \leq \phi(b(u, \nu)) + F(s^q b(fu, f\nu)) \leq F(b(u, \nu)),$$

which implies

$$b(u, \nu) < \frac{1}{s^q} b(u, \nu).$$

Previous inequality is a contradiction, so $b(u, \nu) = 0$ and the fixed point is unique.

Corollary 3.1. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and f be a self-mapping on V. If there exist an increasing function $F : (0, +\infty) \to \mathbb{R}$ and a positive constant τ such that

$$\tau + F(s^q b(f\nu, f\delta)) \leq F(b(\nu, \delta)) \tag{6}$$

for all $\nu, \delta \in V$ with $f\nu \neq f\delta$, and for some $q > 0$, then f has a unique fixed point in V.

Proof. Inequality (1) implies (6) if we set $\phi(r) = \tau > 0$.

Example 3.1. Let $V = [0, +\infty)$ and $b(x, y) = x^2 + y^2 + |x - y|^2$, for all $x, y \in V$. It is clear that b is a b-metric-like on V, with parameter $s = 2$ and (V, b) is complete. Also, b is not a metric-like nor b-metric (nor a metric on V). Consider the self-mapping $f : V \to V$ by $fx = \dfrac{\ln(1+x)}{5}$. For all $x, y \in V$ and constant $q = 2$, we have

$$\begin{aligned}
s^2 b(fx, fy) &= 4\left(f^2 x + f^2 y + |fx - fx|^2\right) \\
&= 4\left(\left(\frac{\ln(x+1)}{5}\right)^2 + \left(\frac{\ln(y+1)}{5}\right)^2 + \left|\frac{\ln(x+1)}{5} - \frac{\ln(y+1)}{5}\right|^2\right) \\
&\leq 4\left[\frac{x^2}{25} + \frac{y^2}{25} + \left|\frac{x}{5} - \frac{y}{5}\right|^2\right] = \frac{4}{25}\left[x^2 + y^2 + |x - y|^2\right] \\
&\leq \frac{1}{5} b(x, y).
\end{aligned}$$

Taking the logarithms in the above inequality and fixing $\tau = \ln 5$ and the function $F(t) = \ln t$ then the conditions of Corollary 3.1 are satisfied and clearly $x = 0$ is a unique fixed point of f.

With the aim of expanding the initiated Definition 2.1 and starting a result that includes Theorem 3.1 and its respective corollaries, we will use a class of implicit relations, which makes simultaneously effective enormous literature on this topic.

Let Γ_4 be the set of all continuous functions $g : [0, +\infty)^4 \to [0, +\infty)$ satisfying

(a) g is non-decreasing with respect to each variable:

(b) $g(t, t, t, t) \leq t$ for $t \in [0, +\infty)$.

Definition 3.2. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and $f : V \to V$ be a self mapping. We say that f is generalized (s, q, ϕ, F)-g-weak contraction, if there exist functions $F : (0, +\infty) \to \mathbb{R}$, $\phi : (0, +\infty) \to (0, +\infty)$ and $g \in \Gamma_4$ such that

(a) F is strictly increasing;

(b) $\liminf_{r \to t^+} \phi(r) > 0$ for all $t > 0$;

(c) $$\phi(b(x, y)) + F(s^q b(fx, fy)) \leq F\left(g\left(b(x, y), b(x, fx), b(y, fy), \frac{b(x, fy) + b(y, fx)}{4s}\right)\right) \tag{7}$$

for all $x, y \in V$ with $fx \neq fy$, and for some $q \geq 1$.

Remark 3.3.

- The above definition reduces to a generalized (s, q, F)-g-weak contraction by setting $\phi : (0, +\infty) \to (0, +\infty)$ to be a constant function $\phi(r) = \tau > 0$.
- Fixing the parameter $s = 1$ we get the definition of (ϕ, F)-g-weak contraction in the setting of metric and metric-like spaces.

- Fixing $s = 1$ and $\phi(r) = \tau > 0$ we get the definition of (F-g)-weak contraction in the setting of metric and metric-like spaces.

Theorem 3.2. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and the self mapping $f : V \to V$ be a generalized (s, q, ϕ, F)-g-weak contraction. If f or F is continuous, then f has a unique fixed point in V.

