

DISPLAY PROBABILITY OF SYMBOL ERRORS FOR MQAM ON RICIAN FADING CHANNEL BASED ON MGF METHOD

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Abstract: We present a new method for calculating the probability of error per symbol (Symbol Error Probability, SEP) of M-ary Quadrature Amplitude Modulation (MQAM) over a slow, flat, identically independently distributed Rician fading channels. Since fading is one of the major constraints in wireless communications, the diversity modulation technique is used for the efficient transfer of message signals. Exact analysis of error probability per symbol for MQAM, transmitted over Rician fading channels, is performed by N branches of diversity reception using maximum ratio of signal-to-noise power (maximal-ratio-combining, MRC), where the information in the channel on the

receiver side is known. We also analyzed the performances of MQAM over Rician fading channels are here also analyzed. Approximate formula is used to represent SEP for MQAM transmitted over Gaussian channels. Boundary condition for the approximation is $M \geq 4$ and $0 \leq \text{SNR} \leq 30$ dB.

Keywords: MQAM (M-ary Quadrature Amplitude Modulation), MRC (Maximal Ratio Combining), MGF (Moment Generating Function), SEP (Symbol Error Probability).

MSC: 65E05, 65D15, 65D18.

1. INTRODUCTION

The advent of high-speed transmission in wireless communications renewed interest in linear M-ary modulation systems, mainly because of their ability to send more bits per transmitted symbol. A common problem encountered in wireless environments is the existence of multiple paths for signal sending from the transmitter to the receiver, which is caused by reflection, diffraction and refraction. The total effect of the existence of multiple signal paths is volatility (fluctuating), i.e. formation of fading signals at the receiver destination, which greatly degrades the reception quality. To combat the effects of fading and to improve the capacity and reliability of wireless communication systems, a technique based on the principle of providing multiple faded replicas of the same information signal sent to the receiver [14], called diversity is used. There are several diversity techniques: transmit/receive diversity [8], time, space and frequency diversity [16], [7], polarization diversity [17]. The best known scheme among all conventional diversity combining schemes is maximal ratio combining (MRC), which gives optimum results [6], [3], [12], [10].

M-ary Quadrature Amplitude Modulation (MQAM) is a well-known modulation technique, and due to its high spectral efficiency, is an attractive technique for wireless communication. For different values of Rician parameter Symbol Error Probability (SEP) is different. When the value of Rician parameter K is varied, the performance varies. Two boundary conditions are: when $K = 0$, then it is called Rayleigh fading channel, and when K tends ∞ , then it is called Gaussian AWGN channel without fading. The same also applies to changes of diversity technique and message signal. We present exact analysis of SEP for MQAM, transmitted over Rician fading channels using the N reception branches of diversity technique combined with MRC [9]. In this analysis of the MQAM, a simple closed form expression for the SEP for diversity order N in Rician fading channel is used. For different conditions, this modulation technique shows different characteristics. The goal of the analysis is to highlight its effect by comparing its characteristics in different working conditions.

2. AVERAGE SIGNAL TO NOISE RATIO

Signal to noise ratio (SNR) is the most common performance for the measurement of the characteristics of digital communication system. SNR is measured at the output of the receiver and is directly connected with the process of data detection. The term noise in

the average signal to noise ratio applies to all current known thermal noise at the receiver input. The word average means that the statistical average value is given over the probability distribution of fading [13] by the equation:

$$\bar{\gamma} = \int_0^{\infty} \gamma \cdot p_{\gamma}(\gamma) \cdot d\gamma \quad (1)$$

where γ denotes the current SNR at the output of the receiver, and $p_{\gamma}(\gamma)$ denotes the probability density function (PDF) of γ .

3. AVERAGE SYMBOL ERROR PROBABILITY

The criterion of average symbol error probability (SEP) describes in the best way the nature of system behaviour. The difficulty in estimating the average SEP is mainly due to the fact that quality of reception is degraded to a large extent. Diversity of conditional SEP is, in general, non-linear function of the current SNR. The nature of nonlinearity comes from the function related to modulation/detection scheme used in the system. The average SEP can be written as [13]:

$$P_s(E) = \int_0^{\infty} P_s\left(\frac{E}{\gamma}\right) \cdot p_{\gamma}(\gamma) \cdot d\gamma \quad (2)$$

where $P_s(E/\gamma)$ is conditional SEP.

