DETERMINATION OF DETONATION PRODUCTS EQUATION OF STATE FROM CYLINDER TEST: ANALYTICAL MODEL AND NUMERICAL ANALYSIS

by

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Contemporary research in the field of explosive applications implies utilization of hydrocode simulations. Validity of these simulations strongly depends on parameters used in the equation of state for high explosives considered. A new analytical model for determination of Jones-Wilkins-Lee equation of state parameters based on the cylinder test is proposed. The model relies on analysis of the metal cylinder expansion by detonation products. Available cylinder test data for five high explosives are used for the calculation of Jones-Wilkins-Lee parameters. Good agreement between results of the model and the literature data is observed, justifying the suggested analytical approach. Numerical finite element model of the cylinder test is created in Abaqus in order to validate the proposed model. Using the analytical model results as the input, it was shown that numerical simulation of the cylinder test accurately reproduces experimental results for all considered high explosives. Therefore, both the analytical method for calculation of Jones-Wilkins-Lee equation of state parameters and numerical Abaqus model of the cylinder test are validated.

Key words: *high explosive, detonation, cylinder test, Jones-Wilkins-Lee equation of state, numerical simulation*

Introduction

The modern approach to research in the field of explosive applications includes the use of hydrocodes [1] – robust programs for numerical simulation of complex, high-energy physical processes involving detonation, shock waves, large strains, high strain rates, *etc*. The accuracy of these simulations depends highly on the equation of state used for the detonation products of the explosive composition considered.

There are two approaches to determination of an equation of state: (1) the approach based on thermo-chemical codes, and (2) methods that imply the use of experimental data. The former approach is more fundamental and uses statistical mechanics and intermolecular potentials to provide equations of state of reactive mixtures. The results are libraries of so-called tabu-

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lated equation of state that consists in pre-coded tables directly implemented in the database of computer codes. We will be here focused on the latter approach which is based on thermo-mechanical analysis of measured data and a subsequent fit of equation of state parameters.

There are a number of proposed equations of state that define the isentrope of detonation products [2, 3]: polytropic expansion law, Williamsburg, Lennard-Jones-Devonshire (LJD), Becker-Kistiakowsky-Wilson (BKW), Jones-Wilkins-Lee (JWL), *etc*. For simplicity, greater accuracy and availability of data for significant number of high explosives, the most frequently used is the empirical JWL equation of state of detonation products [4, 5], which has the form:

$$p = A\mathrm{e}^{-R_1 V} + B\mathrm{e}^{-R_2 B} + CV^{-(1+\omega)} \tag{1}$$

where p is the pressure of detonation products, $V = \rho_0/\rho$ – the expansion ratio of detonation products, while A, B, C, R_1, R_2, and ω are parameters specific for each explosive. The first term of the equation defines the behavior of the detonation products at very high pressures and low expansion ratio, the second addend is related to the intermediate pressure zone, and the third term describes the isentrope in the domain of low pressure, *i. e.* large expansion ratio. In this context, only the parameter ω has a physical meaning and approximately satisfies the relation:

$$\omega = \gamma - 1 \tag{2}$$

where γ is the polytropic constant for the detonation products at pressures close to atmospheric.

There are two ways to determine the JWL parameters of equation of state: (1) by use of a thermo-chemical equilibrium code, and (2) using some of the experimental tests. The former method implies the use of a mentioned semi-empirical computer program that has a JWL fitting procedure built in, *e. g.* [6, 7], and will not be considered here. The latter approach, which is based on detonation products expansion physics, will be further investigated.

The most common source of experimental data to obtain explosive performance parameters is the cylinder test [7-10]. A copper tube is filled with the explosive of interest and the planar detonation wave (normal to the cylinder axis) is generated as shown in fig. 1. As the detonation wave passes through the observation window, the radial displacement of copper tube obscures the backlighting (provided by an argon flash bomb) and the history of displacement is recorded by a streak camera and other techniques like electrical pins, flash X-rays, and laser interferometry. Figure 2 shows a streak camera record from our experimental investigation and a schematic representation of the cylinder test.

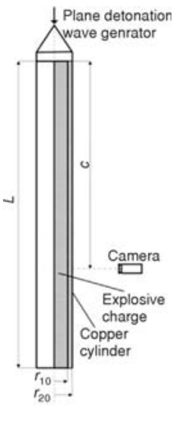

Figure 1. Experimental set-up of a cylinder test

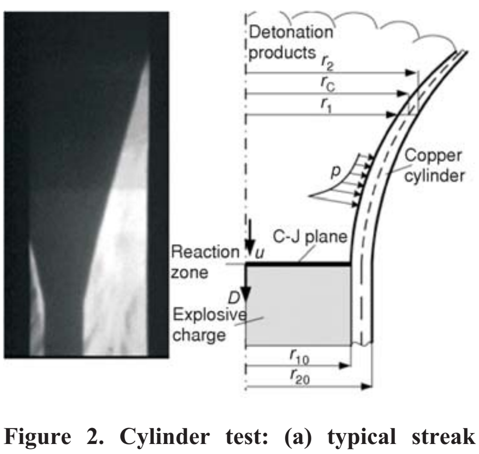

Figure 2. Cylinder test: (a) typical streak camera record, (b) motion of copper tube under the action of detonation products (geometry and notation)

The original method to determine the parameters of JWL equation of state [4] involves the variation of their values in a hydrocode, until a satisfactory correspondence between numerical and experimental results is obtained. Several different methods for calculation of the unknown parameters of equations of state without applying the hydrocodes data have also been used [11-18]. Cylinder test data have been also used for related task – computation of the Gurney energy of explosive [19-21].