Proof. Let $u_0 \in V$ be arbitrary and construct the Picard iterative sequence $\{u_n\}$ as $u_{n+1} = f(u_n)$ for $n \in \{0, 1, 2, \ldots\}$. The proof is clear in the case that there exists $n_0 \in \mathbb{N}$, with $u_{n_0+1} = u_{n_0}$. Therefore, we assume that $u_{n+1} \neq u_n$, which means $fu_m \neq fu_{n-1}$ or $b(fu_n, fu_{n-1}) > 0$ for all $n \in \mathbb{N} \cup \{0\}$. Using (7) for $x = u_n$, $y = u_{n-1}$ we have

$$\begin{aligned}
&\phi(b(u_n, u_{n+1})) + F(s^q b(u_n, u_{n+1})) \\
&= \phi(b(fu_{n-1}, fu_n)) + F(s^q b(fu_{n-1}, fu_n)) \\
&\leq F\left(g\left(b(u_{n-1}, u_n), b(u_{n-1}, fu_{n-1}), b(u_n, fu_n), \frac{b(u_{n-1}, fu_n) + b(u_n, fu_{n-1})}{4s}\right)\right) \\
&= F\left(g\left(b(u_{n-1}, u_n), b(u_{n-1}, u_n), b(u_n, u_{n+1}), \frac{b(u_{n-1}, u_{n+1}) + b(u_n, u_n)}{4s}\right)\right) \\
&\leq F\left(g\left(b(u_{n-1}, u_n), b(u_{n-1}, u_n), b(u_n, u_{n+1}), \frac{sb(u_{n-1}, u_n) + sb(u_n, u_{n+1}) + 2sb(u_{n-1}, u_n)}{4s}\right)\right) \\
&= F\left(g\left(b(u_{n-1}, u_n), b(u_{n-1}, u_n), b(u_n, u_{n+1}), \frac{b(u_n, u_{n+1}) + 3b(u_{n-1}, u_n)}{4}\right)\right).
\end{aligned} \tag{8}$$

If we assume that $b(u_{n-1}, u_n) \leq b(u_n, u_{n+1})$, then inequality (8) yields

$$\begin{aligned}
\phi(b(u_n, u_{n+1})) + F(s^q b(u_n, u_{n+1})) &\leq F\left(g\left(b(u_n, u_{n+1}), b(u_n, u_{n+1}), b(u_n, u_{n+1}), b(u_n, u_{n+1})\right)\right) \\
&\leq F(b(u_n, u_{n+1})),
\end{aligned}$$

for all $n \in \mathbb{N}$. So, we obtain

$$\begin{aligned}
F(s^q b(u_n, u_{n+1}) &\leq F(b(u_n, u_{n+1})) - \phi(b(u_n, u_{n+1})) \\
&< F(b(u_n, u_{n+1})),
\end{aligned}$$

which is a contradiction. Therefore

$$b(u_n, u_{n+1}) < b(u_{n-1}, u_n),$$

for all $n \in \mathbb{N}$. Thus, the sequence $\{b(u_{n-1}, u_n)\}$ is decreasing and bounded below. Consequently, there exists $l \geq 0$ such that $b(u_{n-1}, u_n) \to l$ as $n \to +\infty$. If $l > 0$, then by taking the limit in (8) we get

$$\phi(l) + F(s^q l) \leq F(l),$$

which is a contradiction. Therefore, we conclude that $l = 0$ and

$$\lim_{n \to +\infty} b(u_{n-1}, u_n) = 0. \tag{9}$$

Next, we show that $\lim_{n,m\to\infty} b(u_n, u_m) = 0$. Suppose the opposite, $\lim_{n,m\to\infty} b(u_n, u_m) > 0$. Then by Lemma 2.4, there exist $\varepsilon > 0$ and sequences $\{m_k\}$ and $\{n_k\}$ of positive integers, with $n_k > m_k > k$, such that

$b(u_{m_k}, u_{n_k}) \geq \varepsilon, b(u_{m_k}, u_{n_k-1}) < \varepsilon,$

$\frac{\varepsilon}{s^2} \leq \limsup_{k\to+\infty}, b(u_{m_k-1}, u_{n_k-1}) \leq \varepsilon s,$

$\frac{\varepsilon}{s} \leq \limsup_{k\to+\infty}, b(u_{n_k-1}, u_{m_k}) \leq \varepsilon s^2$ and

$\frac{\varepsilon}{s} \leq \limsup_{k\to+\infty}, b(u_{m_k-1}, u_{n_k}) \leq \varepsilon s^2.$

From condition (7), we get

$$\phi(b(u_{m_k}, u_{n_k})) + F(s^q b(u_{m_k}, u_{n_k}))$$
$$= \phi(b(u_{m_k}, u_{n_k})) + F(s^q b(f u_{m_k-1}, f u_{n_k-1}))$$
$$\leq F(g(b(u_{m_k-1}, u_{n_k-1}), b(u_{m_k-1}, f u_{m_k-1}), b(u_{n_k-1}, f u_{n_k-1}), \frac{b(u_{m_k-1}, f u_{n_k-1}) + b(u_{n_k-1}, f u_{m_k-1})}{4s}))$$
$$= F\left(g\left(b(u_{m_k-1}, u_{n_k-1}), b(u_{m_k-1}, u_{m_k}), b(u_{n_k-1}, u_{n_k}), \frac{b(u_{m_k-1}, u_{n_k}) + b(u_{n_k-1}, u_{m_k})}{4s}\right)\right) \tag{10}$$

