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THERMO-ELASTIC-PLASTIC TRANSITION IN TORSION OF COMPOSITE THICK-WALLED CIRCULAR CYLINDER SUBJECTED TO PRESSURE

TERMO-ELASTIČNO-PLASTIČNI PRELAZNI NAPONI PRI TORZIJI KOMPOZITNOG DEBELOZIDNOG KRUŽNOG CILINDRA POD PRITISKOM

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Abstract

The purpose of this paper is to present study of thermo-elastic-plastic transition in torsion of composite thick-walled circular cylinder subjected to pressure. The concept of transition theory is applied to evaluate shear stresses in a cylinder under torsion. Generalized strain measures are used to simplify the fundamental equations. It has been deduced from the analysis of stresses in the cylinder that in case of torsion, the cylinder of less non-homogeneous material without temperature is a safe design in comparison to the cylinder of highly non-homogeneous material with or without thermal effects, because shear stresses are maximal for the cylinder of less non-homogeneous material without thermal effects than for the non-homogeneous cylinder of high compressibility.

Izvod

U ovom radu je predstavljena studija termo-elastično-plastičnog prelaznih napona pri torziji kompozitnog debelozidog kružnog cilindra pod pritiskom. Koncept teorije prelaznih napona se primenjuje pri oceni smičućih napona u cilindru opterećenim na uvijanje. Generalisane mere deformacije se koriste radi uprošćenja fundamentalnih jednačina. Na osnovu analize napona u cilindru, izveden je zaključak da je slučaj uvijanja cilindra od manje nehomogenog materijala bez temperature bezbedniji u poređenju sa cilindrom od veoma nehomogenih materijala, sa ili bez termičkih uticaja, zbog smičućih napona koji su maksimalni u slučaju cilindra sa manje nehomogenim materijalima bez termičkih uticaja u odnosu na cilindar od nehomogenih materijala velike stišljivosti.

INTRODUCTION

Thick cylinders such as guns, high-pressure hydraulic pipes, bores and driving shafts are very important part of various industries. Wall thicknesses of these cylinders are relatively large and the stress variation across the thickness is also significant. Such types of problems are solved by considering an axisymmetry about the *z*-axis and solving differential equations of stress equilibrium in polar coordinates. Fatigue crack growth is numerically simulated for various internal surface cracks with irregular crack fronts discussed by /1/ and it is found that every defect with any initial shape gets the shape of a semi-elliptical crack after some time. The problem of mixed-mode fracture induced by a semi-elliptical circumferential crack lying at the outer surface of the cross-section of a hollow cylinder is considered, see /2/. The cylinder is subjected to axial force, bending moment and torsion. The finite element analysis method is used to calculate the stress intensity factor along the crack front. Longitudinal cracks in pressurized cylindrical vessels, using finite element method of modelling has been developed by /3/ and it is noticed that the stress intensity factor for external cracks is more than that for internal cracks, and the maximum stress intensity factor, are found at either the deepest- or corner points of the crack, depending on loading and geometry conditions. Eringen and Suhubi, /4/, formulated the constitutive equations of 'simple micro-elastic' solids. Such solids are affected by 'micro' deformations and rotations which have not been faced in the theory of finite elasticity. This theory gave the concept of stress momenta, inertial spin, and other second order effects and their laws of motion. The analytical solution for hollow circular cylinders of homogeneous isotropic micro polar material subjected to torsion is given by /5/. The problem of axially symmetric torsion of a hollow cylinder with an external crack is dealt by /6/ and they determined the stresses, displacement, and stress intensity factors. Finite element simulations of different shear deformations in nonlinear elasticity are proposed, see /7/. The solution is approximated efficiently and guaranteed computational bounds on the magnitude of the Poynting effect in shear and simple torsion. Differential and integral operators are used in solving the nonsymmetric system of equations characterizing the pure torsion of a body of revolution with variable shear moduli by /8/. The torsion solution, based on Michell-Foppl theory for the non-homogeneous circular cylinder having rigid spherical inclusion is given by /9/. It is observed that effect of spherical load on shear stresses is less for homogeneous cylinders. All these authors had considered yield criterion, jump conditions, linear strain measure to determine the stresses using the concept of infinitesimal strain theory. The concept of transition theory, /10/, has been applied to solve the problem. This theory does not require any of the yield criteria and jump conditions, thus it solves the problem using the concept of generalized strain measure. This theory has been applied to many problems, /10-13/, determined transitional stresses for a transversely isotropic circular cylinder subjected to torsion. It is shown that the asymptotic solution through the stress and stress-difference gives the plastic and creep stresses respectively. It is concluded that the value of maximum shear stress for a cylinder of transversely isotropic material is greater than that for a cylinder of isotropic incompressible material. Gupta, Dharmani and Rana, /12/, considered the elastic-plastic stresses for a homogeneous circular cylinder subjected to torsion and it is found that the value of maximum shear stress for a cylinder of compressible material is greater than that for a cylinder of incompressible material. Thakur /14-28/, discuss the creep stresses and strain rates in cylinder and disc under temperature gradient by using Seth's transition theory.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a cylinder made of functionally graded material having internal and external radii as a and b respectively, under the influence of internal and external pressures p_1 and p_2 with temperature Θ on the inner surface. The displacement components are given by

$$u = r(1-\beta),\ v = \eta rz \text{ and } w = dz, \tag{1}$$

where: β is a function of $r=\sqrt{x^2+y^2}$; d is a constant; and η is the angle of twist.

Non-homogeneity is considered as the compressibility of the material in the cylinder

$$C = C_0(r/b)^{-k}, \tag{2}$$

where: $a \le r \le b$, C_0 and k are real values constants.

The generalized strain components are given by:

$$e_{rr}=\frac{1}{n}\left[1-(r\beta'+\beta)^n\right],\ e_{\theta\theta}=\frac{1}{n}\left[1-\beta^n\right],$$

$$e_{zz}=\left(\frac{D^n}{n}\right)\left[1-\left(\frac{\eta r\beta}{D}\right)^n\right],\ e_{\theta z}=\frac{1}{n}(\eta^{n/2}r^{n/2}\beta^n), \tag{3}$$

$$e_{r\theta}=e_{\theta z}=0,$$

where: $D_n = [1-(1-d)^n]$; n is the measure of deformation and $\beta' = d\beta/dr$.