The aim of this paper is to propose a new analytical model for simple and reliable determination of the parameters of JWL equation of state based on the results of the cylinder test. Contribution of the model includes: (1) determination of the expansion ratio of detonation products, (2) treatment of both cylinder dynamics and energy balance, (3) consideration of the metal strength and deformation work, and (4) convergent iterative procedure for determining the JWL parameters. Furthermore, calculated parameters have been used as the input for a numerical model of a cylinder test in order to reproduce corresponding experimental results.

Analytical model

The proposed analytical model follows the ideas of the energetic approach [15] and the concept of the metal cylinder motion due to the detonation products pressure [16, 22]. The usual assumptions have been adopted: (1) the cylinder wall is incompressible, (2) reverberations of the shock wave in the tube are neglected, (3) the detonation wave is planar and in steady state, (4) explosive is instantaneously transformed to detonation products, (5) explosion products are inviscid (behave as ideal fluid), and (6) the flow field is quasi 1-D.

Approximation of the measured cylinder displacement

The result of the experiment is the curve obtained by high-speed photography that represents the history of the cylinder's outer surface displacement:

$$\Delta r_2 = r_2 - r_{20} = f(t) \tag{3}$$

where r_2 and r_{20} are the current and initial values of outer cylinder radius, and t is the time measured from the onset of motion.

This function is represented in discrete form as:

$$[t_i, (\Delta r_2)_i], \quad i = 1, 2, ..., n \tag{4}$$

where n is total number of the measured points from the camera record.

In order to calculate the tube wall velocity and acceleration, it is necessary to approximate experimental results, eq. (4), with a proper function. Analysis of a large number of possible functions showed that two functions describe the experimental results very well. The first function [23] has the form:

$$F_1(t) = \frac{v_\infty t g(t)}{\dfrac{2v_\infty}{a_0} g'(0) + g(t)} \tag{5}$$

where a_0 is the initial cylinder acceleration, v_∞ – the asymptotic radial cylinder velocity, and the function $g(t)$ is defined as:

$$g(t) = (1+t)^\sigma - 1 \tag{6}$$

Parameters a_0, v_∞, and σ are determined to minimize the deviation of the function defined by eq. (5) from experimental results eq. (4) by a least square's fit. The second function, based on the assumption of an exponential drop of detonation products pressure [16], can be written as:

$$F_2(t) = \sum_{i=1}^{2} a_i [b_i t - (1 - \mathrm{e}^{-b_i t})] \tag{7}$$

where a_i, b_i (i = 1, 2) are parameters to be optimized.

For each experimental result, parameters in functions defined by eqs. (5) and (7) are determined, and the function with the better approximation of experiment is used for further calculation, so we have:

$$\Delta r_2 = F(t) \tag{8}$$

We will assume that the cylinder motion is defined by displacement of the central cylinder surface, defined by the relation:

$$r_2^2 - r_c^2 = r_c^2 - r_1^2 = \frac{1}{2}(r_{20}^2 - r_{10}^2) \tag{9}$$

where r_1 is the internal surface radius. The central surface displacement $\Delta r_c = F_c(t)$ and displacement of internal surface Δr_1 are easily obtained from eqs. (8) and (9).

Cylinder velocity and acceleration

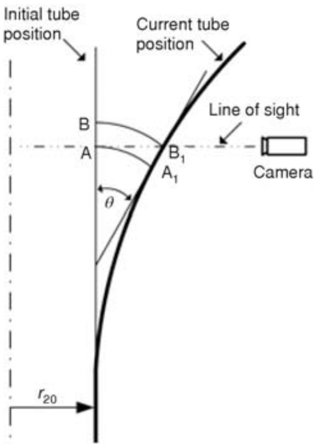

Figure 3. Kinematics of a cylinder test

The kinematics of the system was determined as described in [14, 16]. It is important to note that the streak camera recording $\Delta r_2(t)$ is not a measure of the motion of a particular point on the outer cylinder surface, *i. e.* at any moment t, the displacement of a different point is measured, which is presented in fig. 3. The cylinder motion defined in Eulerian co-ordinate system should be transformed in Lagrangian co-ordinates, where the dynamics of the system can be simply described.