4. MOMENT GENERATING FUNCTION

The distribution of a random variable is often characterized in terms of its moment generating function (MGF), a real function whose derivatives at zero are equal to the moments of the random variable. Moment generating functions have great practical relevance not only because they can be used to easily derive moments, but also because a probability distribution is uniquely determined by its MGF; this fact that, coupled with the analytical tractability of MGFs, makes them a handy tool to solve several problems, such as deriving the distribution of a sum of two or more random variables.

It must be mentioned that not all random variables possess a moment generating function. However, all random variables possess a characteristic function, another transform that enjoys properties similar to those enjoyed by the MGF.

The following is a formal definition.

Definition: Let X be a random variable. If the expected value $E[\exp(tX)]$ exists and is finite for all real numbers t belonging to a closed interval $[-h, h] \supset \mathbb{R}$, with $h > 0$, then we say that X possesses a moment generating function

$$M_X(t) = E[\exp(tX)]$$

and the function is called the moment generating function of X .

Moment generating function (MGF) of the random variable is alternatively represented by a given probability distribution. Therefore, this approach provides an

alternative way to calculate the results compared to direct calculation using probability density functions. There exist simple partial results for the MGF, for non-negative random variable γ with the distribution $p_\gamma(\gamma)$, $\gamma \geq 0$, which is defined as:

$$M_\gamma(s) = \int_0^\infty p_\gamma(\gamma) \cdot e^{-s\gamma} \cdot d\gamma \quad (3)$$

where $p_\gamma(\gamma)$ denotes the probability density function (PDF) of γ .

MGF for common Rician fading distribution with factor K and diversity order N is given as [13], [11]:

$$M_\gamma(s) = \left(\frac{N + K}{N + K - S \cdot \bar{\gamma}} \right)^N \cdot \exp \left(\frac{K \cdot S \cdot \bar{\gamma}}{N + K - S \cdot \bar{\gamma}} \right) \quad (4)$$

5. MOMENT ERROR PROBABILITY FOR MQAM

5.1. Exact formula

The actual SEP for MQAM over Gaussian channels (Additive white Gaussian noise (AWGN) is a channel model in which the only impairment to communication is a linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth), and a Gaussian distribution of amplitude) is given by [11]:

$$P_{AWGN}(\gamma_b) = 1 - \left(1 - \left(2 \cdot \left(1 - \frac{1}{\sqrt{M}} \right) \cdot Q \left(\sqrt{\frac{3 \cdot \gamma}{M - 1}} \right) \right) \right)^2 \quad (5)$$

and is presented in Figure 1.

5.2. Approximate formula

SEP obtained from the approximate formula for MQAM in constellation of the square root over the Gaussian channel is also presented in Figure 1. This approximation is limited to within 1 dB in relation to the exact value for $M \geq 4$ and $0 \leq \text{SNR} \leq 30$ dB, as given in [4]:

$$P_{AWGN}(\gamma_b) = 0.2 \cdot \exp \left(- \frac{1.5 \cdot \gamma}{M - 1} \right). \quad (6)$$

Besides this first approximation, there is the second one, obtained for MQAM in constellation of the square root over the Gaussian channel, which can be seen in Figure 1. This approximation is also limited to within 1 dB in relation to the exact value for $M \geq 4$ and $\text{BER} \leq 10^{-3}$, and is given in [2]:

$$P_{AWGN}(\gamma_b) = 0.2 \cdot \exp \left(- \frac{1.6 \cdot \gamma}{M - 1} \right). \quad (7)$$

From Figure 1, it can be noticed that the difference between approximate and exact values is very small for $\text{BER} \leq 10^{-3}$. In this paper, the first approximate formula [4] will be used because it gives closer values to the actual SEP than the second approximate formula [2].

5.3. Multichannel Communications Link

Let us consider a communication system that uses M-ary signaling through identified independent and identically distributed (i.i.d.) for N channel links. It is assumed that the signal passes through Rician fading with AWGN for each link. It is supposed that the fading is slow and flat compared with the duration and the range of the signal. Rician parameter K_i , which determines the degree of fading on the i link, is defined as reflecting-to-scattering (specular-to-scatter) relation of power for $i = 1, 2, \dots, N$. In addition to the fading in channel, the transmitted signal in each link is also degraded by AWGN with two-sided power spectral density $N_0/2$, for $i = 1, 2, \dots, N$.