Taking the upper limit in (10) as $k \to +\infty$ and using Lemma 2.3, Lemma 2.4 and (9), we get

$$\liminf_{n\to+\infty}, \phi(b(u_{m_k}, u_{n_k})) + F(s^q \varepsilon) \leq \liminf_{n\to+\infty}, \phi\left(b(u_{m_k}, u_{n_k})\right) + F\left(\limsup_{n\to+\infty} s^q b(u_{m_k}, u_{n_k})\right)$$
$$\leq F\left(\limsup_{n\to+\infty} g\left(b(u_{m_k-1}, u_{n_k-1}), b(u_{m_k-1}, u_{m_k}), b(u_{n_k-1}, u_{n_k}), \frac{b(u_{m_k-1}, u_{n_k}) + b(u_{n_k-1}, u_{m_k})}{4s}\right)\right)$$
$$\leq F\left(g\left(\varepsilon, 0, 0, \frac{\varepsilon}{2s}\right)\right)$$
$$\leq F(\varepsilon s).$$

Hence, the acquired inequality

$$\liminf_{n\to+\infty}, \phi(b(u_{m_k}, u_{n_k})) + F(\varepsilon s^q) < F(\varepsilon s),$$

is a contradiction since $\varepsilon > 0$. So $\lim_{n,m\to\infty} b(u_n, u_m) = 0$, and the sequence $\{u_n\}$ is a Cauchy sequence in the complete b-metric-like space (V, b). Thus, there exists $u \in V$, such that

$$\lim_{n\to+\infty} b(u_n, u) = b(u, u) = \lim_{n,m\to+\infty} b(u_n, u_m) = 0. \tag{11}$$

Let $n_0 \in \mathbb{N}$ such that $u_{n+1} \neq fu$ for all $n \geq n_0$ and $u \neq fu$. Now using condition (7) and property F, we have

$$\phi(b(u_n, u)) + F(s^q b(u_{n+1}, fu)) = \phi(b(u_n, u)) + F(s^q b(f u_n, fu))$$
$$\leq F\left(g\left(b(u_n, u), b(u_n, f u_n), b(u, fu), \frac{b(u_n, fu) + b(u, f u_n)}{4s}\right)\right)$$

$$= F\left(g\left(b(u_n, u), b(u_n, u_{n+1}), b(u, fu), \frac{b(u_n, fu) + b(u, u_{n+1})}{4s}\right)\right),$$

which implies

$$\phi(b(u_n, fu)) + s^q b(u_{n+1}, fu) < g\left(b(u_n, u), b(u_n, u_{n+1}), b(u, fu), \frac{b(u, u_{n+1})}{2s}\right). \tag{12}$$

Taking the upper limit in (12), and using Lemma 2.1 and result (9), it follows that

$$\liminf_{n\to+\infty} \phi(b(u_n, fu)) + s^{q-1} b(u, fu) = s^q \cdot \frac{1}{s} b(u, fu) < g(0, 0, b(u, fu), 0) \leq b(u, fu). \tag{13}$$

Since $q \geq 1$, the inequality (13) implies $b(u, fu) = 0$ and therefore $fu = u$. Thus, u is a fixed point and

$$0 = b(u, fu) = b(u, u). \tag{14}$$

Let u and v be two fixed points of f, where $fu = u$ and $fv = v$. Since $u \neq v$, it implies $fu \neq fv$. By (7) we have

$$\begin{aligned}
\phi(u, v) + F(s^q b(u, v)) &= \phi(u, v) + F(s^q b(fu, fv)) \\
&\leq F\left(g\left(b(u, v), b(u, fu), b(v, fv), \frac{b(u, fv) + b(v, fu)}{4s}\right)\right) \\
&= F\left(g\left(b(u, v), b(u, u), b(v, v), \frac{b(u, v) + b(v, u)}{4s}\right)\right) \\
&= F\left(g\left(b(u, v), b(u, u), b(v, v), \frac{b(u, v)}{2s}\right)\right) \\
&= F\left(g\left(b(u, v), 0, 0, \frac{b(u, v)}{2s}\right)\right) \\
&\leq F\left(g\left(b(u, v), b(u, v), b(u, v), b(u, v)\right)\right) \\
&\leq F\left(b(u, v)\right).
\end{aligned} \tag{15}$$

Since this is a contradiction, it implies $b(u, v) = 0$. Therefore, $u = v$ and the fixed point is unique.