Differentiation of the optimized function of the central surface displacement gives the values of the apparent velocity and acceleration of the cylinder:

$$v_\mathrm{a} = \frac{\mathrm{d}F_\mathrm{c}(t)}{\mathrm{d}t}, \quad a_\mathrm{a} = \frac{\mathrm{d}^2 F_\mathrm{c}(t)}{\mathrm{d}t^2} \tag{10}$$

Analysis of the motion of the cylindrical wall shows that the inclination angle of the centerline θ can be determined from relation:

$$v_\mathrm{a} = D \tan\theta \tag{11}$$

where D is the velocity of detonation. Finally, the Lagrangian values of the cylinder velocity and acceleration are:

$$v = 2D \sin\frac{\theta}{2}, \quad a = a_\mathrm{a} \cos^3\theta \tag{12}$$

Pressure of detonation products

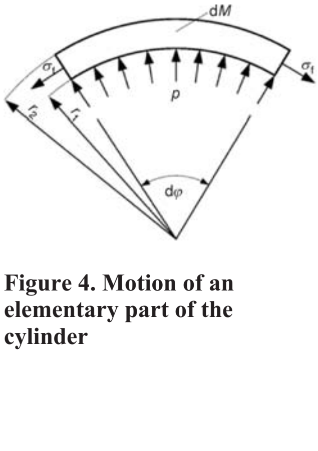

Figure 4. Motion of an elementary part of the cylinder

The equation of motion of a thin ring of the cylinder with central angle dφ, taking into account it's strength (fig. 4), can be written as:

$$\mathrm{d}Ma = pr_1\mathrm{d}\varphi - 2\sigma_f \sin\frac{\mathrm{d}\varphi}{2} \tag{13}$$

Using the calculated cylinder acceleration, the pressure of detonation products p can be obtained from the eq. (13), in the form [24]:

$$p = \frac{Ma}{2\pi r_1} + \sigma_\mathrm{f}\left(\frac{r_2}{r_1} - 1\right) \tag{14}$$

In eqs. (13) and (14), M is the cylinder mass per unit length given by:

$$M = \pi\rho_m (r_{20}^2 - r_{10}^2) \tag{15}$$

where ρ_m is the density of the metal. Flow stress of the cylinder material σ_f is assumed to be constant.

Detonation products expansion ratio

Assuming that the flow of the detonation products is quasi 1-D, the continuity equation can be applied as:

$$\rho_0 A_0 D = \rho A(D - u) \tag{16}$$

where u and ρ are current values of particle velocity and density of the detonation products, and A the channel cross-section area. From the equation of motion (14), neglecting the strength term, one obtains:

$$Mv\frac{\mathrm{d}v}{\mathrm{d}r_1} = 2\pi p r_1 \tag{17}$$

which yields:

$$M\frac{v^2}{2} = 2\pi \int_{r_{10}}^{r_1} p \mathrm{d}\left(\frac{r_1^2}{2}\right) \tag{18}$$

Using the continuity eq. (16) and the Bernoulli's equation in the form:

$$\mathrm{d}p + \rho u \mathrm{d}u = 0 \tag{19}$$

the right-hand side of eq. (18) can be integrated by parts providing [25]:

$$\frac{M}{C}\frac{v^2}{2} = \frac{p}{\rho_0}\frac{A}{A_0} - Du \tag{20}$$

In eq. (20), C is the explosive charge mass per unit length determined by:

$$C = \pi\rho_0 r_{10}^2 \tag{21}$$

Combining eqs. (16) and (20), the expansion ratio can be determined from:

$$V = \frac{\rho_0}{\rho} = \frac{A}{A_0}\left[1 - \frac{A}{A_0}\frac{p}{\rho_0 D^2} + \frac{1}{2}\frac{M}{C}\left(\frac{v}{D}\right)^2\right] \tag{22}$$

where A/A_0 is the geometric expansion ratio:

$$\frac{A}{A_0} = \left(\frac{r_1}{r_{10}}\right)^2 \tag{23}$$

It should be noted that a form-free p-V relation is obtained by eqs. (14) and (22). The JWL parameters in eq. (1) can be fitted to the calculated points in p-V plane, as in [16]. This approach has two main drawbacks: (1) the pressure obtained from the cylinder acceleration (based on the fitted tube displacement) has only approximate value, and (2) the calculated pressure spans over three orders of magnitude, which complicates the fitting procedure. Therefore, we proceed to a consideration of the energy balance.

Energy balance

Energy conservation law for the system consisting of the explosive charge and metallic cylinder can be written as:

$$\rho_0 D A \Delta t \left(U - Q + \frac{u^2}{2} + T_{\text{kin}} + A_{\text{def}} \right) = p u A \Delta t \tag{24}$$

where U is the internal energy of the detonation products per unit mass, Q – the detonation heat per unit mass, u – the particle velocity of the detonation products in the axial direction, T_{kin} – the kinetic energy due to the radial motion of cylinder and gases (*i. e*. Gurney energy) per unit mass of explosive, and A_{def} – the cylinder deformation work per unit mass of explosive charge. Equation (24) can be simplified to the form:

$$E - E_0 + \frac{\rho_0 u^2}{2} + E_{\text{kin}} + W_{\text{def}} = p \frac{u}{D} \tag{25}$$

where $E = \rho_0 U$ and $E_0 = \rho_0 Q$ are the internal energy and detonation heat per unit volume of explosive charge, $E_{\text{kin}} = \rho_0 T_{\text{kin}}$ and $W_{\text{def}} = \rho_0 A_{\text{def}}$ are Gurney energy and deformation work per unit volume of explosive charge.