The technique combining with a maximal-ratio-combining (MRC) is considered for the analysis of linear combinations. Error probability, caused by stationary AWGN, depends only on the current SNR. Thus, as a result of the fading in channel, instantaneous SNR is a random variable in each branch of propagation. In order to evaluate the performance of the system, the random variable γ_i is defined to be the instantaneous SNR in the branch i , for $i = 1, 2, \dots, N$. After combining, the instantaneous SNR at the output is the sum of the current SNR combination of individual branches [1],

$$\gamma = \sum_{i=1}^N \gamma_i . \quad (8)$$

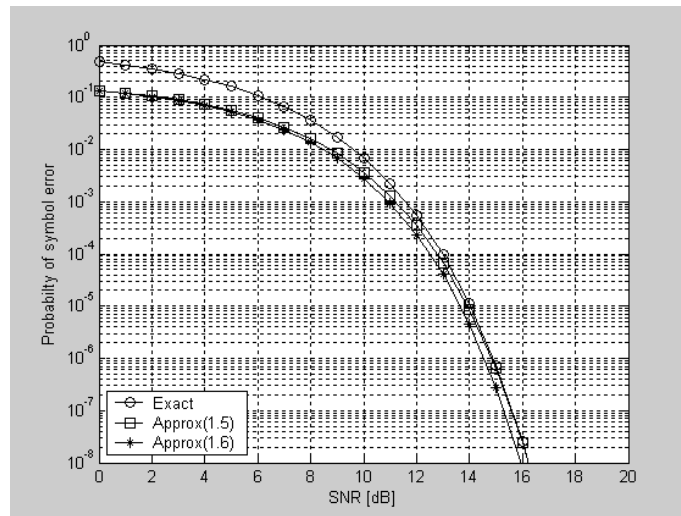


Figure 1: Exact and Approximate SEP for QAM, M = 16 over Gaussian channel

After changing variables and introducing Rician parameter K , it can be shown that the PDF of γ , and SNR after combining diversity, is given by:

$$p_\gamma(\gamma) = \left(\frac{N+K}{\bar{\gamma}} \right) \cdot \left(\frac{(N+K) \cdot \gamma}{K \cdot \bar{\gamma}} \right)^{\frac{N-1}{2}} \cdot \exp\left(-\frac{(N+K) \cdot \gamma + K \cdot \bar{\gamma}}{\bar{\gamma}} \right) \cdot I_{N-1} \left(2 \cdot \sqrt{\frac{K \cdot (N+K) \cdot \gamma}{\bar{\gamma}}} \right) \quad (9)$$

where $\bar{\gamma}$ is the expected value of γ , $K = \sum_{i=1}^N K_i$ and $I_N(\cdot)$ is modified Bessel function of the first kind of the order N .

5.4. M -ary Quadrature Amplitude Modulation

In the MQAM modulation scheme, phase and quadrature components depend on PAM modulation. Constellation diagram for MQAM signal consists of a square lattice, which graphically presents the envelope for each possible state of symbols. SEP in the function of K , $\bar{\gamma}$ and N for the system can be calculated with the average conditional error probability using PDF of γ , expressed by equation (2). Using the approximation from [4], SEP of MQAM over Gaussian channel is given by:

$$P_s \left(\frac{E}{\gamma} \right) = P_{AWGN}(\gamma) = 0.2 \cdot \log_2 M \cdot \exp \left[-\frac{1.5 \cdot \gamma}{(M-1)} \right] \quad (10)$$