The stress-strain relation for isotropic material with temperature is

$$T_{ij}=\lambda\delta_{ij}I_1+2\mu e_{ij}-\xi\Theta\delta_{ij},\ (i,j=1,2,3) \tag{4}$$

where: T_{ij}, e_{ij} are stress and strain tensors respectively; $I_1 = e_{kk}$ are strain invariants; λ, μ are Lame's constant; δ_{ij} is Kronecker's delta; $\xi = \alpha(3\lambda + 2\mu)$, α being the coefficient of thermal expansion, and Θ is temperature. The temperature Θ has to satisfy

$$\Theta,_{ii}=0. \tag{5}$$

The temperature field satisfying Eq.(5) is $\Theta = \Theta_0$ at $r = a$, $\Theta = 0$ at $r = b$, where Θ_0 is constant, given by

$$\Theta=\left(\Theta_0\log\frac{r}{b}\right)\Big/\left(\log\frac{a}{b}\right).$$

The twisting couple M is given by

$$M=2\pi\int_a^b r^2\tau_{\theta z}dr. \tag{6}$$

All equations of equilibrium are satisfied except

$$\frac{d}{dr}(T_{rr})+\frac{(T_{rr}-T_{\theta\theta})}{r}=0. \tag{7}$$

The nonlinear differential equation of transition state is obtained by substituting Eq.(4) in Eq.(7) as:

$$\begin{aligned}
&n\beta^{n+1}P(P+1)^{n-1}\frac{dP}{d\beta}=-(1-C)\Bigg[\left(\frac{2}{3}\eta_1(3-2C)r\beta\right)^{n-1}\frac{2}{3}\eta_1+\\
&+\{-2C'r\beta+(3-2C)\beta(P+1)\}+\beta^n(1-(P+1)^n)C\Bigg]-\\
&-\frac{(2C^2-4C+3)rC'}{C(3-2C)}D^n\left[1-\left(\frac{2\eta_1(3-2C)r\beta}{3D}\right)^n\right]+\\
&+\frac{\alpha\Theta_0 nC'(3-2C)\log\left(\frac{r}{b}\right)}{C^2\log\left(\frac{a}{b}\right)}-\frac{\alpha\Theta_0 n(3-2C)}{Cr\log\left(\frac{a}{b}\right)}-n\beta^n P\times\\
&\times\left[(P+1)^n+(1-C)\right]-\left[\frac{(3-4C)rC'}{C(3-2C)}\right]\left[1-\beta^n(P+1)^n\right]-\\
&-(1-\beta^n)\left[\frac{(2C^2-4C+3)rC'}{C(3-2C)}+C\right],
\end{aligned} \tag{8}$$

where: $r\beta' = \beta P$; $C=\frac{2\mu}{\lambda+2\mu}$; $\bar{\Theta}_0=\frac{\Theta_0}{\log\frac{a}{b}}$; $\xi=\frac{3\alpha}{C}$; and

$\eta=\frac{2\eta_1(3-2C)}{3}$; η_1 is a constant.

The transition points of Eq.(8) are $P \to -1$ and $P \to \pm\infty$. The boundary conditions are

$$T_{rr}=-p_1 \text{ at } r=a \text{ and } T_{rr}=-p_2 \text{ at } r=b. \tag{9}$$

The resultant axial force in the circular cylinder is given by

$$\int_a^b rT_{zz}dr = 0\ . \tag{10}$$

METHOD OF APPROACH

To determine the plastic stresses at the transition point, $P \to \pm\infty$, we define the transition function TR in terms of T_{rr} as follows:

$$TR = T_{rr} + \alpha(3-2C)\Theta - B \equiv \frac{3}{(3-2C)Cn}\left[2-\beta^n\left\{(P+1)^n+(1-C)\right\}\right] + \frac{3(1-C)D^n}{C(3-2C)n}\left\{1-\left(\frac{2\eta_1(3-2C)r\beta}{D}\right)^n\right\} - \alpha\Theta\left(\frac{2C^2-3C+3}{C}\right) - B \tag{11}$$

Taking the logarithmic differentiation of Eq.(11) with respect to r, we have

$$\frac{d}{dr}(\log TR) = \frac{\left[\begin{aligned}&\frac{-3(3-4C)C'}{C^2(3-4C)^2 n}(1-\beta^n(P+1)^n) + \frac{3}{(3-2C)Cn}\left\{\frac{-n\beta^n P(P+1)^n}{r} - \beta^{n+1}n(P+1)^{n-1}\frac{dP}{d\beta}\frac{P}{r}\right\} + \\ &+\frac{3(4C-2C^2-3)C'}{C^2(3-4C)^2 n}(1-\beta^n) - \frac{-3(1-C)\beta^n P}{C(3-2C)r} + \frac{3(4C-2C^2-3)}{C^2(3-2C)^2}\frac{D^n}{n}\left(1-\left(\frac{\alpha\beta r}{D}\right)^n\right) - \\ &-\frac{3(1-C)}{C(3-2C)}(\alpha\beta)^n n r^{n-1}(1+P) - \left(\frac{2C^2-3C+3}{C}\right)\alpha\Theta' - \left(\frac{2C^2-3}{C^2}\right)C'\alpha\Theta\end{aligned}\right]}{\frac{3}{(3-2C)Cn}\left[(1-\beta^n(P+1)^n)+(1-\beta^n)\frac{3(1-C)}{(3-2C)Cn} + \frac{3(1-C)D^n}{(3-2C)Cn}\left(1-\left(\frac{\alpha\beta r}{D}\right)^n\right) - \left(\frac{2C^2-3C+3}{C}\right)\alpha\Theta - B\right]}\ . \tag{12}$$

From Eqs.(8) and (12), we obtain

$$\frac{d}{dr}(\log TR) = -\frac{C}{r}\ . \tag{13}$$

By integrating Eq.(13), we get

$$TR = A\exp f(r)\ , \tag{14}$$

Using Eq.(11) and Eq.(14), we have

$$T_{rr} = A\exp f(r) + B - \alpha\Theta(3-2C)\ . \tag{15}$$

Using boundary conditions of Eq.(9) in Eq.(15), we obtain stresses as:

$$T_{rr} = A[\exp f(r) - \exp f(b)] - p_2 - \alpha\Theta(3-2C)\ , \tag{16}$$

where: $f(r) = -\int Cr^{-1}dr$; and A is a constant of integration.