Detonation heat E_0 can be obtained from the experiment, *e. g*. [17]. The particle velocity of the detonation products is determined by:

$$u = D \left(1 - V \frac{A_0}{A} \right) \tag{26}$$

The specific Gurney energy can be calculated from [24]:

$$E_{\text{kin}} = \frac{\rho_0 v_1^2}{2} \left\{ \frac{M}{C} \left(\frac{r_1}{w} \right)^2 \ln \left[1 + \left(\frac{w}{r_1^2} \right)^2 \right] + \frac{1}{2} \right\} \tag{27}$$

where v_1 is the internal cylinder surface velocity and w is defined as:

$$w^2 = r_{20}^2 - r_{10}^2 \tag{28}$$

Deformation work can be calculated from the relation:

$$W_{\text{def}} = \sigma_{\text{f}} \left[\left(\frac{r_{20}}{r_{10}} \right)^2 - 1 \right] \ln \frac{r_1 + r_2}{r_{10} + r_{20}} \tag{29}$$

Introducing the energy term:

$$E_1 = p \frac{u}{D} - \frac{\rho_0 u^2}{2} \tag{30}$$

one can obtain an energy eq. (25) in the form:

$$E = E_0 + E_1 - E_{\text{kin}} - W_{\text{def}} \tag{31}$$

that allows the internal energy E of the detonation products to be determined.

Internal energy of the detonation products

Assuming adiabatic expansion of the detonation products and considering eq. (1), the internal energy of gases is determined by:

$$E(V) = \int_V^{\infty} p \, \mathrm{d}V = \frac{A}{R_1} \mathrm{e}^{-R_1 V} + \frac{B}{R_2} \mathrm{e}^{-R_2 V} + \frac{C}{\omega} V^{-\omega} \tag{32}$$

Since the internal energy $E(V)$ is calculated from eq. (31), the unknown JWL equation of state parameters A, B, C, R_1, R_2, and ω can be optimized in order to fit eq. (32).

At the same time, the JWL parameters should satisfy three additional conditions:
(1) pressure at the Chapman-Jouget (CJ) state is equal to the experimentally determined value p_{CJ}:

$$A\mathrm{e}^{-R_1V_{CJ}} + B\mathrm{e}^{-R_2V_{CJ}} + CV_{CJ}^{-(1+\omega)} = p_{CJ} \tag{33}$$

(2) internal energy of detonation products at the CJ state is:

$$\frac{A}{R_1}\mathrm{e}^{-R_1V_{CJ}} + \frac{B}{R_2}\mathrm{e}^{-R_2V_{CJ}} + \frac{C}{\omega}V_{CJ}^{-\omega} = E_0 + \frac{\rho_0 u_{CJ}^2}{2} \tag{34}$$

(3) slope of the Rayleigh line is determined by:

$$AR_1\mathrm{e}^{-R_1V_{CJ}} + BR_2\mathrm{e}^{-R_2V_{CJ}} + C(1+\omega)V_{CJ}^{-(2+\omega)} = \rho_0 D^2 \tag{35}$$

Procedure of determining the JWL parameters

The procedure for determination of JWL equation of state parameters from the cylinder test is presented in flowchart (fig. 5). Equation (14) provides the initial detonation products pressure p_{initial}. This value is based on the second derivative of the fitting function $F_c(t)$ and therefore cannot be used as the definitive pressure of detonation products. Instead, p_{initial} is used for calculation of the internal energy E, and then JWL parameters are optimized by a fitting procedure, providing the new value for detonation products pressure $p(V)$. The procedure is repeated with the new value of pressure until the difference between two pressures becomes small enough.

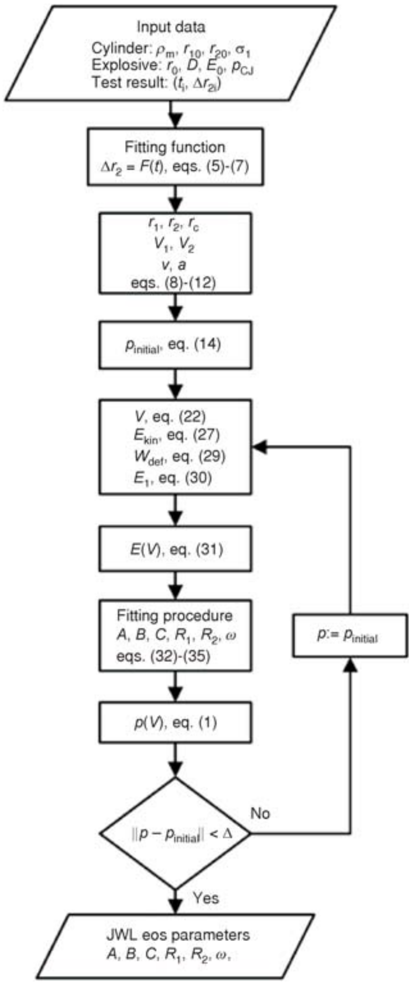

Figure 5. Algorithm for determination of JWL equation of state parameters from the cylinder test