Theorem 3.3. Let (V, b) be $b - m.l.s$ with parameter $s \geq 1$ and $f : V \to V$ be a continuous self-mapping. Assume that there exist the functions $F : (0, +\infty) \to \mathbb{R}$, $g \in \Gamma_4$ and the constant $\tau > 0$ such that

(a) F is strictly increasing;

(b) $\tau + F(s^q b(fx, fy)) \leq F\left(g\left(b(x, y), b(x, fx), b(y, fy), \frac{b(x, fy) + b(y, fx)}{4s}\right)\right)$ for all $x, y \in V$ with $fx \neq fy$, for some $q \geq 1$.

Then f has a unique fixed point in V.

Proof. The proof follows from Theorem 3.2. by setting $\phi(r) = \tau$.

Corollary 3.2. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and $f : V \to V$ be a continuous self-mapping. Assume that there exist the functions $F : (0, +\infty) \to \mathbb{R}$, $\phi : (0, +\infty) \to (0, +\infty)$ such that:

(a) F is strictly increasing;

(b) $\liminf_{r \to t^+} \phi(r) > 0$ for all $t > 0$;

(c) $\phi(b(x, y)) + F(s^q b(fx, fy))$

$$\leq F\left(\max\left(b(x, y), b(x, fx), b(y, fy), \frac{b(x, fy) + b(y, fx)}{4s}\right)\right) \tag{16}$$

for all $x, y \in V$ with $fx \neq fy$, and for some $q \geq 1$. Then f has unique fixed point in V.

Proof. The proof follows from Theorem 3.2 by taking $g \in \Gamma_4$ as $g(t_1, t_2, t_3, t_4) = max\{t_1, t_2, t_3, t_4\}$.

Corollary 3.3. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and f: $V \to V$ be a continuous self-mapping. Assume that there exist functions F: $(0, +\infty) \to \mathbb{R}$, ϕ: $(0, +\infty) \to (0, +\infty)$ such that:

(a) F is strictly increasing;

(b) $\liminf_{r \to t^+} \phi(r) > 0$ for all $t > 0$;

(c) $\phi(b(x, y)) + F(s^q b(fx, fy))$

$$\leq F\left(\max\left(a_1 b(x, y) + a_2 b(x, fx) + a_3 b(y, fy) + a_4 \frac{b(x, fy) + b(y, fx)}{4s}\right)\right) \tag{17}$$

for all $x, y \in V$ with $fx \neq fy$, and for some $q \geq 1$. Then f has unique fixed point in V.

Proof. The proof follows from Theorem 3.2. by taking $g \in \Gamma_4$ as $g(t_1, t_2, t_3, t_4) = a_1 t_1 + a_2 t_2 + a_3 t_3 + a_4 t_4$ with $0 < a_1 + a_2 + a_3 + a_4 < 1$.

Recently, many authors have studied new types of contractions known as interpolative contractions and hybrid contractions. The reader can refer to [3,11,17–21]. The rest of the paper deals with this type of contractions extended in the setting of b-metric-like spaces, which can be obtained from our results as a certain special cases.

Theorem 3.4. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and f: $V \to V$ be a continuous self-mapping. Assume that there exist the functions F: $(0, +\infty) \to \mathbb{R}$, ϕ: $(0, +\infty) \to (0, +\infty)$ such that

(a) F is strictly increasing;

(b) $\liminf_{r \to t^+} \phi(r) > 0$ for all $t > 0$;

(c) $\phi(b(x, y)) + F(s^q b(fx, fy))$

$$\leq F\left(\left[a_1 (b(x, y))^p + a_2 (b(x, fx))^p + a_3 (b(y, fy))^p + a_4 \left(\frac{b(x, fy) + b(y, fx)}{4s}\right)^p\right]^{\frac{1}{p}}\right) \tag{18}$$

for all $x, y \in V$ with $fx \neq fy$, and for some $q \geq 1$. Then f has unique fixed point in V.

Proof. The proof follows from Theorem 3.2 by taking $g \in \Gamma_4$ as

$g(t_1, t_2, t_3, t_4) = \left[a_1 t_1^p + a_2 t_2^p + a_3 t_3^p + a_4 t_4^p\right]^{\frac{1}{p}}, p > 0,$

where $0 < a_1 + a_2 + a_3 + a_4 < 1$.