After replacing (10) in (2) and putting in (11) by:

$$p_\gamma(\gamma) = \left(\frac{N+K}{\bar{\gamma}} \right) \cdot \left(\frac{(N+K) \cdot \gamma}{K \cdot \bar{\gamma}} \right)^{\frac{N-1}{2}} \cdot \exp\left(-\frac{(N+K) \cdot \gamma + K \cdot \bar{\gamma}}{\bar{\gamma}} \right) \cdot I_{N-1} \left(2 \cdot \sqrt{\frac{K \cdot (N+K) \cdot \gamma}{\bar{\gamma}}} \right) \quad (11)$$

we get:

$$p_\gamma(\gamma) = \int_0^\infty \left(\frac{N+K}{\bar{\gamma}} \right) \cdot \left(\frac{(N+K) \cdot \gamma}{K \cdot \bar{\gamma}} \right)^{\frac{N-1}{2}} \cdot \exp\left(-\frac{(N+K) \cdot \gamma + K \cdot \bar{\gamma}}{\bar{\gamma}} \right) \cdot I_{N-1} \left(2 \cdot \sqrt{\frac{K \cdot (N+K) \cdot \gamma}{\bar{\gamma}}} \right) \cdot 0.2 \cdot \log_2 M \cdot \exp \left[-\frac{1.5 \cdot \gamma}{(M-1)} \right] \quad (12)$$

Using the relation that is obtained from [5]

$$\int_0^\infty x^\nu \cdot e^{-\alpha x} \cdot I_{2\nu} \left(2 \cdot \beta \cdot \sqrt{x} \right) dx = \alpha^{-(2\nu+1)} \cdot \beta^{2\nu} \cdot \exp \left[\frac{\beta^2}{\alpha} \right] \quad (13)$$

SEP of MQAM over independent and identically distributed (i.i.d.) Rician fading channel with Rician parameter K , N diversity, and the average value per symbol SNR, $\bar{\gamma}$, are given by the following expression:

$$p(\gamma) \approx 0.2 \cdot \log_2 M \cdot \left[\frac{(N+K) \cdot (M-1)}{(N+K) \cdot (M-1) + 1.5 \cdot \bar{\gamma}} \right]^N \cdot \exp\left(-\frac{1.5 \cdot K \cdot \bar{\gamma}}{(N+K) \cdot (M-1) + 1.5 \cdot \bar{\gamma}}\right) \quad (14)$$

Comparing SEP expressions for MPSK and MDPSK [15] with the expression for SEP (14) obtained by mathematical analysis for MQAM over slow, flat, independent and identically distributed (i.i.d.) Rician fading channels when MRC technique is applied in the receiver, we can easily see that the expressions for SEP in the case of MPSK and MDPSK are given in integral form, while the expression for SEP for MQAM is given in a simple closed form and contains only exponential functions.

If we introduce a substitution $N = 1$ in equation (14), we get the expression:

$$p(\gamma) \approx 0.2 \cdot \log_2 M \cdot \left[\frac{(1+K) \cdot (M-1)}{(1+K) \cdot (M-1) + 1.5 \cdot \bar{\gamma}} \right] \cdot \exp\left(-\frac{1.5 \cdot K \cdot \bar{\gamma}}{(1+K) \cdot (M-1) + 1.5 \cdot \bar{\gamma}}\right) \quad (15)$$

Moreover, one can also show that if K tends to zero, than change the $K=0$ in (14) is obtained by a symbol error probability of M-ary QAM over Rayleigh fading channels with diversity N

$$p(\gamma) \approx 0.2 \cdot \log_2 M \cdot \left[\frac{N \cdot (M-1)}{N \cdot (M-1) + 1.5 \cdot \bar{\gamma}} \right]^N \quad (16)$$

It is easy to show that if K tends to infinity, i.e. if $K = \infty$ in (14), we obtain SEP of MQAM over Gaussian channels as:

$$p_{AWGN}(\gamma) = 0.2 \cdot \log_2 M \cdot \exp\left(-\frac{1.5 \cdot \bar{\gamma}}{(M-1)}\right) \quad (17)$$

6. RESULTS AND DISCUSSION

In Figure 2, error performances of MQAM over Rician fading channel with MRC diversity combining for range of Rician fading parameter $K = 0$ and 2, diversity order $N = 1, 2$ and 10, are presented, while in Figure 3, Rician fading parameter $K = 0, 6$ and 12, diversity order $N = 1, 4$ and 10, and modulation order $M = 16$.

In Figure 4 error performances of MQAM over Rician fading channel with MRC diversity combining for range of Rician fading parameter $K = 0$ and 2, diversity order $N = 1, 2$ and 10, are presented while in Figure 5 Rician fading parameter $K = 0, 6$ and 12, diversity order $N = 1, 4$ and 10, and modulation order $M=8$.