$$T_{\theta\theta} = A\left[(1-C)\exp f(r) - \exp f(b)\right] - \alpha\bar{\Theta}_0\left[(3-2C)\left(1+\log\frac{r}{b}\right) - 2rC'\log\frac{r}{b}\right] - p_2\ , \tag{17}$$

$$T_{zz} = \frac{1-C}{2-C}(T_{rr}+T_{\theta\theta}) + \frac{3}{2-C}e_{zz} - \frac{3\alpha}{2-C}\Theta\ , \tag{18}$$

$$T_{\theta z} = \frac{3\left(\frac{2\eta_1(3-2C)}{3}r\right)^{n/2}}{(3-2C)n}\left[1-\frac{nC(3-2C)}{3+6C-4C^2}\left\{\frac{(3-2C)}{C}(T_{\theta\theta}-T_{rr}) + \frac{1}{C}T_{\theta\theta} + \frac{3\alpha\Theta}{(2-C)}\right\}\right]\ , \tag{19}$$

where: $e_{zz} = \frac{D^n}{n}\left[1-\left(\frac{2\eta_1(3-2C)r}{3D}\right)^n\left\{1-\frac{nC(3-2C)}{3+6C-4C^2}\left(\frac{3-2C}{C}(T_{\theta\theta}+T_{rr}) + \frac{1}{C}T_{\theta\theta} + \frac{3\alpha\Theta}{2-C}\right)\right\}\right]$, and

$$D^n = \frac{\int_a^b\left[\frac{3\left(\frac{2\eta_1(3-2C)}{3}\right)^n r^{n+1}}{(2-C)}\left\{1-\frac{n(3-2C)^2}{3+6C-4C^2}(T_{\theta\theta}+T_{rr}) - \frac{n(3-2C)}{3+6C-4C^2}T_{\theta\theta} + \frac{nC(3-2C)\alpha\Theta}{(2-C)(3+6C-4C^2)}\right\}\right]dr}{\frac{1}{n}\int_a^b\frac{3r}{(2-C)}dr}\ .$$

Considering the non-homogeneity due to variable compressibility, $C = C_0\left(\frac{r}{b}\right)^{-k}$, Eqs.(16)-(19) become:

$$T_{rr} = A_1\left[\exp\left(\frac{C_0}{k}\left(\frac{r}{b}\right)^{-k}\right) - \exp\left(\frac{C_0}{k}\right)\right] - p_2 - \alpha\Theta\left[3-2C_0\left(\frac{r}{b}\right)^{-k}\right], \tag{20}$$

$$T_{\theta\theta} = A_1\left[\left(1-C_0r^{-k}\right)\exp\left(\frac{C_0}{k}\left(\frac{r}{b}\right)^{-k}\right) - \exp\left(\frac{C_0}{k}\right)\right] - \alpha\bar{\Theta}_0[(3-2C_0r^{-k})(1+\log\frac{r}{b}) + 2C_0k\left(\frac{r}{b}\right)^{-k}\log\frac{r}{b} - p_2\ , \tag{21}$$

$$T_{zz}=\frac{(1-C_0R_1)}{2-C_0R_1}(T_{rr}+T_{\theta\theta})+\frac{3}{2-C_0R_1}e_{zz}-\frac{3\alpha\Theta}{2-C_0R_1};\quad R_1=(r/b)^{-k}, \tag{22}$$

$$T_{\theta z}=\frac{3\left(\frac{2\eta_1(3-2C_0R_1)}{3}\right)r^{n/2}}{(3-2C_0R_1)n}\left[1-\frac{nC_0R_1(3-2C_0R_1)}{3+6C_0R_1-\left(4C_0(r/b)\right)^2}\left\{\frac{(3-2C_0R_1)}{C_0R_1}(T_{\theta\theta}-T_{rr})+\frac{1}{C_0R_1}T_{\theta\theta}+\frac{3\alpha\Theta}{(2-C_0R_1)}\right\}\right], \tag{23}$$

$$M=2\pi\int_a^b r^2T_{\theta z}dr, \tag{24}$$

where: $e_{zz}=\frac{D^n}{n}\left[1-\left(\frac{2\eta_1(3-2C_0R_1r)}{3D}\right)^n\left\{1-\frac{nC_0R_1(3-2C_0R_1)}{3+6C_0R_1-4(C_0R_1)^2}\left(\frac{3-2C_0R_1}{C_0R_1}(T_{\theta\theta}+T_{rr})+\frac{1}{C_0R_1}T_{\theta\theta}+\frac{3\alpha\Theta}{2-C_0R_1}\right)\right\}\right]$,

$$D^n=\frac{\int_a^b\left[\frac{3\left(\frac{2\eta_1(3-2C_0R_1)}{3}\right)^n r^{n+1}}{2-C_0R_1}\left\{1-\frac{n(3-2C_0R_1)^2}{3+6C_0R_1-4(C_0R_1)^2}(T_{\theta\theta}+T_{rr})-\frac{n(3-2C_0R_1)}{3+6C_0R_1-4(C_0R_1)^2}T_{\theta\theta}+\frac{nC_0R_1(3-2C_0R_1)\alpha\Theta}{(2-C_0R_1)(3+6C_0R_1-4(C_0R_1)^2)}\right\}\right]dr}{\frac{1}{n}\int_a^b\frac{3r}{(2-C_0R_1)}dr}.$$

Equations (20)-(24) are thermal transitional radial, circumferential, axial, shear stresses and the twisting couple for torsion of functionally graded thick-walled circular cylinder under internal and external pressure. From Eqs.(20) and (21), we have Tresca's yield criterion as:

$$T_{\theta\theta}-T_{rr}=-A_1C_0R_1\exp(C_0R_1/k)-2k\alpha C_0\bar{\Theta}_0R_1\log R_1- \\ -\alpha\bar{\Theta}_0(3-2C_0R_1), \tag{25}$$

It can be seen from Eq.(25) that $|T_{\theta\theta}-T_{rr}|$ is maximum at $r=(e^2b^k)^{\frac{1}{k}}=r_1$, therefore, we have

$$\left|T_{\theta\theta}-T_{rr}\right|_{r=r_1}=\left|-A_1C_0R_1\exp(C_0R_1/k)-2kC_0\alpha\bar{\Theta}_0R_1- \\ -\alpha\bar{\Theta}_0(3-2C_0R_1)\right|\equiv Y\ (\text{say}), \tag{26}$$

where: Y is yield stress; $A_1=\frac{\alpha\Theta_0(3-2C_0R_2)+(p_1-p_2)}{\exp(C_0R_2/k)-\exp(C_0/k)}$; and $R_2=(a/b)^{-k}$.