Theorem 3.5. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and f: $V \to V$ be a continuous self-mapping. Assume that there exist the functions F: $(0, +\infty) \to \mathbb{R}$, ϕ: $(0, +\infty) \to (0, +\infty)$ such that

(a) F is strictly increasing;

(b) $\liminf_{r \to t^+} \phi(r) > 0$ for all $t > 0$;

(c) $\phi(b(x, y)) + F(s^q b(fx, fy))$

$$\leq F\left(\left[\max\left\{(b(x, y))^p, (b(x, fx))^p, (b(y, fy))^p, \left(\frac{b(x, fy) + b(y, fx)}{4s}\right)^p\right\}\right]^{\frac{1}{p}}\right) \tag{19}$$

for all $x, y \in V$ with $fx \neq fy$, and for some $q \geq 1$.

Then f has unique fixed point in V.

Proof. The proof follows from Theorem 3.2 by taking $g \in \Gamma_4$ as
$g(t_1, t_2, t_3, t_4) = \left[\max\left\{t_1^p, t_2^p, t_3^p, t_4^p\right\}\right]^{\frac{1}{p}}, p > 0.$

Theorem 3.6. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and f: $V \to V$ be a continuous self-mapping. Assume that there exist the functions F: $(0, +\infty) \to \mathbb{R}$, ϕ: $(0, +\infty) \to (0, +\infty)$ such that:

(a) F is strictly increasing;

(b) $\liminf_{r \to t^+} \phi(r) > 0$ for all $t > 0$;

(c) $(b(x, y)) + F(s^q b(fx, fy))$

$$\leq F\left((b(x, y))^{a_1} (b(x, fx))^{a_2} (b(y, fy))^{a_3} \left(\frac{b(x, fy) + b(y, fx)}{4s}\right)^{1-(a_1+a_2+a_3)}\right) \tag{20}$$

for all $x, y \in V$ with $fx \neq fy$, and for some $q \geq 1$.

Then f has unique fixed point in V.

Proof. The proof follows from Theorem 3.2 by taking $g \in \Gamma_4$ as
$g(t_1, t_2, t_3, t_4) = t_1^{a_1} \cdot t_2^{a_2} \cdot t_3^{a_3} \cdot t_4^{1-(a_1+a_2+a_3)}$,
where $a_1, a_2, a_3 \in (0, 1)$ and $a_1 + a_2 + a_3 < 1$.

Theorem 3.7. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and $f : V \to V$ be a continuous self-mapping. Assume that there exist the functions $F : (0, +\infty) \to \mathbb{R}$, $\phi : (0, +\infty) \to (0, +\infty)$ and $\lambda \in (0, 1)$ such that

(a) F is strictly increasing;

(b) $\liminf_{r \to t^+} \phi(r) > 0$ for all $t > 0$;

(c) $\phi(b(x, y)) + F(s^q b(fx, fy))$

$$\leq F\left(\left[\lambda \max\left\{(b(x, y))^p, (b(x, fx))^p, (b(y, fy))^p, \left(\frac{b(x, fy) + b(y, fx)}{4s}\right)^p\right\}\right]^{\frac{1}{p}}\right) \tag{21}$$

for all $x, y \in V$ with $fx \neq fy$, and for some $q \geq 1$.

Then f has unique fixed point in V.

Proof. The proof follows from Theorem 3.2 by taking $g \in \Gamma_4$ as

$$g(t_1, t_2, t_3, t_4) = \left[\lambda \max\left\{t_1^p, t_2^p, t_3^p, t_4^p\right\}\right]^{\frac{1}{p}}, p > 0, \lambda \in (0, 1).$$

Corollary 3.4. Let (V, b) be a $b - m.l.s$ with parameter $s \geq 1$ and $f : V \to V$ be a continuous self-mapping. Assume that there exist the functions $F : (0, +\infty) \to \mathbb{R}$, $\phi : (0, +\infty) \to (0, +\infty)$ such that

(a) F is strictly increasing;

(b) $\liminf_{r \to t^+} \phi(r) > 0$ for all $t > 0$;

(c) $\phi(b(x, y)) + F(s^q b(fx, fy)) \leq F\left((b(x, fx))^{a_1} (b(y, fy))^{1-a_1}\right)$ (22)

for all $x, y \in V \backslash F(fix(f))$ with $fx \neq fy$, for some $q \geq 1$.

Then f has unique fixed point in V.

Proof. The proof follows from Theorem 3.2 by taking $g \in \Gamma_4$ as $g(t_1, t_2, t_3, t_4) = t_2^a \cdot t_3^{1-a}$, where $a \in (0, 1)$.

Remark 3.4.

- Varieties of further results can be obtained by extending the set Γ_4 to Γ_5, Γ_6, Γ_7, etc.
- Many significant fixed point theorems that were established for types of interpolative and hybrid contractive conditions essentially belong to the class of generalized (ϕ, s, q, F)-g-contractions.