Model results and comparison with experimental data

The present model is applied to determination of the parameters of JWL equation of state for five explosives: TNT, Composition B, PBX-9404, HMX, and FH-5. The characteristics of the tested explosives are given in tab. 1. All data were taken from [26], except for the phlegmatized hexogen FH-5 (with nominal composition – 95% RDX and 5% montan wax) [18]. The corresponding experimental cylinder test data, t_i, $(\Delta r_2)_i$ are also obtained from [26] and [18], respectively. Properties of the copper cylinder are listed in tab. 2 [27]. It should be noted that in the test with FH-5, a cylinder with smaller diameter was used.

An analysis of the results for a cylinder test with TNT is given as the representative example of the present method. The energy balance is shown as a function of the expansion ratio in fig. 6. The given specific detonation energy E_0, the computed values of kinetic en-

Table 1. Detonation properties of the examined high explosives

Explosive composition	Density	Detonation velocity	Pressure at CJ state	Detonation heat
	ρ_0 [kgm^{-3}]	D [ms^{-1}]	p_{CJ} [GPa]	E_0 [GPa]
TNT	1630	6930	21.0	7.0
Composition B (RDX/TNT-64/36)	1717	7980	29.5	8.5
PBX-9404 (HMX/NC/CEF-94/3/3)	1840	8800	37.0	10.2
HMX	1891	9110	42.0	10.5
FH-5 (RDX/wax-95/5)	1600	7930	24.96	8.7

Table 2. Properties of the copper cylinder

Density	Dimensions*		Flow stress
ρ_m [kgm^{-3}]	r_{10} [mm]	r_{20} [mm]	σ_f [MPa]
8940	12.70	15.30	89.63

* Cylinder dimensions for FH-5 test: r_{10} = 10.20 mm, r_{20} = 12.70 mm

ergy E_{kin}, deformation work W_{def} and specific energy E_1 enable the determination of specific internal energy $E(V)$. The specific internal energy of detonation products $E(V)$ obtained by the proposed model is compared with the literature curve [26] in fig. 7. Good agreement between model and literature curve can be observed.

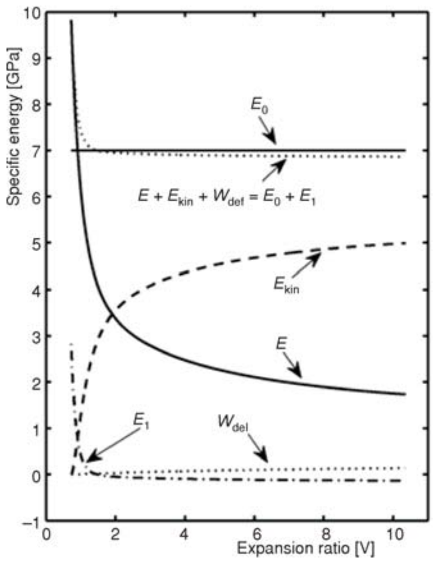

Figure 6. Balance of specific energies involved in the process of detonation product expansion in cylinder test

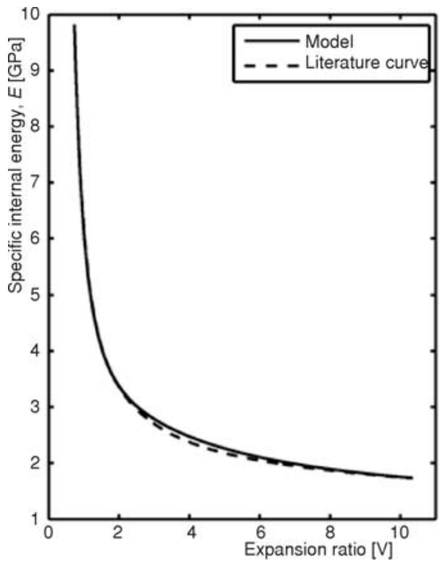

Figure 7. Comparison of the calculated specific internal energy of the detonation products with the data from [26]

Figure 8 shows the contribution of three terms of the JWL equation of state to the total specific internal energy, after the fitting procedure. The third term in eq. (29) can be confirmed as the entire internal energy for large expansions ($V > 6$), and the first term can indeed be neglected for expansions $V > 2.5$. This fact is used to simplify the fitting procedure.