Effective pressure required for initial yielding is given from Eq.(26) as:

$$\left|P_iA_2+\Theta_1A_3\right|=1, \tag{27}$$

where

$$P_i=\frac{p_1}{Y}-\frac{p_2}{Y}=P_{i1}-P_{i2},\ A_2=\frac{k}{e\{\exp(C_0R_2/k)-\exp(C_0/k)\}}$$

and

$$A_3=\frac{3+2k}{\log(a/b)}-\frac{(3-2C_0R_2)k}{e\{\exp(C_0R_2/k)-\exp(C_0/k)\}}-2k^2\log(r_1/b).$$

For fully plastic state i.e. $C_0\to 0$, Eq.(23) becomes

$$\left|T_{\theta\theta}-T_{rr}\right|_{r=b}= \\ =\left|\frac{\left[(p_1-p_2)-3\alpha\Theta_0\right]kb^{-k}-3\alpha\bar{\Theta}_0(a^{-k}-b^{-k})}{(a^{-k}-b^{-k})}\right|\equiv Y_1\ (\text{say}), \tag{28}$$

where Y_1 is the yield stress in the fully plastic state.

Effective pressure required for fully plastic state is given by

$$P_f=\left|\frac{1}{A_4}\right|+\Theta_1\left|\frac{A_5}{A_4}\right|, \tag{29}$$

where: $P_f=\frac{p_1}{Y_1}-\frac{p_2}{Y_1}=P_{f1}-P_{f2}$, $\Theta_1=\frac{\alpha\Theta_0}{Y_1}$,

$A_4=\frac{k}{b^k(a^{-k}-b^{-k})}$ and $A_5=3\left[\frac{k}{b^k(a^{-k}-b^{-k})}+\frac{1}{\log(a/b)}\right]$.

Now components in non-dimensional form are

$R=(r/b),\ R_0=(a/b),\ \sigma_{rr}=[T_{rr}/Y],\ \sigma_{\theta\theta}=[T_{\theta\theta}/Y],$

$\sigma_{zz}=[T_{zz}/Y],\ \sigma_{\theta z}=[T_{\theta z}/Y],\ M_1=\frac{M}{Y}.$

Now the necessary effective pressure required for initial yielding in non-dimensional form is given from Eq.(27) by

$$|P_i|=\frac{1}{|A_6|}-\Theta_1\frac{|A_7|}{|A_6|} \tag{30}$$

where: $A_6=\frac{k}{e\left[\exp(-e^2R_0^{-k})-\exp(-e^2)\right]}$,

$A_7=\frac{3+2k}{\log R_0}-\frac{(3+2ke^2R_0^{-k})k}{e\left[\exp(-e^2R_0^{-k})-\exp(-e^2)\right]}-4k$.

The thermal radial, circumferential, axial, shear stresses and twisting couple in the transition state from Eqs.(20)-(24) in non-dimensional form are given by following equations:

$$\sigma_{rr}=\left[\Theta_1(3-2C_0(R_0)^{-k})-P_i\right]\times \\ \times\left\{\left[\exp\left(\frac{C_0R^{-k}}{k}\right)-\exp\left(\frac{C_0}{k}\right)\right]/R_3\right\}- \\ -P_{i2}-\Theta_1\frac{\log R}{\log R_0}(3-2C_0(R)^{-k}), \tag{31}$$

$$\sigma_{\theta\theta} = \left[\left\{\Theta_1(3-2C_0(R_0)^{-k}) - P_i\right\}/R_3\right]\left[(1-C_0R^{-k})\exp(C_0R^{-k}/k) - \exp(C_0/k)\right] -$$

$$-\frac{\Theta_1}{\log R_0}\left[(3-2C_0R^{-k})(1+\log R) + 2C_0kR^{-k}\log R\right] - P_{i2}, \tag{32}$$

$$\sigma_{zz} = \frac{1-C_0R^{-k}}{2-C_0R^{-k}}(\sigma_{rr}+\sigma_{\theta\theta}) + \frac{3}{2-C_0R^{-k}}e_{zz} - \frac{3\Theta_1\log R}{(2-C_0R^{-k})\log R_0}, \tag{33}$$

$$\sigma_{\theta z} = \frac{3\left[2\eta_1(3-2C_0(Rb)^{-k})/3\right](Rb)^{n/2}}{(3-2C_0R^{-k})n}\left[1-\frac{nC_0R^{-k}(3-2C_0R^{-k})}{R_4}\left\{\frac{(3-2C_0R^{-k})}{C_0R^{-k}}(\sigma_{\theta\theta}-\sigma_{rr}) + \frac{1}{C_0R^{-k}}T_{\theta\theta} + \frac{3\Theta_1\log R}{(2-C_0R^{-k})\log R_0}\right\}\right], \tag{34}$$

$$M_1 = 2\pi\int_{R_0}^{1} R^2b^3\sigma_{\theta z}dR, \tag{35}$$

where: $R_3 = \left\{\exp\left(\frac{C_0R_0^{-k}}{k}\right) - \exp\left(\frac{C_0}{k}\right)\right\}$, $R_4 = 3+6C_0R^{-k} - 4(C_0R^{-k})^2$,

$$e_{zz} = \frac{D^n}{n}\left\{1-\left[\frac{2\eta_1(3-2C_0(Rb)^{-k})Rb}{3D}\right]^n\left\{1-\frac{nC_0R^{-k}(3-2C_0R^{-k})}{3+6C_0R^{-k}-4C_0^2R^{-2k}}\left[\frac{(3-2C_0R^{-k})}{C_0R^{-k}}(\sigma_{\theta\theta}-\sigma_{rr}) + \frac{\sigma_{\theta\theta}}{C_0R^{-k}} + \frac{3\Theta_1\log R}{(2-C_0R^{-k})\log R_0}\right]\right\}\right\},$$

$$D^n = \frac{\displaystyle\int_{R_0}^{1}\left[\frac{3\left[2\eta_1(3-2C_0(Rb)^{-k})/3\right]^n(Rb)^n}{2-C_0R^{-k}}\left\{\begin{array}{l}1-\left[n(3-2C_0R^{-k})^2/R_4\right](\sigma_{\theta\theta}+\sigma_{rr}) - \left[n(3-2C_0R^{-k})/R_4\right]T_{\theta\theta} + \\ +\left[nC_0R^{-k}(3-2C_0R^{-k})\alpha\Theta/(2-C_0R^{-k})R_4\right]\end{array}\right\}\right]dR}{\displaystyle\frac{1}{n}\int_{R_0}^{1}\frac{3bR}{2-C_0R^{-k}}dR}.$$