4 Application

The study of the existence, nonexistence and uniqueness of the solution of differential and integral equations, plays a fundamental role in the research on nonlinear analysis and engineering mathematics. One of the main tools developed in this area consists of the application of a fixed point method.

Let us study the existence of solution for the nonlinear integral equation

$$v(t) = \lambda_1 \int_0^t G_1(t, \rho) H_1(\rho, v(\rho)) d\rho + \lambda_2 \int_0^k G_2(t, \rho) H_2(\rho, v(\rho)) d\rho; \quad t, k \in [0, 1], \tag{23}$$

where λ_i are positive constants and functions $G_i : [0, 1] \times [0, 1] \to \mathbb{R}^+$, $H_i : [0, 1] \times \mathbb{R} \to \mathbb{R}$ for $i = 1, 2$ are given.

Let $V = C([0, 1])$ be the set of real continuous functions defined on [0, 1] endowed with the b-metric-like

$$b(v, u) = \sup_{\rho \in [0,1]} |v(\rho) + u(\rho)|^m \quad \text{forall} \quad v, u \in V, m \in \mathbb{N}. \tag{24}$$

It is obvious that (V, b) is a complete b-metric-like space with parameter $s = 2^{m-1}$.

Consider the mapping $f : V \to V$ by

$$fv(t) = \lambda_1 \int_0^t G_1(t, \rho) H_1(\rho, v(\rho)) d\rho + \lambda_2 \int_0^k G_2(t, \rho) H_2(\rho, v(\rho)) d\rho;$$

for all $v \in C[0, 1]$ and $t, k \in [0, 1]$.

Theorem 4.1. Consider the integral Eq. (1) via the following assertions:

i. The mapping $f : V \to V$ is continuous;

ii. $H_i : [0,1] \times \mathbb{R} \to \mathbb{R}$ are continuous and there exist constants A_i satisfying

$$H_i(\rho, v(\rho)) + H_i(\rho, u(\rho)) \leq A_i |v(\rho) + u(\rho)|$$

for $i = 1, 2$ and $t, \rho, k \in [0,1]$;

iii. The constants λ_i, A_i and functions G_i, for $i = 1, 2$ satisfy condition

$$0 < \lambda_1 A_1 \int_0^t G_1(t,\rho) d\rho + \lambda_2 A_2 \int_0^k G_2(t,\rho) < \frac{1}{\sqrt[m]{s^{q+1}}}$$

for $t, k \in (0,1)$ and $q \geq 1$. Then the integral Eq. (23) has a unique solution $v(t) \in V$.

Proof. For all $t \in [0,1]$, and $v, u \in V$ we have

$$\begin{aligned}
&s^q \sigma_b(fv(t), fu(t)) = s^q |fv(t) + fu(t)|^m \\
&= s^q \Big| \lambda_1 \int_0^t G_1(t,\rho) H_1(\rho, v(\rho)) d\rho + \lambda_2 \int_0^k G_2(t,\rho) H_2(\rho, v(\rho)) d\rho \\
&\quad + \lambda_1 \int_0^t G_1(t,\rho) H_1(\rho, u(\rho)) d\rho + \lambda_2 \int_0^k G_2(t,\rho) H_2(\rho, u(\rho)) d\rho \Big|^m \\
&= s^q \Big| \lambda_1 \int_0^t G_1(t,\rho) \left(H_1(\rho, v(\rho)) + H_1(\rho, u(\rho))\right) d\rho \\
&\quad + \lambda_2 \int_0^k G_2(t,\rho) \left(H_2(\rho, v(\rho)) + H_2(\rho, u(\rho))\right) d\rho \Big|^m \\
&\leq s^q \left| \lambda_1 \int_0^t G_1(t,\rho) A_1 \left(|v(\rho) + u(\rho)|\right) d\rho + \lambda_2 \int_0^k G_2(t,\rho) A_2 \left(|v(\rho) + u(\rho)|\right) d\rho \right|^m \\
&= s^q \left| \lambda_1 \int_0^t G_1(t,\rho) A_1 \left(|v(\rho) + u(\rho)|^m\right)^{\frac{1}{m}} d\rho + \lambda_2 \int_0^k G_2(t,\rho) A_2 \left(|v(\rho) + u(\rho)|^m\right)^{\frac{1}{m}} d\rho \right|^m \\
&\leq s^q \left| \lambda_1 \int_0^t G_1(t,\rho) A_1 \left(b(v,u)\right)^{\frac{1}{m}} d\rho + \lambda_2 \int_0^k G_2(t,\rho) A_2 \left(b(v,u)\right)^{\frac{1}{m}} d\rho \right|^m \\
&= s^q \left| \lambda_1 \left(b(v,u)\right)^{\frac{1}{m}} \int_0^t A_1 G_1(t,\rho) d\rho + \lambda_2 \left(b(v,u)\right)^{\frac{1}{m}} \int_0^k A_2 G_2(t,\rho) d\rho \right|^m \\
&= s^q \left| \left(b(v,u)\right)^{\frac{1}{m}} \left(A_1 \lambda_1 \int_0^t G_1(t,\rho) d\rho + A_2 \lambda_2 \int_0^k G_2(t,\rho) d\rho \right) \right|^m \\
&\leq s^q \left| \frac{1}{\sqrt[m]{s^{q+1}}} \left(b(v,u)\right)^{\frac{1}{m}} \right|^m \\
&= \frac{b(v,u)}{s}.
\end{aligned} \tag{25}$$