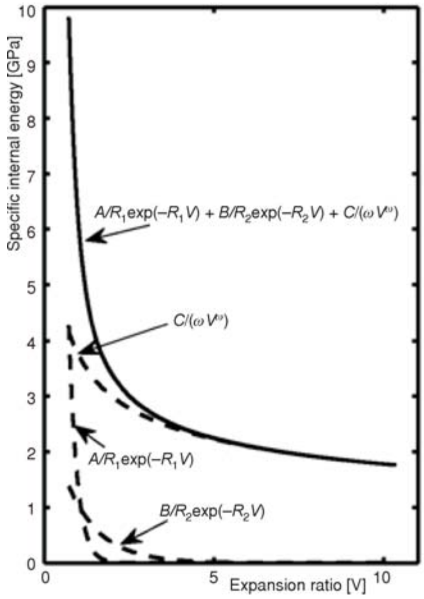

Figure 8. Specific internal energy of TNT detonation products as the sum of three terms of the JWL model

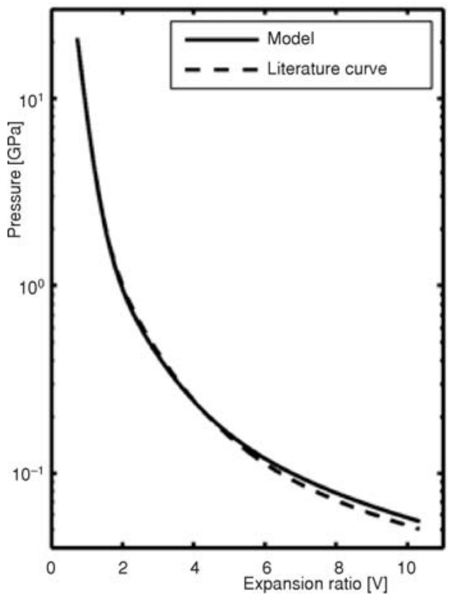

Figure 9. Comparison of *p-V* curves for TNT detonation products obtained by presented method and from [26]

The parameters obtained are used to determine detonation products pressure curve which is compared with the data from [26] (fig. 9). Very good agreement of these results is found.

Calculated JWL parameters for all considered high explosives are presented in tab. 3, along with literature data. Good agreement of the model and the literature *p-V* curves is found. It is important to note that the JWL equation of state has six parameters and each parameter calculated by our method is different from the corresponding value from [26] (especially parameters A, B, and C). However, the resulting *p*(V) and *E*(V) functions are close to the referent curves, because the large number of parameters allows that virtually the same functions can be defined by different sets of parameters.

Table 3. JWL parameters for five explosives - a comparison of model results with literature data

Explosive	Method	R_1	R_2	ω	A [GPa]	B [GPa]	C [GPa]
TNT	Model	4.1245	0.9436	0.3135	366.42	2.6983	1.1480
	Ref. [26]	4.15	0.95	0.3	371.21	3.2306	1.0453
Comp. B	Model	4.0489	0.7833	0.3460	497.08	3.4246	1.1260
	Ref. [26]	4.2	1.1	0.34	524.23	7.6783	1.0082
HMX	Model	4.2018	1.1078	0.3072	768.74	1.2131	7.8843
	Ref. [26]	4.2	1.0	0.30	778.28	7.0714	0.6430
PBX-9404	Model	4.5403	1.3255	0.3007	832.26	1.7768	9.9479
	Ref. [26]	4.60	1.30	0.38	852.40	18.020	1.2070
FH-5	Model	4.2750	0.3175	0.2178	573.43	0.96006	0.82373

Numerical analysis of cylinder test

As previously shown, the proposed analytical model provides results that are in good agreement with literature data. Additional validation of the calculated JWL equation of state parameters is performed using a numerical approach. This is especially important for the test with FH-5 for which there are no reliable reference data.

A considerable number of numerical studies have been published investigating various explosive effects using different hydrocodes, *e. g.* [27, 28]. A numerical analysis of the cylinder test is performed with the commercial FEM based software Abaqus [29]. The solver Abaqus/Explicit configured for simulation of transient non-linear dynamic events is used. The coupled Eulerian Lagrangian (CEL) capability of Abaqus is employed enabling interaction between highly deformable material (detonation products) and relatively stiff bodies (metallic cylinder). The Abaqus/CAE pre- and post-processor is used for model creation, editing, monitoring, and result visualization.

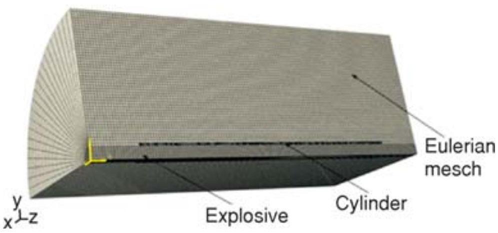

Figure 10. Model of cylinder test in Abaqus/CAE

After a convergence study, an optimal quarter symmetry finite element model was adopted, as shown in fig. 10. The copper cylinder is modeled as a Lagrangian solid using about 8.000 hexahedral (C3D8R) elements. The Eulerian domain, including the explosive charge, is modeled using approximately 200.000 (EC3D8R) elements. The plane-wave generator was not considered. Instead, the plane detonation wave was prescribed by appropriate initiation conditions. The explosive charge is extended on both sides for 50 mm, providing a stable detonation process and minimizing end effects.

The Johnson-Cook plasticity model [30], as well as the Johnson-Cook damage model [31] has been used for the copper cylinder. A linear U_s-U_p equation of state is employed. The geometry of the cylinder and the material properties of copper used in the Abaqus model are listed in tab. 4.