These curves provide the necessary information to assess the transferred energy to achieve desired SEP.

The features that are obtained for MQAM for different values of the modulation order M and diversity order $N = 2$ are compared in Figure 6, with Rician fading parameter $K = 2$, in Figure 7, with Rician fading parameter $K = 6$, and in Figure 8, with Rician fading parameter $K = 12$.

Characteristics presented in these Figures correspond to the value of the modulation order $M = 4, 8, 16$ and 32. As expected, for $M = 32$, MQAM has the worst performances. Degradation of performances is noticeable when M is varied from 4 to 32. But, from

Figures 6, 7 and 8, we can conclude that high data rate can be achieved by increasing the value of the modulation order M without reducing system performance.

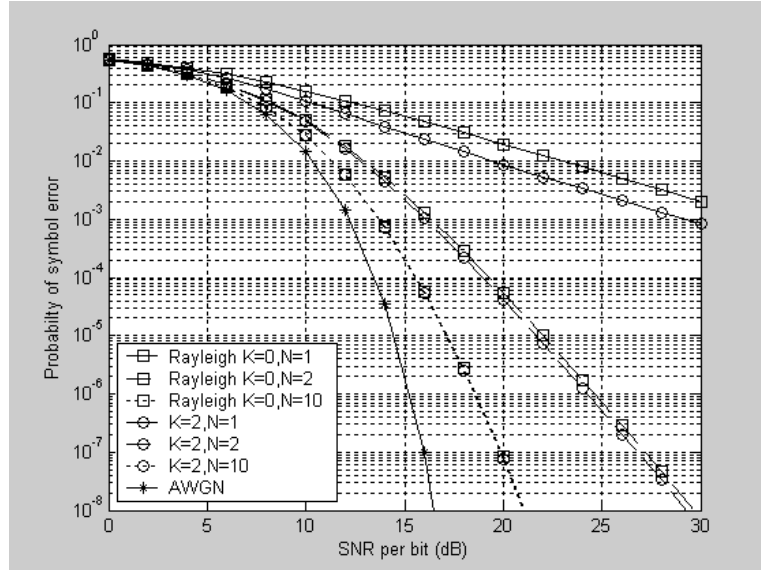


Figure 2: SEP for MQAM over Rician fading channel for $M = 16$ with N diversity: a solid line for $N = 1$, a dashed line for $N = 2$, and a line with dots for $N = 10$

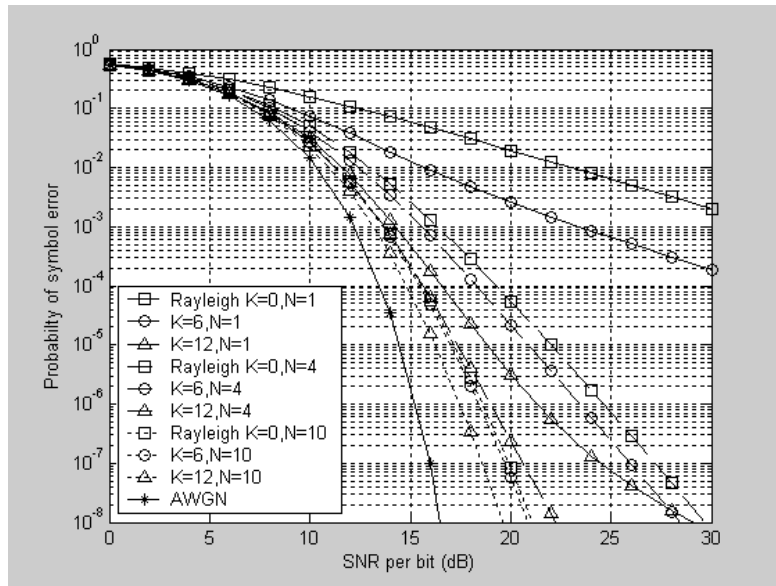


Figure 3: SEP for MQAM over Rician fading channel for $M = 16$ with N diversity: a solid line for $N = 1$, a dashed line for $N = 4$, and a line with dots for $N = 10$