The effective pressure required for fully plasticity in non-dimensional form from Eq.(29) is given by:

$$\left|P_f\right| = \frac{1}{|A_8|} + \Theta_1\frac{|A_9|}{|A_8|}, \tag{36}$$

where: $A_8 = \frac{k}{R_0^{-k}-1}$, $A_9 = 3\left[\frac{k}{R_0^{-k}-1} + \frac{1}{\log R_0}\right]$.

Now stresses for full plasticity from Eqs.(31)-(35) in non-dimensional from are obtained by taking $C_0 \to 0$ as:

$$\sigma_{rr} = (3\Theta_1 - P_f)\left(\frac{R^{-k}-1}{R_0^{-k}-1}\right) - P_{f2} - 3\Theta_1\frac{\log R}{\log R_0}, \tag{37}$$

$$\sigma_{\theta\theta} = (3\Theta_1 - P_f)\left[\frac{(1-k)R^{-k}-1}{R_0^{-k}-1}\right] - P_{f2} - 3\Theta_1\frac{1+\log R}{\log R_0}, \tag{38}$$

$$\sigma_{zz} = \frac{1}{2}(\sigma_{rr}+\sigma_{\theta\theta}) + \frac{3}{2}e_{zz} - \frac{3}{2}\Theta_1\frac{\log R}{\log R_0}, \tag{39}$$

where: $[e_{zz}]_{C_0\to 0}$

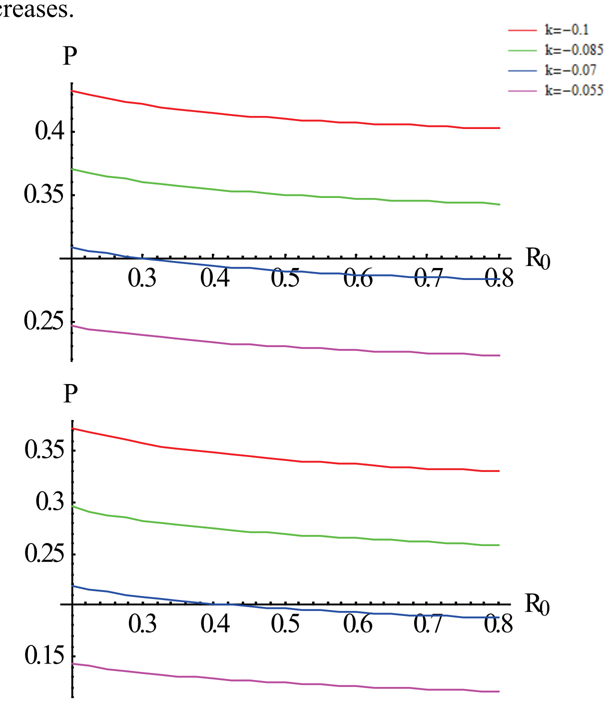
$$\sigma_{\theta z} = \frac{(2\eta_1 Rb)^{n/2}}{n}(1-4n\sigma_{\theta\theta}+3n\sigma_{rr}), \tag{40}$$

$$M_1 = 2\pi\int_{R_0}^{1}\frac{R^2b^2(\alpha Rb)^{n/2}}{n}(1-4n\sigma_{\theta\theta}+3n\sigma_{rr})dR. \tag{41}$$

Equations (37)-(41) are thermal radial, circumferential, axial, shear stresses, and twisting couple in non-dimensional form for the fully plastic state.

NUMERICAL RESULTS AND DISCUSSION

In order to find the pressure required for the initial yielding state against various radii ratios, Fig. 1 is drawn. From Fig. 1, the effective pressure required for initial yielding is maximum at the internal surface of the cylinder. Also, the effective pressure necessary for initial yielding goes on increasing with the non-homogeneity of the cylinder. As the temperature is applied, the pressure for initial yielding decreases.



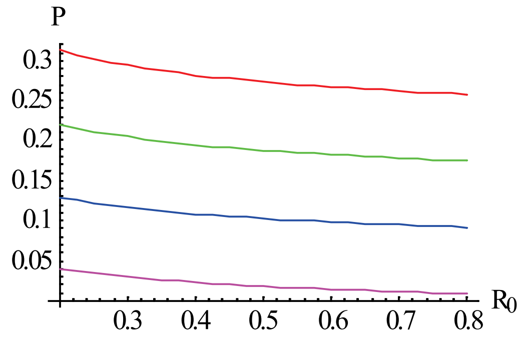


Figure 1. Effective pressure required for initial yielding at various temperatures (0; 0.05; and 0.1).

A shear stress is generated when a structural component is twisted. Due to torsion, shear stress generates in addition to principal stresses. The localized parallel shear forces will be highest when the normal forces are high. Due to combined loading, i.e. axial and torsional loading, stresses generated in functionally graded material are intersecting the stresses generated in the homogeneous material. As the cylinder is subjected to different normal forces on each side and thus when the internal pressure is greater than the external pressure, forces on the outer side are small and on the inner side are high. Non-uniform distribution of pressure and twist creates shear stress that tries to distort the cube. Shear stress act as support force, and support forces are not evenly distributed because of the non-uniform distribution of pressure. The graphs of transitional shear stresses are drawn with the angle of twist $\eta_1 = 50$ in Figs. 2-7.