Table 4. Properties of the copper cylinder used in the Abaqus model

Cylinder (*copper*)			
Dimensions	Internal radius [mm]		12.70
	External radius [mm]		15.30
	Length [mm]		300
Density [kgm^{-3}]			8940
Specific heat [Jkg^{-1}K^{-1}]			383.5
Johnson-Cook plasticity model	Yield stress [MPa]		89.63
	Hardening coefficient [MPa]		291.6
	Hardening exponent		0.31
	Strain-rate coefficient		0.025
	Thermal softening coefficient		1.09
	Initial temperature [K]		294
	Melt temperature [K]		1356
Johnson-Cook damage model	Parameters	d_1	0.3
		d_2	0.28
		d_3	–3.03
		d_4	0.014
		d_5	1.12
EOS model	Sound speed [ms^{-1}]		3940
	Slope		1.49
	Gruneisen coefficient		1.96

The explosive material is modeled by a JWL equation of state, eq. (1).

Explosive is detonated in stable regime, and pressure and energy are calculated from the equation of state, without consideration of intermediate species and non-equilibrium processes in the reaction zone. The explosive density and detonation velocity given in tab. 1 are used in the model, as well as the calculated JWL equation parameters listed in tab. 3.

A comparison between the radial cylinder displacements calculated using Abaqus CEL model and the experimental data from [26] is shown for TNT, Composition B, PBX, and HMX in fig. 11. The simple relation between the time τ from the onset of motion of a node on the outer cylinder surface, and the time t when the node is captured by camera is given by:

$$t = \tau - \frac{z(\tau)}{D} \tag{36}$$

and provides a comparison of the experimental and the numerical results. In eq. (36), $z(\tau)$ is the axial displacement of the node. Excellent agreement between numerical and experimental results can be noted. Hence the numerical model of the cylinder test based on CEL technique in Abaqus/Explicit has been validated.

Velocities of the cylinder (*i. e.* node at the outer cylinder surface) as a function of time for four explosives are given in fig. 12. Velocity oscillations are primarily related to reverberations of shock waves in the tube wall. The cylinder velocities according to Gurney model [19] are also shown, as given by:

$$v_{\mathrm{lim}} = v_{\mathrm{G}}\left(\frac{1}{2}+\frac{M}{C}\right)^{-1/2} \tag{37}$$

In eq. (37), v_{G} is the Gurney velocity which characterizes the explosive performance as described by Kennedy [32]. Final cylinder velocity is close to the Gurney limit velocity (within the error of ±3%) confirming the validity of this simple approach.

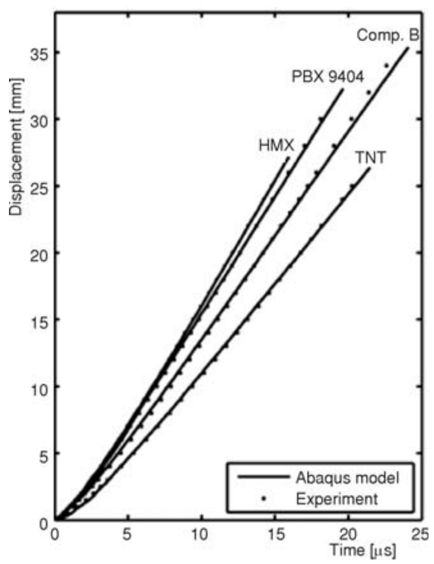

Figure 11. Comparison of numerically determined cylinder displacement with experimental data [26] for TNT, Composition B, PBX 9404, and HMX

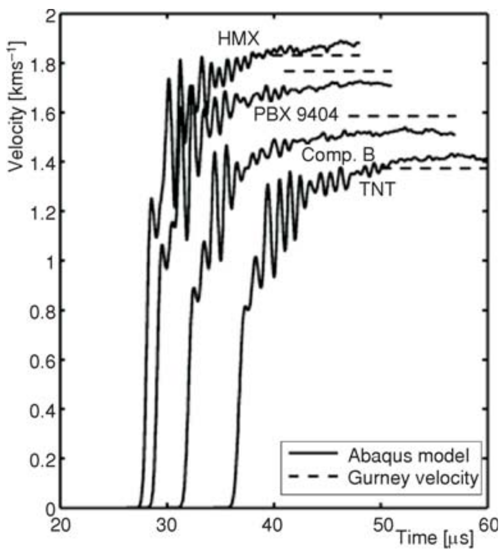

Figure 12. History of cylinder velocity from a numerical model for TNT, Composition B, PBX 9404, and HMX, compared with calculated Gurney limit velocities

The simulation of a cylinder test with an explosive charge of FH-5 is performed using the approach previously described, except with different cylinder dimensions (r_{10} = 10.20 mm, r_{20} = 12.70 mm). A comparison of the calculated and measured cylinder displacement is shown in fig. 13. Very good agreement between these results can be noted. Also, cylinder's outer surface velocity is shown in fig. 14, along with the corresponding Gurney limit velocity. The calculated Gurney velocity of FH-5 is 2.63 km/s [33]. It can be concluded that numerical simulation provides realistic results for the case of FH-5 explosive charge.