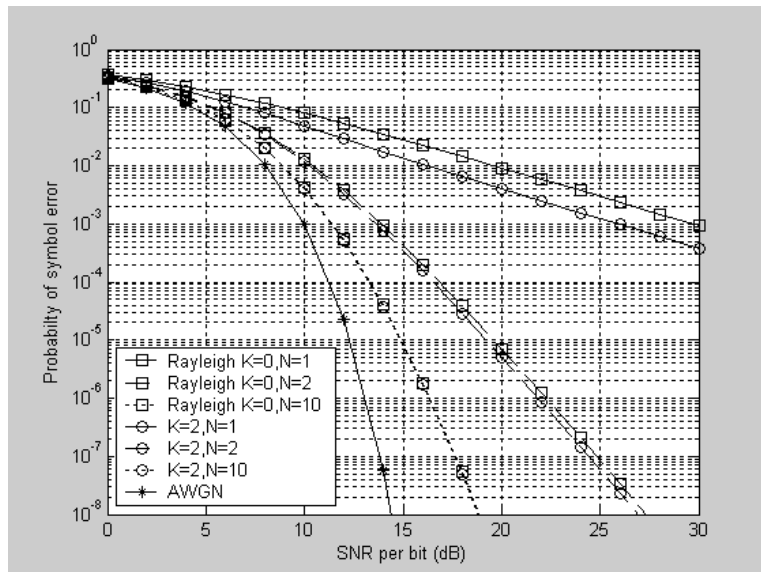


Figure 4: SEP for MQAM over Rician fading channel for $M = 8$ with N diversity: a solid line for $N = 1$, a dashed line for $N = 2$, and a line with dots for $N = 10$

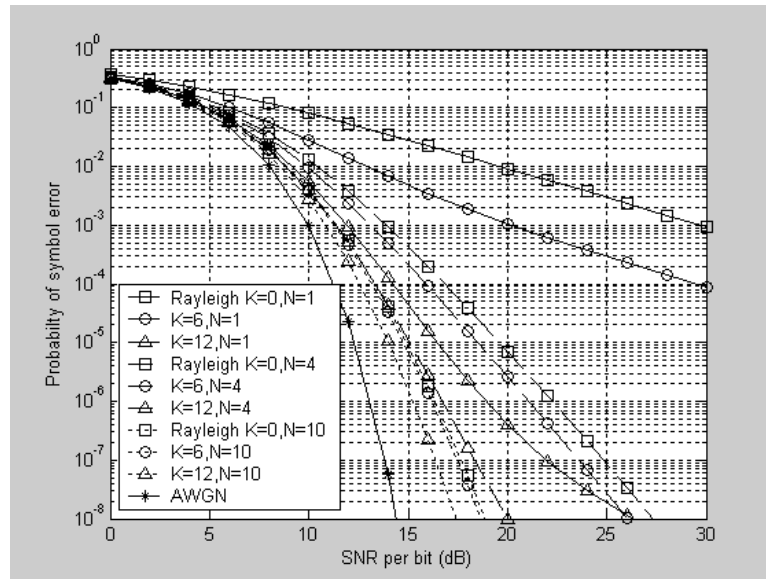


Figure 5: SEP for MQAM over Rician fading channel for $M = 8$ with N diversity: a solid line for $N = 1$, a dashed line for $N = 4$, and a line with dots for $N = 10$

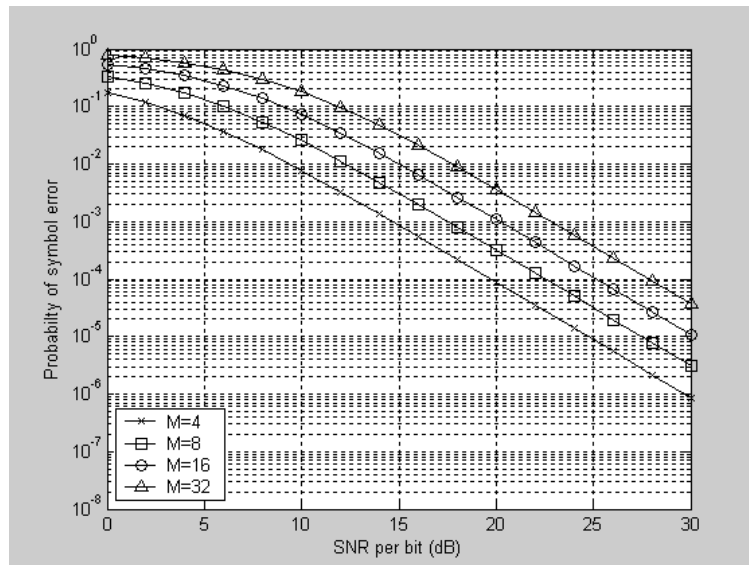


Figure 6: SEP for MQAM sent over Rician fading channel for different values of M and for the $K = 2$, $N = 2$