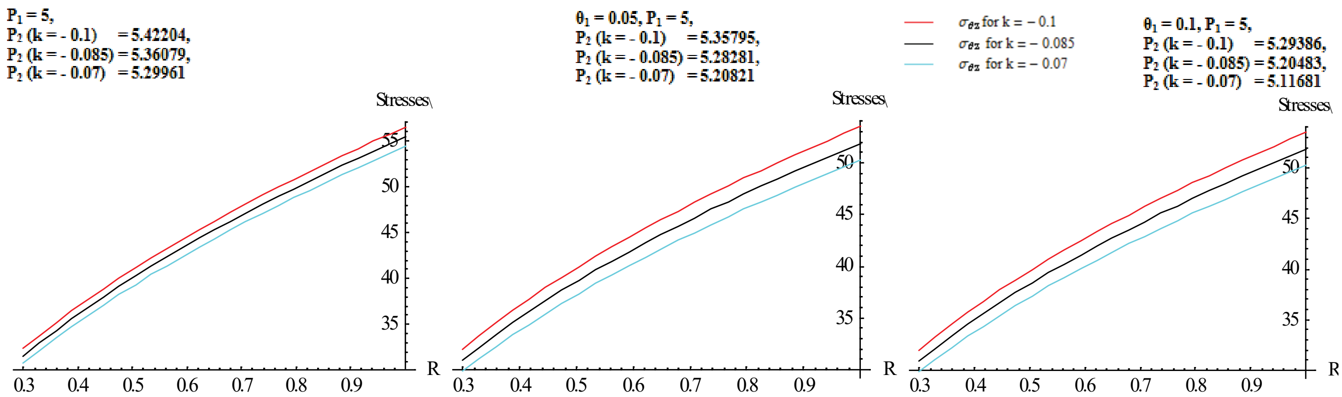


Figure 2. Transitional shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

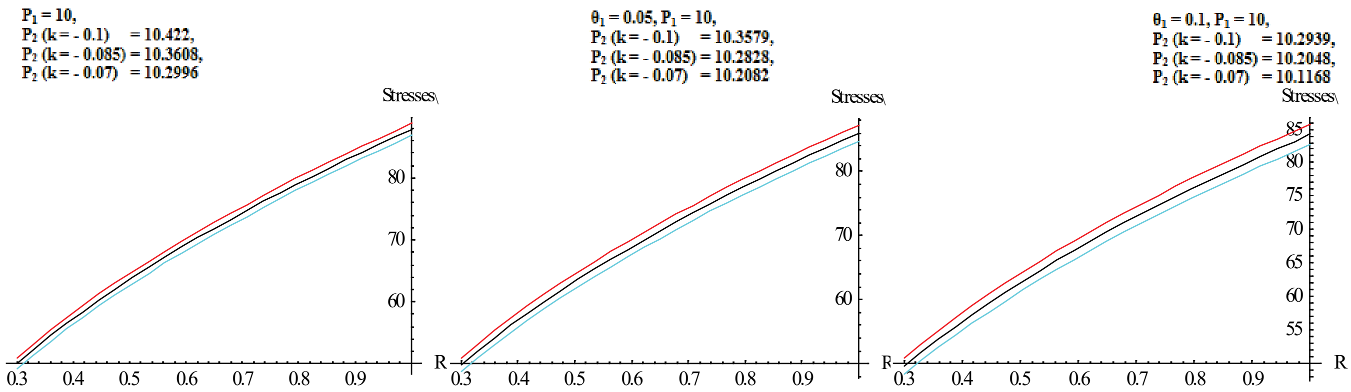


Figure 3. Transitional shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

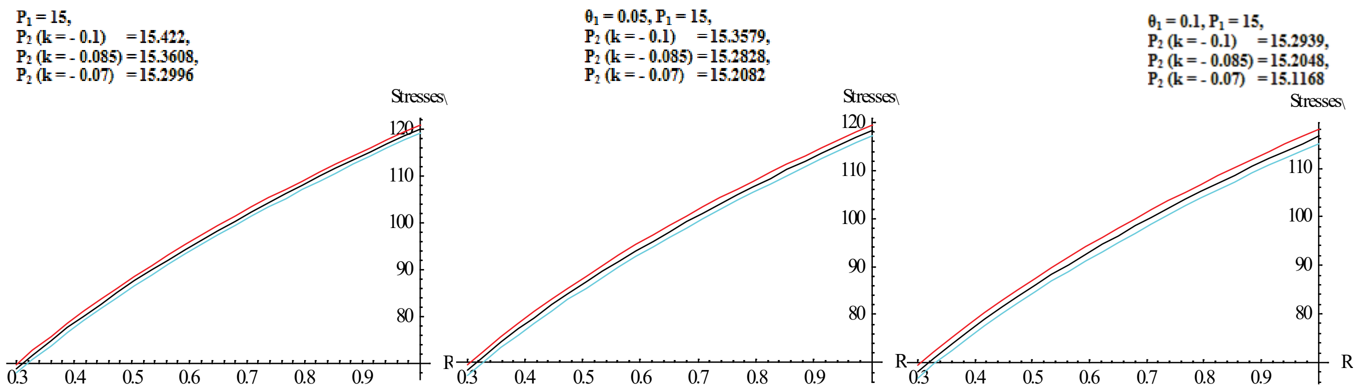


Figure 4. Transitional shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

It is observed that when external pressure is greater than internal pressure, the shear stresses in transitional state are tensile and maximal at the external surface of the cylinder. These stresses increase with thermal effects. As pressure increases, there is a remarkable increase in stresses as can be seen in Figs. 3 and 4.