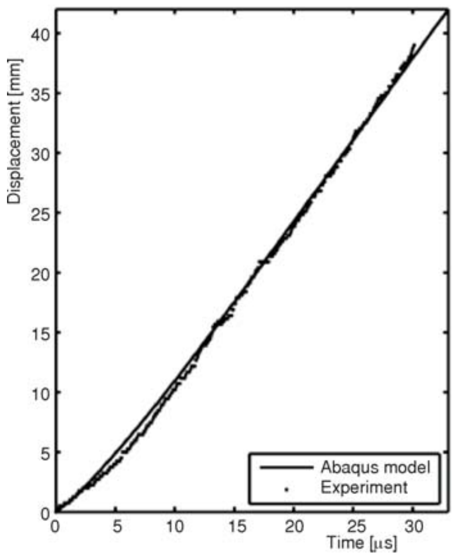

Figure 13. Comparison of numerically--determined cylinder displacement with experimental data for FH-5

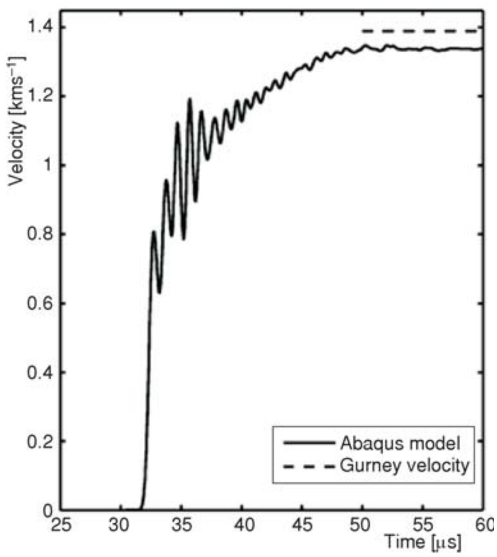

Figure 14. History of cylinder velocity from a numerical model of an FH-5 explosive charge and corresponding Gurney limit velocity

Conclusions

The present paper considers the problem of determining the detonation product JWL equation of state parameters from cylinder test data. To solve this problem, a new analytical model has been proposed. The model is based on: (1) fitting the experimental data with analytical function, (2) cylinder kinematics, (3) cylinder motion dynamics, (4) detonation products expansion analysis, (5) energy balance, and (6) final fitting of the detonation products' internal energy. A computer program based on the model has been developed.

Cylinder test data for five high explosives are used for calculation of JWL parameters. There are reliable literature values of JWL parameters for four of the analyzed explosives (TNT, Composition B, PBX-9404, and HMX), and no data for FH-5 which we have experimentally investigated. Extensive analysis indicates good compatibility between the results of the model and the available literature data.

A numerical finite element model of the cylinder test was created in Abaqus/Explicit to validate analytical results. Using the analytical model results as the input, it was shown that a numerical simulation of the cylinder test in Abaqus accurately reproduces experimental results for all the considered high explosives. Hence both the analytical method of calculation of equation of state parameters and the numerical Abaqus model of the cylinder test were validated.

In further research, the analytical and numerical models developed here will be applied for other high explosives. Also, the CEL capability of Abaqus/Explicit can be applied to other explosive test methods, as well as to various explosive applications.

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Nomenclature

A – channel cross-section area, [cm^2]
A, B, C – parameters in JWL equation of state, [GPa]
a – acceleration of cylinder, [kms^{-2}]
C – explosive charge mass per unit length, [kgm^{-1}]; in eq. 20
C-J – Chapman-Jouget state
D – detonation velocity, [kms^{-1}]
E – internal energy of detonation products per unit volume, [GPa]
E_{kin} – kinetic energy per unit volume of explosive, [GPa]
E_0 – detonation heat per unit volume, [GPa]
M – cylinder mass per unit length, [kgm^{-1}]
p – pressure of detonation products, [GPa]
R_1, R_2 – parameters in JWL equation of state, [–]
r – cylinder radius, [mm]
T_{kin} – kinetic energy per unit mass of explosive, [MJkg^{-1}]
t – time, [μs]
u – velocity of detonation products, [kms^{-1}]
V – expansion ratio of detonation products, [–]
v – cylinder velocity, [kms^{-1}]

Greek symbols

γ – polytropic exponent of detonation products, [–]
θ – cylinder inclination angle, [rad]
ρ – density of detonation products, [kgm^{-3}]
ρ_{m} – density of cylinder material, [kgm^{-3}]
ρ_0 – density of explosive, [gcm^{-3}]
σ_{f} – flow stress of cylinder material, [MPa]
φ – central angle of a ring sector [rad]
ω – parameter in JWL equation of state, [–]

Subscripts

a – apparent values
c – cylinder centerline
p – particle
s – shock wave
0 – initial values
1 – internal cylinder surface
2 – external cylinder surface

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