Hence, by taking logarithms in inequality (25) we get

$$\ln s + \ln(s^q b(fv, fu)) \leq \ln(b(v,u)).$$

Further, fixing $F(\zeta) = \ln(\zeta)$, $\tau = \ln s$ and taking $g \in \Gamma_4$ as $g(t_1, t_2, t_3, t_4) = t_1$ we obtain

$$\tau + F\left(s^q b(fv, fu)\right) \leq F\left(g\left(b(v, u), b(v, fv), b(u, fu), \frac{b(v, fu) + b(u, fv)}{2s}\right)\right).$$

Therefore, f is a (s, q, F)-g-contraction on V and all conditions of Theorem 3.3 are satisfied. Thus, $v(t)$ is the unique fixed point of f, i.e., the solution of the integral Eq. (23).

5 Conclusion

The Definitions 2.1 and 3.2 not only a large class of contractions in terms of ϕ, s, q, g and F in the metric, b-metric, metric-like, partial metric, but also have a unifying power for both linear and nonlinear contractions in the framework of b-metric-like spaces.

Funding Statement: The authors received no specific funding for this study.

Conflicts of Interest: The author declares that they have no conflicts of interest to report regarding the present study.

References

1. Wardowski, D. (2012). Fixed points of a new type of contractive mappings in complete metric spaces. *Fixed Point Theory and Applications, 2012,* 94. DOI 10.1186/1687-1812-2012-94.
2. Wardowski, D. (2018). Solving existence problems via F-contractions. *Proceedings of the American Mathematical Society, 146(4),* 1585–1598. DOI 10.1090/proc/13808.
3. Karapinar, E. (2018). Revisiting the kannan type contractions via interpolation. *Advances in the Theory of Nonlinear Analysis and its Application, 2,* 85–87. DOI 10.31197/atnaa.431135.
4. Younis, M., Singh, D., Abdou, A. A. N. (2022). A fixed point approach for tuning circuit problem in dislocated b-metric spaces. *Mathematical Methods in the Applied Sciences, 45(4),* 2234–2253. DOI 10.1002/mma.7922.
5. Younis, M., Singh, D. (2022). On the existence of the solution of hammerstein integral equations and fractional differential equations. *Journal of Applied Mathematics and Computing, 68,* 1087–1105. DOI 10.1007/s12190-021-01558-1.
6. Ding, H. S., Imdad, M., Radenović, S., Vujaković, J. (2016). On some fixed point results in b-metric, rectangular and b-rectangular metric spaces. *Arab Journal of Mathematical Sciences, 22(2),* 151–164. DOI 10.3390/math8040488.
7. Ege, O. (2015). Complex valued rectangular b-metric spaces and an application to linear equations. *Journal of Nonlinear Science and Applications, 8(6),* 1014–1021. DOI 10.22436/jnsa.008.06.12.
8. Bajović, D., Mitrović, Z. D., Saha, M. (2021). Remark on contraction principle in cone$_{tvs}b$-metric spaces. *The Journal of Analysis, 29(1),* 273–280.
9. de la Sen, M., Nikolić, N., Došenović, T., Pavlović, M., Radenović, S. (2019). Some results on (s-q)-graphic contraction mappings in b-metric-like spaces. *Mathematics, 7,* 1190–1197.
10. Zoto, K., Radenović, S., Ansari, H. A. (2018). On some fixed point results for (s, p, a)-contractive mappings in b-metric-like spaces and applications to integral equations. *Open Mathematics, 16,* 235–249.
11. Karapinar, E., Aydi, H., Fulga, A. (2020). On hybrid-wardowski contractions. *Journal of Mathematics, 2020,* 526–534. DOI 10.1155/2020/1632526.
12. Zoto, K., Radenović, S., Dine, J., Vardhami, I. (2017). Fixed points of generalized (ψ, s, α)-contractive mappings in dislocated and b-dislocated metric spaces. *Communications in Optimization Theory, 2017,* 1–16. DOI 10.23952/cot.2017.22.