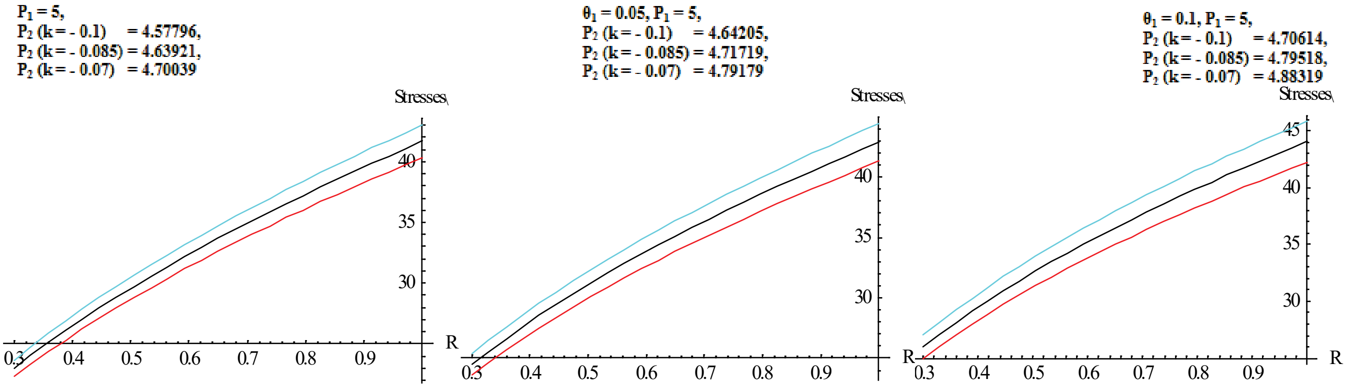


Figure 5. Transitional shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

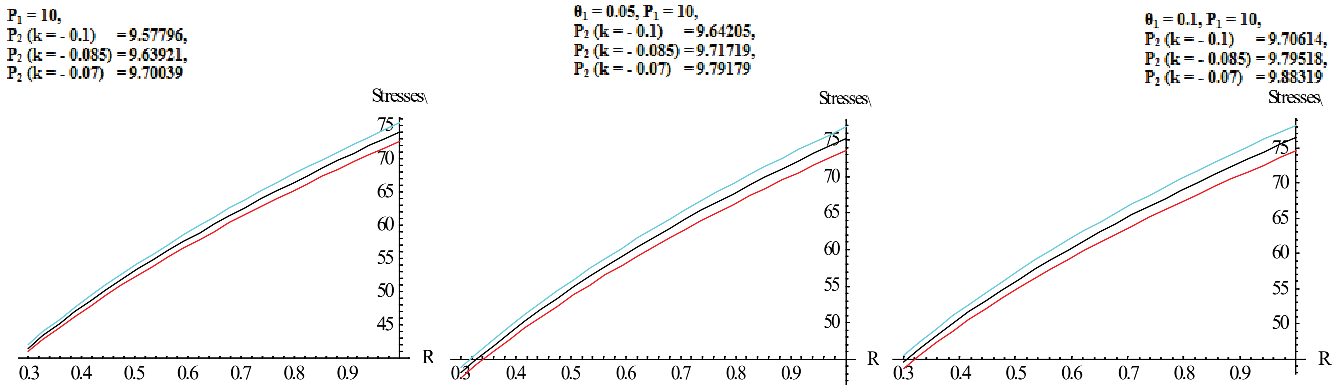


Figure 6. Transitional shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

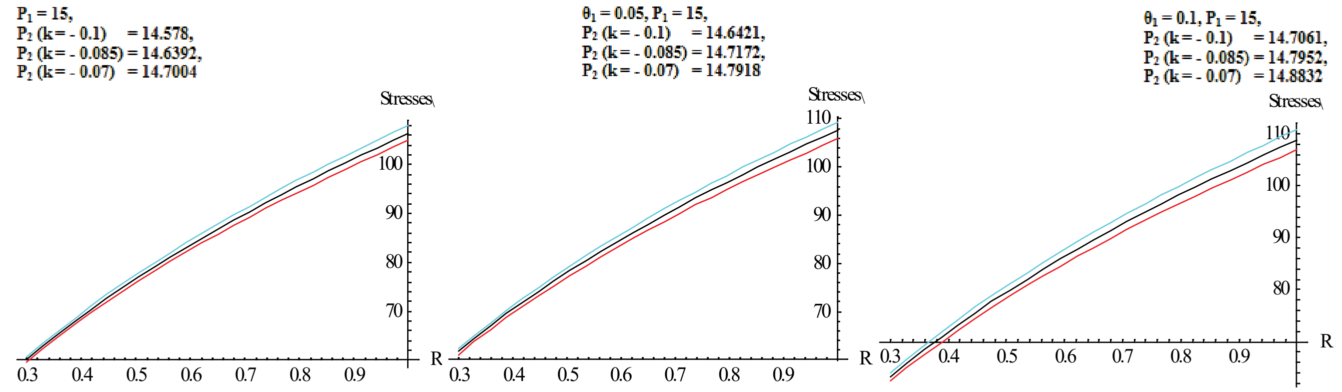


Figure 7. Transitional shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

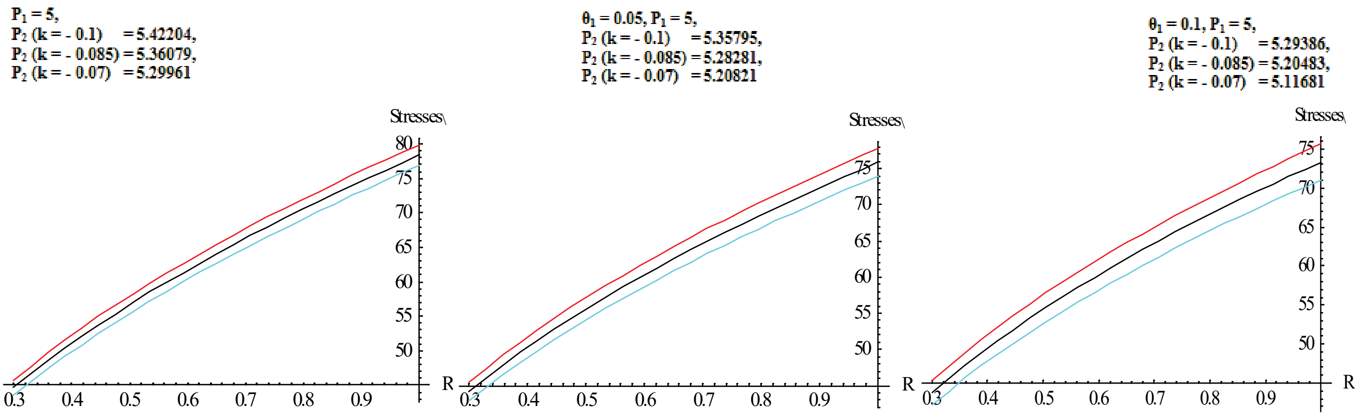


Figure 8. Transitional shear stresses for internal and external pressure at various temperatures with $\eta_1 = 100$.