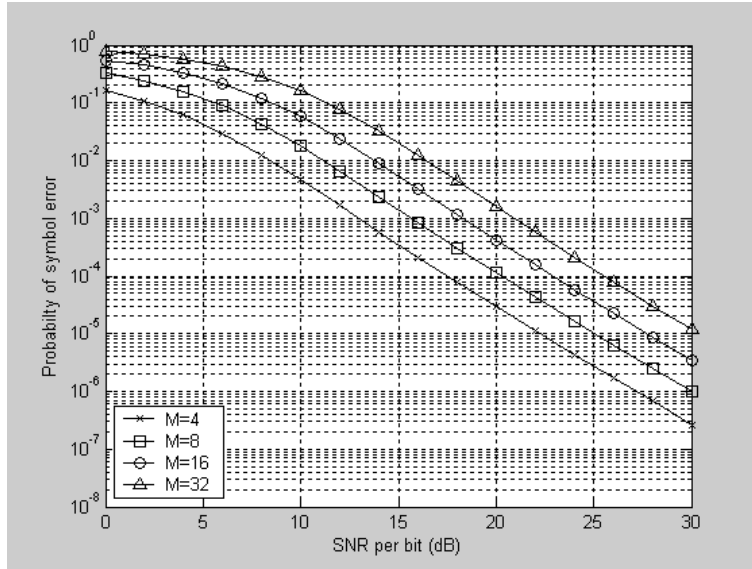


Figure 7: SEP for MQAM sent over Rician fading channel for different values of M and for the $K = 6, N = 2$

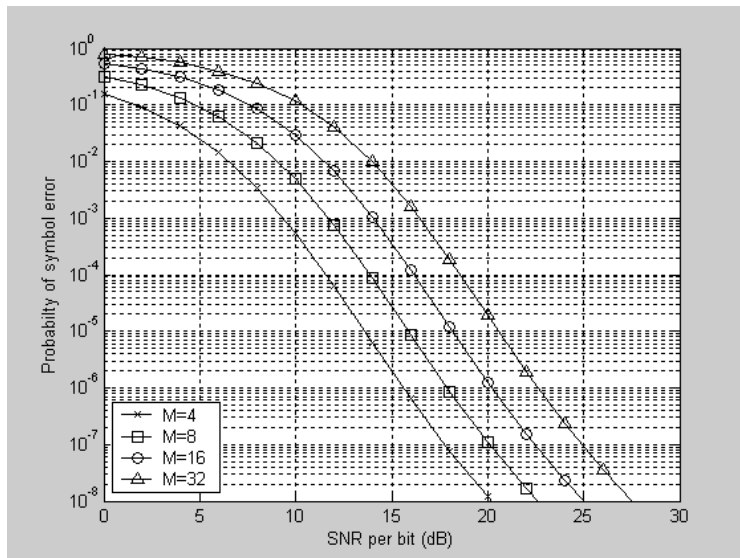


Figure 8: SEP for MQAM sent over Rician fading channel for different values of M and for the $K = 12, N = 2$

7. CONCLUSION

MQAM modulation technique is the most appropriate technique to reduce the combined fading in wireless communications. For the effective transfer of information it shows good performance in lowering SEP, as SEP should be small for perfect data transmission from the transmitter to the receiver.

In this paper, we used the first approximate formula [4] because it gives quite good (with very little deviation [9]) values to the actual SEP. For MQAM over Rayleigh fading and maximal ratio combining (MRC), we found that the maximum difference between exact formula and approximate formula is 0.8 dB at SER of 10^{-3} at ($M = 16$, $N = 1$) and ($M = 16$, $N = 10$), [9]. This approximation reflects quite well the behavior of M-QAM modulation for different Rician fading in the channel [9].