13. Aleksić, S., Mitrović, Z. D., Radenović, S. (2020). Picard sequences in b-metric spaces. *Fixed Point Theory, 21,* 35–46.
14. Czerwik, S. (1993). Contraction mappings in b-metric spaces. *Acta Mathematica et Informatica Universitatis Ostraviensis, 1,* 5–11.
15. Alghmandi, A. M., Hussain, N., Salimi, P. (2013). Fixed point and coupled fixed point theorems on b-metric-like spaces. *Journal of Inequalities and Applications, 2013,* 1–25. DOI 10.1186/1029-242x-2013-402.
16. Fabiano, N., Došenović, T., Rakić, D., Radenović, S. (2020). Some new results on (s,q)-Dass-Gupta-Jaggi type contractive mappings in b-metric-like spaces. *Filomat, 34(12),* 4015–4026. DOI 10.2298/fil2012015f.
17. Agarwal, P. R., Karapinar, E. (2019). Interpolative Rus-Reich-Ćirić type contractions via simulation functions. *Analele Stiintifice ale Universitatii Ovidius Constanta, Seria Matematica, 27,* 137–152. DOI 10.2478/auom-2019-0038.
18. Karapinar, E., Algahtani, O., Aydi, H. (2018). On interpolative Hardy-Rogers type contractions. *Symmetry, 11(1),* 32–40. DOI 10.3390/sym11010008.
19. Aydi, H., Karapinar, E., de Hierro, A. F. R. L. (2019). ω-interpolative Ćirić-Reich-Rus-type contractions. *Mathematics, 7(1),* 1–8. DOI 10.3390/math7010057.
20. Aydi, H., Chen, M. C., Karapinar, E. (2019). Interpolative Ćirić-Reich-Rus type contractions via the branciari distance. *Proceedings of the American Mathematical Society, 7(1),* 1–7. DOI 10.3390/math7010084.
21. Karapinar, E., Agarwal, R., Aydi, H. (2018). Interpolative Reich-Rus-Ćirić type contractions on partial metric spaces. *Mathematics, 6(11),* 256–267. DOI 10.3390/math6110256.
22. Zoto, K., Rhoades, E. B., Radenović, S. (2019). Common fixed point theorems for a class of (s,q)–contractive mappings in b– metric-like spaces and applications to integral equations. *Mathematica Slovaca, 69(1),* 1585–1598. DOI 10.1515/ms-2017-0217.
23. Zoto, K., Rhoades, B. E., Radenović, S. (2017). Some generalizations for contractions in b-metric-like spaces and application. *Fixed Point Theory and Applications, 26,* 1–20. DOI 10.1186/s13663-017-0620-1.
24. Karapinar, E., Abdeljawad, T., Jarad, F. (2019). Applying new fixed point theorems on fractional and ordinary differential equations. *Advances in Difference Equations, 2019*, 421. DOI 10.1186/s13662-019-2354-3.
25. Vujaković, J., Aydi, H., Radenović, S., Mukheimer, A. (2019). Some remarks and new results in ordered partial b-metric spaces. *Mathematics, 7,* 334. DOI 10.3390/math7040334.
26. Hussain, N., Radenović, S., Zoto, K. (2018). Common fixed point results of $(\alpha - \psi, \phi)$ -contractions for a pair of mappings and applications. *Mathematics, 6(19),* 1–17. DOI 10.3390/math6100182.
27. Kadelburg, Z., Radenović, S. (2018). Notes on some recent papers concerning F-contractions in b-metric spaces. *Constructive Mathematical Analysis, 1,* 108–112. DOI 10.33205/cma.468813.
28. Popescu, O., Stan, G. (2020). Two fixed point theorems concerning F-contraction in complete metric spaces. *Symmetry, 12(58),* 1–10. DOI 10.3390/sym12010058.
29. Mitrović, S., Parvaneh, V., de La Sen, M., Vujaković, J., Radenović, S. (2021). Some new results for Jaggi-W-contraction-type mappings on b-metric-like spaces. *Mathematics, 9,* 1921. DOI 10.3390/math9161921.
30. Vetro, F. (2016). F-contractions of Hardy-Rogers-type and application to multistage decision. *Nonlinear Analysis: Modelling and Control, 21(4),* 531–546. DOI 10.15388/na.2016.4.7.
31. Cosentino, M., Vetro, P. (2014). Fixed point result for F-contractive mappings of Hardy-Rogers type. *Filomat, 28,* 715–722. DOI 10.2298/fil1404715c.