It is observed from Fig. 5 that transitional shear stresses are tensile and maximal on the external surface of the cylinder when external pressure is less than internal pressure. With the introduction of thermal effects, these shear stresses increase accordingly, with a further increase with thermal effects. Also, these transitional shear stresses show a significant increase with internal and external pressure as can be seen in Figs. 6 and 7.

The transitional shear stresses increase with the increase in the value of the angle of a twist as can be observed from Fig. 8.

From Fig. 9, it is noticed that shear stresses in fully plastic state are tensile when the internal pressure is greater than the external pressure and these stresses are maximal at the internal surface. With the introduction of thermal effects, these fully plastic shear stresses observe a significant decrease which further decreases with temperature. The considerable increase is noticed in the shear stresses with the change in the internal pressure as can be seen from Figs. 10 and 11. Also, more significant changes have been noticed in highly non-homogeneous cylinder at different temperatures.

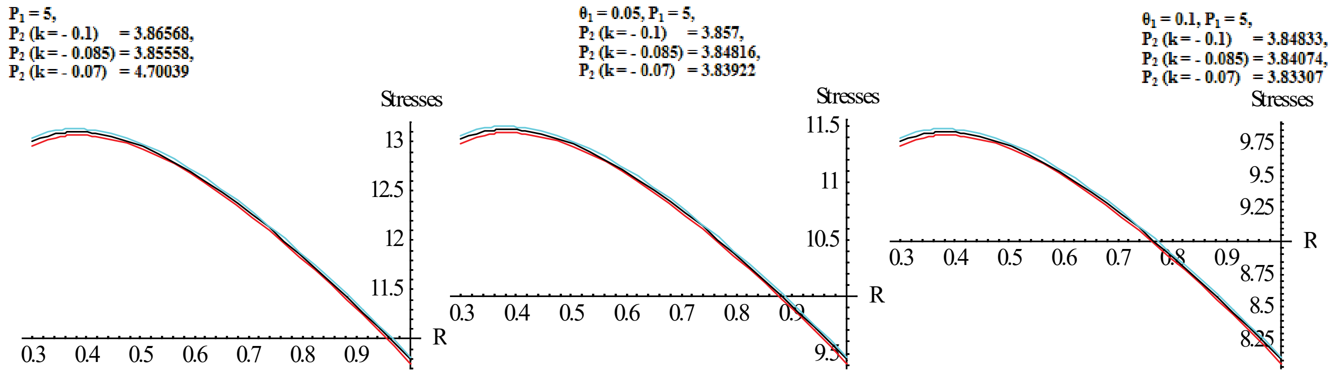


Figure 9. Fully plastic shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

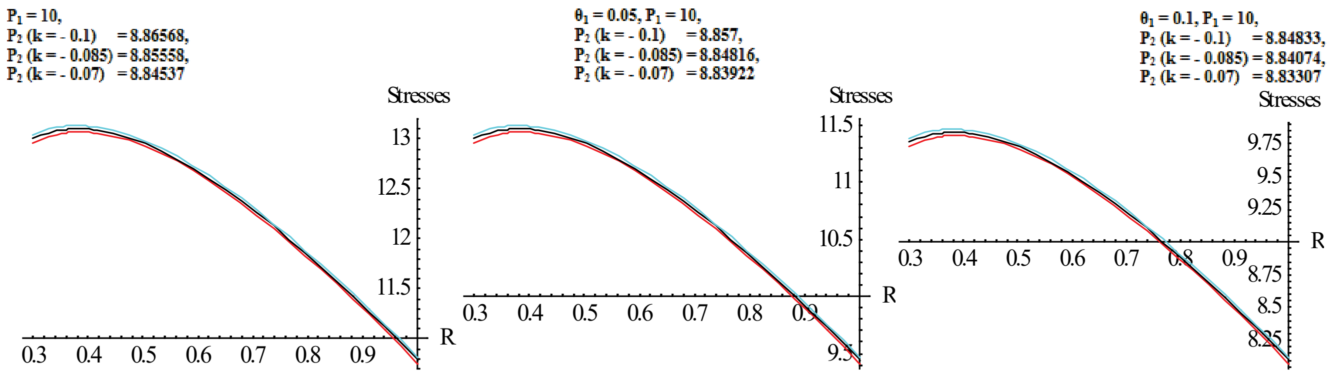


Figure 10. Fully plastic shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

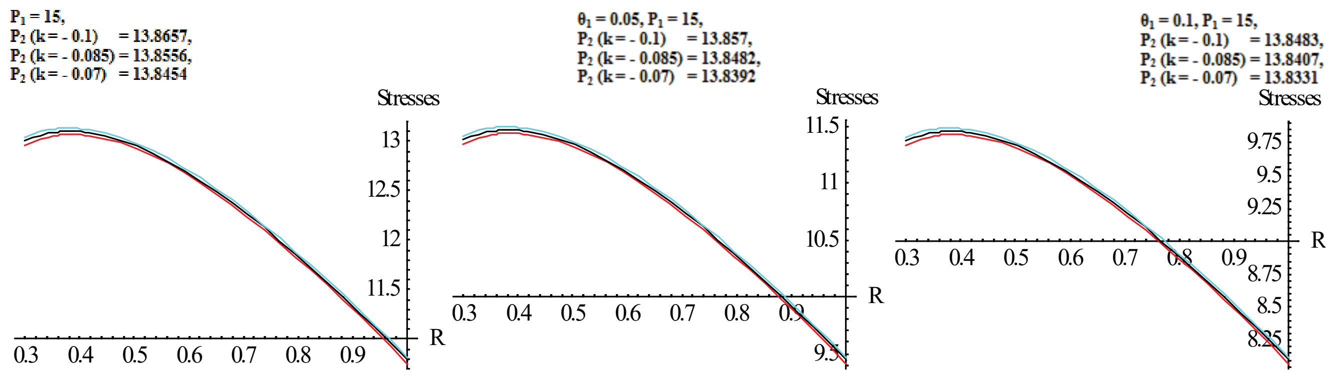


Figure 11. Fully plastic shear stresses for internal and external pressure at various temperatures with $\eta_1 = 50$.

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