We did the comparisons of error characteristics are presented for MQAM parameters, $M = 8$ and $M = 16$, for different Rician fading in the channel. Analyzing the graphical representation of SEP for these two modulation orders M , we found that the SEP of MQAM, for modulation order $M = 8$, was lower than for the modulation order $M = 16$. For MQAM modulation with N diversity branches, under the assumption that the channel information is known, and with the maximal-ratio-combining (MRC) in Rician fading channel, we obtained a simple closed form expression for the SEP (14). Expression (14) is correct for all values of K , N , and $M \geq 4$ when it is $0 < \text{SNR} < 30$ dB [4].

This idea can be used for different types of combining the availability or unavailability of channel state information at the transmitter (CSIT). Another possible usage of this analysis is to obtain accurate bit-error rate (BER) for MQAM over slow, flat, Rician fading channels when the linear combination of diversity is applied to reduce the degradation due to fading.

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REFERENCES

- [1] Brennan, D.G., "Linear Diversity Combining Techniques," *Proc. IEEE*, 91 (2) (2003).
- [2] Chung, S.T., and Goldsmith, A.J., "Degrees of Freedom in Adaptive Modulation: A Unified View," *IEEE Transactions Communications*, 49 (9) (2001).
- [3] Fangming, H., Hong, M. and Wei, W., "Maximal Ratio Diversity Combining Enhanced Security," *IEEE Communications Letters*, 15 (5) (2011) 509–511.
- [4] Goldsmith, A.J., and Chua, S.G., "Variable-Rate Variable-Power M-QAM for Fading Channels," *IEEE Transactions Communications*, 45 (1997) 1218–1230.
- [5] Gradshteyn, I.S., and Ryzhik, I.M., *Table of Integrals, Series and Products*, San Diego, Fifth edition, CA: Academic, 1994.
- [6] Haewoon, N., Young, C.K., Womack, B.F., "Performance analysis of OT-MRC over I.I.D. Nakagami and non-I.I.D. Rayleigh Fading Channels," *Vehicular Technology, IEEE Transactions on Vehicular Technology*, 55 (6) (2006).
- [7] Hafeth, H., "An Overview of Diversity Techniques in Wireless Communication Systems," *S-72.333 Postgraduate Course in Radio Communications*, (2004/2005) 1-5.

- [8] Jameel, A.J., "Performance enhancement of wireless communication systems using transmit and receive diversity", *7th International Multi-Conference on Systems Signals and Devices*, IEEE (2010).
- [9] Khalid Ibrahim Alkhudairi, *Diversity Analyses for M-QAM in Rician Fading Channels*, Department of Electrical Engineering College of Engineering King Saud University, 2005.
- [10] Moradi, H., Refai, H.H., LoPresti, P.G. and Atiquzzaman, M., "Selection diversity for wireless optical communications with non-coherent detection without CSI", *IEEE GLOBECOM Workshops (GC Wkshps)*, (2010) 1010–1014.
- [11] Proakis, J.G., *Digital Communications*, McGraw-Hill, Singapore, Fourth edition, 2001.
- [12] Rizvi, U.H., Yilmaz, F., Alouini, M.S., Janssen, G.J.M. and Weber, J.H., "Performance of Equal Gain Combining with Quantized Phases in Rayleigh Fading Channels", *IEEE Transactions on Communications*, 59 (1) (2011) 13–18.
- [13] Simon, M.K., and Alouini, M.S., *Digital Communications over fading channels*, Second Edition, Wiley-IEEE Press, 2004.
- [14] Stüber, L.G., *Principle of mobile communication*, Second edition, Springer international edition, (2000).
- [15] Sun, J., and Reed, I., "Linear diversity analyses for M-PSK in Rician fading channels," *IEEE Transactions Communications*, 51 (11) (2003) 1749-1753.
- [16] Yihua, D., Xiangyu, Y. and Shengli, X., "A New Scheme for Jointed Space-Polarization Diversity and Its Performance Analysis, *11th IEEE Singapore International Conference on Communication Systems*, (2008).
- [17] Yujiang, W., Zaiping, N., "On Improving the Performance of Polarization Diversity in Wireless Communication Systems", *WiCOM 2006. International Conference on Wireless Communications, Networking and Mobile Computing*, IEEE (2006).